

## 18.330 :: Homework 8 :: Spring 2012 :: Not due

Fourier exercises (05/15):

1. Show that a  $C^p$  periodic function ( $p \geq 1$ ) has Fourier series coefficients that decay like  $|k|^{-p}$ .
2. Show that the error of the trapezoidal rule for integrating a  $C^p$  periodic function is  $O(h^p)$ .
3. Show that the  $L^2$  error of bandlimited differentiation of a  $C^p$  periodic function is  $O(h^{p-3/2})$ .
4. Let  $f(x) = \frac{1}{1+x^2}$ , for which  $\hat{f}(k) = \pi e^{-|k|}$ . For the purpose of this exercise, assume  $x \in \mathbb{R}$  and  $k \in \mathbb{R}$  (no boundaries.)
  - Find the decay rate (as a function of  $N$ ) of the error  $\|f - f_N\|_2$  of best approximation by a function  $f_N$  with Fourier transform supported in  $[-N, N]$ .
  - Find the decay rate (as a function of  $h$ ) of the error  $\|f - p\|_2$  or bandlimited interpolation form the samples on the grid  $x_j = jh$ .
  - Same question for the error of bandlimited differentiation, and the error of the trapezoidal rule.
5. Formulate and prove the discrete convolution theorem.

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