2024 FRM®
Exam Prep

SchweserNotes™
Financial Markets and Products

PART I BOOK 3

KAPLAN SCHWESER

Book 3: Financial Markets and Products

SchweserNotes™ 2024

FRM Part I

KAPLAN SCHWESER

SCHWESERNOTES™ 2024 FRM® PART I BOOK 3: FINANCIAL MARKETS AND PRODUCTS

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27. Banks

Global Association of Risk Professionals. *Financial Markets and Products*. New York, NY: Pearson, 2023. Chapter 1.

After completing this reading, you should be able to:

a. identify the major risks faced by banks and explain how these risks can arise.
b. distinguish between economic capital and regulatory capital.
c. summarize the Basel Committee regulations for regulatory capital and their motivations.
d. explain how deposit insurance gives rise to a moral hazard problem.
e. describe investment banking financing arrangements, including private placement, public offering, best efforts, firm commitment, and Dutch auction approaches.
f. describe the potential conflicts of interest among commercial banking, securities services, and investment banking divisions of a bank, and recommend solutions to these conflict of interest problems.
g. describe the distinctions between the banking book and the trading book of a bank.
h. explain the originate-to-distribute banking model and discuss its benefits and drawbacks.

28. Insurance Companies and Pension Plans

Global Association of Risk Professionals. *Financial Markets and Products*. New York, NY: Pearson, 2023. Chapter 2.

After completing this reading, you should be able to:

a. describe the key features of the various categories of insurance companies and identify the risks facing insurance companies.
b. describe the use of mortality tables and calculate the premium payments for a policy holder.
c. distinguish between mortality risk and longevity risk and describe how to hedge these risks.
d. describe defined benefit plans and defined contribution plans and explain the differences between them.
e. compare the various types of life insurance policies.
f. calculate and interpret loss ratio, expense ratio, combined ratio, and operating ratio for a property-casualty insurance company.
g. describe moral hazard and adverse selection risks facing insurance companies, provide examples of each, and describe how to overcome these risks.
h. evaluate the capital requirements for life insurance and property-casualty insurance companies.
i. compare the guaranty system and the regulatory requirements for insurance companies with those for banks.

29. Fund Management

Global Association of Risk Professionals. *Financial Markets and Products*. New York, NY: Pearson, 2023. Chapter 3.

After completing this reading, you should be able to:

a. differentiate among open-end mutual funds, closed-end mutual funds, and exchange-traded funds (ETFs).
b. identify and describe potential undesirable trading behaviors at mutual funds.
c. explain the concept of net asset value (NAV) of an open-end mutual fund and how it relates to share price.
d. explain the key differences between hedge funds and mutual funds.
e. calculate the return on a hedge fund investment and explain the incentive fee structure of a hedge fund including the terms hurdle rate, high-water mark, and clawback.
f. describe various hedge fund strategies including long-short equity, dedicated short, distressed securities, merger arbitrage, convertible arbitrage, fixed income arbitrage, emerging markets, global macro, and managed futures, and identify the risks faced by hedge funds.

g. describe characteristics of mutual fund and hedge fund performance and explain the effect of measurement biases on performance measurement.

30. Introduction to Derivatives

Global Association of Risk Professionals. *Financial Markets and Products*. New York, NY: Pearson, 2023. Chapter 4.

After completing this reading, you should be able to:

a. define derivatives, describe the features and uses of derivatives, and compare linear and non-linear derivatives.
b. describe the specifics of exchange-traded and over-the-counter markets, and evaluate the advantages and disadvantages of each.
c. differentiate between options, forwards, and futures contracts.
d. identify and calculate option and forward contract payoffs.
e. differentiate among the broad categories of traders: hedgers, speculators, and arbitrageurs.
f. calculate and compare the payoffs from hedging strategies involving forward contracts and options.
g. calculate and compare the payoffs from speculative strategies involving futures and options.
h. describe arbitrageurs' strategy and calculate an arbitrage payoff.
i. describe some of the risks that can arise from the use of derivatives.

31. Exchanges and OTC Markets

Global Association of Risk Professionals. *Financial Markets and Products*. New York, NY: Pearson, 2023. Chapter 5.

After completing this reading, you should be able to:

a. describe how exchanges can be used to alleviate counterparty risk.
b. explain the developments in clearing that reduce risk.
c. define netting and describe a netting process.
d. describe the implementation of a margining process, explain the determinants of and calculate initial and variation margin requirements.
e. describe the process of buying stock on margin without using a CCP and calculate margin requirements.
f. compare exchange-traded and OTC markets and describe their uses.
g. identify risks associated with OTC markets and explain how these risks can be mitigated.
h. describe the role of collateralization in the OTC market and compare it to the margining system.
i. explain the use of special purpose vehicles (SPVs) in the OTC derivatives market.

32. Central Clearing

Global Association of Risk Professionals. *Financial Markets and Products*. New York, NY: Pearson, 2023. Chapter 6.

After completing this reading, you should be able to:

a. provide examples of the mechanics of a central counterparty (CCP).
b. describe the role of CCPs and distinguish between bilateral and centralized clearing.
c. describe advantages and disadvantages of central clearing of OTC derivatives.
d. explain regulatory initiatives for the OTC derivatives market and their impact on central clearing.
e. compare margin requirements in centrally cleared and bilateral markets and explain how margin can mitigate risk.
f. compare netting in bilateral markets vs centrally cleared markets.
g. assess the impact of central clearing on the broader financial markets.
h. identify and explain the types of risks faced by CCPs.
i. identify and distinguish between the risks to clearing members and to non-members.

STUDY SESSION 9

33. Futures Markets

Global Association of Risk Professionals. *Financial Markets and Products*. New York, NY:

Pearson, 2023. Chapter 7.

After completing this reading, you should be able to:

a. define and describe the key features and specifications of a futures contract, including the underlying asset, the contract price and size, trading volume, open interest, delivery, and limits.
b. explain the convergence of futures and spot prices.
c. describe the role of an exchange in futures transactions.
d. explain the differences between a normal and an inverted futures market.
e. describe the mechanics of the delivery process and contrast it with cash settlement.
f. describe and compare different trading order types.
g. describe the application of marking to market and hedge accounting for futures.
h. compare and contrast forward and futures contracts.

34. Using Futures for Hedging

Global Association of Risk Professionals. *Financial Markets and Products*. New York, NY: Pearson, 2023. Chapter 8.

After completing this reading, you should be able to:

a. define and differentiate between short and long hedges and identify their appropriate uses.
b. describe the arguments for and against hedging and the potential impact of hedging on firm profitability.
c. define and calculate the basis, discuss various sources of basis risk, and explain how basis risks arise when hedging with futures.
d. define cross hedging and compute and interpret hedge ratio and hedge effectiveness.
e. calculate the profit and loss on a short or a long hedge.
f. compute the optimal number of futures contracts needed to hedge an exposure and explain and calculate the "tailing the hedge" adjustment.
g. explain how to use stock index futures contracts to change a stock portfolio's beta.
h. explain how to create a long-term hedge using a stack-and-roll strategy and describe some of the risks that arise from this strategy.

35. Foreign Exchange Markets

Global Association of Risk Professionals. *Financial Markets and Products*. New York, NY: Pearson, 2023. Chapter 9.

After completing this reading, you should be able to:

a. explain and describe the mechanics of spot quotes, forward quotes, and futures quotes in the foreign exchange markets; distinguish between bid and ask exchange rates.
b. calculate a bid-ask spread and explain why the bid-ask spread for spot quotes may be different from the bid-ask spread for forward quotes.
c. compare outright (forward) and swap transactions.
d. define, compare, and contrast transaction risk, translation risk, and economic risk.
e. describe examples of transaction, translation, and economic risks and explain how to hedge these risks.
f. describe the rationale for multi-currency hedging using options.
g. identify and explain the factors that determine exchange rates.
h. calculate and explain the effect of an appreciation/depreciation of one currency relative to another.
i. explain the purchasing power parity theorem and use this theorem to calculate the appreciation or depreciation of a foreign currency.
j. describe the relationship between nominal and real interest rates.
k. describe how a non-arbitrage assumption in the foreign exchange markets leads to the interest rate parity theorem and use this theorem to calculate forward foreign exchange rates.
l. distinguish between covered and uncovered interest rate parity conditions.

36. Pricing Financial Forwards and Futures

Global Association of Risk Professionals. *Financial Markets and Products*. New York, NY: Pearson, 2023. Chapter 10.

After completing this reading, you should be able to:

a. define and describe financial assets.
b. define short-selling and calculate the net profit of a short sale of a dividend-paying stock.

c. describe the differences between forward and futures contracts and explain the relationship between forward and spot prices.
d. calculate the forward price given the underlying asset's spot price and describe an arbitrage argument between spot and forward prices.
e. distinguish between the forward price and the value of a forward contract.
f. calculate the value of a forward contract on a financial asset that does or does not provide income or yield.
g. explain the relationship between forward and futures prices.
h. calculate the value of a stock index futures contract and explain the concept of index arbitrage.

37. Commodity Forwards and Futures

Global Association of Risk Professionals. *Financial Markets and Products*. New York, NY: Pearson, 2023. Chapter 11.

After completing this reading, you should be able to:
a. explain the key differences between commodities and financial assets.
b. define and apply commodity concepts such as storage costs, carry markets, lease rate, and convenience yield.
c. identify factors that impact prices on agricultural commodities, metals, energy, and weather derivatives.
d. explain the formula for pricing commodity forwards.
e. describe an arbitrage transaction in commodity forwards and compute the potential arbitrage profit.
f. define the lease rate and explain how it determines the no-arbitrage values for commodity forwards and futures.
g. describe the cost of carry model and determine the impact of storage costs and convenience yields on commodity forward prices and no-arbitrage bounds.
h. compute the forward price of a commodity with storage costs.
i. explain how to create a synthetic commodity position and use it to explain the relationship between the forward price and the expected future spot price.
j. explain the impact of systematic and nonsystematic risk on current futures prices and expected future spot prices.
k. define and interpret normal backwardation and contango.

STUDY SESSION 10

38. Options Markets

Global Association of Risk Professionals. *Financial Markets and Products*. New York, NY: Pearson, 2023. Chapter 12.

After completing this reading, you should be able to:
a. describe the various types and uses of options; define moneyness.
b. explain the payoff function and calculate the profit and loss from an options position.
c. explain how dividends and stock splits can impact the terms of a stock option.
d. describe the application of commissions, margin requirements, and exercise procedures to exchange-traded options, and explain the trading characteristics of these options.
e. define and describe warrants, convertible bonds, and employee stock options.

39. Properties of Options

Global Association of Risk Professionals. *Financial Markets and Products*. New York, NY: Pearson, 2023. Chapter 13.

After completing this reading, you should be able to:
a. identify the six factors that affect an option's price.
b. identify and compute upper and lower bounds for option prices on non-dividend and dividend paying stocks.
c. explain put-call parity and apply it to the valuation of European and American stock options, with dividends and without dividends, and express it in terms of forward prices.
d. explain and assess potential rationales for using the early exercise features of American call and put options.

40. Trading Strategies

Global Association of Risk Professionals. *Financial Markets and Products*. New York, NY: Pearson, 2023. Chapter 14.

After completing this reading, you should be able to:

a. explain the motivation to initiate a covered call or a protective put strategy.
b. describe principal protected notes (PPNs) and explain necessary conditions to create them.
c. describe the use and calculate the payoffs of various spread strategies.
d. describe the use and explain the payoff functions of combination strategies.

41. Exotic Options

Global Association of Risk Professionals. *Financial Markets and Products*. New York, NY: Pearson, 2023. Chapter 15.

After completing this reading, you should be able to:

a. define and contrast exotic derivatives and plain vanilla derivatives.
b. describe some of the reasons that drive the development of exotic derivative products.
c. explain how any derivative can be converted into a zero-cost product.
d. describe how standard American options can be transformed into nonstandard American options.
e. identify and describe the characteristics and payoff structures of the following exotic options: gap, forward start, compound, chooser, barrier, binary, lookback, Asian, exchange, and basket options.
f. describe and contrast volatility swaps and variance swaps.
g. explain the basic premise of static option replication and how it can be applied to hedging exotic options.

STUDY SESSION 11

42. Properties of Interest Rates

Global Association of Risk Professionals. *Financial Markets and Products*. New York, NY: Pearson, 2023. Chapter 16.

After completing this reading, you should be able to:

a. describe Treasury rates, LIBOR, Secured Overnight Financing Rate (SOFR), and repo rates, and explain what is meant by the "risk-free" rate.
b. calculate the value of an investment using different compounding frequencies.
c. convert interest rates based on different compounding frequencies.
d. calculate the theoretical price of a bond using spot rates.
e. calculate the Macaulay duration, modified duration, and dollar duration of a bond.
f. evaluate the limitations of duration and explain how convexity addresses some of them.
g. calculate the change in a bond's price given its duration, its convexity, and a change in interest rates.
h. derive forward interest rates from a set of spot rates.
i. derive the value of the cash flows from a forward rate agreement (FRA).
j. calculate zero-coupon rates using the bootstrap method.
k. compare and contrast the major theories of the term structure of interest rates.

43. Corporate Bonds

Global Association of Risk Professionals. *Financial Markets and Products*. New York, NY: Pearson, 2023. Chapter 17.

After completing this reading, you should be able to:

a. describe features of bond trading and explain the behavior of bond yield.
b. describe a bond indenture and explain the role of the corporate trustee in a bond indenture.
c. define high-yield bonds and describe types of high-yield bond issuers and some of the payment features unique to high-yield bonds.
d. differentiate between credit default risk and credit spread risk.
e. describe event risk and explain what may cause it to manifest in corporate bonds.
f. describe different characteristics of bonds such as issuer, maturity, interest rate, and collateral.
g. describe the mechanisms by which corporate bonds can be retired before maturity.

h. define recovery rate and default rate, and differentiate between an issue default rate and a dollar default rate.
i. evaluate the expected return from a bond investment and identify the components of the bond's expected return.

44. Mortgages and Mortgage-Backed Securities

Global Association of Risk Professionals. *Financial Markets and Products*. New York, NY: Pearson, 2023. Chapter 18.

After completing this reading, you should be able to:

a. describe the various types of residential mortgage products.
b. calculate a fixed-rate mortgage payment and its principal and interest components.
c. summarize the securitization process of mortgage-backed securities (MBS), particularly the formation of mortgage pools, including specific pools and to-be-announced (TBA).
d. calculate the weighted average coupon, weighted average maturity, single monthly mortality rate (SMM), and conditional prepayment rate (CPR) for a mortgage pool.
e. describe the process of trading pass-through agency MBS.
f. explain the mechanics of different types of agency MBS products, including collateralized mortgage obligations (CMOs), interest-only securities (IOs), and principal-only securities (POs).
g. describe a dollar roll transaction and how to value a dollar roll.
h. describe the mortgage prepayment option and factors that affect it; explain prepayment modeling and its four components: refinancing, turnover, defaults, and curtailments.
i. describe the steps in valuing an MBS using Monte Carlo simulation.
j. define Option-Adjusted Spread (OAS) and explain its uses and challenges.

45. Interest Rate Futures

Global Association of Risk Professionals. *Financial Markets and Products*. New York, NY: Pearson, 2023. Chapter 19.

After completing this reading, you should be able to:

a. identify the most commonly used day count conventions, describe the markets that each one is typically used in, and apply each to an interest calculation.
b. calculate the conversion of a discount rate to a price for a U.S. Treasury bill.
c. differentiate between the clean and dirty price for a U.S. Treasury bond; calculate the accrued interest and dirty price on a U.S. Treasury bond.
d. explain and calculate a U.S. Treasury bond futures contract conversion factor.
e. calculate the cost of delivering a bond into a Treasury bond futures contract.
f. describe the impact of the level and shape of the yield curve on the cheapest-to-deliver Treasury bond decision.
g. calculate the theoretical futures price for a Treasury bond futures contract.
h. calculate the final contract price on a Eurodollar futures contract and compare Eurodollar futures to FRAs.
i. describe and compute the Eurodollar futures contract convexity adjustment.
j. calculate the duration-based hedge ratio and create a duration-based hedging strategy using interest rate futures.
k. explain the limitations of using a duration-based hedging strategy.

46. Swaps

Global Association of Risk Professionals. *Financial Markets and Products*. New York, NY: Pearson, 2023. Chapter 20.

After completing this reading, you should be able to:

a. explain the mechanics of a plain vanilla interest rate swap and compute its cash flows.
b. explain how a plain vanilla interest rate swap can be used to transform an asset or a liability and calculate the resulting cash flows.
c. explain the role of financial intermediaries in the swaps market.
d. describe the role of the confirmation in a swap transaction.
e. describe the comparative advantage argument for the existence of interest rate swaps and evaluate some of the criticisms of this argument.
f. explain how the discount rates in a plain vanilla interest rate swap are computed.
g. calculate the value of a plain vanilla interest rate swap based on two simultaneous bond positions.
h. calculate the value of a plain vanilla interest rate swap from a sequence of FRAs.

i. explain how a currency swap can be used to transform an asset or liability and calculate the resulting cash flows.
j. calculate the value of a currency swap based on two simultaneous bond positions.
k. calculate the value of a currency swap based on a sequence of forward exchange rates.
l. identify and describe other types of swaps, including commodity, volatility, credit default, and exotic swaps.
m. describe the credit risk exposure in a swap position.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Financial Markets and Products, Chapter 1.

READING 27

BANKS

Study Session 8

EXAM FOCUS

This reading introduces a number of concepts about banks that are developed more fully elsewhere in the FRM curriculum. For the exam, focus on understanding the major types of risk a bank faces and how they are addressed, both by banks themselves and by bank regulators. Be prepared to explain the differences between commercial banking and investment banking as well as the conflicts that exist in an organization that performs both of these services. Also, understand the distinctions between the lending and trading operations of a bank. Finally, be able to describe the implications of banks originating loans and distributing them to other parties.

MODULE 27.1: BANKS

When we speak of banks, we include financial institutions that provide a variety of services. Banks can be categorized by the functions they perform and the customers they serve.

Commercial banks are those that take deposits and make loans. Commercial banks include **retail banks**, which primarily serve individuals and small businesses, and **wholesale banks**, which primarily serve corporate and institutional customers.

Investment banks are those that assist in raising capital for their customers (e.g., by managing the issuance of debt and equity securities) and advising them on corporate finance matters such as mergers and restructurings.

Whether a bank or bank holding company engages in both commercial banking and investment banking or must only do one or the other depends on the regulations where it does business.

Major Risks Faced by Banks

LO 27.a: Identify the major risks faced by banks and explain how these risks can arise.

The main risks faced by a bank include credit risk, market risk, and operational risk.

Credit risk refers to the risk that borrowers do not repay their loans or that counterparties to contracts such as derivatives may default on their obligations when the contract has negative value to the counterparty (and positive value to the bank). The bank's trading of derivatives also introduces market risk since the derivatives contract is dependent on the price of the underlying asset. Regarding loans, the interest rate charged by banks on loans takes into account the expected losses; for example, assuming a 2% differential in average interest rate charged and cost of funds, expected losses of 0.6% would leave 1.4% remaining for operating costs and profit.

Market risk refers to the risk of losses from a bank's trading activities, such as declines in the value of securities the bank owns. Specific examples of market risk factors include changes in interest rates, exchange rates, and stock prices. Banks allow their larger investors to trade in a variety of financial contracts where the bank acts as a market maker. In those instances, the bank has controlled (but not zero) exposures to market risk factors.

Operational risk refers to the possibility of losses arising from external events (e.g., cyberattacks or physical asset damage) or failures of a bank's internal controls (e.g., employee defalcation, business interruption, IT failures, and human error). Operationally, banks are most exposed to legal, compliance, and cyber risks.

Economic Capital vs. Regulatory Capital

LO 27.b: Distinguish between economic capital and regulatory capital.

To mitigate the risk of bank failures caused by losses on loans or trading assets, banks must be funded by adequate sources of capital. Equity capital is needed to shield against possible losses and to maintain solvency. Banks may also issue long-term debt (debt capital) to bolster their capital. This debt is subordinated to the claims of depositors if a bank faces financial distress.

Equity capital can be thought of as going concern capital since it is meant to cover losses when the bank continues to operate as a business. In contrast, debt capital can be thought as gone concern capital since it is meant to cover losses only once the bank ceases to operate as a business.

Banks and their regulators may have different views about how much capital is sufficient in light of the risks a bank faces. **Regulatory capital** refers to the minimum amount required and is determined by bank regulators.

Economic capital refers to the amount of capital that a bank believes is adequate based on its own risk models. Both regulatory and economic capital refer to funds that are set aside to be used to cover unexpected losses. The amount of required capital will correspond to the amount of potential losses.

Basel Committee Regulations

LO 27.c: Summarize the Basel Committee regulations for regulatory capital and their motivations.

The Basel Committee regulations began as capital requirements to account for loan and derivatives contracts defaults (i.e., credit risk only). Over time, the capital requirements evolved and added amounts for market risk and operational risk.

Models are used to compute regulatory capital, specifically standardized models developed by the Basel Committee and internals models developed by the banks. After the credit crisis of 2007 to 2009, the Basel Committee has allowed less use of bank internal models. As of now, all three risks (credit, market, and operational) must be computed using a standardized model. However, if a bank is approved by its national regulator, then it may use an internal model for market and credit risks only. The internal models calculate a required capital amount based on the greater of the capital computed by the internal model, and 72.5% of the capital computed by the standardized model. Note that the 72.5% is a figure that will apply by 2027.

The credit crisis of 2007 to 2009 highlighted that many of the problems arose due to a liquidity shortage as opposed to a capital shortage. As a result, the Basel Committee introduced two liquidity ratio requirements. The **liquidity coverage ratio (LCR)** is meant to ensure that banks have enough funding sources to remain viable for 30 days in the event of minor financial stress periods. The **net stable funding ratio (NSFR)** attempts to control the maturity mismatches between the bank's assets and liabilities.

Deposit Insurance and Moral Hazard

LO 27.d: Explain how deposit insurance gives rise to a moral hazard problem.

To increase public confidence in the banking system and prevent runs on banks, most countries have established systems of **deposit insurance**. Typically, a depositor's funds are guaranteed up to some maximum amount if a bank fails. These systems are funded by insurance premiums paid by banks.

Like other forms of insurance, deposit insurance brings an element of **moral hazard**. Moral hazard is the observed phenomenon that insured parties take greater risks than they would normally take if they were not insured. In the banking context, with deposit insurance in place, the moral hazard arises when depositors pay less attention to banks' financial health than they otherwise would. This allows banks to offer higher interest rates on deposits and make higher-risk loans with the funds they attract. Losses on such loans contributed to increased bank failures in the United States in the 1980s and 2000s.

One way of mitigating moral hazard is by making insurance premiums risk-based. For example, in recent years, poorly capitalized banks have been required to pay higher deposit insurance premiums than well-capitalized banks.

Investment Banking Financing Arrangements

LO 27.e: Describe investment banking financing arrangements, including private placement, public offering, best efforts, firm commitment, and Dutch auction approaches.

When an investment bank arranges a securities issuance for a customer, it may try to place the entire issue with a particular buyer or group of buyers or sell the issue in the public market.

In a **private placement**, securities are sold directly to qualified investors with substantial wealth and investment knowledge. The investment bank earns fee income for arranging a private placement.

If the securities are sold to the investing public at large, the issuance is referred to as a **public offering**. Investment banks have two methods of assisting with a public offering. With a **firm commitment**, the investment bank agrees to purchase the entire issue at a price that is negotiated between the issuer and bank. The investment bank earns income by selling the issue to the public at a spread above the price it paid the issuer. An investment bank can also agree to distribute an issue on a **best efforts** basis rather than agreeing to purchase the whole issue, which is less risky for the bank. If only part of the issue can be sold, the bank is not obligated to buy the unsold portion. As with a private placement, the investment bank earns fee income for its services.

First-time issues of stock by firms whose shares are not currently publicly traded are called **initial public offerings (IPOs)**. Since the shares are not yet traded, it is challenging to determine a reasonable post-IPO share price. An investment bank can assist in determining an IPO price by analyzing the value of the issuer.

An IPO price may also be discovered through a **Dutch auction** process. A Dutch auction begins with a price greater than what any bidder will pay, and this price is reduced until a bidder agrees to pay it. Bidders may specify how many units they will purchase when accepting a price. The price continues to be reduced until bidders have accepted all the shares that the seller wants to sell. The price at which the last of the shares can be sold becomes the price paid by all successful bidders. Assuming all potential bidders participate, that price is the equilibrium price where demand and supply intersect.

Potential Conflicts of Interest

LO 27.f: Describe the potential conflicts of interest among commercial banking, securities services, and investment banking divisions of a bank, and recommend

solutions to these conflict of interest problems.

If a bank or a bank holding company provides commercial banking, investment banking, and securities services, several conflicts of interest may arise. For example, an investment banking division that is trying to sell newly issued stocks or bonds might want the securities division to sell these to their clients. The investment bankers may press the securities division's financial analysts to maintain buy recommendations, or press its financial advisors to allocate these stocks and bonds to customer accounts. Such pressure may interfere with analysts' independence and objectivity or conflict with advisors' duties to clients.

Another clear conflict of interest among banking departments involves material nonpublic information. A commercial banking or investment banking division may acquire nonpublic information about a company when negotiating a loan or arranging a securities issuance. Other parts of the banking company, such as its trading desk, may benefit unfairly if they gain access to this information.

Because of these inherent conflicts, most bank regulators require some degree of separation among commercial banking, securities services, and investment banking. In some cases, they have prohibited firms from engaging in more than one of these activities, as was true in the United States when the Glass-Steagall Act was in force. Where banking firms are permitted to have commercial banking, securities, and investment banking units, the firms must implement **Chinese walls**, which are internal controls to prevent information from being shared among these units.

Banking Book vs. Trading Book

LO 27.g: Describe the distinctions between the banking book and the trading book of a bank.

Distinctions between banking book and trading book are required when computing regulatory capital.

The **banking book** refers to assets and liabilities that are meant to be held to maturity. For example, it would include loans made, which are the primary assets of a commercial bank. In calculating regulatory capital, credit risk capital calculations apply to the banking book.

The **trading book** refers to assets and liabilities related to a bank's trading activities. Unlike other assets and liabilities, trading book items are marked to market daily. In calculating regulatory capital, market risk capital computations apply to the trading book, which often result in lower capital requirements than the banking book.

In general, the default classification for a given financial instrument is the banking book. However, if a bank dedicates a desk to trade a given instrument, then it would likely be classified in the trading book.

The Originate-to-Distribute Model

LO 27.h: Explain the originate-to-distribute banking model and discuss its benefits and drawbacks.

In contrast to a bank making loans and keeping them as assets, the **originate-to-distribute model** involves making loans and selling them to other parties. Many mortgage lenders in the United States operate on the originate-to-distribute model. Government agencies such as Ginnie Mae (GNMA), Fannie Mae (FNMA), and Freddie Mac (FHLMC) purchase mortgage loans from banks and issue securities backed by the cash flows from these mortgages.

The benefit of the originate-to-distribute model is that it increases liquidity in the sectors of the lending market where it is used. In addition to the residential mortgage market, this model has been applied in other areas such as student loans, credit card balances, and commercial loans and mortgages. For the banks that originate the loans, selling them to other parties is a way of freeing up capital with which they can meet regulatory requirements or make new loans.

A drawback of this model is that, in some cases, it has led banks to loosen lending standards. This was one of the factors that led to the credit crisis in the United States from 2007 to 2009.

MODULE QUIZ 27.1

1. The minimum level of capital a bank needs to maintain, according to its own estimates, models, and risk assessments, is best described as its:
   A. equity capital.
   B. financial capital.
   C. economic capital.
   D. regulatory capital.

2. Which of the following actions in the banking system is most likely intended to address the problem of moral hazard?
   A. Deposit insurers charge risk-based premiums.
   B. Banks increase loans to higher-risk borrowers.
   C. Governments implement deposit insurance programs.
   D. Banks increase the interest rates they offer to depositors.

3. An investment bank is most likely to earn a trading profit from buying and selling securities if it arranges a:
   A. Dutch auction.
   B. private placement.
   C. best efforts offering.
   D. firm commitment offering.

4. The purpose of a Chinese wall in banking is to:
   A. prevent a bank failure from endangering other banks.
   B. prevent a bank's departments from sharing information.
   C. restrict companies from offering both banking and securities services.

D. restrict companies from engaging in both commercial and investment banking.

5. A drawback of the originate-to-distribute banking model is that it has led to:
   A. too little liquidity in certain sectors.
   B. too much liquidity in certain sectors.
   C. looser credit standards in certain sectors.
   D. tighter credit standards in certain sectors.

KEY CONCEPTS

LO 27.a

The major risks faced by a bank include the following.

- Credit risk from defaults on loans or by counterparties.
- Market risk from declines in the value of trading book assets.
- Operational risk from external events or failure of internal controls.

LO 27.b

To mitigate the risk of bank failures caused by losses on loans or trading assets, banks must be funded by adequate sources of capital. Banks and their regulators may have different views about how much capital is sufficient in light of the risks a bank faces.

LO 27.c

Regulatory capital is the amount of capital that regulators require a bank to hold. Economic capital is the amount of capital a bank believes it needs to hold based on its own models.

LO 27.d

Deposit insurance exists to increase public trust in the banking system. However, it gives rise to moral hazard by decreasing the attention depositors pay to a bank's financial health and increasing the level of risk a bank is willing to take when its depositors are insured.

LO 27.e

In a private placement, securities are sold directly to qualified investors. In a public offering, securities are sold to the investing public.

When assisting a securities issuer on a best-efforts basis, an investment bank sells as much of the issue to the public as it can. In a firm commitment, an investment bank buys an entire issue of securities from the issuer for one price and resells the securities to the public for a higher price. A Dutch auction process may be used to determine a price for an initial public offering.

LO 27.f

Within a firm that provides commercial banking, investment banking, and securities services, inherent conflicts of interest exist. Information may be acquired in a commercial banking or investment banking transaction that would give the other units an unfair advantage. An investment bank's task of selling newly issued stocks and bonds may conflict with a securities unit's duties to act in the best interests of its clients and recommend trading actions independently.

Bank regulators generally require commercial banking, investment banking, and securities activities to be kept separate, either by preventing firms from engaging in more than one of these activities or by requiring Chinese walls between these units of a bank.

LO 27.g

The banking book refers to a bank's assets and liabilities to be held to maturity (e.g., loans made by a bank for a specific term). The trading book refers to assets and liabilities related to a bank's trading activities. Regulatory capital requirements are generally higher for the banking book than for the trading book.

LO 27.h

The originate-to-distribute model involves banks making loans and selling them to other parties, many of which pool the loans and issue securities backed by their cash flows. This model frees up capital for the originating banks and may increase liquidity in sectors of the loan market. However, it has also led to decreased lending standards and lower credit quality of the loans sold.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 27.1

1. **C** Economic capital refers to a bank's own assessment of the minimum level of capital it needs to maintain. Economic capital is often less than regulatory capital, which is the minimum level a bank must maintain to comply with capital adequacy regulations. (LO 27.b)

2. **A** Charging risk-based premiums is a measure intended to address the problem of moral hazard, which exists when insured parties take greater risks than they would take in the absence of insurance. (LO 27.d)

3. **D** With a firm commitment offering, an investment bank buys an entire issue of securities from the issuer and attempts to sell them to the public at a higher price. In a private placement or a best efforts offering, an investment bank earns fee income rather than trading income. A Dutch auction is a method of price discovery for an initial public offering that does not involve buying and reselling shares. (LO 27.e)

4. **B** Chinese walls are internal controls to prevent a banking company's commercial banking, securities, and investment banking operations from sharing information. (LO 27.f)

5. **C** One drawback to the originate-to-distribute model is that it has led to looser credit standards in certain sectors, such as residential mortgages. A benefit of the model is that it has increased liquidity in certain sectors. (LO 27.h)

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Financial Markets and Products, Chapter 2.

READING 28

INSURANCE COMPANIES AND PENSION PLANS

Study Session 8

EXAM FOCUS

The focus of this reading is primarily on concepts related to life insurance and nonlife (property and casualty) insurance, such as moral hazard, adverse selection, mortality risk, and longevity risk. For the exam, be able to apply mortality tables to perform life expectancy computations and breakeven premium computations for life insurance companies, and be able to compute ratios relevant to property and casualty insurance companies. In addition, understand the risks facing insurance companies and be able to discuss specific ways to mitigate them.

MODULE 28.1: INSURANCE COMPANIES AND PENSION PLANS

LO 28.a: Describe the key features of the various categories of insurance companies and identify the risks facing insurance companies.

LO 28.e: Compare the various types of life insurance policies.

Insurance companies protect policyholders from specific loss events in exchange for the payment of periodic premiums. Three categories of insurance companies include life insurance, property and casualty (nonlife) insurance, and health insurance.

Life Insurance

Life insurance companies usually provide long-term coverage and make a specified payment to the policyholder’s beneficiaries upon the natural death (i.e., certain event) of the policyholder during the policy term. Coverage is also available for accidental death (i.e., uncertain event).

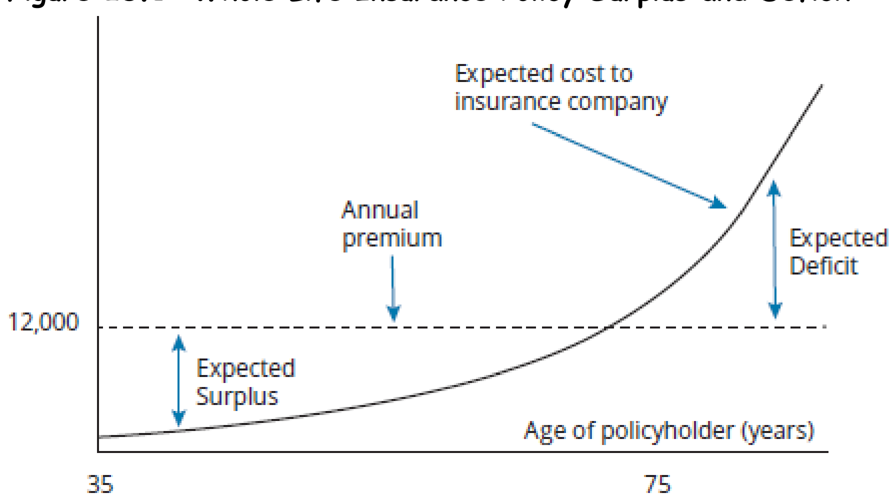
Term (temporary) **life insurance** provides a specified amount of insurance coverage for a fixed period. No payments are made to the policyholder’s beneficiaries if the policyholder survives the term of the policy; therefore, payment is not certain. Payment is only made if the policyholder dies during the policy term. The use of mortality tables to calculate breakeven premiums is discussed later. **Endowment life insurance** is a subset of term insurance that has a payout at the stated contract maturity. If the policyholder dies before maturity, then there will be a payout at death. A with-profits endowment policy involves a higher payout assuming the insurance company's underlying investments perform well. A unit-linked endowment policy involves the policyholder choosing an investment and having the payout amount linked to the performance of the investment.

Employers on behalf of their employees usually arrange **group life insurance**. It involves the pooling of risks for a large number of individuals. Because medical examinations are often not required for group insurance, there will be some good risks and some bad risks taken by the insurance company.

Whole (permanent) **life insurance** provides a specified amount of insurance coverage for the life of the policyholder so payment will occur upon death, but there is uncertainty as to the timing. For both term and whole life insurance, it is most common for premiums and the amount of coverage to be fixed for the entire period in question.

In analyzing the relationship between the cost of one year of life insurance and whole life insurance premiums, assume a 30-year-old male purchases a $2 million whole life policy with an annual premium of $12,000. Based on mortality tables (as shown in LO 28.b), the probability of death within the year of a 30-year-old male is 0.001467, so the premium for one year of insurance should be $2,934. The excess of $9,066 is a surplus premium that is not required to cover the risk of a payout and is therefore invested by the insurance company for the policyholder. The process continues year after year while the cost of a one-year policy increases as the policyholder ages. Later in the policyholder's life, the one-year policy cost will exceed the annual premium ($12,000). From an overall perspective, the surplus in the earlier years is offset by the deficit in the later years. This concept is illustrated graphically in Figure 28.1.

Figure 28.1: Whole Life Insurance Policy Surplus and Deficit



Some variations of whole life insurance include variable life insurance whereby the final payout may be increased if the underlying investments outperform and universal life insurance whereby the premium may be reduced in exchange for a reduced final payout.

Annuity contracts are the opposite of life insurance contracts. In general, an initial lump sum payment is made by the annuitant to the insurance company in return for a stream of future payments from the insurance company to the annuitant for the remainder of life. Some annuities begin immediately while others start an agreed-upon number of years later (e.g., deferred annuities). Some deferred annuities have a guaranteed minimum amount of payments. The funds invested in the annuity will earn investment income; the total amount of the principal and income less the total payments made to the annuitant is equal to the accumulation value. Depending on the terms of the contract, the accumulation value may be withdrawn prematurely but likely with penalties.

Property and Casualty (P&C) Insurance

P&C insurance companies usually provide annual and renewable coverage against loss events. The premiums may increase or decrease based on any changes in estimates of expected payout. **Property insurance** covers property losses such as fire and theft. The risks can be managed in some instances because the expected payouts on claims can be estimated with a high degree of confidence if many policies are written on thousands of independent events (e.g., automobile insurance). However, property insurers may be subject to catastrophe risks arising from many large claims due to natural disasters, or they may benefit if there are no natural disasters, hence, the all-or-nothing nature of catastrophe risks. Such risks could be managed using geographical, seismographic, and meteorological information to determine the probability and severity of catastrophic events. **Casualty** (liability) **insurance** covers third-party liability for injuries sustained while on a policyholder's premises or caused by the policyholder's use of a vehicle, for example.

In general, for P&C insurance companies, property damage claims from natural disasters, and liability insurance claims are subject to fluctuating payouts and are very challenging to predict.

Health Insurance

Health insurance companies provide coverage to policyholders for medical services that are not covered under a publicly funded health care system. Policyholders pay ongoing premiums and the insurance company will make payments for events such as necessary hospital treatment or prescription medication. Premiums may increase due to general increases in health care costs (similar to automobile insurance), but they typically will not increase due to the worsening of the policyholder's health (similar to life insurance). In some cases, insurance coverage may not be denied to individuals with preexisting (but unknown) medical conditions.

Risks Facing Insurance Companies

Major risks facing insurance companies include the following:

- *Insufficient funds to satisfy policyholders' claims.* The liability computations often provide a significant cushion, but it is always possible to have a sudden surge of payouts in a short time (e.g., mortality risk and catastrophe risk) or payouts that continue for longer than expected (e.g., longevity risk).
- *Poor return (market risk) on investments.* Insurance companies often invest in fixed-income securities and if defaults suddenly increase, insurance companies will incur losses. Diversification of investments by industry sector and geography can help mitigate such losses.
- *Credit risk.* By transacting with banks and reinsurance companies, insurance companies face credit risk if the counterparty defaults on its obligations.
- *Operational risk.* Similar to banks, an insurance company faces losses due to failure of its systems and procedures or from external events outside the company's control (e.g., computer failure and human error).

Mortality Tables

LO 28.b: Describe the use of mortality tables and calculate the premium payments for a policy holder.

An excerpt from mortality tables estimated by the U.S. Social Security Administration for 2013 is provided in Figure 28.2.

As an example, examine the row for a male aged 40. The second column indicates that the probability of a 40-year-old male dying within the next year is 0.002092 (or 0.2092%). The third column indicates that the probability of a male surviving to age 40 is 0.95908 (or 95.908%). The fourth column indicates that a 40-year-old male has a remaining life expectancy of 38.53 years so that, on average, he will live to age 78.53. The remaining three columns show the same estimates for a female, and they appear slightly better than for a male.

Figure 28.2: Partial Mortality Table

Age (Years)	Male			Female		
	Probability of Death Within 1 Year	Survival Probability	Life Expectancy	Probability of Death Within 1 Year	Survival Probability	Life Expectancy
0	0.006519	1	76.28	0.005377	1	81.05
1	0.000462	0.99348	75.78	0.000379	0.99462	80.49
2	0.000291	0.99302	74.82	0.000221	0.99425	79.52
3	0.000209	0.99273	73.84	0.000162	0.99403	78.54
30	0.001467	0.97519	47.82	0.000664	0.98635	52.01
40	0.002092	0.95908	38.53	0.001287	0.97753	42.43
41	0.00224	0.95708	37.61	0.001393	0.97627	41.48
42	0.002418	0.95493	36.7	0.001517	0.97491	40.54
43	0.002629	0.95262	35.78	0.001662	0.97343	39.6
50	0.005038	0.9294	29.58	0.003182	0.95829	33.16
51	0.00552	0.92472	28.73	0.003473	0.95524	32.27
52	0.006036	0.91961	27.89	0.003767	0.95193	31.38
53	0.006587	0.91406	27.05	0.004058	0.94834	30.49
60	0.011197	0.86112	21.48	0.006545	0.91526	24.46
61	0.012009	0.85147	20.72	0.007034	0.90927	23.62
62	0.012867	0.84125	19.97	0.007607	0.90287	22.78
63	0.013772	0.83042	19.22	0.008281	0.896	21.95
70	0.023528	0.73461	14.24	0.015728	0.82864	16.43
71	0.025693	0.71732	13.57	0.017338	0.81561	15.68
72	0.028041	0.69889	12.92	0.019108	0.80147	14.95
73	0.030567	0.6793	12.27	0.021041	0.78616	14.23
80	0.059403	0.50629	8.2	0.043289	0.6388	9.64
90	0.167291	0.17735	4.03	0.132206	0.29104	4.8

Source: Social Security Administration, https://www.ssa.gov/OACT/STATS/table4c6_2013.html

When examining the full table, the probability of death during the following year is a decreasing function of age until age 10 and then it increases. For an 80-year-old male, the probability of death within the next year is about 5.9% and increases to about 16.7% at age 90.

Some probabilities can be computed indirectly using other numbers in the table. For example, in the third column, the probability of a male surviving to age 70 is 0.73461 and the probability of the male surviving to age 71 is 0.71732. Therefore, the probability of death of a male between age 70 and 71 is 0.73461 − 0.71732 = 0.01729 (or about 1.73%). Given that a male reaches age 70, the probability of death within the following year is 0.01729 / 0.73461 = 0.023536 (or about 2.35%), which is consistent with the number in the second column.

Going further, the probability of the death of a 70-year-old male in the second year (between ages 71 and 72) is the probability that he does not die in the first year times the probability that he does die in the second year. Using the numbers in the second column, the probability is (1 − 0.023528) × 0.025693 = 0.025088 (or about 2.51%).

With the information in the mortality tables, we can calculate the breakeven premium payment by equating the present value of the expected payout to the present value of the expected premium payments.

EXAMPLE: Breakeven premium payments

The relevant interest rate for insurance contracts is 3% per annum (semiannual compounding applies), and all premiums are paid annually at the beginning of the year. A $500,000 term insurance contract is being proposed for a 60-year-old male in average health. Assuming that payouts occur halfway throughout the year, **calculate** the insurance company's breakeven premium for a one-year term and a two-year term.

Answer:

One-year term:

The expected payout for a one-year term is 0.011197 × $500,000 = $5,598.50. Assuming the payout occurs in six months, the breakeven premium is: $5,598.50 / 1.015 = $5,515.76.

Two-year term:

The expected payout for a two-year term is the sum of the expected payouts in both the first year and the second year. The probability of death in the second year is (1 − 0.011197) × 0.012009 = 0.011874, so the expected payout in the second year is 0.011874 × $500,000 = $5,937.27. If the payout occurs in 18 months, then the present value is $5,937.27 / $(1.015)^3$ = $5,677.91. The total present value of the payouts is then $5,515.76 + $5,677.91 = $11,193.67.

The first premium payment occurs immediately (i.e., beginning of the first year) so it is certain to be received. However, the probability of the second premium payment being made at the beginning of the second year is the probability of not dying in the first year, which is 1 − 0.011197 = 0.988803. The present value of the premium payments (using Y as the breakeven premium) = Y + (0.988803Y / 1.015^2) = 1.959793Y.

Computing the breakeven annual premium equates the present value of the payouts and the premium payments as follows: 11,193.67 = 1.959793Y. Solving for *Y*, the breakeven annual premium is $5,711.66.

P&C Insurance Ratios

LO 28.f: Calculate and interpret loss ratio, expense ratio, combined ratio, and operating ratio for a property-casualty insurance company.

Property and casualty insurance companies compute the following ratios:

- The **loss ratio** for a given year is the percentage of payouts versus premiums generated. Assuming a loss ratio of 65%, then for every $100 of generated premiums, $65 will be paid out in claims and $35 is left to pay expenses and possibly earn profits.
- The **expense ratio** for a given year is the percentage of expenses versus premiums generated. The largest expenses are usually loss adjustments (e.g., claims investigation and assessing payout amounts) and selling (e.g., broker commissions).
- The **combined ratio** for a given year is equal to the sum of the loss ratio and the expense ratio.
- The **combined ratio after dividends** for a given year is equal to the combined ratio plus the payment of dividends to policyholders as a percentage of premiums (if applicable).
- The **operating ratio** for a given year is the combined ratio (after dividends) less investment income as a percentage of premiums. The mismatch of the cash inflows (generally earlier) and outflows (generally later) for many insurance companies allows them to earn interest income. For example, policyholders tend to pay their premiums upfront at the beginning of the year, but insurance companies tend to pay out claims throughout the year or after year-end.

Moral Hazard and Adverse Selection

LO 28.g: Describe moral hazard and adverse selection risks facing insurance companies, provide examples of each, and describe how to overcome these risks.

Moral hazard describes the risk to the insurance company that having insurance will lead the policyholder to act more recklessly than if the policyholder did not have insurance.

An example of moral hazard would be the existence of collision and liability coverage with automobile insurance. By having such coverage, some drivers would be willing to drive over the speed limits knowing that if an accident occurs, they would be covered for damage to the car and any resulting injury to a third party. Another example would be the existence of health insurance. By having such coverage, some policyholders may request more health services than necessary.

Methods to mitigate against moral hazard include deductibles (e.g., policyholder is responsible for a fixed amount of the loss), coinsurance provisions (e.g., insurance company will only pay a fixed percentage of losses), and policy limits (e.g., fixed maximum payout).

Adverse selection describes the situation where an insurer is unable to differentiate between a good risk and a bad risk. By charging the same premiums to all policyholders, the insurer may end up insuring more bad risks (e.g., careless drivers, sick individuals).

Methods to mitigate against adverse selection include greater initial due diligence (e.g., mandatory physical examinations for life insurance, researching driving records for automobile insurance) and ongoing due diligence (e.g., updating driving records and adjusting premiums to reflect changing risk).

Mortality Risk vs. Longevity Risk

LO 28.c: Distinguish between mortality risk and longevity risk and describe how to hedge these risks.

Mortality risk refers to the risk of policyholders dying earlier than expected due to illness or disease, for example. From the perspective of the insurance company, the risk of losses increases due to the earlier-than-expected life insurance payout.

Longevity risk refers to the risk of policyholders living longer than expected due to better healthcare and healthier lifestyle choices, for example. From the perspective of the insurance company, the risk of losses increases due to the longer-than-expected annuity payout period.

There is a natural hedge (or offset) for insurance companies that deal with both life insurance products and annuity products. For example, longevity risk is bad for the annuity business but is good for the life insurance business due to the delayed payout (or no payout if the policyholder has term insurance and dies after the policy expires). Mortality risk is bad for the life insurance business but is good for the annuity business because of the earlier-than-expected termination of payouts. The offset is not likely to be even, thereby resulting in some residual exposure that could be controlled with longevity derivatives, for example.

Longevity derivatives are used to hedge longevity risk inherent in annuity contracts and defined benefit pensions. An example of a payoff could be calculated as (prestated fixed mortality rate – actual mortality rate) × principal amount. Another example would be a longevity bond (or a survivor bond) whereby the bond coupon is set to an amount that is linked to the number of people in a defined population group that are still alive.

Capital Requirements for Insurance Companies

LO 28.h: Evaluate the capital requirements for life insurance and property-casualty insurance companies.

No global capital requirements exist for insurance companies; however, Solvency II is a set of regulations that is applicable in the European Union (EU). Under Solvency II, there is a minimum capital requirement (MCR) and a solvency capital requirement (SCR):

- If capital < SCR, capital must increase above the SCR.
- If capital < MCR, business operations may become significantly restricted.
- MCR is usually 25% to 45% of SCR.

SCR and MCR are calculated based on the sum of charges for

- investment risk (assets), which includes credit and market risk,
- underwriting risk (liabilities), and
- operational risk.

There is substantially more equity capital required for a P&C insurance company than for a life insurance company due to the potentially catastrophic nature and amount of claims for P&C insurance contracts. In contrast, the risks are lesser for life insurance companies that face exposure to more predictable longevity and mortality risks.

Guaranty System for Insurance Companies

LO 28.i: Compare the guaranty system and the regulatory requirements for insurance companies with those for banks.

In the United States, a **guaranty system** exists for both insurance companies and banks. Insurance companies are regulated at the state level while banks are regulated at the federal level.

For insurance companies, every insurer must be a member of the guaranty association in the state(s) in which it operates. If an insurance company becomes insolvent in a state, each of the other insurance companies must contribute an amount to the state guaranty fund (based on the amount of premium income it earns in that state). The guaranty fund proceeds are distributed to the policyholders of the insolvent company. In some cases, limits may apply on claims and there may be delays in settlement.

In contrast, the guaranty system for banks is a permanent fund to protect depositors and consists of amounts remitted by banks to the Federal Deposit Insurance Corporation (FDIC). No such permanent fund generally exists for insurance companies; therefore, insurance companies must make contributions whenever a default occurs.

Pension Plans

LO 28.d: Describe defined benefit plans and defined contribution plans and explain the differences between them.

Many companies establish pension plans on behalf of their employees with contributions being made by both parties. Upon retirement, the employee will receive periodic pension payments until death.

Defined benefit plans (i.e., employee benefit known, employer contribution unknown) explicitly state the amount of the pension that the employee will receive upon retirement. It is usually calculated as a fixed percentage times the number of years of employment times the annual salary for a specific period. There is significant risk borne by the employer because it is obligated to fund the benefit to the employee; therefore, when the present value of the pension obligation exceeds the market value of the pension assets, the employer must cover the deficiency. As a result, there is no risk borne by the employee (in theory). The computation of the pension liability is highly sensitive to the discount rate used and generally must be equal to the yield on AA rated bonds. Additionally, some defined benefit plans may include indexation of pension amounts to account for inflation and/or continued pension payments (likely on a reduced basis) to the surviving spouse upon the death of a retired employee.

Defined contribution plans (i.e., employer contribution known, employee benefit unknown) involve both employer and employee contributions being invested in one or more investment options selected by the employee. Upon retirement, the employee could opt to receive a lifetime pension (based on the ending value of the contributions) in the form of an annuity or, in some cases, simply to receive a lump sum. In contrast to defined benefit plans, there is virtually no risk borne by the employer because it is obligated simply to make a set contribution and no more. The risk of underperformance of the plan's investments is borne solely by the employee.

A defined contribution plan involves one individual account associated with one employee. The individual pension is computed based only on the funds in that account. In contrast, a defined benefit plan involves one pooled account for all employees; all contributions go into and all payments come out of the one account.

MODULE QUIZ 28.1

1. The relevant interest rate for insurance contracts is 2% per annum (semiannual compounding applies) and all premiums are paid annually at the beginning of the year. A $2,000,000 term insurance contract is being proposed for a 40-year-old male in average health. Assume that payouts occur halfway throughout the year. Using the mortality rates estimated by the U.S. Social Security Administration (in Figure 28.2), which of the following amounts is closest to the insurance company's breakeven premium for a two-year term?
   A. $4,246.
   B. $4,287.
   C. $4,332.
   D. $8,482.

2. The following information pertains to a property and casualty (P&C) insurance company:

Investment income	5%
Dividends	2%
Loss ratio	74%
Expense ratio	23%

   Based on the information provided, what is this company's operating ratio?
   A. 90%.
   B. 94%.
   C. 97%.
   D. 99%.

3. Which of the following problems would most likely be a concern for life insurance companies that are worried about differentiating between good risks and bad risks?
   A. Adverse selection.
   B. Catastrophe risk.
   C. Longevity risk.

D. Moral hazard.

4. Which of the following statements regarding the capital requirements and regulation of insurance companies is correct?
   A. Insurance companies are regulated at both the state and federal level.
   B. The guaranty system for insurance companies consists of a permanent fund created from premiums paid by insurers.
   C. If an insurance company's capital falls below the solvency capital requirement (SCR), then its business operations may become significantly restricted.
   D. The amount of equity required on the balance sheet of a life insurance company is typically lower than that of a P&C insurance company.

5. A new hire is researching the differences between a defined benefit plan and a defined contribution plan. Which of the following statements within the company's policies would indicate that the pension plan is a defined benefit plan? A defined benefit plan:
   A. involves one individual account associated with one employee.
   B. risks underperformance of the plan's investments, and this risk is borne solely by the employee.
   C. does not explicitly state the amount of the pension that the employee will receive upon retirement.
   D. involves one pooled account for all employees as all contributions go into and all payments come out of the one account.

KEY CONCEPTS

LO 28.a

Three categories of insurance companies include life insurance, nonlife (P&C) insurance, and health insurance. Life insurance companies usually provide long-term coverage and will make a specified payment to the policyholder's beneficiaries upon the death of the policyholder during the policy term.

Risks facing insurance companies include insufficient funds to satisfy policyholders' claims, poor return (market risk) on investments, credit risk, and operational risk.

LO 28.b

Mortality tables can be used to compute life insurance premiums. Mortality tables include information related to the probability of an individual dying within the next year, the probability of an individual surviving to a specific age, and the remaining life expectancy of an individual of a specific age.

LO 28.c

Mortality risk refers to the risk of policyholders dying earlier than expected. For the insurance company, the risk of losses increases due to the earlier-than-expected life insurance payouts. Longevity risk refers to the risk of policyholders living longer than expected. For the insurance company, the risk of losses increases due to the longer-than-expected annuity payout period. There is a natural hedge (or offset) for insurance companies that deal with both life insurance products and annuity products because longevity risk is bad for the annuity business but good for the life insurance business, and mortality risk is bad for the life insurance business but good for the annuity business.

LO 28.d

Defined benefit plans explicitly state the amount of the pension that the employee will receive upon retirement. It is usually calculated as a fixed percentage times the number of years of employment times the annual salary for a specific period. There is significant risk borne by the employer because it is obligated to fund the benefit to the employee.

Defined contribution plans involve both employer and employee contributions being invested in one or more investment options selected by the employee. There is virtually no risk borne by the employer because it is obligated simply to make a set contribution and no more. The risk of underperformance of the plan's investments is borne solely by the employee.

LO 28.e

Term (temporary) life insurance provides a specified amount of insurance coverage for a fixed period. Endowment life insurance is a subset of term insurance that has a payout at the stated contract maturity.

Whole (permanent) life insurance provides a specified amount of insurance coverage for the life of the policyholder.

LO 28.f

P&C insurance companies compute the following ratios:

- loss ratio + expense ratio = combined ratio
- combined ratio + dividends = combined ratio after dividends
- combined ratio after dividends − investment income = operating ratio

LO 28.g

Moral hazard describes the risk to the insurance company that having insurance will lead the policyholder to act more recklessly than if the policyholder did not have insurance. Methods to mitigate moral hazard include deductibles, coinsurance, and policy limits.

Adverse selection describes the situation where an insurer is unable to differentiate between a good risk and a bad risk. Methods to mitigate adverse selection include greater initial due diligence and ongoing due diligence.

LO 28.h

Under Solvency II, there is an MCR and a SCR:

- If capital < SCR, capital must increase above the SCR.
- If capital < MCR, business operations may become significantly restricted.
- MCR is usually 25% to 45% of SCR.

For a P&C insurance company, there is substantially more equity capital required than for a life insurance company because of the highly unpredictable nature of claims for P&C insurance contracts.

LO 28.i

For insurance companies in the United States, every insurer must be a member of the guaranty association in the state(s) in which it operates. If an insurance company becomes insolvent in a state, then each of the other insurance companies must contribute an amount to the state guaranty fund based on the amount of premium income it earns in that state.

The guaranty system for banks in the United States is a permanent fund to protect depositors that consists of amounts remitted by banks to the FDIC. No such permanent fund exists for insurance companies.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 28.1

1. **B** One-year term:

   The expected payout for a one-year term is 0.002092 × \$2,000,000 = \$4,184. Assuming the payout occurs in six months, the breakeven premium is \$4,184 / 1.01 = \$4,142.57.

   Two-year term:

   The expected payout for a two-year term is the sum of the expected payouts in both the first year and the second year. The probability of death in the second year is (1 − 0.002092) × 0.00224 = 0.0022353, so the expected payout in the second year is 0.0022353 × \$2,000,000 = \$4,470.63. If the payout occurs in 18 months, then the present value is \$4,470.63 / $(1.01)^3$ = \$4,339.15. The total present value of the payouts is then \$4,142.57 + \$4,339.15 = \$8,481.72.

   The first premium payment occurs immediately (i.e., beginning of the first year) so it is certain to be received. However, the probability of the second premium payment being made at the beginning of the second year is the probability of not dying in the first year, which is 1 − 0.002092 = 0.997908. The present value of the premium payments is as follows (using Y as the breakeven premium): Y + (0.997908Y / 1.01^2) = 1.978245Y.

   Computing the breakeven annual premium equates the present value of the payouts and the premium payments as follows: 8,481.72 = 1.978245Y. Solving for Y, the breakeven annual premium is \$4,287.50.

   Response A (\$4,246) is not correct because it performs the computation on the assumption that all payouts occur at the end of the year instead of halfway throughout the year. Response C (\$4,332) is not correct because it did not apply any discounting (at the 1% semiannual rate). Response D (\$8,482) is not correct because it is simply the total present value of the payouts. (LO 28.b)

2. **B** The operating ratio is computed as follows:

loss ratio (74%) + expense ratio (23%) + dividends (2%) − investment income (5%) = 94%

The combined ratio is computed as follows:

loss ratio (74%) + expense ratio (23%) = 97%

The combined ratio after dividends is computed as follows:

loss ratio (74%) + expense ratio (23%) + dividends (2%) = 99%

(LO 28.f)

3. **A** Adverse selection describes the situation where an insurer is unable to differentiate between a good risk and a bad risk. In the context of life insurance, by charging the same premiums to all policyholders (healthy and unhealthy individuals), the insurer may end up insuring more bad risks (e.g., unhealthy individuals). To mitigate adverse selection, a life insurance company might require physical examinations before providing coverage. (LO 28.g)

4. **D** Property and casualty insurance companies typically have a greater amount of equity than a life insurance company because of the highly unpredictable nature of P&C claims (both timing and amount).

   Insurance companies are regulated at the state level only (and banks are regulated at the federal level only). The guaranty system for insurance companies is not a permanent fund; in contrast, banks have a permanent fund created from premiums paid by banks to the FDIC. On the liability side of a P&C insurance company's balance sheet, there are unearned premiums that represent prepaid insurance contracts whereby amounts are received but the coverage applies to future periods. If an insurance company's capital falls below the MCR, then its business operations may become significantly restricted. If capital falls below the SCR, then plans must be made to increase capital above the SCR. (LO 28.h)

5. **D** A defined contribution plan involves one individual account associated with one employee. The individual pension is computed based only on the funds in that account. In contrast, a defined benefit plan involves one pooled account for all employees; all contributions go into and all payments come out of the one account. Defined benefit plans explicitly state the amount of the pension that the employee will receive upon retirement. (LO 28.d)

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Financial Markets and Products, Chapter 3.

READING 29

FUND MANAGEMENT

Study Session 8

EXAM FOCUS

Not every investor has the time or the skill to manage their own financial assets. For this reason, investors will sometimes hire a professional manager in the form of a mutual fund or perhaps a hedge fund. These pooled investment vehicles offer instant diversification and professional management to their investors. Smaller investors often use mutual funds while hedge funds are tools for wealthy individuals. Because hedge funds are limited only to those who can afford to lose their investment, they are subject to much less regulation. For the exam, be able to describe the various types of mutual funds and hedge funds along with their regulatory environments and typical fee structures.

MODULE 29.1: MUTUAL FUNDS AND EXCHANGE-TRADED FUNDS

LO 29.a: Differentiate among open-end mutual funds, closed-end mutual funds, and exchange-traded funds (ETFs).

Mutual funds are pooled investment vehicles that offer instant diversification for their investors. This diversification is very important because it spreads out risk to different sectors and asset classes. Most investors either do not have the time or the skill to properly diversify on their own. For this reason, investment vehicles like open-end mutual funds, closed-end mutual funds, and exchange-traded funds (ETFs) were created. There has been significant growth in mutual fund assets in the United States over the years, from $0.5 billion in 1940 to about $19 trillion in 2017.

Open-End Mutual Funds

Open-end mutual funds, which are often simply called mutual funds, are the most common pooled investment vehicle (more than 98% of U.S. mutual fund assets). Essentially, investors are commingling their funds to be better diversified, to save on transaction fees, and to hire a professional management team. The professional management team will conduct research and ultimately invest commingled assets on behalf of their investors. These investors begin their investment by purchasing a set dollar amount of an open-end mutual fund and then they receive a proportional ownership interest (in the form of shares) in the mutual fund. This means that the number of shares goes up as new investors arrive and goes down as investors withdraw assets. When investors decide that they want to exit their investment in an open-end mutual fund, they can redeem their shares directly from the fund company, who will promptly send them either a check or a digital transfer of the value of their investment.

At a high level, open-end mutual funds are broken down into four main categories: money market funds, equity funds, bond funds, and hybrid funds. Money market funds invest in short-term interest-bearing instruments, such as Treasury bills, commercial paper, and banker's acceptances. Money market investors are typically risk averse. This category is an alternative to interest-bearing bank accounts and is often the "cash" portion of an investor's asset allocation mix. Equity funds invest solely in stocks. Within this category, one can find index funds that track a broad market index, such as the S&P 500 Index, funds that follow a certain style, such as medium company value funds, or sector funds, such as a health care sector fund. The amount of tracking error is indicative of the success of a fund in tracking the desired index. Bond funds invest only in fixed-income instruments, such as sovereign debt, corporate bonds, and asset-backed securities. Hybrid funds will blend stock and bond ownership into the same fund.

Open-end funds trade at the fund's **net asset value (NAV)**, which is essentially the sum of all assets owned minus any liabilities of the fund then divided by the shares outstanding. When investors decide they want to buy shares of an open-end mutual fund, they will transact at the next available NAV, which is not calculated until after the market closes at 4:00 pm in New York City. An investor who decides at 10:00 am that they want to buy shares will enter a buy order for a set dollar amount, but they will not know the price at which they will transact until after the market closes. For this reason, we say that open-end fund investors have poor price visibility. Since shares are transacted at an unknown price, investors cannot use stop orders or limit orders. They must place a market order to transact in shares of an open-end mutual fund.

Taxes are levied against open-end mutual fund investors as if they owned the diversified fund's holdings outright. If the underlying investment pays a dividend, then the investors must pay taxes on their proportional ownership interest in that dividend. The open-end fund may also buy and sell underlying investments and generate taxable short-term or long-term capital gains. These taxable events are also passed on to investors. Dividends and capital gains are distributed to investors typically toward the end of the calendar year, but they can be automatically reinvested in the fund to purchase more shares. Investors often choose reinvestment if they do not need the cash flow for current consumption.

The cost of investing is also a major consideration for any investment category. Open-end mutual funds have a management fee and potentially a sales charge. The management fee covers the operational costs of the open-end mutual fund company, including the salaries of the management team. The expense ratio is calculated as the annual management fee divided by the assets under management. Management fees vary depending on country but are significantly lower for index funds compared to actively managed funds, due to the increased complexity of the actively managed funds. Sales charges are commonly called loads. A **front-end load** is a set percentage that is charged to the investor upon initial investment in the fund. Alternatively, some funds impose a sales charge when the investor sells an investment in the fund. This is called a **back-end load**.

Closed-End Mutual Funds

Closed-end mutual funds are a similar concept to open-end funds with a few notable differences.

The first difference is that a purchase of shares in an open-end mutual fund will increase the number of shares outstanding because new shares are created, but a closed-end fund's number of shares remains static. Investors who desire to purchase or sell shares of a closed-end fund do not transact directly with the fund company but rather with other investors. Recall that investors who want to close their investment position in an open-end fund can simply redeem their shares from the fund company. This is where the fund gets the name open-end.

The second difference is that closed-end fund shares can be bought or sold at any time and both long and short positions can be taken; open-end fund shares can only be bought or sold at specific times and only long positions can be taken.

The third difference is that, while open-end funds always transact at the next available NAV, a closed-end fund can transact at a price other than NAV. It is very common for a closed-end fund to trade at a discount to its actual NAV. In the case of a discount, an argument can be made that the discount arises because of management fees.

PROFESSOR'S NOTE

In terms of trading, a closed-end fund behaves much like an individual stock. Investors can trade closed-end funds throughout the trading day, which means they have better price visibility and can utilize stop orders and limit orders if they so choose.

Exchange-Traded Funds

Exchange-traded funds (ETFs) are created by depositing shares with an ETF and then receiving shares of the ETF. They represent an innovative twist on the open-end mutual fund. They enable instant diversification like an open-end fund, but they are exchange-traded, which means they trade throughout the day on the open market just as a closed-end fund does. Because they trade throughout the day, investors can utilize stop orders, limit orders, and even short selling in some cases.

Unlike a closed-end fund, ETFs typically trade at their NAV. Many ETFs are passively managed index funds, although some new actively managed ETFs are beginning to come to market. One of the most widely known ETFs is the SPDR S&P 500 (SPY).

ETFs must disclose their holdings twice each day, which enables investors to have tremendous visibility into their underlying investments. Open-end mutual funds, on the other hand, disclose their holdings very infrequently, perhaps as delayed as once per quarter.

Undesirable Trading Behaviors

LO 29.b: Identify and describe potential undesirable trading behaviors at mutual funds.

Potential undesirable trading behaviors among mutual funds include late trading, market timing, front running, and directed brokerage.

Late trading occurs when orders are accepted after the 4:00 pm cut off trading time (in the United States for open-end mutual funds). It is possible for significant market events to occur shortly after 4:00 pm that would cause previously submitted trades to be reversed. Therefore, the acceptance of orders after 4:00 pm would be considered illegal and subject to prosecution.

Market timing occurs because some fund assets are not actively traded, thereby resulting in stale pricing when calculating NAV. If market prices are rising (falling) shortly before the 4:00 pm cut off, it may be profitable to buy (sell) at the NAV at 4:00 pm since the stale pricing means that the value of the shares is likely higher (lower) than the NAV. Market timing trades may result in sudden fluctuations in the fund's size that will require the fund to maintain greater liquidity to satisfy redemptions (holding liquid assets such as cash would lead to reduced fund returns since cash generates little or no investment income). Although of potential concern to regulators if trading exceptions are made for market timing, the act of market timing is not illegal.

Front running involves trading ahead of a likely price increase or decrease due to a known upcoming trade to be made by the fund. It may involve the trader's own account or favored clients or employees. Like late trading, front running is illegal, and is subject to prosecution.

Directed brokerage involves a quid pro quo whereby a mutual fund will direct trades to a broker in exchange for the broker investing its clients in the mutual fund. Although not illegal, it is a strongly discouraged practice.

Net Asset Value

LO 29.c: Explain the concept of net asset value (NAV) of an open-end mutual fund and how it relates to share price.

To calculate the net asset value (NAV), the fund needs to know the current value of all investment holdings (including cash positions), any liabilities such as management fees payable, and the total number of shares outstanding. Calculation of the NAV is shown as follows:

$$\text{NAV} = \frac{\text{fund assets} - \text{fund liabilities}}{\text{total shares outstanding}}$$

EXAMPLE: Computing NAV

Consider an open-end mutual fund that owns $1.1 billion in equities, $350 million in bonds, and $35 million in cash. They owe $1.85 million in management fees payable at this point in the quarter and they have 39.635 million shares outstanding. **Calculate** this fund's NAV.

Answer:

$$\$37.42 = \frac{(\$1{,}100 + \$350 + \$35) - 1.85}{39.635}$$

Investors who wish to buy or sell this fund will transact at exactly $37.42 per share, which is not calculated until after the market closes on the trading day in question. If they wanted to invest $25,000, then they would buy exactly 668.092 (= $25,000 / $37.42) shares after the market closes on the relevant trading day.

Recall that the NAV for an open-end mutual fund is only calculated after the close of trading on any given day, while the NAV for closed-end funds and ETFs is calculated continuously throughout the day. Also, note that it's possible for ETF share price and NAV to diverge slightly in the short-term. However, since ETFs must disclose assets twice each day, any differences are quickly arbitraged away by institutional investors.

MODULE QUIZ 29.1

1. Which of the following statements is not correct regarding investment funds available to all investors?
   A. Open-end mutual funds always transact at the next available NAV.
   B. Stop orders can be used on closed-end funds.
   C. Open-end mutual funds can be purchased with a limit order.
   D. Short selling is available for some ETFs.

MODULE 29.2: HEDGE FUNDS

LO 29.d: Explain the key differences between hedge funds and mutual funds.

Mutual funds are marketed to all investors, while hedge funds are restricted to only wealthy and sophisticated investors and institutions. As a result, hedge funds escape certain regulations that apply to mutual funds (although hedge funds may face some restrictions posed by their prime broker, the bank that provides the hedge fund with financing and trade processing). Specifically, they do not need to provide the redemption of shares at any time the investor chooses, a daily calculated NAV, or the full disclosure of their investment policies and strategies. Hedge funds are also permitted to use leverage while mutual funds are not. Because hedge funds can use leverage and are permitted to use both long and short investment strategies, they are considered to be an alternative investment class. Finally, hedge funds charge an additional incentive fee while mutual funds do not.

The term hedge fund implies that the fund is hedging some form of risk. This may be the case if the fund is using both long and short positions (long positions for expected outperformers and short positions for expected underperformers), but not all hedge funds focus on risk reduction. Some involve no hedging and focus on risk enhancement.

Since hedge funds are not required to redeem shares any time an investor requests, they have lockup periods. The **lockup period** is a certain amount of time (often one year) in which the investor is not able to withdraw his funds.

PROFESSOR'S NOTE

A lockup period exists for one key reason—many hedge fund investments are not easy to unwind on short notice. Some hedge fund investments are illiquid, which means managers cannot sell them quickly and retain a proper value. In addition, some hedge fund investments are bets on certain asset mispricing, and those trades can take time to unwind.

Hedge Fund Expected Returns and Fee Structures

LO 29.e: Calculate the return on a hedge fund investment and explain the incentive fee structure of a hedge fund including the terms hurdle rate, high-water mark, and clawback.

While mutual funds charge fees as a set percentage of assets under management (AUM), hedge funds deploy a more complex compensation structure centered around incentive fees. These **incentive fees** are engineered to give hedge fund managers significant payouts based on their performance. The typical hedge fund fee structure is known as **2 plus 20%**, which means that they charge a flat 2% of all assets that they manage plus an additional 20% of all profits above a specified benchmark. An assumption needs to be made as to whether management fees are computed based on beginning or end of year assets and whether incentive fees are computed before or after deducting management fees.

Using the more conservative assumptions of management fees being computed based on beginning of year assets (A) and incentive fees being computed after deducting management fees, the incentive fee is computed as follows:

$$0.2 \times \max(R \times A - 0.02 \times A, 0)$$

where:
R = return on assets for the year

The incentive fee structure can be thought of as a call option on the dollar asset return with a strike price of 2% of AUM. For the managers, there is significant upside potential with no corresponding downside risk.

Hedge funds do soften the incentive fee structure with a few safeguards for investors. The first safeguard is the **hurdle rate**, which is the benchmark that must be beaten before incentive fees can be charged.

The second safeguard is a **high-water mark clause**, which essentially states that previous losses must first be recouped and hurdle rates surpassed before incentive fees once again apply. Consider a hedge fund that just began with $100 million in assets from investors. Their hurdle rate is the 10-year Treasury, which is currently yielding

1.5%. In the first year of operation, this hedge fund made some bad decisions and ended up losing \$10 million (ending balance of \$90 million). This means that the managers get to charge the 2% flat fee, but no incentive fees apply. Incentive fees would only have applied to any profits earned above a 1.5% return, meaning that only an ending balance higher than \$101.5 million would have triggered the 20% incentive fee. In year two, this hedge fund would need to get its fund up above \$103 million (two years of beating Treasuries) in order for incentive fees to apply. In this case, the high-water mark for year one is \$101.5 million, and for year two it is \$103 million.

The third safeguard for investors is a **clawback clause**, which enables investors to retain a portion of previously paid incentive fees in an escrow account that is used to offset subsequent investment losses should they occur.

The incentive fee structure of a hedge fund certainly encourages hedge fund managers to reach for profits, but this comes at the expense of also encouraging them to take risks. A hedge fund manager essentially owns a call option against the assets of the hedge fund and payoff for options are higher if volatility is higher. Consider an example where a hedge fund manager is presented with an opportunity that offers a 40% probability of returning 50% and a 60% probability of losing 50%. The expected return of the fund can be calculated as follows:

$$(0.4 \times 50\%) + (0.6 \times -50\%) = -10\%$$

In this example, the hedge fund manager might be willing to take a big risk (60% probability) of losing money, which would end in him only collecting his 2% flat fee. The alternative is that if he were to end up making a huge return with the lower probability event, then he would potentially earn a substantial incentive fee. If this hedge fund generates a 50% profit, then he could potentially earn fees of 11.6% [= 2% (flat fee) + 0.20 × 48% (incentive fee on return above the 2% flat fee)]. The expected payoff for fees then becomes 5.84%:

$$(0.4 \times 11.6\%) + (0.6 \times 2\%) = 5.84\%$$

From the investor's perspective, the expected payoff is −15.84%:

$$[0.4 \times (50\% - 11.6\%)] + [0.6 \times (-50\% - 2\%)] = 0.1536 - 0.312 = -15.84\%$$

The expected return for the hedge fund is 5.84% and the expected return for the hedge fund investor is −15.84%. When these two numbers are added together, we arrive back at the original return of −10%. This shows the disproportionate payoff for the hedge fund manager. Why would investors be willing to make this investment? Clearly they are hoping that the incentive fees will motivate the hedge fund manager to do everything within their power to produce significant returns for both the investor and the hedge fund manager.

Hedge Fund Strategies

LO 29.f: Describe various hedge fund strategies including long-short equity, dedicated short, distressed securities, merger arbitrage, convertible arbitrage, fixed income arbitrage, emerging markets, global macro, and managed futures,

and identify the risks faced by hedge funds.

Hedge funds deploy numerous different strategies in their attempt to capture incentive fees. Not all hedge funds fall easily into a specific category, but the discussion in this section follows the classification system used by the Dow Jones, which provides indices to track various hedge fund strategies. Throughout this section, you will see the term **arbitrage**, which (in the hedge fund context) involves short selling an asset that is believed to be overvalued and buying an asset that is believed to be undervalued in an attempt to exploit a pricing differential.

Long/Short Equity

Long/short equity hedge funds endeavor to find mispriced securities. Managers of a long/short equity fund spend a great deal of time conducting fundamental analysis on stocks that are largely ignored by most analysts, in an attempt to find mispricings. They will buy (go long) a stock that they believe to be undervalued and short sell (go short) a stock that they believe to be overvalued. Sometimes funds can have a net long bias or a net short bias depending on what opportunities they see in the markets. Funds can also be sector neutral, where they net long and short positions that cancel out sector exposure (e.g., automobile stocks General Motors and Ford).

Dedicated Short

Dedicated short hedge funds are focused exclusively on finding a company that they think is overvalued and then short selling the stock. Traditionally, short sellers are looking for companies with weak financial performance that has not been captured by the market. Due to the lack of hedging of overall markets, dedicated short funds do not perform well when markets are performing well.

Distressed Debt

Bonds with a CCC rating are considered distressed. Distressed bonds usually trade at deep discounts to par value. **Distressed debt hedge funds** are searching for distressed bonds with the potential to turn things around. Many of these distressed companies are in or close to being in bankruptcy proceedings. Some distressed bond investors take an active approach to influencing the target company's reorganization to their advantage. For example, if they a large enough position of any class of a bond, then they can block any reorganization plan that is not in their best interest.

Merger Arbitrage

Merger arbitrage hedge funds try to find arbitrage opportunities after mergers are announced, but before the deal is closed (i.e., the period where it is not completely certain that the merger will occur).

Consider an all-cash deal where Company A announces that it will buy Company B at $50 per share. Preannouncement, Company B was trading at $37.50 and postannouncement Company B will typically be trading somewhere near $48. A merger arbitrage fund would buy the shares of Company B and wait for the full $50 (or better) price to be achieved.

Now consider an all-stock deal where Company A offers one share of its stock for every four shares of Company B's stock. This could be a realistic ratio if Company B had a considerably lower market capitalization than Company A. In this case, a merger arbitrage fund would buy a certain amount of Company B's shares and, at the same time, they would short sell one-quarter of this number of shares in Company A's stock. This is because the acquirer usually pays too much and their stock usually goes down after a merger.

The underlying assumption here is that any assessment of a merger arbitrage strategy takes into account public (and not insider) information.

Convertible Arbitrage

Some hedge funds invest using convertible bonds, which are fixed-income instruments that can be converted into shares of stock if the stock price rises above a prespecified value. **Convertible arbitrage hedge funds** develop a sophisticated model to value convertible bonds. The idea is that if the market price is not the same as the model price, then profits can be made if the two prices converge.

Fixed-Income Arbitrage

Fixed-income arbitrage hedge funds attempt to exploit perceived mispricing—long positions in underpriced bonds and short positions in overpriced bonds. The positions are usually highly leveraged to be profitable.

Emerging Market

Emerging market hedge funds focus on investments in developing countries; the managers often expend great effort to research their investments in less-known securities. Some hedge funds invest using American depository receipts (ADRs), which are certificates issued in the United States that provide ownership in foreign countries coupled with currency exposure. There are occasionally pricing discrepancies between the ADR and the underlying asset that an adept hedge fund manager can exploit as well. If managers decide to invest using emerging market debt, then they need to consider default risk because some countries have defaulted numerous times (e.g., Russia, Argentina, Brazil, and Venezuela).

Global Macro

Global macro hedge funds attempt to profit from a macroeconomic trend that they feel is in disequilibrium (i.e., not currently priced correctly and rationally). They will place very large dollar bets on the equilibrium being reestablished. For example, the investment focus may be on foreign exchange rates, interest rates, or inflation. A deviation from equilibrium could take a long time to correct itself and some hedge funds will not be able to wait out the trend.

Managed Futures

Managed futures hedge funds involve strategies that try to predict future movements in commodity prices and invest according to those predictions. Fund managers will often backtest their trading rules using historical data. A key drawback of backtesting is that there is no distinction made between strategies that truly worked based on proper fundamental analysis or strategies that were successful strictly because of luck and subsequently may might not have repeated success.

Hedge Fund Performance and Measurement Bias

LO 29.g: Describe characteristics of mutual fund and hedge fund performance and explain the effect of measurement biases on performance measurement.

Empirical research has suggested that on average, actively managed mutual funds underperform the market once expenses are accounted for. That has led many investors to invest in index funds, which are passively managed and have lower fees. In addition, the few actively managed funds that do beat the market in a given year do not have an increased probability of doing so in subsequent years (i.e., low persistence).

Hedge fund performance is not as easy to assess as mutual fund performance, which is readily available and accurately reported by numerous independent parties. Participation in hedge fund indices is voluntary. If the fund had good performance, then they will report their results to the index vendor. If they did not have good results, then they simply do not report their results to the index. In the Barclay's Hedge Fund Index, the data for August 2016 had 2,914 funds reporting information, while September 2016 only had 617. Based on that example, it is clear that there is **measurement bias** in hedge fund index reporting. When returns are reported by a hedge fund, the database is then backfilled with the fund's previous returns. This is known as **backfill bias** and it creates an issue with reliability for hedge fund benchmarks. It is common for a hedge fund to have a string of several good years and then have a meltdown.

MODULE QUIZ 29.2

1. Which of the following characteristics is a key differentiator between mutual funds and hedge funds?
   A. Professional asset management.
   B. Immediate access to withdrawals from the fund.
   C. Charging a fee for providing investment services.
   D. Easy diversification for an investor.

2. What is the expected payoff for fees if a hedge fund uses a standard 2 plus 20% incentive fee structure with an investment that has a 35% probability of making 55% and a 65% probability of losing 45%?
   A. 3.78%.
   B. 5.28%.
   C. 5.71%.
   D. 6.12%.

3. Which type of hedge fund focuses on isolating mispricings in foreign exchange markets?
   A. Fixed-income arbitrage hedge funds.
   B. Global macro hedge funds.
   C. Managed futures hedge funds.

D. Convertible arbitrage hedge funds.

4. Which of the following statements is/are possible regarding hedge fund performance reporting?
   I. When a hedge fund's performance is recorded in an index, all of its prior results are also included.
   II. Hedge funds are permitted to self-select if their performance is reported in index averages.
   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

KEY CONCEPTS

LO 29.a

There are three primary types of commingled pools of investments that are available to investors. They are open-end mutual funds, closed-end mutual funds, and ETFs. Open-end funds transact at the next available NAV, which occurs after the market has closed for the day. Shares may be redeemed directly from the fund company with an open-end fund. Closed-end funds transact throughout the trading day, but shares cannot be redeemed at the fund company and their price may differ substantially from their NAV—the shares must be bought or sold by other investors. ETFs also trade throughout the day, but their shares do trade at the NAV. ETFs usually have the lowest internal fees, which is a big component of investment returns.

LO 29.b

Undesirable trading behaviors may occur despite rules designed to prevent fraud and conflicts of interest. These trading behaviors include late trading, market timing, front running, and directed brokerage.

LO 29.c

The NAV is easily calculated as the total invested assets of the fund minus any liabilities (typically management fees payable) all divided by the total shares outstanding. The NAV for an open-end fund is set after the trading day is over, while the NAV for a closed-end fund and an exchange-traded fund is calculated continuously throughout the trading day. The NAV is used to determine the number of shares purchased or sold in a fund.

LO 29.d

Both mutual funds and hedge funds offer professional management, instant diversification, and the ability to commingle funds with other investors. However, there are some notable differences between mutual funds and hedge funds. Hedge funds are only marketed to wealthy and sophisticated investors. Because of this, hedge funds escape certain regulatory oversight, which enables them to avoid allowing investors to redeem shares at any time they want, calculating the NAV daily, and disclosing investment policies and strategies. They are also permitted to use leverage and short selling, which are commonly not permitted for mutual funds. In addition, hedge funds use lock-up periods to prevent investor withdrawals at the wrong time for the fund.

LO 29.e

Hedge funds commonly deploy a 2% and 20% incentive fee structure, where they earn management fees for investment results relative to a given hurdle rate. Investors are partially protected with the use of high-water marks and clawback clauses.

LO 29.f

There are many different types of hedge fund strategies. They all search for perceived mispricings in different corners of the markets and then try to exploit them for profit.

Long/short equity funds take both long and short positions in the equity markets, diversifying, or hedging across sectors, regions, or market capitalizations, and have directional exposure to the overall market.

Dedicated short funds tend to take net short positions in equities, and their returns are negatively correlated with equities.

Distressed hedge funds invest across the capital structure of firms that are under financial or operational distress or are in the middle of bankruptcy. These hedge fund managers try to profit from an issuer's ability to improve its operation or come out of a bankruptcy successfully.

Merger arbitrage funds bet on spreads related to proposed merger and acquisition transactions.

Convertible arbitrage funds attempt to profit from the purchase of convertible securities and the shorting of corresponding stock.

Fixed-income arbitrage funds try to obtain profits by exploiting inefficiencies and price anomalies between related fixed-income securities.

Emerging market funds invest in currencies, debt, equities, and other instruments in countries with emerging or developing markets.

Global macro managers make large bets on directional movements in interest rates, exchange rates, commodities, and stock indices, and do better during extreme moves in the currency markets.

Managed futures funds attempt to predict future movements in commodity prices based on either technical analysis or fundamental analysis.

LO 29.g

Mutual fund performance is generally easy and objective to measure. Actively managed funds have generally underperformed the market and led to the increased popularity of passive mutual funds (e.g., index funds) with lower fees. Hedge fund benchmarks are problematic due to measurement bias and backfill bias.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 29.1

1. **C** Open-end mutual funds have very low price transparency because they trade at the next available NAV, which is not calculated until after the market closes. As such, they can only be bought or sold using a market order. Closed-end funds can be bought or sold using stop orders and limit orders. In some cases, ETFs can be sold short. (LO 29.a)

Module Quiz 29.2

1. **B** Mutual funds must offer immediate access to withdrawals from their fund. This is an SEC requirement. Hedge funds have advance notification and lock-up periods, which prevent immediate access to withdrawals from the fund. (LO 29.d)

2. **C** The hedge fund could potentially earn fees of 12.6% [2% (flat fee) + 0.20 × 53% (incentive fee on return above the 2% flat fee)]. The expected payoff for fees then becomes 5.71% computed as follows:

   $(0.35 \times 12.6\%) + (0.65 \times 2\%) = 5.71\%$

   (LO 29.e)

3. **B** Global macro funds focus on finding mispricings at the level of the global macro economy. They materialize in foreign exchange pricing and interest rates. Fixed-income arbitrage funds focus on various mispricings with fixed-income securities. Managed futures funds focus on forecasting commodity prices. Convertible arbitrage funds focus on valuing convertible bonds. (LO 29.f)

4. **C** Statement I describes backfill bias and Statement II describes measurement bias. Backfill bias arises when the database is backfilled with the fund’s previous returns. Measurement bias indicates that not all hedge funds report their performance to index providers. (LO 29.g)

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Financial Markets and Products, Chapter 4.

READING 30

INTRODUCTION TO DERIVATIVES

Study Session 8

EXAM FOCUS

In this reading, we present the basic concepts of derivatives securities and derivatives markets. For the exam, know the basic derivatives terms as well as the terms related to derivatives markets. Also, be able to compute payoffs for the different derivatives securities. Finally, be able to create a hedge and know how to take advantage of an arbitrage situation.

MODULE 30.1: DERIVATIVES MARKETS AND SECURITIES

LO 30.a: Define derivatives, describe the features and uses of derivatives, and compare linear and non-linear derivatives.

A **derivative** security is a financial security whose value is derived in part from another security's characteristics or value. This other security is referred to as the **underlying asset**. A derivative effectively derives its price from some other variable.

Derivatives can be used for financial risk management (i.e., hedging), for speculation, for diversification of exposures, as added features to a bond (e.g., convertible, callable), as employee compensation in the case of stock options, within a capital project as an embedded option (e.g., real or abandonment options).

Linear derivatives, such as forward and futures contracts, have a linear payoff that is directly related to the value of the underlying. The contracts specify the buying or selling of an underlying asset for a stated price at a stated time in the future. They are essentially zero-sum games where one party wins the same amount that the other party loses. In contrast, **nonlinear derivatives**, such as options, involve the option purchaser (holder) having the right but not being obligated to buy or sell an underlying asset at a stated time in the future. Therefore, the payoff is nonlinear in relation to the value of the underlying.

LO 30.b: Describe the specifics of exchange-traded and over-the-counter markets, and evaluate the advantages and disadvantages of each.

Traditional derivatives exchanges use both an open outcry system and an electronic trading system to match buyers with sellers. The open outcry system (e.g., the Chicago Board of Trade [CBOT]) is the more traditional system, which involves traders indicating their trades through hand signals and shouting. Electronic trading, which represents most of the trading done today, does not involve a physical exchange location, but rather involves matching buyers and sellers electronically via computers (e.g., NASDAQ). Algorithmic trading is a form of electronic trading, which executes trades without human involvement.

An **over-the-counter (OTC) market** differs from a traditional exchange in that the end users and dealers would contact each other either directly or through a broker; dealers frequently use interdealer brokers to transact with other dealers. A dealer maintains bid and offer prices in a security and stands ready to buy or sell lots of the given security. The OTC market typically involves much larger trades than traditional exchanges.

The OTC market is several times the size of the traditional exchange market. For example, in 2017, the OTC market was over $530 trillion, while the exchange-traded market was over $80 trillion.

Advantages of OTC trading:

- Terms are not set by any exchange (i.e., not standardized so customization is possible).
- Some new regulations since the credit crisis (e.g., standardized OTC derivatives now traded on swap execution facilities, a central counterparty is now required for standardized trades, and trades are now required to be reported to a central registry).
- Greater anonymity (e.g., an interdealer broker only identifies the client at the conclusion of the trade).

Disadvantages of OTC trading:

- OTC trading has more credit risk than exchange trading when it comes to nonstandardized transactions. Exchanges are organized in such a way that credit risk is eliminated.

LO 30.c: Differentiate between options, forwards, and futures contracts.

An **option contract** is a contract that, in exchange for paying an option premium, gives the option buyer the right, but not the obligation, to buy (sell) an asset at the prespecified exercise (strike) price from (to) the option seller within a specified time period, or depending on the type of option, a precise date (i.e., expiration date). A call option gives the option holder the right to purchase the underlying asset by a certain specified date at the exercise price. A put option gives the option holder the right to sell the underlying asset by a selected date at the exercise price. An **American-style option** contract can be exercised any time between issue date and expiration date. In contrast, a **European-style option** contract may be exercised only on the actual expiration date. American options will be worth more than European options when the right to early exercise is valuable, and they will have equal value when it is not.

A **forward contract** is a contract that specifies the price and quantity of an asset to be delivered sometime in the future. There is no standardization for forward contracts, and these contracts are traded in the OTC market. One party takes the long position, agreeing to purchase the underlying asset at a future date for a specified price, while the other party is the short, agreeing to sell the asset on that same date for that same price. Forward contracts are often used in foreign exchange situations as these contracts can be used to hedge foreign currency risk.

A **futures contract** is a more formalized, legally binding agreement to buy or sell a commodity or financial instrument in a predesignated month in the future, at a price agreed upon today by the buyer/seller. Futures contracts are highly standardized regarding quality, quantity, delivery time, and location for each specific commodity. These contracts are typically traded on an exchange.

LO 30.d: Identify and calculate option and forward contract payoffs.

Call Option Payoff

The payoff on a **call option** to the option buyer is calculated as follows:

$C_T = \max(0, S_T - X)$

where:
C_T = payoff on call option
S_T = stock price at maturity
X = strike price of option

The payoff to the option seller is $-C_T$ [i.e., $-\max(0, S_T - X)$]. We should note that $\max(0, S_t - X)$, where time, *t*, is between 0 and T, is the payoff if the owner decides to exercise the call option early (in the case of an American option as we will discuss later).

The price paid for the call option, C_0, is referred to as the **call premium**. Thus, the profit to the option buyer is calculated as follows:

$\text{profit} = C_T - C_0$

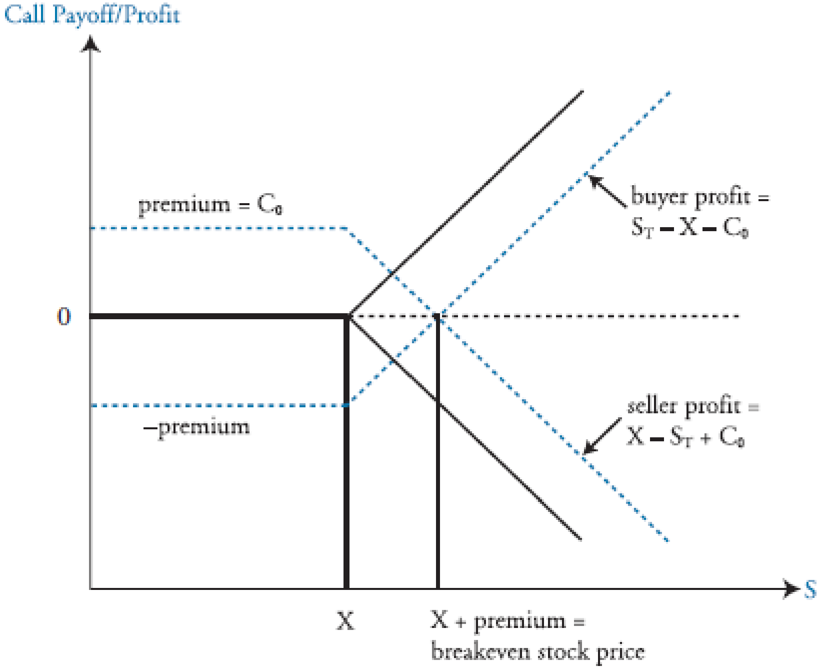
where:
C_T = payoff on call option
C_0 = call premium

Conversely, the profit to the option seller is:

$\text{profit} = C_0 - C_T$

Figure 30.1 depicts the payoff and profit for the buyer and seller of a call option.

Figure 30.1: Profit Diagram for a Call at Expiration



Put Option Payoff

The payoff on a **put option** is calculated as follows:

$$P_T = \max(0, X - S_T)$$

where:
P_T = payoff on put option
S_T = stock price at maturity
X = strike price of option

The payoff to the option seller is $-P_T$ [i.e., $-\max(0, X - S_T)$]. We should note that $\max(0, X - S_t)$, where $0 < t < T$, is also the payoff if the owner decides to exercise the put option early.

The price paid for the put option, P_0, is referred to as the **put premium**. Thus, the profit to the option buyer is calculated as follows:

$$\text{profit} = P_T - P_0$$

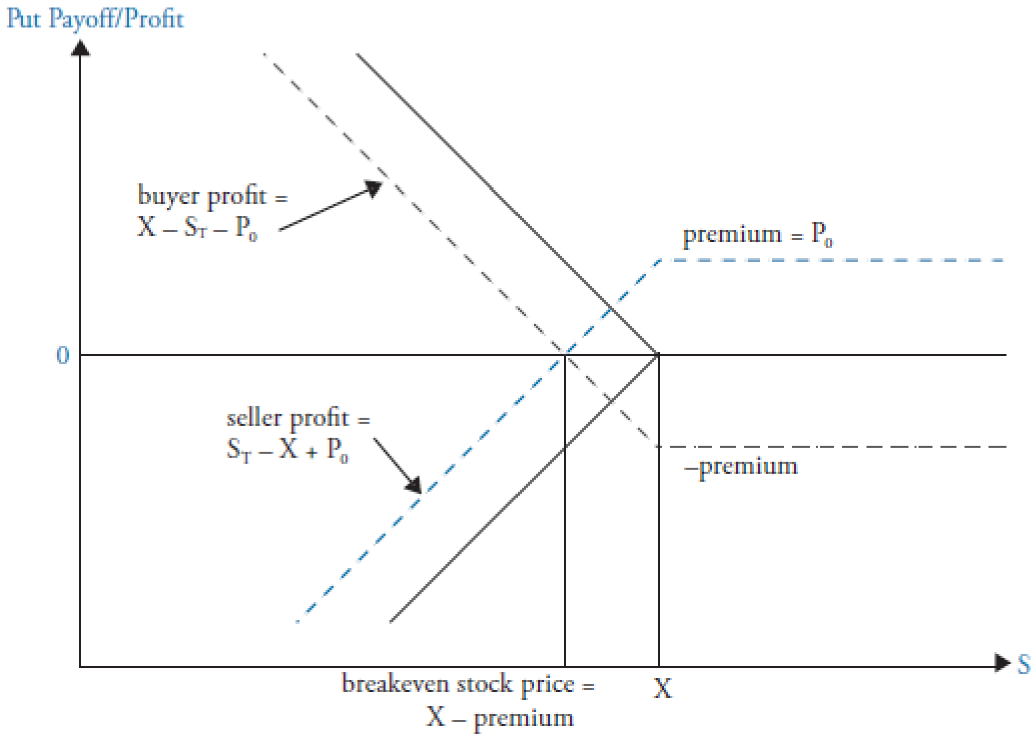
where:
P_T = payoff on put option
P_0 = put premium

The profit to the option seller is:

$$\text{profit} = P_0 - P_T$$

Figure 30.2 depicts the payoff and profit for the buyer and writer of a put option.

Figure 30.2: Profit Diagram for a Put at Expiration



EXAMPLE: Calculating payoffs and profits from options

Compute the payoff and profit to a call buyer, a call writer, put buyer, and put writer if the strike price for both the put and the call is \$45, the stock price is \$50, the call premium is \$3.50, and the put premium is \$2.50.

Answer:

Call buyer:

$$\text{payoff} = C_T = \max(0, S_T - X) = \max(0, \$50 - \$45) = \$5$$
$$\text{profit} = C_T - C_0 = \$5 - \$3.50 = \$1.50$$

Call writer:

$$\text{payoff} = -C_T = -\max(0, S_T - X) = -\max(0, \$50 - \$45) = -\$5$$
$$\text{profit} = C_0 - C_T = \$3.50 - \$5 = -\$1.50$$

Put buyer:

$$\text{payoff} = P_T = \max(0, X - S_T) = \max(0, \$45 - \$50) = \$0$$
$$\text{profit} = P_T - P_0 = \$0 - \$2.50 = -\$2.50$$

Put writer:

$$\text{payoff} = -P_T = -\max(0, X - S_T) = -\max(0, \$45 - \$50) = \$0$$
$$\text{profit} = P_0 - P_T = \$2.50 - \$0 = \$2.50$$

Forward Contract Payoff

The payoff to a long position in a forward contract is calculated as follows:

$$\text{payoff} = S_T - K$$

where:

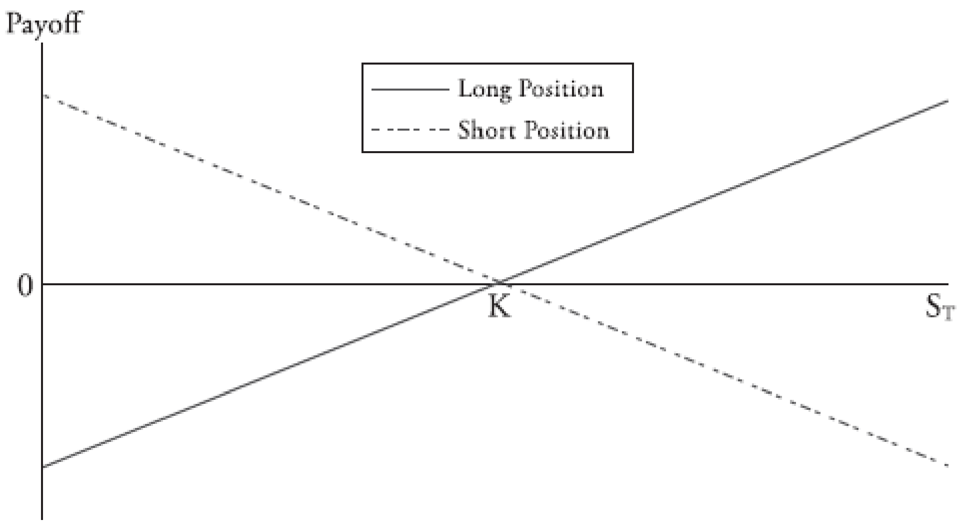
S_T = spot price at maturity
K = delivery price

Conversely, the payoff to a short position in a forward contract is calculated as follows:

$$\text{payoff} = K - S_T$$

Figure 30.3 depicts the payoff for the long and short positions in a forward contract.

Figure 30.3: Forward Contract Payoff



EXAMPLE: Calculating forward contract payoffs

Compute the payoff to the long and short positions in a forward contract given that the forward price is \$25 and the spot price at maturity is \$30.

Answer:

Payoff to long position:

payoff = $S_T - K$ = \$30 − \$25 = \$5

Payoff to short position:

payoff = $K - S_T$ = \$25 − \$30 = −\$5

MODULE QUIZ 30.1

1. Which of the following statements is an advantage that is specific only to exchange trading compared to over-the-counter (OTC) trading? On an exchange system:
   A. terms are not specified.
   B. trades are made in such a way as to reduce credit risk.
   C. participants have flexibility to negotiate.

D. there is greater anonymity.

2. An agreement sold over an exchange to buy/sell a commodity or financial instrument at a designated future date is known as a(n):
   A. spot contract.
   B. option contract.
   C. futures contract.
   D. forward contract.

MODULE 30.2: DERIVATIVES TRADERS

LO 30.e: Differentiate among the broad categories of traders: hedgers, speculators, and arbitrageurs.

Hedgers typically reduce their risks with forward contracts or options. By using forward contracts (at no cost), the trader is attempting to neutralize risk by fixing the price the hedger will pay or receive for the underlying asset. Option contracts, in contrast, are more of an insurance policy that require the payment of a premium, but will protect against downside risk while keeping some of the upside.

Speculators are effectively betting on future price movement. When a speculator uses the underlying asset, any potential gain or loss arises only on the differential between the share purchase price and the future share price. When a speculator uses options, the potential gain is magnified (assuming the same initial dollar investment in shares as options) and the maximum loss is the dollar investment in options.

Arbitrageurs take offsetting positions in financial instruments to lock in a riskless profit on the assumption that there are mispricings in the same asset in different markets.

Hedging Strategies

LO 30.f: Calculate and compare the payoffs from hedging strategies involving forward contracts and options.

Hedgers use forward contracts and options to reduce or eliminate financial exposure. An investor or business with a long exposure to an asset can hedge exposure by either entering into a short futures contract or by buying a put option. An investor or business with a short exposure to an asset can hedge exposure by either entering into a long futures contract or by buying a call option.

Hedgers use forward contracts to lock in the price of the underlying security. Forward contracts do not require an initial investment, but hedgers give up any price movement that may have had positive results in the event that the position was left unhedged. Option contracts on the other hand function as insurance for the underlying by providing the downside protection that the hedger seeks and allowing for price movement in the direction that could yield positive results. This insurance does not come without a cost, as we described earlier, since hedgers are required to pay a premium to purchase options.

EXAMPLE: Hedging with a forward contract

Suppose that a company based in the United States will receive a payment of €10M in three months. The company is worried that the euro will depreciate and is contemplating using a forward contract to hedge this risk. **Compute** the following:

1. The value of the €10M in U.S. dollars at maturity given that the company hedges the exchange rate risk with a forward contract at 1.25 $/€ (i.e., 1.25 dollars per 1 euro).
2. The value of the €10M in U.S. dollars at maturity given that the company did not hedge the exchange rate risk and the spot rate at maturity is 1.20 $/€.

Answer:

1. The value at maturity for the hedged position is:

   €10,000,000 × 1.25 $/€ = $12,500,000

2. The value at maturity for the unhedged position is:

   €10,000,000 × 1.20 $/€ = $12,000,000

EXAMPLE: Hedging with a put option

Suppose that an investor owns one share of ABC stock currently priced at $30. The investor is worried about the possibility of a drop in share price over the next three months and is contemplating purchasing put options to hedge this risk. **Compute** the following:

1. The profit on the unhedged position if the stock price in three months is $25.
2. The profit on the unhedged position if the stock price in three months is $35.
3. The profit for a hedged stock position if the stock price in three months is $25, the strike price on the put is $30, and the put premium is $1.50.
4. The profit for a hedged stock position if the stock price in three months is $35, the strike price on the put is $30, and the put premium is $1.50.

Answer:

1. Profit = $S_T - S_0$ = \$25 − \$30 = −\$5
2. Profit = $S_T - S_0$ = \$35 − \$30 = \$5
3. Profit = $S_T - S_0 + \max(0, X - S_T) - P_0$

   $= \$25 - \$30 + \max(0, \$30 - \$25) - \$1.50 = -\$1.50$

4. Profit = $S_T - S_0 + \max(0, X - S_T) - P_0$

   $= \$35 - \$30 + \max(0, \$30 - \$35) - \$1.50 = \$3.50$

PROFESSOR'S NOTE

Notice that the max term is $5 in Case #3 and $0 in Case #4.

Speculative Strategies

LO 30.g: Calculate and compare the payoffs from speculative strategies involving futures and options.

Speculators have a different motivation for using derivatives than hedgers. They use derivatives to make bets on the market, while hedgers try to eliminate exposures.

The motivation for using futures in speculation is that the limited amount of initial investment creates significant leverage. The amount of investment required for futures is the amount of the initial margin required by the exchange. This is generally a small percentage of the notional value of the underlying, and Treasury securities can typically be posted as margin. Futures contracts can result in large gains or large losses, and contract payoffs are symmetrical.

Options also create significant leverage, as investors only need to pay the option premium to purchase an option instead of the face value of the underlying. Options differ from futures in that options have asymmetrical payoffs. Gains can be quite large from going long options, but losses from long option positions are limited to the option premium.

EXAMPLE: Speculating with futures

An investor believes that the euro will strengthen against the dollar over the next three months and would like to take a position with a value of €250,000. He could purchase euros in the spot market at $1.25 per 1 euro or purchase two futures contracts at $1.20 per 1 euro with an initial margin of $10,000. **Compute** the profit from the following:

1. Purchasing euros in the spot market if the spot rate in three months is 1.15 $/€.
2. Purchasing euros in the spot market if the spot rate in three months is 1.30 $/€.
3. Purchasing the futures contract if the spot rate in three months is 1.15 $/€.
4. Purchasing the futures contract if the spot rate in three months is 1.30 $/€.

Answer:

1. Profit = €250,000 × (1.15 $/€ − 1.25 $/€) = −$25,000
2. Profit = €250,000 × (1.30 $/€ − 1.25 $/€) = $12,500
3. Profit = €250,000 × (1.15 $/€ − 1.20 $/€) = −$12,500
4. Profit = €250,000 × (1.30 $/€ − 1.20 $/€) = $25,000

A summary of these four transactions is as follows:

	Purchase Euros in Spot Market	Purchase Long Forward Position
Investment	$312,500	$10,000
Profit if spot at maturity = 1.15 $/€	–$25,000	–$12,500
Profit if spot at maturity = 1.30 $/€	$12,500	$25,000

EXAMPLE: Speculating with options

An investor who has $30,000 to invest believes that the price of stock XYZ will increase over the next three months. The current price of the stock is $30. The investor could directly invest in the stock, or she could purchase 3-month call options with a strike price of $35 for $3. **Compute** the profit from the following:

1. Investing directly in the stock if the price of the stock is $45 in three months.
2. Investing directly in the stock if the price of the stock is $25 in three months.
3. Purchasing call options if the price of the stock is $45 in three months.
4. Purchasing call options if the price of the stock is $25 in three months.

Answer:

1. Number of stocks to purchase = $30,000 / $30 = 1,000
   Profit = 1,000 × ($45 − $30) = $15,000
2. Profit = 1,000 × ($25 − $30) = −$5,000
3. Number of call options to purchase = $30,000 / $3 = 10,000
   Profit = 10,000 × [max(0, $45 − $35) − $3] = $70,000
4. Profit = 10,000 × [max(0, $25 − $35) − $3] = −$30,000

PROFESSOR'S NOTE

Since option contracts are traded in amounts of 100 options, the transactions in #3 and #4 here would entail the purchase of 100 call option contracts (i.e., 10,000 / 100 = 100).

A summary of these four transactions is as follows:

	Purchase Stock	Purchase Call Option
# Shares/call option	1,000	10,000
Profit if stock at maturity = $45	$15,000	$70,000
Profit if spot at maturity = $25	–$5,000	–$30,000

Arbitrage Opportunities

LO 30.h: Describe arbitrageurs' strategy and calculate an arbitrage payoff.

Arbitrageurs are also frequent users of derivatives. Arbitrageurs seek to earn a risk-free profit in excess of the risk-free rate through the discovery and manipulation of mispriced securities. They earn a riskless profit by entering into equivalent offsetting positions in one or more markets. Arbitrage opportunities typically do not last long as supply and demand forces will adjust prices to quickly eliminate the arbitrage situation.

EXAMPLE: Arbitrage of stock trading on two exchanges

Assume stock DEF trades on the New York Stock Exchange (NYSE) and the Tokyo Stock Exchange (TSE). The stock currently trades on the NYSE for \$27 and on the TSE for ¥2,880. Given the current exchange rate is \$0.009 per 1 yen, **determine** if an arbitrage profit is possible.

Answer:

Value in dollars of DEF on TSE = ¥2,880 × \$0.009/¥ = \$25.92

Arbitrageur could purchase DEF on TSE for \$25.92 and sell on NYSE for \$27.

Profit per share = \$27 − \$25.92 = \$1.08

Risks From Using Derivatives

LO 30.i: Describe some of the risks that can arise from the use of derivatives.

Derivatives are versatile and can be used for hedging, arbitrage, and pure speculation. If, however, the bet one makes starts going in the wrong direction, the results can be catastrophic (e.g., Barings Bank). Additionally, the risk exists that a trader with instructions to hedge a position may use derivatives to speculate due to the massive potential payoffs if speculation succeeds. This risk is known as an operational risk when it is done in an unauthorized manner. Controls need to be carefully established and monitored within both financial and nonfinancial corporations to prevent misuse of derivatives. Risk limits should be set, and adherence to risk limits should be monitored.

MODULE QUIZ 30.2

1. Which of the following statements regarding futures contracts is most likely correct? A business with a long exposure to an asset would hedge this exposure by either entering into a:
   A. long futures contract or by buying a call option.
   B. long futures contract or by buying a put option.
   C. short futures contract or by buying a call option.

D. short futures contract or by buying a put option.

2. Which of the following statements is most likely correct regarding the use of derivatives?
   A. Although a legitimate risk for all financial institutions, the misuse of derivatives is a relatively small risk overall.
   B. Speculating using derivatives should be banned to prevent catastrophic losses.
   C. Due to leverage inherent in derivatives, profits can be magnified while losses will always be minimized.
   D. It can be difficult to differentiate between arbitrage activities and speculative activities.

3. An individual that maintains bid and offer prices in a given security and stands ready to buy or sell lots of said security is a(n):
   A. hedger.
   B. arbitrageur.
   C. speculator.
   D. dealer.

KEY CONCEPTS

LO 30.a

A derivative security is a financial security whose value is derived in part from another security's characteristics or value (i.e., an underlying asset).

Linear derivatives, such as forward and futures contracts, have a linear payoff that is directly related to the value of the underlying. In contrast, nonlinear derivatives, such as options, have a payoff that is nonlinear in relation to the value of the underlying.

LO 30.b

The over-the-counter (OTC) market is often used for large trades where terms are not set by an exchange, giving traders more flexibility to negotiate mutually agreeable terms. The OTC market has more credit risk for nonstandard transactions, but some regulations have been introduced for more standardized transactions. In contrast, exchanges are organized to eliminate credit risk.

LO 30.c

A call option gives its holder the right to buy a specified number of shares of the underlying security at the given strike price, on or before the option contract's expiration date, while a put option is the right to sell a fixed number of shares at a fixed price within a given prespecified time period.

A forward contract is an agreement to buy or sell an asset at a preselected future time for a certain price.

A futures contract is a more formalized, legally binding agreement to buy or sell a commodity or financial asset in a predesignated month in the future, at a price agreed upon today by the buyer/seller.

LO 30.d

The payoff on a call option to the option buyer is calculated as follows:

$$\text{Call}_T = \max(0, S_T - X)$$

where:
S_T = stock price at maturity
X = strike price of option

The payoff on a put option is calculated as follows:

$$\text{Put}_T = \max(0, X - S_T)$$

where:
S_T = stock price at maturity
X = strike price of option

The payoff to a long position in a forward contract is calculated as follows:

$$\text{payoff} = S_T - K$$

where:
S_T = spot price at maturity
K = delivery price

LO 30.e

There are three broad types of traders: hedgers, speculators, and arbitrageurs.

Hedging is used for risk management. The hedger has a risk associated with the underlying commodity or financial instrument. The use of futures helps mitigate those risks.

Speculating does not mitigate risk but is risk-taking. Profit is the motive of the speculator since he has no risk before entering into the futures transactions.

Arbitrage ensures that futures and cash markets stay in balance. Buying in the cheaper market and selling in the overpriced market will bring markets back into alignment and provide a riskless profit for an arbitrageur.

LO 30.f

Hedgers use derivatives to control or eliminate a financial exposure. Futures lock in the price of the underlying security and do not allow for any upside potential. Options hedge negative price movements and allow for upside potential since they have asymmetric payouts.

LO 30.g

Speculators use derivatives to make bets on the market. Futures require a small initial investment, which is the initial margin requirement. Futures contracts can result in large gains or large losses as futures have a symmetrical payout function.

LO 30.h

Arbitrageurs seek to earn a riskless profit through the discovery and manipulation of mispriced securities. Riskless profit is earned by entering into equivalent offsetting positions in one or more markets. Arbitrage opportunities do not last long as the act of arbitrage brings prices back into equilibrium quickly.

LO 30.i

Derivatives are versatile instruments and can be used for hedging, arbitrage, and pure speculation. Controls need to be carefully established to prevent misuse of derivatives. Risk limits must be carefully established and scrupulously enforced.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 30.1

1. **B** Exchanges are organized to reduce credit risk. The other answer choices are advantages of over-the-counter trading. (LO 30.b)

2. **C** A futures contract is an agreement sold on an exchange to buy/sell a commodity or financial instrument in a designated future month. (LO 30.c)

Module Quiz 30.2

1. **D** A business with a long exposure to an asset would hedge the exposure by either entering into a short futures contract or by buying a put option. (LO 30.f)

2. **D** A misuse of derivatives could occur when a trader makes speculative activities look like arbitrage activities (i.e., operational risk). This misuse is a significant risk for financial institutions. Speculating is important for a financial institution to maximize profits; therefore, it should be controlled but not banned. Leverage magnifies both losses and gains; purchasing an option allows for huge potential upside and minimal downside in the form of the loss of the option premium paid. (LO 30.i)

3. **D** A dealer maintains bid and offer prices in a security and stands ready to buy or sell lots of the given security. (LO 30.e)

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Financial Markets and Products, Chapter 5.

READING 31

EXCHANGES AND OTC MARKETS

Study Session 8

EXAM FOCUS

In this reading, we look at the role of exchanges and the differences between exchange-traded derivatives and over-the-counter (OTC) derivatives trading. We then examine the role of the central counterparty (CCP) in clearing and mitigating counterparty risk. For the exam, be able to compare and contrast exchange-traded and OTC derivatives. Also, be familiar with the development of central clearing, including the various mechanisms that exist to manage risks, including special purpose vehicles (SPVs).

MODULE 31.1: EXCHANGE-TRADED DERIVATIVES

LO 31.a: Describe how exchanges can be used to alleviate counterparty risk.

An **exchange** is a central market where standardized futures, options, and other derivatives contracts can be traded. They have a long history and have evolved from simple trading forums (e.g., contract definition, dispute resolution) to sophisticated financial centers with settlement and counterparty risk management functions.

Clearing

LO 31.b: Explain the developments in clearing that reduce risk.

Clearing is the process of reconciling and matching contracts between counterparties from the time the commitments are made until settlement. Clearing, along with the developments of margining and netting, are important counterparty risk mitigants. *Margining* involves posting both initial and variation margins from one counterparty to another. **Initial margin** represents upfront funds posted to mitigate against counterparty default, while **variation margin** represents the daily transfer of funds (cash or other assets) to cover position gains and losses (mark-to-market process). **Netting** refers to consolidating multiple offsetting positions (e.g., long and short) between counterparties into a single payment.

Exchanges use **central counterparties (CCPs)** to clear trades between two members. For example, Entity X and Entity Y agree to a specific trade and then the CCP acts as the counterparty to both X and Y. As a result, neither entity will have concerns about the creditworthiness of the other and may not even know the identity of the other. The CCP will subsequently clear the transaction. In addition, the existence of the CCP allows for a simpler process in closing out positions. Without the CCP, if Entity X wants to close out the position, it would need to do so with Entity Y or another member and that might not be possible.

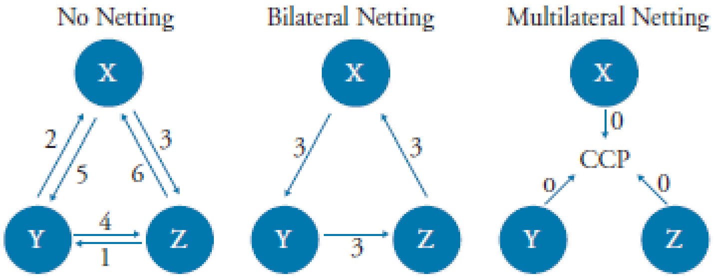
Netting

LO 31.c: Define netting and describe a netting process.

As mentioned, netting involves combining long and short positions into a single exposure. A simple example of netting is as follows: Entity X enters into a buy transaction of gold with Entity Y for $500,000 settling in December. Entity Y enters into a separate buy transaction of gold with Entity X for $475,000 settling in December. The two contracts could be netted with Entity Y paying Entity X $25,000 in December.

Figure 31.1 illustrates how counterparty risk exposures are reduced through netting and the use of a CCP. Suppose X, Y, and Z represent three different counterparties. The arrows in the figure represent the amount of money owed (i.e., counterparty risk exposure) between the entities. For example, under a no netting framework, Entity X has an exposure of two to Entity Y, and Entity Y has an exposure of five to Entity X.

Figure 31.1: Reduction of Risk Exposure Through Multilateral Netting



Suppose Entity X defaults under the *no netting* framework. This will result in a loss of five for Entity Y and a loss of three to Entity Z. Under the no netting framework, Entity X still claims a total of eight from both entities Y and Z (i.e., 2 + 6). With *bilateral netting*, trades between two entities are cleared, reducing the total exposure in the market. If Entity X defaults under the bilateral framework, then Entity Y will have a reduced loss of three and Entity Z will have no loss because there is no risk exposure remaining from Entity X for Entity Z.

If Entity X defaults under the **multilateral netting** framework, entities Y and Z do not have losses because there is no outstanding counterparty risk exposure from other CCP members. All trades are cleared through the CCP in the multilateral netting framework. Without the CCP, there could be problems if the entities have different credit qualities and it may involve margin arrangements and margin transfers, which could add tremendous complexity. Hence, the market development to have CCPs clear transactions between members has avoided any concerns about creditworthiness.

Margining

LO 31.d: Describe the implementation of a margining process, explain the determinants of and calculate initial and variation margin requirements.

Variation margin and **daily settlement** are interrelated. Futures contracts are settled each day during the life of the contract rather than at the very end. Assume Entity X contracts to buy (long position) 100 units of gold for $1,850 per unit from Entity Y. The price of gold falls at the close of trading the next day to $1,840. Entity X has lost $1,000 (= 100 units × $10/unit) and must pay $1,000 to the CCP. The CCP will then pay $1,000 to Entity Y. Alternatively, if the price of gold rose from $1,850 to $1,870 the next day, then Entity X has gained $2,000 and the CCP will pay $2,000 to Entity X (that was paid to the CCP by Entity Y). Such payments are called variation margin.

From the CCP's perspective, there are an equal number of long and short positions so that the CCP experiences net cash flows of zero. Daily settlement facilitates the closing out of futures contracts because everything has been accounted for up to the close of trading on the previous day so only the incremental gain/loss for the current day needs to be considered in closing out the position.

CCPs also have **initial margin** requirements. Initial margin is intended to cover any potential losses from a member who fails to make a variation margin payment and for any further losses that may occur from unfavorable price changes arising from any delays involved with the CCP closing out the member's position. For example, a member with a net short position may fail to make a variation margin payment. The CCP would then need to close out that position by taking a long position in the underlying. If there is a subsequent price increase in the underlying prior to closing out the long position, there would be a further loss to the CCP. The CCP's losses are meant to be recovered from the initial margin previously paid by the defaulting member. However, if the amount of initial margin is insufficient, the **default fund contributions** (a separate requirement of members) of that particular member are applied to cover the difference. If there is still a shortage, then the default fund contributions of other members are applied.

Initial margins are determined by the exchange and are a function of futures price volatility; initial margins may vary depending on market conditions. CCPs will pay interest on initial margin (but not variation margin). As an alternative to cash margin, a member could provide securities (e.g., T-bills). The equivalent cash margin would be a discounted amount of the securities' value, with the discount known as a haircut. Haircuts increase and decrease accordingly with the price volatility of the underlying asset.

The initial and variation margins may be impacted when there is more than one derivatives contract on the same asset. Assume a short August contract and a long November contract held by one member. A variation margin receipt on the August contract may be offset by a variation margin payment on the November contract, which is a built-in netting mechanism. In addition, there are likely to be provisions to allow for the total initial margin of the short August and long November contracts to be less than the sum of the initial margins for the contracts if they were determined independently.

MODULE QUIZ 31.1

1. Which of the following methods of managing credit risk is not utilized by an exchange?
   A. Daily settlement.
   B. Default fund contributions.
   C. Netting.
   D. Physical collateral.

2. Which of the following statements regarding the margining process on an exchange is correct?
   A. The initial margin is calculated as a function of the futures price.
   B. CCPs pay interest on both the initial margin and the variation margin.
   C. The initial margin or a futures contract is negotiated between the two parties directly.
   D. When providing noncash margin, the haircut is positively correlated with the price volatility of the underlying asset.

MODULE 31.2: OVER-THE-COUNTER DERIVATIVES

LO 31.f: Compare exchange-traded and OTC markets and describe their uses.

Exchange-traded derivatives are standardized contracts with a liquid, active, and regulated market, with the exchange or CCP acting as the central counterparty to trades. In contrast, **OTC derivatives** are privately negotiated bilateral contracts transacted in a market with little or no regulation. OTC derivatives have historically been traded between an end user and a dealer. The terms, settlement, and documentation are bilaterally negotiated. This allows for contracts to be tailored to the specific needs of counterparties and includes a high level of customization.

Clearing and settlement on exchanges are functions carried out centrally by CCPs to mitigate risk. For OTC derivatives, clearing and settlement have traditionally been done bilaterally (e.g., determination of future cash flows), which did not generally mitigate risk. However, there has been more use of CCPs for OTC derivatives (multilateral) over the past 10 years.

The following table provides a comparison of the general differences between exchange-traded and OTC derivatives.

Figure 31.2: Comparing Exchange-Traded and OTC Markets

	Exchange-Traded Derivatives	OTC Derivatives
Terms	Standardized	Custom, negotiable
Maturity	Standardized	Negotiable, nonstandard
Liquidity	Strong	Weak
Credit risk	Little (CCP guarantee)	High (bilateral)

Classes of OTC Derivatives

OTC derivatives comprise of five broad classes: interest rate, foreign exchange, equity, commodity, and credit default swaps. Interest rate derivatives dominate the five classes. As of December 2017, interest rate derivatives had a notional principal (underlying assets) outstanding of $426.6 trillion, which represented 80% of the total notional principal of OTC derivatives of $531.9 trillion. The second and third largest categories were foreign exchange derivatives and credit default swaps, respectively, followed by equity and commodity derivatives.

As mentioned, interest rate derivatives dominate the market by notional principal outstanding of contracts. However, measuring OTC derivatives exposure through notional principal can be misleading. A basic fixed-for-floating coupon interest rate swap, for example, does not have principal risk because only the coupon cash flows are exchanged at each settlement. Furthermore, even coupon risk is lower, because only the net cash flows are exchanged. When considering cash flows, the swap may have a negative value to a party when its counterparty defaults. As a result, transaction value is often seen as a more useful measure for OTC derivatives, including the ratio of transaction value to notional principal value. The ratio is typically relatively small, and was close to 2% (at December 2017) for interest rate and foreign exchange derivatives.

Mitigating Risks of OTC Derivatives

LO 31.e: Describe the process of buying stock on margin without using a CCP and calculate margin requirements.

LO 31.g: Identify risks associated with OTC markets and explain how these risks can be mitigated.

Credit risk is the key risk with OTC markets. Various mitigants exist to contain or reduce the risk of the default, including netting, margining, and default fund contributions as discussed previously.

We will now consider three specific examples of risks in the OTC markets that are mitigated with margin accounts: (1) options on stocks, (2) short sales, and (3) buying on margin.

Options on Stocks

Unlike long positions in option contracts, short positions expose the trader to the risk of having to buy or sell the underlying asset at a price that is relatively too high or low, respectively, upon exercise of the options. To mitigate the risk arising from short positions, the CCP requires traders to post margin equal to 100% of the option value plus some spread that is a function of the underlying stock price or the option exercise price.

For example, assume 200 put options on a stock are sold for $3 per option when the stock price is $40. The margin requirement is the greater of: (1) 100% of the option value + 20% of the underlying stock price − the amount that the option is out-of-the-money (OTM), or (2) 100% of the option value + 10% of the exercise price. If the exercise price is $35, then the option is $5 OTM and the required margin is the greater of

- [3 + (0.20 × 40) − 5] × 200 = $1,200, or
- [3 + (0.10 × 35)] × 200 = $1,300.

Therefore, the required margin is $1,300. With changes to the underlying stock price and option price, the margin requirements could change and the trader could be asked for additional margin. In contrast to futures, options are often settled at maturity. Interest on cash margin balances is paid by the CCP to the trader.

Short Sales

Short sales require the borrowing of shares that the short seller will repurchase later and return. Often a margin of 150% of the stock price applies, which means that the trader must post additional margin equal to 50% of the stock price. A decrease (increase) in the stock price increases (decreases) the margin balance. In addition, a **maintenance margin** is often set at 125% of the stock price so if the margin balance becomes less than the maintenance margin, then a margin call will be made to replenish the balance to the maintenance margin.

PROFESSOR'S NOTE

The maintenance margin is the minimum margin account balance required to retain the position. When the margin account balance falls below the maintenance margin, the investor gets a margin call. With short sales, a margin call results in bringing the balance up to the maintenance margin. This differs from futures contracts where a margin call results in bringing the balance up to the initial margin.

For example, assume a short sale of 100 shares of a stock trading at $25 that generates $2,500. Based on a 150% margin requirement, the additional margin to be posted is $1,250 (= 50% × $2,500) for a total of $3,750. If the stock price rises to $28, then the shorted shares are worth $2,800 and the maintenance margin becomes $3,500 (= 1.25 × $2,800). The initial margin of $3,750 is sufficient to cover the maintenance margin. However, a subsequent stock price increase to $32 results in a maintenance margin of $4,000 (= 1.25 × $3,200), which means there is a $250 margin call (= $4,000 − $3,750). Interest on margin balances is paid to the short seller.

Buying on Margin

To take advantage of leverage, an investor may borrow funds from a broker to invest. Assume the purchase of 500 shares for $40 per share (total cost $20,000) with a 50% initial margin and 25% maintenance margin requirement. Therefore, the initial margin to be deposited with the broker is $10,000 and the remaining $10,000 is borrowed from the broker. The broker holds the shares as collateral.

The margin account balance begins as $10,000 (share value less amount borrowed from broker) and is increased by gains on the shares and decreased by losses on the shares and interest charged by the broker. Assuming a $3 fall in the stock price, the value of the shares declines by $1,500 to $18,500 and the margin account balance declines to $8,500. Ignoring interest, the margin to share value ratio is just under 46%. That exceeds the 25% maintenance margin, which avoids a margin call. However, if there is a further $12 fall in the stock price, the value of the shares declines to $12,500 and the margin account balance declines to $2,500. Unfortunately, the margin to share value ratio is now 20% and because it is less than 25%, then a margin call will be required. The additional margin required will be $625 [= (25% × $25 × 500) − $2,500]. Upon receipt of the additional margin, the margin account balance becomes $3,125 (= $2,500 + $625) and the amount borrowed from the broker becomes $9,375 (= $10,000 − $625).

Collateralization

LO 31.h: Describe the role of collateralization in the OTC market and compare it to the margining system.

Bilateral clearing usually includes a master agreement with a credit support annex, which outlines the use of collateral between parties. Providing collateral is a means of reducing credit risk in OTC markets. This collateralization is basically a marked-to-market feature for the OTC market where any loss is settled in cash at the end of the trading day. A cash payment is made to the counterparty with a positive account balance. This is a similar system to trading on margin, where the futures trader needs to restore funds if the value of the contract drops below the maintenance margin.

Special Purpose Vehicles

LO 31.i: Explain the use of special purpose vehicles (SPVs) in the OTC derivatives market.

SPVs are also known as special purpose entities (SPEs). They are bankruptcy remote legal entities set up by a parent firm to shield the SPV from any financial distress of the firm. The firm transfers assets to the SPV, which in turn issues structured products to investors to finance a particular project. The primary benefit of using an SPV is to obtain a strong credit rating, typically AAA. The SPV's rating is therefore stronger than the firm's credit rating. As a result, issuing securities through the SPV is more beneficial (i.e., lower cost of funding) than if the firm issued securities directly in the market.

In context of the structured products, SPVs and SPEs pool mortgages or other loans into derivatives securities. Investors of the derivatives will receive cash flows based on the performance of the underlying mortgages and loans. For example, any defaults on the underlying may reduce cash flows.

MODULE QUIZ 31.2

1. A trader sells short 1,000 shares of Stock A, which is currently trading at $40 per share. A margin requirement of 140% applies as well as a maintenance margin of 125%. If the share price rises to $55, the amount of the margin call is closest to:
   A. $0.
   B. $1,000.
   C. $13,000.
   D. $15,000.

2. Which of the following classes of OTC derivatives accounts for the smallest portion of the OTC market based on underlying asset value?
   A. Commodity derivatives.
   B. Credit default swaps.
   C. Foreign exchange derivatives.
   D. Interest rate derivatives.

KEY CONCEPTS

LO 31.a

An exchange is a central market where standardized contracts can be traded.

LO 31.b

Clearing, margining, and netting are important counterparty risk mitigants. Clearing is the process of reconciling and matching contracts between counterparties. Margining represents both upfront funds posted to mitigate against counterparty default (initial margin), and daily transfer of funds to cover position gains and losses (variation margin). Netting refers to consolidating multiple offsetting positions between counterparties into a single payment.

LO 31.c

Netting refers to consolidating multiple offsetting positions (e.g., long and short) between counterparties into a single payment.

With bilateral netting, trades between two entities are cleared, reducing the total exposure in the market.

Multilateral netting refers to creating a single net obligation between each participant and the CCP from the various bilateral OTC trades (which typically include redundant trades). Netting reduces total risk and minimizes contagion from a member default.

LO 31.d

Variation margin and daily settlement are interrelated. Futures contracts are settled each day during the life of the contract rather than at the very end. Assume Entity X contracts to buy (long position) units of gold for a fixed amount per unit from Entity Y.

If the price of gold falls at the close of trading the next day, Entity X has a loss and must pay an amount to the CCP. The CCP will then pay that amount to Entity Y. Alternatively, if the price of gold rose the next day, then Entity X has a gain and the CCP will pay an amount to Entity X (that was paid to the CCP by Entity Y). Such payments are called variation margin.

CCPs also have initial margin requirements. Initial margin is intended to cover any potential losses from a member who fails to make a variation margin payment and for any further losses that may occur from unfavorable price changes arising from any delays involved with the CCP closing out the member's position.

LO 31.e

To take advantage of leverage, an investor may borrow funds from a broker to invest. An initial margin is to be deposited with the broker and the remaining amount is borrowed from the broker. The shares are held by the broker as collateral. The margin account balance begins as the share value less the amount borrowed from broker and is increased by gains on the shares and decreased by losses on the shares and interest charged by the broker.

LO 31.f

The following table provides a comparison of the general differences between exchange-traded and OTC derivatives.

	Exchange-Traded Derivatives	OTC Derivatives
Terms	Standardized	Custom, negotiable
Maturity	Standardized	Negotiable, nonstandard
Liquidity	Strong	Weak
Credit risk	Little (CCP guarantee)	High (bilateral)

OTC derivatives comprise of five broad classes of derivatives: interest rate, foreign exchange, equity, commodity, and credit derivatives. Interest rate derivatives comprise the largest class, followed by foreign exchange derivatives and credit derivatives.

Measuring OTC derivatives exposure through notional principal can be misleading. Because only net cash flows are exchanged for most bonds and swaps, the notional principal overstates the amount at risk. As a result, transaction value is often seen as a more useful measure for OTC derivatives, including the ratio of transaction value to notional principal value.

LO 31.g

Three specific examples of risks in the OTC markets that are mitigated with margin accounts are: (1) options on stocks, (2) short sales, and (3) buying on margin.

LO 31.h

With the daily mark-to-market of derivatives, any change in the net value generally requires the losing party to post additional collateral equal to the amount of the change.

LO 31.i

SPVs are bankruptcy remote legal entities set up by a parent firm to shield the SPV from any financial distress of the firm. The firm transfers assets to the SPV, which in turn issues structured products to investors to finance projects.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 31.1

1. **D** Daily settlement, default fund contributions, and netting are explicitly used by exchanges. However, physical collateral is not, although initial margin and variation margin are a form collateral but they are not physical collateral per se. (LO 31.b)

2. **D** Haircuts increase and decrease accordingly with the price volatility of the underlying asset.

   The initial margin is calculated as a function of futures price *volatility* and is determined by the exchange (not by the two parties). CCPs pay interest on *initial margin only*. (LO 31.d)

Module Quiz 31.2

1. **C** The short sale of 1,000 shares of Stock A trading at $40 generates $40,000. Based on a 140% margin requirement, the additional margin to be posted is $16,000 (= 40% × $40,000) for a total of $56,000. If the stock price rises to $55, then the shorted shares are worth $55,000 and the maintenance margin becomes $68,750 (= 1.25 × $55,000). The initial margin of $56,000 is insufficient to cover the maintenance margin, which means there is a $12,750 margin call (= $68,750 – $56,000). (LO 31.g)

2. **A** As of December 2017, commodity derivatives had underlying assets with a value of about $1.9 billion, which represented about 0.35% of the OTC market.

   As a percentage of the OTC market, credit default swaps accounted for about 1.8%, followed by foreign exchange derivatives at 16.4%, and finally interest rate derivatives at 80.2%. (LO 31.f)

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Financial Markets and Products, Chapter 6.

READING 32

CENTRAL CLEARING

Study Session 8

EXAM FOCUS

This reading covers the principles of central clearing, including the functions and mechanics of a central counterparty (CCP). For the exam, be able to describe these functions and understand the related terminology, including the three key regulatory initiatives. Clearing, novation, and netting are also important concepts for the exam. In addition, understand the advantages and disadvantages of CCPs, and be able to discuss common terms, including loss mutualization, moral hazard, adverse selection, and procyclicality.

MODULE 32.1: PRINCIPLES OF CENTRAL CLEARING

LO 32.a: Provide examples of the mechanics of a central counterparty (CCP).

Clearing and Settlement

Clearing refers to the processes (including margining and netting) between the period from trade execution until settlement. This period is typically short (a few days or months) for classically cleared non-over-the-counter (OTC) derivatives. In contrast, for OTC derivatives, this time period could extend to years or even decades. **Settlement** of a trade occurs when the trade is completed and all payments have been made and legal obligations satisfied.

Key functions of a central counterparty (CCP) related to the clearing process include margining, novation, netting, managing the auction process, and loss mutualization. We will discuss these functions throughout this reading.

CCPs clear trades very similarly in OTC markets versus exchanges in terms of requirements from members for initial margin, variation margin, and default fund contributions. Initial margin is based on the risk of the transaction and not the credit risk of the member. CCPs will usually set the initial margin so that there is only about a 1% chance that the initial margin will be insufficient to cover losses. From there, default fund contributions are utilized. Clearing members typically include large players only, including large banks and global financial institutions.

Auctions and Defaults

When a member defaults, rather than closing out the trades at market value, the CCP typically auctions off the trades to the surviving members through an auctioning process. Participating in the auctioning process is in the best interest of the members in order to minimize their losses that would otherwise occur with lower market prices or with the use of default fund contributions.

Failed auctions may allow the CCP to assign losses to those who have had recent gains. Alternatively, the CCP could tear up transactions by closing out the defaulting member's trades that will result in the nondefaulting members incurring some losses. **Loss mutualization** is a form of insurance and refers to members' contributions to a default fund to cover future losses from member defaults. Since all members must contribute to the fund, the potential losses from the default of any given member are contained. When a member does default, any amounts that cannot be covered from the member's own resources are covered from the fund. Given that losses are spread among surviving members, it is possible that a member will suffer losses even if it never traded with the defaulting counterparty or had no positions with the CCP.

CCPs in OTC Transactions

LO 32.b: Describe the role of CCPs and distinguish between bilateral and centralized clearing.

A CCP plays an important role in the clearing and settlement of transactions following the initial trade execution. Its primary function is to simplify the operational processes and reduce counterparty risk that exists in the bilateral market (e.g., without the CCP, the trades would need to be cleared between the counterparties, which may involve significant credit risk). When a CCP interjects itself as the central counterparty for standard OTC trades and acts as the seller to each buyer and the buyer to each seller, it reduces the interconnectedness of trades and of participants, and reduces the risk of default or nonpayment by a counterparty. At the same time, the process of centralized clearing improves trade liquidity and transparency.

CCPs operate in a similar fashion to clearinghouses on futures exchanges. After two parties (X and Y) negotiate an OTC agreement, it is submitted to the CCP for acceptance. Assuming the transaction is accepted, the CCP will become the counterparty to both parties X and Y. Thus, it assumes the credit risk of both parties in an OTC transaction. This risk is managed by requiring the parties to post initial margin and any variation margin on a daily basis.

LO 32.c: Describe advantages and disadvantages of central clearing of OTC derivatives.

Advantages of Central Clearing

Central clearing through CCPs has the following advantages:

- *Default management (counterparty risk)*: CCPs act as the counterparty to each trade, which reduces counterparty risk. Unlike bilateral clearing, there is no need to negotiate a closeout with the counterparty; the possibility of an unfavorable closeout increases counterparty risk.
- *Loss mutualization*: A member's losses are distributed among all surviving members, which spread the impact of losses, reduce costs, and minimize market impact and systemic risk.
- *Legal and operational efficiency*: The centralized role of CCPs in the clearing (margining, netting) and settlement process improves operational efficiency while reducing costs.
- *Liquidity*: CCPs may greatly facilitate the netting of and closing of trades, which increases liquidity.
- *Standardized documentation*: CCPs promote the use of standardized documentation with OTC derivatives.
- *Increased transparency*: In OTC markets, parties typically do not see all outstanding trades between the various counterparties. CCPs have a consolidated view of trading positions and can therefore better react to extreme events.

Disadvantages of Central Clearing

While we noted loss mutualization as an advantage of the central clearing process, it can lead to potential problems, including moral hazard and adverse selection:

- *Moral hazard*: Moral hazard is the risk that one party will take on higher risk knowing that another party bears the costs of this risk. In central clearing, the risk is that members will have less incentive to monitor risk knowing that the CCP takes on most of the risks.
- *Adverse selection*: Adverse selection is the risk that participants with a better understanding of product risks and pricing will trade more products whose risks the CCP underprices, and will trade fewer products whose risks the CCP overprices.
- *Procyclicality*: Procyclicality essentially reflects the downside of margining. It reflects a scenario where a CCP increases margin requirements (initial margin) in volatile markets or during a crisis, which may aggravate systemic risk.
- *Credit risk*: CCP members' (not the CCP itself) ability to determine credit risk is challenging because of the lack of transparency regarding other members' trades, which puts members' default fund contributions and variation margin gains at risk should another member default. That is unlike bilateral clearing, where the risks are more transparent but more concentrated.

Regulatory Initiatives for OTC Markets

LO 32.d: Explain regulatory initiatives for the OTC derivatives market and their impact on central clearing.

Regulators have pushed for the use of CCPs in OTC markets to try to reduce systemic risk, which is the risk that a failure by a significant financial institution will impact other institutions (due to the extensive interconnections of derivatives dealers) and potentially lead to a collapse of the overall financial system. Prior to the 2007–2009 credit crisis, the OTC derivatives market was essentially unregulated and it is thought that OTC derivatives in subprime mortgages contributed to the crisis.

Three key regulations emerged from the G-20 leaders meeting in September 2009: (1) standardized OTC derivatives must be cleared through CCPs, (2) standardized OTC derivatives must be traded on electronic platforms, and (3) all OTC trades must be reported to a central trade repository.

Standardized derivatives would primarily consist of plain vanilla interest rate swaps and credit default swaps on indices. By clearing all transactions through CCPs, there are much fewer interconnections between dealers, thereby reducing systemic risk. By transacting on electronic platforms, there is much greater knowledge of trading prices by all members, thereby increasing price transparency. By reporting all transactions centrally, regulators are given key inputs in determining the risks involved with OTC derivatives.

The first two regulations only apply to trades between two financial institutions, which exempts dealers when transacting with their nonfinancial entities. However, CCPs must be used for interdealer transactions for standardized products, which has greatly increased the amount of central clearing of OTC derivatives.

Margining

LO 32.e: Compare margin requirements in centrally cleared and bilateral markets and explain how margin can mitigate risk.

Central clearing applies to standard transactions only (e.g., interest rate swaps or credit default swaps on indices). Standard transactions involving a given security have four key attributes:

- Standard legal and economic terms
- Widespread models to easily value the security
- Active trading of the security to ensure ease of unwinding positions and obtaining valuations
- Long price history of the security to determine initial margin

In contrast, nonstandard transactions are cleared bilaterally and are deemed uncleared. Just like centrally cleared transactions, uncleared transactions between two financial institutions are also required to post initial and variation margin, which is a notable departure from the past regarding initial margins.

To mitigate risk, initial margin is made up of cash or liquid assets transferred by a member at trade inception to cover a worst-case loss in the event of a member default. CCPs set margin requirements based only on the risks of the members' transactions, and the credit quality of the member is typically not a consideration for initial margin. Variation margin is typically cash posted by a member to cover the daily net change of the member's position. Initial margin on uncleared trades is forwarded to a third party in trust while variation margin can be forwarded between counterparties.

Novation and Netting

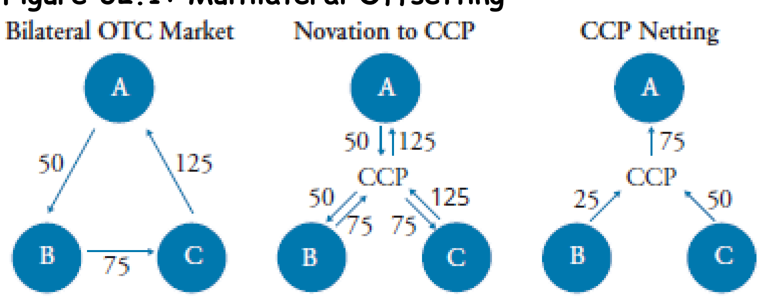
LO 32.f: Compare netting in bilateral markets vs centrally cleared markets.

The legal process of interposing the CCP between the seller and the buyer is called **novation**. Through novation, one contract (the bilateral contract between OTC participants) is replaced with another contract (or contracts) with the CCP. This is important because novation transforms the process from bilateral trading to trading with a CCP, where the CCP is the insurer of counterparty risk. Assuming the legal enforceability of novation, the old contracts cease to exist and the original bilateral parties have no further obligations to each other. At the same time, because all trades are centralized, the CCP maintains a matched book of trades with no net market risk. The CCP does have conditional credit risk from a member's potential default.

Market participants often prefer to offset rather than to terminate trades, which creates redundant trades. When trades are novated to a CCP, these redundant trades become a single net obligation between each participant and the CCP. This process is called **netting**. Netting reduces total risk and minimizes the potential of a domino effect stemming from the default of a participant.

The process of moving from bilateral to novation to netted positions is illustrated in Figure 32.1.

Figure 32.1: Multilateral Offsetting



Impact of Central Clearing

LO 32.g: Assess the impact of central clearing on the broader financial markets.

By now it should be evident that central clearing through a CCP has significant advantages, but it is not without its challenges. When a CCP is included in the clearing process, systemic risk in the financial markets is reduced, but can be increased at the same time. Systemic risk is reduced because CCPs reduce counterparty risk by offsetting positions (novation and netting), provide transparency for the market, and improve liquidity. However, the potential requirement that members post higher initial margin during times of increased market volatility could increase systemic risk. In addition, concentrating all trades in a single place exposes the market to the risk of CCP failure and heightened systemic risk.

The long-term maturity of OTC derivatives contracts, often years or decades, also poses challenges for CCPs. It is not yet evident that they are effective in clearing these long-dated, more complex, and illiquid trades.

MODULE QUIZ 32.1

1. Which of the following statements on central clearing is accurate?
   I. The composition of clearing members typically includes a combination of large global banks and smaller banks and nonfinancial institutions.
   II. In the auction process, a CCP normally does not closeout trades at their market value.
   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

2. Alex Dell, a derivatives trader, has some reservations about the central clearing of OTC derivatives with a central counterparty (CCP). Specifically, he is worried that clearing members' willingness to monitor credit risk may decline since the CCP assumes most of the risks, and that CCPs may increase margin requirements during a period of market stress. Which of the following concepts best describe Dell's reservations?

	Decline in Willingness	Higher Margin Requirements
A.	Moral hazard	Procyclicality
B.	Adverse selection	Netting
C.	Moral hazard	Netting
D.	Adverse selections	Procyclicality

3. In a recently released report to management, a credit analyst indicates that the level of initial margin set by a central counterparty (CCP) is dependent on the risk of the member that is required to post it, and on the risk of the specific derivatives transactions. The analyst is correct with respect to these:

	Dependent on Risk of Member	Dependent on Risk of Transactions
A.	Yes	Yes
B.	Yes	No
C.	No	Yes
D.	No	No

4. Alpha Bank recently noted that its bilateral OTC trade obligations with Beta Bank ceased to exist and the bank now directly faces a central counterparty (CCP) for its trade obligations. Which of the following concepts best identify this scenario?
   A. Netting.
   B. Novation.
   C. Margining.
   D. Multilateral netting.

MODULE 32.2: RISKS FACED BY CENTRAL COUNTERPARTIES

LO 32.h: Identify and explain the types of risks faced by CCPs.

Default Risk

The default of a clearing member and its flow through effects is the most significant risk for a CCP. Because of a default, there may be the default or distress of other clearing members given that default correlation is likely to be high among OTC derivatives market participants.

With CCPs, all members are considered equal credit risks when computing initial margin and default fund contributions. That is in direct contrast with bilateral clearing where credit quality is explicitly considered.

In the event of a failed auction or an insufficient number of bids, the CCP will be required to pass on the defaulting member’s losses through rights of assessment, loss allocation methods, or both. Passing on losses to other clearing members may result in defaults by those members. The loss allocation methods may be considered unfair because some of them, such as variation margin gains haircutting (VMGH) and tear-ups, impose losses on winning positions. With VMGH, members whose positions increased in value (i.e., they are owed variation margin) will likely not receive the full amount for their gains (i.e., haircutting). Members who instead owe money to the CCP will still be required to pay the full margin amount to the CCP. In a tear-up, the CCP terminates the unmatched position, and may balance resources by drawing from both the defaulter’s initial margin and the default fund.

Some clearing members may resign from the CCP after the default of another clearing member. The initial resignation may result in a negative reputational impact to the CCP as witnessed by further resignations of clearing members.

Model Risk

OTC derivatives are not priced by the market but are instead priced using valuation models that perform the mark-to-market function, which subjects CCPs to model risk.

Especially sensitive to model risk would be a CCP's determination of initial margins. In that context, model risk could arise due to errors pertaining to volatility and in that regard, initial margins should be amended frequently to correspond with changes in volatility.

Liquidity Risk

There are large amounts of cash inflows and outflows flowing through the CCP due to initial margins and margin calls. As a result, CCPs are exposed to liquidity risk. The CCP attempts to earn the greatest return possible on the funds it holds without incurring too much credit or liquidity risk, thereby resulting in a trade-off between more liquid securities (e.g., treasuries) and less liquid securities (e.g., corporate bonds).

Should there be a default by one or more members, the CCP is still required to meet the obligations of the other members. In such a case, the CCP's investments must be quickly and easily convertible to cash. In that regard, investments must be analyzed under stressed market conditions since it is most likely that defaults will occur in such conditions that are contemporaneous with lower liquidity levels.

Operational Risk

Due to the centralization of some functions within a CCP to increase efficiency, additional risks arise that affect counterparties due to concentration at the CCP. CCPs face operational risks that are common to all entities such as business interruption due to information systems failures and internal or external fraud. However, a systems failure within a CCP could have a disastrous impact on many counterparties, especially if they hold large positions.

Legal Risk

Legal risks in the form of litigation or claims may arise due to differing laws in different jurisdictions or laws that are inconsistent with the CCP's regulations. A good example would involve the segregation and movement of margin and positions (i.e., netting) through a CCP.

Investment Risk

Investment risk occurs in the form of losses of margin funds resulting from investment actions performed within or outside of the stated investment policy.

Overall, it is probable that the various loss events will be correlated and will affect the CCP at the same time. In the case of a default, there will probably be a major market impact that increases the probability of operational and investment issues. Additionally, in a default scenario, there is usually a wide spread between gain and loss positions that increase legal and fraud risks.

Risks to Clearing Members and Non-Members

LO 32.i: Identify and distinguish between the risks to clearing members and to non-members.

Non-members face exposure from CCPs, clearing members, and other non-members. If a CCP fails, a non-member may be able to avoid losses so long as its counterparty (a clearing member) is solvent. Unlike clearing members, non-members are not required to contribute to default funds so, therefore, non-members are not exposed to losses that result from CCP failures.

Furthermore, the extent of non-members' losses due to defaults of CCPs and clearing members lies with the initial margins and whether they are segregated, guaranteed, or both. In addition, non-members face the risk of not being able to port their trades should the counterparty member default. As a result, such trades may have to be closed out at a loss.

Finally, one has to consider non-members' liability with respect to CCP loss allocation rules. It is possible that CCPs are able to pass on losses to non-members through tear-up transactions, which would reduce the gains of non-members.

MODULE QUIZ 32.2

1. Which of the following statements regarding risks facing a CCP is correct?
   A. OTC derivatives are priced by the market.
   B. A good example of legal risk would involve netting arrangements.
   C. Default correlations tend to be low among OTC derivatives market participants.
   D. Investment risk refers to the risk of losses of margin funds resulting from investment actions performed outside of the stated investment policy.

2. Erin Parker and Nate James are analysts at a large financial institution. During one of their recent discussions on OTC derivatives and central clearing with a central counterparty (CCP), Parker states, "CCPs are beneficial because they convert operational and legal risk into counterparty risk." James adds to that statement by suggesting, "When requiring higher margin in turbulent times, CCPs reduce systemic risk."

   With respect to the statements made:
   A. only Parker is correct.
   B. only James is correct.
   C. both Parker and James are correct.
   D. neither Parker nor James is correct.

KEY CONCEPTS

LO 32.a

CCPs are involved in the clearing and settlement of transactions. Clearing refers to the processes between the period from trade execution until settlement. Settlement refers to the satisfaction of legal obligations and trade completion.

Functions of a CCP include novation, netting, margining, managing the auction process, and loss mutualization. Auctioning refers to selling off the defaulted member's trades to the surviving members through an auctioning process. Loss mutualization refers to members' contributions to a default fund to cover future losses from member defaults.

LO 32.b

A central counterparty CCP's primary function is to simplify the operational processes and reduce counterparty risk that exists in the bilateral market (e.g. without the CCP, the trades would need to be cleared between the counterparties, which may involve significant credit risk).

CCPs operate in a similar fashion to clearinghouses on futures exchanges. After two parties (X and Y) negotiate an OTC agreement, it is submitted to the CCP for acceptance. Assuming the transaction is accepted, the CCP will become the counterparty to both parties X and Y. Thus, it assumes the credit risk of both parties in an OTC transaction.

LO 32.c

Advantages of CCPs include: default management, loss mutualization, legal and operational efficiency, liquidity, standardized documentation, and increased transparency.

Disadvantages of CCPs include: moral hazard, adverse selection, procyclicality of margin requirements, and credit risk (from the perspective of members, not the CCP itself).

LO 32.d

Three key regulations emerged from the G-20 leaders meeting in September 2009: (1) standardized OTC derivatives must be cleared through CCPs, (2) standardized OTC derivatives must be traded on electronic platforms, and (3) all OTC trades must be reported to a central trade repository.

LO 32.e

Central clearing applies to standard transactions only. In contrast, nonstandard transactions are cleared bilaterally and are deemed uncleared. Just like centrally cleared transactions, uncleared transactions between two financial institutions are also required to post initial and variation margins, which is a notable departure from the past regarding initial margins.

CCPs set margin requirements based only on the risks of the members' transactions, and the credit quality of the member is typically not a consideration for initial margin.

LO 32.f

Novation refers to replacing a bilateral OTC contract with another contract (or contracts) with the CCP, where the CCP is the insurer of counterparty risk. The CCP maintains a matched book of trades with no net market risk.

Netting refers to creating a single net obligation between each participant and the CCP from the various bilateral OTC trades (which typically include redundant trades). Netting reduces total risk and minimizes contagion from a member default.

LO 32.g

By including a CCP in the clearing process, systemic risk can be both reduced and increased. Systemic risk is reduced because counterparty risk is reduced, and transparency and liquidity improve. Systemic risk is increased because higher initial margin during times of stress would heighten market risk, and the failure of a CCP may lead to a catastrophic event.

LO 32.h

CCPs face six major risks: default risk, model risk, liquidity risk, operational risk, legal risk, and investment risk. The default of a clearing member and its flow through effects is the most significant risk for a CCP. Because of a default, there may be the default or distress of other clearing members given that default correlation is likely to be high among OTC derivatives market participants.

LO 32.i

Non-members face exposure from CCPs, clearing members, and other non-members. If a CCP fails, a non-member may be able to avoid losses so long as its counterparty is solvent. Non-members are not required to contribute to default funds so they are not exposed to losses that result from CCP failures. The extent of non-members' losses lies with the initial margins and whether they are segregated, guaranteed, or both. Non-members face the risk of not being able to port their trades should the counterparty member default.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 32.1

1. **B** Clearing members typically include large players only, including large banks and global financial institutions. In the auction process, a CCP normally does not closeout trades at their market value. Instead, trades are auctioned to existing members. (LO 32.a)

2. **A** Dell's reservations describe moral hazard and procyclicality, respectively. In central clearing, moral hazard is the risk that members have less incentive to monitor risk knowing that the CCP assumes most of the risks of the transactions. Procyclicality describes a scenario where a CCP increases margin requirements (initial margin) in volatile markets or during a crisis, which may aggravate systemic risk.

   Netting describes the elimination of duplicate bilateral contracts by transacting through a CCP, which improves flexibility and reduces costs. Adverse selection is the risk that participants with a better understanding of product risks and pricing will trade more products whose risks the CCP underprices, and fewer products whose risks the CCP overprices. (LO 32.f)

3. **C** CCPs set initial margin requirements based on the risk of the transactions, but not on the risk of the members. (LO 32.a)

4. **B** Novation describes the process where one contract (the bilateral contract between OTC participants) is replaced with another contract (or contracts) with the CCP. As a result, counterparties' bilateral obligations with each other cease to exist.

   Netting refers to creating, from the various bilateral OTC trades, a single net obligation between each participant and the CCP.

   Margining is the process of posting some form of collateral, typically cash or marketable securities, to cover member defaults (initial margin) or security mark-to-market movements (variation margin). (LO 32.f)

Module Quiz 32.2

1. **B** Legal risks in the form of litigation or claims may arise due to differing laws in different jurisdictions or laws that are inconsistent with the CCP's regulations. A good example would involve the segregation and movement of margin and positions (i.e., netting) through a CCP.

   Response C is not correct, because default correlation is likely to be *high* among OTC derivatives market participants. Response A is not correct, because OTC derivatives are not priced by the market but are instead priced using valuation models. Response D is not correct, because investment risk refers to the risk of losses of margin funds resulting from investment actions performed *within or outside* of the stated investment policy. (LO 32.h)

2. **D** Neither Parker nor James is correct. CCPs convert counterparty risk into operational and legal risk. When CCPs require higher margin in turbulent times, CCPs can increase systemic risk. This risk is known as procyclicality. (LO 32.h)

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Financial Markets and Products, Chapter 7.

READING 33

FUTURES MARKETS

Study Session 9

EXAM FOCUS

In this reading, candidates should focus on the terminology of futures markets, how futures differ from forwards, the mechanics of margin deposits, and the process of marking to market. Limit price moves, delivery options, and convergence of spot prices to futures prices are also likely exam concepts. It is also important to understand the ways a futures position can be terminated before contract expiration and how cash settlement is accomplished by the final mark-to-market at contract expiration.

MODULE 33.1: FUTURES CHARACTERISTICS

LO 33.a: Define and describe the key features and specifications of a futures contract, including the underlying asset, the contract price and size, trading volume, open interest, delivery, and limits.

Futures contracts are exchange-traded obligations to buy or sell a certain amount of an underlying good at a specified price and date. The underlying asset varies from agricultural products to stock indices. Most futures positions are not held to take delivery of the underlying good. Instead, they are closed out or reversed before the settlement date.

The purchaser of a futures contract is said to have gone long or taken a **long position**, while the seller of a futures contract is said to have gone short or taken a **short position**. For each contract traded, there is a buyer and a seller. The long has contracted to buy the asset at the contract price at contract expiration, and the short has an obligation to sell at that price.

Open interest is the total number of long positions in a given futures contract. It also equals the total number of short positions in a futures contract. An open interest of 200 would imply that there are 200 short positions in existence and 200 long positions in existence. It is possible, on any given day, for the trading volume on a contract to be higher than its open interest. **Trading volume** is the number of contracts traded daily and may exceed open interest when there are many positions being closed out or if there is excessive **day trading** occurring (i.e., trades entered and closed out in one day).

Key Features of a Futures Contract

When a new futures contract is introduced to the marketplace, the futures exchange must specify the exact features of the contract, which include the following:

- *Underlying asset*. When the underlying asset for the contract is a financial asset, such as Japanese yen, the definition of the asset is straightforward. However, when the underlying asset is a commodity, there may be different levels of quality for that good available in the marketplace (e.g., different qualities of wheat). The futures exchange stipulates the quality of a good that will be acceptable for settling the contract.
- *Contract size*. The contract size specifies the quantity of the asset that must be delivered to settle a futures contract (e.g., one grain contract = 5,000 bushels).
- *Delivery location*. The exchange specifies the place where delivery will take place.
- *Delivery time*. Futures contracts are referred to by the month in which delivery is to take place (e.g., a December corn contract). Some contracts are not settled by delivery but by payment in cash, based on the difference between the futures price and the market price at settlement.
- *Price quotes*. The exchange determines how the price of a contract will be quoted as well as the minimum price fluctuation for the contract, which is referred to as the tick size. For example, grain is quoted in dollars per bushel, and the minimum tick size is ¼ cent per bushel. Since a grain contract consists of 5,000 bushels, the minimum tick size is $12.50 (= 5,000 × $0.0025) per contract.
- *Price limits*. The exchange sets the maximum price movement for a contract during a day. For example, wheat cannot move more than $0.20 from its close the preceding day, for a daily price limit of $1,000. When a contract moves down by its daily price limit, it is said to be **limit down**. When the contract moves up by its price limit, it is said to be **limit up**.
- *Position limits*. The exchange sets a maximum number of contracts that a speculator may hold to prevent speculators from having an undue influence on the market.

Futures/Spot Convergence

LO 33.b: Explain the convergence of futures and spot prices.

The spot (cash) price of a commodity or financial asset is the price for immediate delivery. The futures price is the price today for delivery at some future point in time (i.e., the maturity date). The **basis** is the difference between the spot price and the futures price.

basis = spot price − futures price

As the maturity date nears, the basis converges toward zero. Before maturity, the futures price may be above or below the spot price. At expiration, the spot price must equal the futures price because the futures price has become the price today for delivery today, which is the same as the spot. Arbitrage will force the prices to be the same at contract expiration.

EXAMPLE: Why the futures price must equal the spot price at expiration

Suppose the current spot price of silver is \$24.65. **Demonstrate** by arbitrage that the futures price of a futures silver contract that expires in one minute must equal the spot price.

Answer:

Suppose the futures price was \$24.70. We could buy the silver at the spot price of \$24.65, sell the futures contract, and deliver the silver under the contract at \$24.70. Our profit would be \$24.70 − \$24.65 = \$0.05. Because the contract matures in one minute, there is virtually no risk to this arbitrage trade.

Suppose instead the futures price was \$24.61. Now we would buy the silver contract, take delivery of the silver by paying \$24.61, and then sell the silver at the spot price of \$24.65. Our profit is \$24.65 − \$24.61 = \$0.04. Once again, this is a riskless arbitrage trade.

Therefore, to prevent arbitrage, the futures price at the maturity of the contract must be equal to the spot price of \$24.65.

Margin Requirements

As discussed in previous readings, margin is cash or highly liquid collateral placed in an account to ensure that any trading losses will be met. Marking to market is the daily procedure of adjusting the margin account balance for daily movements in the futures price. The amount required to open a futures position is called the initial margin. Recall that the maintenance margin is the minimum margin account balance required. An investor will receive a margin call if the margin account balance falls below the maintenance margin. In this case, the investor must bring the margin account back to the initial margin amount (with variation margin).

EXAMPLE: Margin trading

Assume an investor has a long gold contract. The investor entered the position at \$1,850. Each contract controls 100 troy ounces for a current market value of \$185,000. Assume that the initial margin is \$2,500, the maintenance margin is \$2,000, and the futures price drops to \$1,848 at the end of the first day and \$1,840 on the end of the second day. **Compute** the amount in the margin account at the end of each day for the long position and any variation margin needed.

Answer:

At the end of the first day, the loss is computed as (\$1,848 − \$1,850)100 = −\$200, so when the account is marked to market, \$200 is withdrawn from the buyer's margin account and \$200 deposited in the seller's margin account. The buyer's (long) margin account balance is now \$2,300 (= \$2,500 − \$200). The margin account balance for the short position is now \$2,700 (= \$2,500 + \$200).

At the end of the second day, the daily loss is ($1,840 − $1,848)100 = −$800, and the buyer's margin account balance is reduced to $1,500 (= $2,300 − $800). At $1,500, the investor will get a margin call since the margin account balance is less than the maintenance margin. The variation margin is the amount necessary to bring the margin account back up to the initial margin. In this case, it is $1,000 (= $2,500 − $1,500).

MODULE QUIZ 33.1

1. When an investor is obligated to buy the underlying asset in a futures position, it is a:
   A. basis trade.
   B. long-futures position.
   C. short-futures position.
   D. hedged-futures position.

2. Which of the following are characteristics specified by a futures contract?
   I. Asset quality and asset quantity.
   II. Delivery arrangements and delivery time.
   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

3. An investor enters into a short position in a gold futures contract with the following characteristics:
   - The initial margin is $3,000.
   - The maintenance margin is $2,250.
   - The contract price is $1,900.
   - Each contract controls 100 troy ounces.

   If the price drops to $1,895 at the end of the first day and $1,890 at the end of the second day, which of the following is closest to the variation margin required at the end of the second day?
   A. $0.
   B. $250.
   C. $500.
   D. $1,000.

MODULE 33.2: FUTURES MARKETS

Exchanges in Futures Transactions

LO 33.c: Describe the role of an exchange in futures transactions.

As discussed in previous readings, the exchange guarantees that traders in the futures markets will honor their obligations. The exchange does this by splitting each trade once it is made and acting as the opposite side of each position. The exchange acts as the buyer to every seller and the seller to every buyer. By doing this, the exchange allows either side of the trade to reverse positions at a future date without having to contact the other side of the initial trade. This allows traders to enter the market knowing that they will be able to reverse their position. Traders are also freed from having to worry about the counterparty defaulting since the counterparty is now the exchange.

Futures Market Quotes

LO 33.d: Explain the differences between a normal and an inverted futures market.

Futures quotes can be found from exchanges as well as various online websites. Figure 33.1 contains a subset of gold futures quotes posted on the CME Group website (www.cmegroup.com). The first column indicates the maturity month for a given futures contract. Recall that each gold futures contract represents 100 ounces and is priced in U.S. dollars per ounce.

Figure 33.1: Gold Futures Quotes

Month	Last Trade	Change	Prior Settlement	Open	High	Low	Volume
Dec-21	1808.1	+11.8	1796.3	1794.2	1811.5	1793.0	147,708
Feb-22	1810.0	+11.7	1798.3	1796.4	1813.3	1795.3	5,354
Apr-22	1811.7	+11.7	1800.0	1798.7	1814.6	1798.7	772
Jun-22	1813.0	+11.3	1801.7	1799.3	1815.8	1799.3	1,878
Dec-22	1818.9	+12.0	1806.9	1810.4	1819.9	1808.0	247

The current trading price of a given futures contract is shown in the second column. The change between the previous day's settlement price and the last trade is reflected in the third column. The settlement price is typically computed as the price right before the end of the previous trading day. This price is used for computing daily gains and losses as well as determining margin requirements.

Figure 33.1 also indicates the opening prices for a given day and the highest and lowest prices during the trading day. As you can see, the December 2021 gold futures contract for this particular day opened at $1,794.2/oz., increased to a high of $1,811.5/oz. and decreased to a low of $1,793/oz. This contract last traded at $1,808.1/oz., which indicates a change of +11.8/oz. over the previous day's settlement price.

The last column in Figure 33.1 reflects the trading volume in each futures contract, which is the number of contracts that have been traded on a given day. This amount differs from open interest, discussed earlier, which is the total number of long (or short) positions in a given futures contract. Trading volume could potentially be higher than open interest on any particular day if a large amount of day trading took place, where traders open and close a futures position on the same day.

Figure 33.1 also shows the pattern of futures prices as a function of contract maturity. Depending on the direction of futures settlement prices, the market may be normal or inverted. In this case, gold futures contract prices are moving higher with increasing time horizons. Increasing settlement prices over time would indicate a **normal futures**

market. Conversely, decreasing settlement prices over time would indicate an **inverted futures market**.

The Delivery Process

LO 33.e: Describe the mechanics of the delivery process and contrast it with cash settlement.

Delivery of the underlying asset occurs very infrequently with futures contracts. The most common way to terminate futures contracts before delivery is to make a reverse or offsetting trade in the futures market.

Now let's assume the short will deliver the goods. When the long (e.g., central counterparty [CCP]) accepts this delivery, the long pays the contract price to the short. This is called **delivery**. The location for delivery (for physical assets), terms of delivery, and details of exactly what is to be delivered are all specified in the **notice of intention to deliver** document. Each exchange has specific rules as to the conditions for making an intent to deliver. However, the price paid or received will be the most recent settlement price. The **first notice day** is the first day when a notice to deliver can be made and the **last notice day** is the last day when it can be made. The **last trading day** is often several days before the last notice day.

In a **cash-settlement contract**, delivery is not an option. The futures account is marked to market based on the settlement price on the last day of trading.

Types of Trading Orders

LO 33.f: Describe and compare different trading order types.

There are several different types of orders in the marketplace.

Market orders are orders to buy or sell at the best price available. The key problem is that the transaction price may be significantly higher or lower than planned. A **discretionary order** is a market order where the broker has the option to delay transaction in search of a better price.

Limit orders are orders to buy or sell away from the current market price. A limit buy order is placed below the current price. A limit sell order is placed above the current price. Limit orders have a time limit, such as instantaneous, one day, one week, one month, or good until canceled. Limit orders are turned over to the specialist by the commission broker.

Stop-loss orders are used to prevent losses or to protect profits. Suppose you own a stock currently selling for $40. You are afraid that it may drop in price, and if it does, you want your broker to sell it, thereby limiting your losses. You would place a stop-loss-sell order at a specific price (e.g., $35); if the stock price drops to this level, your broker will place a sell market order. A stop-loss-buy order is usually combined with a short sale to limit losses. If the stock price rises to the stop price, the broker enters a market order to buy the stock.

Variations on these order types also exist. **Stop-limit orders** are a combination of a stop and limit order. The stop price and limit price must be specified, so that once the stop level is reached, or bettered, the order would turn into a limit order and hopefully transact at the limit price. **Market-if-touched orders**, or MIT orders, are orders that would become market orders once a specified price is reached in the marketplace.

For those orders that remain outstanding until the designated price range is reached, the trader making the order needs to indicate the time for the order (time-of-day order). **Good-til-canceled (GTC) orders** (a.k.a. **open orders**) are orders that remain open until they either transact or are canceled. A popular method of submitting a limit order is to have it automatically canceled at the end of the trading day in which it was submitted. **Fill-or-kill orders** must be executed immediately or the trade will not take place.

Marking to Market and Hedge Accounting

LO 33.g: Describe the application of marking to market and hedge accounting for futures.

Futures contracts are settled daily, therefore, all gains/losses for a given year would be considered realized and, therefore, must recorded each year (i.e., marked to market). The **mark-to-market process** will most likely increase a firm's earnings volatility.

However, if futures are used to hedge a transaction that will not occur for several years into the future then hedge accounting may apply. In general, **hedge accounting** requires the hedge to be fully documented and for the hedge to be effective (i.e., reasonable economic relationship between hedging instrument and hedged item). Assuming a transaction qualifies for hedge accounting, the accounting rules permit gains and losses from the futures (that would otherwise be reported annually) to be deferred and reported simultaneously with the gains/losses on the hedged items.

There are corresponding rules for hedges for tax purposes and because the rules may differ significantly between jurisdictions, a given transaction might qualify as a hedge for accounting purposes but not for tax purposes and vice versa.

Forwards and Futures Contracts

LO 33.h: Compare and contrast forward and futures contracts.

Forward and futures contracts are agreements to purchase or sell an underlying asset at a stated time in the future.

However, forward and futures contracts differ in the following ways:

- Forwards are private (OTC) transactions between two parties; futures are traded on organized exchanges.
- Forwards are customizable to satisfy both parties; futures are standardized for underlying asset, size, and maturity.

- Forwards are bilateral agreements with counterparty risk; futures trade with exchanges and have no counterparty risk.
- It is difficult to offset or cancel a forward contract because trading and liquidity are low; it is easy to offset or cancel futures because the market is active and provides good liquidity.
- Forwards settle at expiration; futures are marked to market and settle daily.

MODULE QUIZ 33.2

1. Which of the following items are functions of the exchange?
   I. Determine which contracts trade.
   II. Receive margin deposits from brokers.
   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

2. Which of the following types of orders must always be executed?
   A. Limit orders.
   B. Market-if-touched (MIT) orders.
   C. Stop-limit orders.
   D. Market orders.

KEY CONCEPTS

LO 33.a

A long (short) futures position obligates the owner to buy (sell) the underlying asset at a specified price and date. Most futures positions are reversed (or closed out) as opposed to satisfying the contract by making (or taking) delivery.

LO 33.b

The spot price of a commodity or financial asset is the price for immediate delivery. The futures price is the price today for delivery at some future point in time (i.e., the maturity date). The basis is the difference between the spot price and the futures price. As the maturity date nears, the basis converges toward zero. Arbitrage will force the spot and futures prices to be the same at contract expiration.

LO 33.c

The exchange maintains an orderly and liquid market by acting as the counterparty to each long or short-futures position. In the OTC markets, the central counterparty (i.e., exchange) becomes the counterparty to both parties in an OTC transaction.

LO 33.d

Increasing settlement prices over time indicate a normal market, while decreasing settlement prices over time indicate an inverted market.

LO 33.e

A short can terminate the futures contract by delivering the goods. When the long accepts this delivery, the long pays the contract price to the short. This is known as the delivery process. In a cash-settlement contract, delivery is not an option and the settlement amount is based on a mark-to-market process.

LO 33.f

Several different types of orders exist in the marketplace, including market, limit, stop-loss, stop-limit, and MIT orders. Market orders are orders to buy or sell at the best price available. Limit orders are orders to buy or sell away from the current market price. Stop-loss orders are used to prevent losses or to protect profits. Stop-limit orders are a combination of a stop and limit order. MIT orders are orders that would become market orders once a specified price is reached.

LO 33.g

Futures contracts are settled daily, therefore, all gains/losses for a given year would be considered realized and, therefore, must recorded each year (i.e., mark to market).

Assuming a transaction qualifies for hedge accounting, the accounting rules permit gains/losses from the futures (that would otherwise be reported annually) to be deferred and reported simultaneously with the gains/losses on the hedged items.

LO 33.h

Futures contracts are similar to forward contracts in that both allow for a transaction to take place at a future date at a price agreed upon today. The key difference between the two is that forward contracts are private, customized contracts with more credit risk, while futures trade on an organized exchange and have terms that are highly standardized.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 33.1

1. **B** When an investor is obligated to buy the underlying asset in a futures position, it is a long-futures position. (LO 33.a)

2. **C** Delivery time, asset quality, asset quantity, and delivery arrangements are all characteristics specified by the futures contract. (LO 33.a)

3. **A** Note that the investor in this question has a short position that profits from price declines. The short position margin account has increased by $1,000 over the two days, so there is no variation margin required. (LO 33.b)

Module Quiz 33.2

1. **C** The exchange acts as buyer to every seller and seller to every buyer, thus virtually eliminating default risk. It also collects margin payments from clearing members

(brokers). Determining which contracts will trade is also a function of the exchange. (LO 33.c)

2. **D** Market orders are orders to buy or sell at the best price available. The other orders require that the price reach a certain range before being activated. If the price never reaches that range, the order will never be activated. (LO 33.f)

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Financial Markets and Products, Chapter 8.

READING 34

USING FUTURES FOR HEDGING

Study Session 9

EXAM FOCUS

Futures contracts are used extensively for implementing hedging strategies. This reading presents the calculations for determining the optimal hedge ratio and shows how to use it to determine the number of futures contracts necessary to hedge a spot market exposure. This reading also addresses basis risk, the change in the relationship between spot prices and futures prices over a hedge horizon. Basis risk arises because an asset being hedged may not be exactly the same as the asset underlying the futures contract.

MODULE 34.1: PRINCIPLES OF HEDGING

LO 34.a: Define and differentiate between short and long hedges and identify their appropriate uses.

LO 34.e: Calculate the profit and loss on a short or a long hedge.

A **short hedge** occurs when the hedger shorts (sells) a futures contract to hedge against a price decrease in the existing long position. When the price of the hedged asset decreases, the short futures position realizes a positive return, offsetting the decline in asset value. Therefore, a short hedge is appropriate when you have a long position and expect prices to decline.

EXAMPLE: Computing the profit/loss on a short hedge

Assume an investor will receive 50,000 troy ounces of silver in six months. The current spot price is \$24.70/ounce. In order to hedge this entire position, the investor will short six-month futures contracts priced at \$25. **Compute** the profit on the overall position assuming the spot price at time of delivery is either \$24.25 or \$26.75. Suppose each futures contract represents 5,000 troy ounces of silver.

Answer:

If the spot price at delivery is 24.25, the investor will receive a market price of: 50,000 × 24.25 = 1,212,500. At this price, the short futures contract will show a gain of: 10 contracts × (25 – 24.25) × 5,000 = 37,500. Therefore, the overall profit will equal: 1,212,500 + 37,500 = $1,250,000.

If the spot price at delivery is 26.75, the investor will receive a market price of: 50,000 × 26.75 = 1,337,500. At this price, the short futures contract will show a loss of: 10 contracts × (25 – 26.75) × 5,000 = –87,500. Therefore, the overall profit will equal: 1,337,500 – 87,500 = $1,250,000.

A **long hedge** occurs when the hedger buys a futures contract to hedge against an increase in the value of the asset that underlies a short position. In this case, an increase in the value of the shorted asset will result in a loss to the short seller. The objective of the long hedge is to offset the loss in the short position with a gain from the long futures position. A long hedge is therefore appropriate when you have a short position and expect prices to rise.

EXAMPLE: Computing the profit/loss on a long hedge

Assume an investor will need to buy 75,000 pounds of copper in three months. The current spot price is $3.80/pound. In order to hedge this entire position, the investor will buy three-month futures contracts priced at $4.50. **Compute** the payment on the overall position assuming the spot price at time of delivery is either $4.00 or $5.25. Suppose each futures contract represents 25,000 pounds of copper.

Answer:

If the spot price at delivery is $4.00, the investor will pay a market price of: 75,000 × 4.00 = 300,000. At this price, the long futures contract will show a loss of: 3 contracts × (4.00 – 4.50) × 25,000 = –37,500. Therefore, the overall payment will equal: 300,000 + 37,500 = 337,500. Note that when the spot price goes down it increases the net cost.

If the spot price at delivery is $5.25, the investor will pay a market price of: 75,000 × 5.25 = 393,750. At this price, the long futures contract will show a gain of: 3 contracts × (5.25 – 4.50) × 25,000 = 56,250. Therefore, the overall payment will equal: 393,750 – 56,250 = $337,500. Note that when the spot price goes up it decreases the net cost.

Advantages and Disadvantages of Hedging

LO 34.b: Describe the arguments for and against hedging and the potential impact of hedging on firm profitability.

The objective of hedging with futures contracts is to reduce or eliminate the price risk of an asset or a portfolio. For example, a farmer with a large corn crop that will be harvested in a few months could wait until the end of the growing season and sell his corn at the prevailing spot price, or he could sell corn futures and “lock in” the price of his corn at a predetermined rate. By taking a short position in a corn futures contract, the farmer eliminates—or at least reduces—exposure to fluctuating corn prices. This is an example of a short hedge, where the user locks in a future selling price.

Alternatively, a cereal company will need to purchase corn in the future. The company could wait to buy corn in the spot market and face the volatility of future corn spot prices or lock in its purchase price by buying corn futures in advance. This demonstrates an anticipatory hedge. The cereal company has an anticipated need for corn and buys corn futures to lock in the price of those future corn purchases. This is an example of a long hedge, where the user locks in a future purchasing price.

It is easy to see that the benefit from hedging leads to less uncertainty regarding future profitability and, hence, the reduction in volatility of earnings. However, despite the outcome being more certain with hedging, basis risk still exists (which will be discussed in the next section). There are some other arguments against hedging. The main issue is that hedging can lead to less profitability if the asset being hedged ends up increasing in value. The increase in value will be offset by a corresponding loss in the futures contract used for the hedge. Of course, it can also lead to more profitability in the opposite situation (i.e., hedged asset decreases in value but is offset by a gain in the futures contract).

Another argument against hedging is the questionable benefit that accrues to shareholders. Clearly, hedging reduces risk for a company and its shareholders, but there is reason to believe that shareholders can more easily hedge risk on their own by diversifying their investments in terms of industry and/or geography, for example. A third argument deals with the nature of the hedging company's industry. For example, assume that prices in an industry frequently adjust for changes in input prices and exchange rates and, therefore, there is virtually no exposure. If competitors do not hedge, then there is an incentive for a given firm to keep the status quo and not hedge either. In this way, the company ensures that profitability will remain more stable than if it were to hedge frequent changes. In other words, hedging could work if industry prices do not adjust to the changes, but hedging could lose money if industry prices do adjust and, therefore, the hedge is unnecessary.

Basis Risk

LO 34.c: Define and calculate the basis, discuss various sources of basis risk, and explain how basis risks arise when hedging with futures.

LO 34.d: Define cross hedging and compute and interpret hedge ratio and hedge effectiveness.

When all the existing position characteristics match perfectly with those of the futures contract specifications, we have a perfect hedge. With a perfect hedge, the loss on a hedged position will be perfectly offset by the gain on the futures position. Perfect hedges are not very common. There are two major reasons why this is so: (1) the asset in the existing position is often not the same as that underlying the futures (e.g., we may be hedging a corporate bond portfolio with a futures contract on a U.S. Treasury bond), and (2) the hedging horizon may not match perfectly with the maturity of the futures contract. With respect to (2), if the futures position is unwound before maturity, the return to the futures position could be different from the return to the cash position. The existence of either (1) or (2) leads to what is called **basis risk**.

The basis in a hedge is defined as the difference between the spot price on a hedged asset and the futures price of the hedging instrument (e.g., futures contract). Basis is calculated as:

basis = spot price of asset being hedged − futures price of contract used in hedge

When the hedged asset and the asset underlying the hedging instrument are the same, the basis will be zero at maturity. Sometimes it may be more efficient to **cross hedge** or hedge a cash position with a hedge asset that is closely related but different from the cash asset (e.g., different types of corn).

PROFESSOR'S NOTE

This is the typical definition for basis (where basis equals spot price minus futures price). However, basis is also sometimes defined as futures price minus spot price, mostly when dealing with financial asset futures.

When hedging, a change in basis is unavoidable. The change in basis over the hedge horizon is basis risk, and it can work either for or against a hedger.

To minimize basis risk, hedgers should select the contract on an asset that is most highly correlated with the spot position and a contract maturity that is closest to the hedging horizon.

The Optimal Hedge Ratio

We can account for an imperfect relationship between the spot and futures positions by calculating an **optimal hedge ratio** that incorporates the degree of correlation between the rates.

A hedge ratio is the ratio of the size of the futures position relative to the spot position. The optimal hedge ratio, which minimizes the variance of the combined hedge position, is defined as follows:

$$HR = \rho_{S,F}\frac{\sigma_S}{\sigma_F}$$

This is also the beta of spot prices with respect to futures contract prices since:

$$\rho = \frac{Cov_{S,F}}{\sigma_S\sigma_F} \text{ and } \frac{Cov_{S,F}}{\sigma_S\sigma_F}\times\frac{\sigma_S}{\sigma_F} = \frac{Cov_{S,F}}{\sigma_F^2} = \beta_{S,F}$$

where:

$\rho_{S,F}$ = correlation between the spot prices and the futures prices

σ_S = standard deviation of the spot price

σ_F = standard deviation of the futures price

EXAMPLE: Hedge ratio

Suppose a currency trader computed the correlation between the spot and futures to be 0.925, the annual standard deviation of the spot price to be $0.10, and the annual standard deviation of the futures price to be $0.125. **Compute** the hedge ratio.

Answer:

$$HR = 0.925 \times \frac{0.100}{0.125} = 0.74$$

The ratio of the size of the futures to the spot should be 0.74.

The *effectiveness of the hedge* measures the variance that is reduced by implementing the optimal hedge. This effectiveness can be evaluated with a coefficient of determination (R^2) term where the independent variable is the change in futures prices and the dependent variable is the change in spot prices. Recall that R^2 measures the goodness-of-fit of a regression. As shown previously, the beta of spot prices with respect to futures prices is equal to the hedge ratio, which is also the slope of this regression. The R^2 measure for this simple linear regression is the square of the correlation coefficient (ρ^2) between spot and futures prices.

MODULE QUIZ 34.1

1. Which of the following situations describe a hedger with exposure to basis risk?
   I. A portfolio manager for a large-cap growth fund knows he will be receiving a significant cash investment from a client within the next month and wants to pre-invest the cash using stock index futures.
   II. A farmer has a large crop of corn he is looking to sell before June 30. The farmer uses a June futures contract to lock in his sales price.
   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

2. The standard deviation of price changes in a wheat futures contract is 0.6, while the standard deviation of changes in the price of wheat is 0.75. The covariance between the spot price changes and the futures price changes is 0.3825. Which of the following amounts is closest to the optimal hedge ratio?
   A. 0.478.
   B. 0.850.
   C. 1.063.

D. 1.250.

MODULE 34.2: HEDGING WITH STOCK INDEX FUTURES

LO 34.f: Compute the optimal number of futures contracts needed to hedge an exposure and explain and calculate the "tailing the hedge" adjustment.

A common hedging application is the hedging of equity portfolios using futures contracts on stock indices (index futures). In this application, it is important to remember that the hedged portfolio's beta serves as a hedge ratio when determining the correct number of contracts to purchase or sell. The number of futures contracts required to completely hedge an equity position is determined with the following formula:

$$\text{number of contracts} = \beta_{\text{portfolio}} \times \left(\frac{\text{portfolio value}}{\text{value of futures contract}}\right)$$

$$= \beta_{\text{portfolio}} \times \left(\frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}}\right)$$

EXAMPLE: Hedging with stock index futures

You are a portfolio manager with a \$20 million growth portfolio that has a beta of 1.4, relative to the S&P 500. The S&P 500 futures are trading at 4,150, and the multiplier is 250. You would like to hedge your exposure to market risk over the next few months. **Identify** whether a long or short hedge is appropriate, and **determine** the number of S&P 500 contracts you need to implement the hedge.

Answer:

You are long the S&P 500, so you should construct a short hedge and sell the futures contract. The number of contracts to sell is equal to:

$$1.4 \times \frac{\$20{,}000{,}000}{4{,}150 \times 250} \approx 27 \text{ contracts}$$

Tailing the Hedge

A hedger may actually overhedge the underlying exposure if daily settlement is not properly accounted for. To correct for the possibility of overhedging, a hedger can implement a **tailing the hedge** strategy. The extra step needed to carry out this strategy is to multiply the hedge ratio by the daily spot-price-to-futures-price ratio. In practice, it is not efficient to adjust the hedge for every daily change in the spot-to-futures ratio.

EXAMPLE: Tailing the hedge

Suppose that you would like to make a tailing the hedge adjustment to the number of contracts needed in the previous example. Assume that when evaluating the next daily settlement period, you find that the S&P 500 spot price is 4,095 and the futures price is now 4,260. **Determine** the number of S&P 500 contracts needed after making a tailing the hedge adjustment.

Answer:

The number of contracts to sell is equal to:

$$1.4 \times [(\$20{,}000{,}000) / (4{,}150 \times 250)] \times (4{,}095 / 4{,}260) = 26 \text{ contracts}$$

Adjusting the Portfolio Beta

LO 34.g: Explain how to use stock index futures contracts to change a stock portfolio's beta.

Hedging an existing equity portfolio with index futures is an attempt to reduce the *systematic risk* of the portfolio. If the beta of the capital asset pricing model is used as the systematic risk measure, then hedging boils down to a reduction of the portfolio beta. Let β be our portfolio beta, β^* be our target beta after we implement the strategy with index futures, P be our portfolio value, and A be the value of the underlying asset (i.e., the stock index futures contract). To compute the appropriate number of futures, we use the following equation:

$$\text{number of contracts} = (\beta^* - \beta)\frac{P}{A}$$

This equation can result in either positive or negative values. Negative values indicate selling futures (decreasing systematic risk), and positive values indicate buying futures contracts (increasing systematic risk).

EXAMPLE: Adjusting portfolio beta

Suppose we have a well-diversified \$100 million equity portfolio. The portfolio beta relative to the S&P 500 is 1.2. The current value of the 3-month S&P 500 Index is 4,480. The portfolio manager wants to completely hedge the systematic risk of the portfolio over the next three months using S&P 500 Index futures. **Demonstrate** how to adjust the portfolio's beta.

Answer:

In this instance, our target beta, β^*, is 0, because a complete hedge is desired.

$$\text{number of contracts} = (0 - 1.2)\frac{100{,}000{,}000}{4{,}480 \times 250} = -107.14$$

The negative sign tells us we need to sell 107 contracts.

Rolling a Hedge Forward

LO 34.h: Explain how to create a long-term hedge using a stack-and-roll strategy and describe some of the risks that arise from this strategy.

When the hedging horizon is long relative to the maturity of the futures used in the hedging strategy, hedges have to be rolled forward as the futures contracts in the hedge come to maturity or expiration. Typically, as a maturity date approaches, the hedger must close out the existing position and replace it with another contract with a later maturity. This is called **rolling the hedge forward** or **stack-and-roll** strategy.

When rolling a hedge forward, hedgers are not only exposed to the basis risk of the original hedge, they are also exposed to the basis risk of a new position each time the hedge is rolled forward. This is called rollover basis risk.

MODULE QUIZ 34.2

Use the following information to answer Questions 1 and 2.

An equity portfolio is worth $100 million with the benchmark of the Dow Jones Industrial Average (Dow). The Dow is currently at 35,000, and the corresponding portfolio beta is 1.2. The futures multiplier for the Dow is 10.

1. Which of the following is the closest to the number of contracts needed to double the portfolio beta?
   A. 300.
   B. 316.
   C. 321.
   D. 343.

2. To cut the beta in half, the correct trade is:
   A. long 171 contracts.
   B. short 171 contracts.
   C. long 300 contracts.
   D. short 343 contracts.

3. A large-cap value equity manager has a $6,500,000 equity portfolio with a beta of 0.92. An S&P 500 futures contract is available with a current value of 4,175 and a multiplier of 250. What position should the manager take to completely hedge the portfolio's market risk?
   A. Short 6 contracts.
   B. Short 8 contracts.
   C. Short 10 contracts.
   D. Long 8 contracts.

KEY CONCEPTS

LO 34.a

Hedging may be achieved by shorting futures to protect an underlying position against price deterioration or by buying futures to hedge against unanticipated price increases in an underlying asset.

LO 34.b

Investors hedge with futures contracts to reduce or eliminate the price risk of an asset or a portfolio. The key advantage of hedging is that it leads to less uncertainty regarding future profitability. The key disadvantage of hedging (assuming a short hedge) is that it can lead to less profitability if the asset being hedged ends up increasing in value.

LO 34.c

Basis risk is the risk that a difference may occur between the spot price of a hedged asset and the futures price of the contract used to implement the hedge. Basis risk is zero only when there is a perfect match between the hedged asset and the contract's underlying instrument in terms of maturity and asset type.

LO 34.d

Sometimes it may be more efficient to cross hedge or hedge a cash position with a hedge asset that is closely related but different from the cash asset.

A hedge ratio is the ratio of the size of the futures position relative to the spot position necessary to provide a desired level of protection.

$$\text{HR} = \rho_{\text{spot, futures}} \times \frac{\sigma_{\text{spot}}}{\sigma_{\text{futures}}}$$

The effectiveness of the hedge measures the variance that is reduced by implementing the optimal hedge.

LO 34.e

By using either a short or long hedge, an investor can lock in a price equal to the current futures price.

LO 34.f

A common hedging application is the hedging of equity portfolios using futures contracts on stock indices (index futures). The number of futures contracts required to completely hedge an equity position is determined as follows:

$$\#\text{ of contracts} = \beta_{\text{portfolio}} \times \left(\frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}}\right)$$

To correct for the possibility of overhedging, a hedger can implement a tailing the hedge strategy by multiplying the hedge ratio by the daily spot-price-to-futures-price ratio.

LO 34.g

When hedging an equity portfolio with a short position in stock index futures, the beta of the portfolio is reduced. To change a stock portfolio's beta, use the following formula:

$$\text{number of contracts} = (\beta^* - \beta) \times \frac{\text{portfolio value}}{\text{value of futures contract}}$$

LO 34.h

When the hedging horizon is longer than the maturity of the futures, the hedge must be rolled forward to retain the hedge. This exposes the hedger to rollover risk, the basis risk when the hedge is reestablished.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 34.1

1. **C** Both of these situations describe exposure to basis risk—the risk that the difference between the spot price and the futures delivery price will change. The portfolio manager using futures to pre-invest the cash does not know the exact date he will receive the cash and may need to sell or hold the futures contract for a longer time period than intended. The farmer may need to sell his June futures contract early if he sells his corn earlier than the June futures expiration date. (LO 34.c)

2. **C** Notice in this problem, we were given the covariance but not the correlation. We can calculate the correlation as follows:

$$\rho = \frac{\text{Cov}_{S,F}}{(\sigma_S)(\sigma_F)} = \frac{0.3825}{(0.75)(0.60)} = 0.85$$

Now that we have our correlation value, we can calculate the minimum hedge ratio as:

$$0.85\left(\frac{0.75}{0.60}\right) = 1.0625, \text{ or, directly, } = \frac{\text{Cov}_{S,F}}{\sigma_F^2} = \frac{0.3825}{0.6^2} = 1.0625$$

(LO 34.d)

Module Quiz 34.2

1. **D** $(2.4 - 1.2)\dfrac{100{,}000{,}000}{35{,}000 \times 10} = 1.2 \times 285.71 = 343$

where beta = 1.2, target beta = 2.4, A = 10 × 35,000, P = $100 million (LO 34.g)

2. **B** $(0.6 - 1.2)\dfrac{100{,}000{,}000}{35{,}000 \times 10} = -0.6 \times 285.71 = -171$

where beta = 1.2, target beta = 0.6, A = 10 × 35,000, P = $100 million (LO 34.g)

3. **A** $0.92 \times \dfrac{6{,}500{,}000}{4{,}175 \times 250} \approx 6 \text{ contracts}$

Because the manager has a long position in the market, she will want to take a short position in the futures. (LO 34.f)

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Financial Markets and Products, Chapter 9.

READING 35

FOREIGN EXCHANGE MARKETS

Study Session 9

EXAM FOCUS

Exposure to foreign exchange risks (transaction, translation, and economic) is a natural result of the globalization of financial institutions. These risks arise when foreign currency trading and/or foreign asset-liability positions are mismatched in individual currencies. Unexpected foreign currency volatility can generate significant losses for the firm, which could, in turn, threaten profitability or even solvency. These risks can be mitigated by direct hedging through matching foreign asset-liability books of business, hedging through forward contracts, and other nonfinancial operational decisions. For the exam, understand foreign exchange quotes and be able to perform the necessary calculations. Fundamental concepts such as purchasing power parity and covered interest rate parity are also highly testable.

MODULE 35.1: FOREIGN EXCHANGE QUOTES AND RISKS

Spot, Forward, and Futures Quotes

LO 35.a: Explain and describe the mechanics of spot quotes, forward quotes, and futures quotes in the foreign exchange markets; distinguish between bid and ask exchange rates.

LO 35.b: Calculate a bid-ask spread and explain why the bid-ask spread for spot quotes may be different from the bid-ask spread for forward quotes.

Foreign exchange (FX) quotes have a base currency and a quote currency. They state how much of the quote currency is necessary to obtain one unit of the base currency. Quotes are usually displayed as XXXYYY where XXX is the base currency and YYY is the quote currency. For example, a quote between Canadian and U.S. dollars could be CADUSD 0.75, which means it takes 0.75 U.S. dollars to obtain one Canadian dollar.

Spot quotes are for an immediate currency trade and have four decimal places. For example, assume the CADUSD spot quote is bid 0.7535 and ask 0.7541, for a spot bid-ask spread of 0.0006. The **bid exchange rate** is lower, and it is the rate the dealer would pay if buying the currency from the investor (or the rate the investor would receive if selling the currency to the dealer). The **ask exchange rate** is higher, and it is the rate the dealer would receive if selling the currency to the investor (or the rate the investor would pay if buying the currency from the dealer). Bid-ask spreads are wider (narrower) when there are smaller (larger) amounts of the currencies being traded.

Forward quotes are for a future currency trade and are quoted as points that are multiplied by 0.0001 and then added to or subtracted from the spot quote. For example, a CADUSD three-month forward quote may be bid 90.11 and ask 93.91. Therefore, the three-month forward bid quote is 0.7535 + 0.009011 = 0.762511 and the three-month forward ask quote is 0.7541 + 0.009391 = 0.763491. To conclude the analysis, the forward bid-ask spread is 0.0006 + 0.00038 = 0.00098 or 0.763491 − 0.762511 = 0.00098. Typically, the bid-ask spread will widen as the term of the forward contract increases, which accounts for greater risk/uncertainty compared with a spot transaction that would occur immediately with certainty.

PROFESSOR'S NOTE

If the bid/ask points are negative, then the forward rate will be less than the spot rate (and the negative ask points will be smaller in magnitude than the negative bid points so that the bid-ask spread is always wider for forward quotes than for spot quotes).

FX futures traded by the CME Group always use the USD as the quote currency (i.e., XXXUSD). Thus, in cases where the forward uses USD as the base currency (i.e., USDYYY), the futures quote would be the inverse of the forward quote. For example, a forward quote for USDEUR of 0.8700 would be quoted as a futures quote of 1.1494 (= 1 / 0.8700) USD per EUR.

LO 35.c: Compare outright (forward) and swap transactions.

An outright (forward) FX transaction occurs when two parties agree to a transaction on a given date in the future.

An FX swap transaction is more complex because it involves two steps: (1) FX buy/(sell) in spot market, and (2) FX sell/(buy) in forward market. To illustrate, consider a Canadian company funding its U.S. operations by borrowing in Canadian dollars (CAD) and buying USD today. At the same time, company is selling USD for CAD three months from now. The buy USD and sell USD transactions cancel out, with the end result being the financing of the U.S. operations in the domestic CAD currency.

Transaction, Translation, and Economic Risk

LO 35.d: Define, compare, and contrast transaction risk, translation risk, and

economic risk.

LO 35.e: Describe examples of transaction, translation, and economic risks and explain how to hedge these risks.

Transaction Risk

Transaction risk occurs when cash flow of one currency must be exchanged for another at a future date to settle a specific transaction. For a firm, it can occur within the context of a receivable or a payable.

An importer must pay the foreign currency in exchange for goods or services at a future date and is at risk if that currency appreciates. For example, a U.K. company must pay CAD 100 million in six months. The spot exchange rate is GBPCAD 1.7165, and the six-month forward exchange rate is GBPCAD 1.7350. The cost of buying the CAD forward is lower than in the spot market (i.e., it takes more CAD to buy 1 GBP), but the spot market is not relevant because the CAD are not yet needed. The forward market does allow the U.K.-based company to lock in the GBP cost of the transaction at 1.7350.

An exporter will receive foreign currency in payment for goods or services at a future date and is at risk if that currency depreciates. For example, a U.S. exporter has contracted to sell EUR 9 million to a European trade partner in three months. The spot exchange rate is USDEUR 0.8700, and the three-month forward exchange rate is USDEUR 0.8750. The spot market is not relevant because the EURs are not yet in hand, but the forward rate can be locked in by selling the EUR forward at 0.8750.

The transactions in this section can be hedged with outright forward transactions. In the first example, paying the foreign currency is a short position in the foreign currency that can be hedged by buying a forward contract. In the second example, receiving the foreign currency is a long position in the foreign currency that can be hedged by selling a forward contract.

Translation Risk

Translation risk occurs when the financial statements in a foreign currency must be converted to a (different) domestic currency. Depending on the translation method used (based on the accounting rules), there will be foreign exchange gains or losses. Although they will have an effect on reported earnings, they do not necessarily reflect real economic gain or loss (i.e., no effect on cash flows).

The principal on foreign borrowings is subject to foreign exchange gains and losses. For example, assume a U.S. company borrows from a Canadian bank. If the CADUSD exchange rate decreases during the year, then it means the USD has appreciated (and the CAD has depreciated) and the company has incurred a foreign exchange gain. If the CADUSD exchange rate increases, then the appreciation of the CAD means the company has incurred a foreign exchange loss.

Hedging translation risk should be done on a specific future date. The best way to do it is to ensure that assets in a foreign country should be financed by borrowings in the same country. That way, all the gains (losses) on assets will be canceled by losses (gains) on the liabilities, with a net result of zero translation gain or loss.

Economic Risk

Economic risk is less directly measurable than the other two risks. Economic risk occurs when currency volatility affects the firm's cash flows or its competitive standing within the domestic market. For example, for cash flows, if Disney World in the United States incurs all revenue and expenses in USD, a decline in the USD relative to all currencies may be beneficial if it induces more foreign travelers to visit the United States and Disney World. From a competitive standpoint, if there is a Canadian firm that has domestic sales and production only, changes in the Canadian exchange rate may make it advantageous for a foreign competitor to enter the Canadian market and negatively impact the Canadian firm. Hedging economic risk may occur from an operational perspective by relocating production facilities or expanding sales overseas, for example.

Multicurrency Hedging

LO 35.f: Describe the rationale for multi-currency hedging using options.

A multinational firm faces risk from multiple currencies, which is a good thing in a sense because the changes in the different currencies have a correlation of less than +1. So portfolio currency risk is reduced compared with the risk of a single currency. The use of options versus forwards for hedging is because of the two-sided nature of options: (1) downside protection from undesirable changes in exchange rate, and (2) upward potential from desirable changes. Forwards would simply "neutralize" any potential profit or loss.

One hedging alternative involves long positions on options for each currency for which the firm faces risk, which is relatively costly. A less costly alternative is the purchase of a basket option in the over-the-counter (OTC) market; a basket option targets specific currencies desired by the firm.

In terms of timing, multinational firms often have monthly exchange rate risks, so purchasing options with monthly maturities is one alternative, which is relatively costly. A less costly alternative is to purchase Asian options, which have strike prices based on the average exchange rate during the year.

MODULE QUIZ 35.1

1. A multinational firm faces various risks, and it attempts to mitigate such risks with hedging techniques. Which type of risk would the firm find the most challenging to hedge?
   A. Economic risk.
   B. Financial risk.
   C. Transaction risk.
   D. Translation risk.
2. Assume the GBPUSD spot quote is bid 1.2944 and ask 1.2952. The three-month forward points quote is bid 56.34 and ask 58.85. The forward bid-ask spread is

closest to:

A. 0.000251.
B. 0.0008.
C. 0.001051.
D. 0.006685.

MODULE 35.2: INTEREST RATES, INFLATION, AND EXCHANGE RATES

Exchange Rate Drivers

LO 35.g: Identify and explain the factors that determine exchange rates.

Exchange rates are primarily driven by (1) balance of payments and trade flows, (2) monetary policy, and (3) inflation. Inflation will be discussed later in context of purchasing power parity.

Balance of payments and trade flows: When comparing the imports and exports of Country X and Country Y, we can make the following general conclusions:

- If Country X exports > Country Y exports, then Country X's currency will appreciate due to the increased demand for X's currency.
- If Country X imports > Country Y imports, then Country Y's currency will appreciate due to the increased demand for Y's currency (e.g., Country X needs more of Country Y's currency to pay for the imported items).

To understand the full picture, in the first instance, although there is an appreciation of Country X's currency, it also means that its exports are now relatively more expensive for Country Y's residents. This will eventually reduce Country Y's demand for those exported items.

Monetary policy: All things being equal, an increase in a country's money supply will result in a depreciation of that country's currency. The idea is that more of that country's currency is required to buy the same amount of items.

Currency Appreciation/Depreciation

LO 35.h: Calculate and explain the effect of an appreciation/depreciation of one currency relative to another.

Assume the EURUSD exchange rate has changed from EURUSD 1.1500 to 1.1300. The percentage change in the USD price of a euro is simply:

$$1.1300 / 1.1500 - 1 = -0.0174 = -1.74\%$$

Because the USD price of a euro has fallen, the euro has depreciated relative to the dollar, and a euro now buys 1.74% fewer U.S. dollars. It is correct to say that the EUR has depreciated by 1.74% relative to the USD.

On the other hand, it is not correct to say that the USD has appreciated by 1.74%. To calculate the percentage appreciation of the USD, we need to convert the quotes to USDEUR. Our beginning quote of EURUSD 1.1500 must be flipped to get USDEUR and we get USDEUR 0.8696. Our ending quote of EURUSD 1.1300 must be flipped to get USDEUR and we get USDEUR 0.8850. Using those exchange rates, we can calculate the change in the euro price of a USD as 0.8850 / 0.8696 − 1 = 0.0177 = +1.77%. In this case, it is correct to say that the USD has appreciated 1.77% with respect to the EUR. For the same quotes, the percentage appreciation of the USD is not the same as the percentage depreciation in the EUR.

Purchasing Power Parity

LO 35.i: Explain the purchasing power parity theorem and use this theorem to calculate the appreciation or depreciation of a foreign currency.

Purchasing power parity (PPP) states that changes in exchange rates should exactly offset the price effects of any inflation differential between the two countries. Simply put, if (over a 1-year period) Country A has a 6% inflation rate and Country B has a 4% inflation rate, then Country A's currency should depreciate by approximately 2% relative to Country B's currency over the period. Practically speaking, PPP may be correct in the long term, but there are major discrepancies from PPP in the short term.

The equation for PPP is as follows:

$$\%\Delta S = \text{inflation(foreign)} - \text{inflation(domestic)}$$

where:

$\%\Delta S$ = change in domestic spot rate

EXAMPLE: Purchasing power parity

Assume the expected annual inflation rate is 2% in Europe and 1% in the United States. Also assume that the current spot exchange rate is EURUSD 1.1500. If PPP holds, **compute** the expected exchange rate in one year.

Answer:

Because inflation in Europe is higher than the inflation in the United States by 1%, the euro is expected to depreciate by 1% annually against the dollar.

The current spot rate is EURUSD 1.1500.

Expected exchange rate in 1 year = 1.1500(0.99) = EURUSD 1.1385.

Nominal and Real Interest Rates

LO 35.j: Describe the relationship between nominal and real interest rates.

Each domestic and foreign nominal interest rate consists of two components. The first component is the **real interest rate**, which reflects a given currency's real demand and supply for its funds. Differences in real interest rates will cause a flow of capital into those countries with the highest available *real* rates of interest. Therefore, there will be an increased demand for those currencies, and they will appreciate relative to the currencies of countries whose available real rate of return is low.

The second component is the **expected inflation rate**, which reflects the amount of compensation required by investors to offset the expected erosion of real value over time due to inflation. Differences in inflation rates will cause the residents of the country with the highest inflation rate to demand more imported (cheaper) goods. For example, if prices in the United States are rising twice as fast as in Australia, U.S. residents will increase their demand for Australian goods (because Australian goods are now cheaper relative to domestic goods). If a country's inflation rate is higher than that of its trading partners, the demand for the country's currency will be low, and the currency will depreciate.

The **nominal interest rate**, *r*, is the compounded sum of the real interest rate, real *r*, and the expected rate of inflation, E(i), over an estimation horizon. This relationship is often called the Fisher equation:

exact methodology: $(1 + r) = (1 + \text{real } r) \times [1 + E(i)]$

linear approximation: $r \approx \text{real } r + E(i)$

We can use the Fisher equation to explain why we might expect the changes in value of foreign currencies to be less than perfectly positively correlated, and consequently why diversification in multicurrency portfolios might reduce foreign exchange risk.

If real interest rates and expected inflation are perfectly correlated across countries (which is unlikely), then global markets are perfectly economically integrated, and there would be no diversification benefit of holding multicurrency portfolios. However, to the extent global markets aren't perfectly integrated, foreign exchange rates are positively but not perfectly positively correlated, and there are risk-reduction benefits from holding multicurrency portfolios.

Interest Rate Parity

LO 35.k: Describe how a non-arbitrage assumption in the foreign exchange markets leads to the interest rate parity theorem and use this theorem to calculate forward foreign exchange rates.

If we assume that hedged Swiss loans (using forward contracts) offer a higher return than the U.S. loans, it makes sense for a bank to focus its activities on making hedged Swiss loans. However, as more is invested in Swiss loans, the bank must buy more Swiss francs. This will continually reduce the forward rate spread until no additional profits could be made by making the forward contract-hedged investments.

As the bank moves into more Swiss loans, the spot exchange rate for buying francs will rise. In equilibrium, the forward exchange rate would have to fall to completely eliminate the attractiveness of the Swiss investments.

This relationship is called **interest rate parity (IRP)** because the discounted spread between domestic and foreign interest rates equals the percentage spread between forward and spot exchange rates. In other words, the hedged dollar return on foreign investments should be equal to the return on domestic investments. IRP implies that in a competitive market, a firm should not be able to make excess profits from foreign investments (i.e., a higher domestic currency return from lending in a foreign currency and locking in the forward rate of exchange).

For the exam, you should know the exact IRP equation:

$$\text{forward} = \text{spot} \times \left[\frac{(1 + r_{YYY})}{(1 + r_{XXX})}\right]^T$$

where:
r_{YYY} = quote currency rate
r_{XXX} = base currency rate

If this equality does not hold, an arbitrage opportunity exists. For example, if the forward rate is *less* than the rate suggested by the preceding equation, then an arbitrageur could borrow at the (low) base currency rate, convert the funds to the quote currency, invest at the (high) quote currency rate, and enter into a forward contract to convert the (higher) amount back to base currency funds, pay off the (lower) base currency borrowings, and retain a riskless arbitrage profit.

Alternatively, if the forward rate is *more* than the rate suggested by the equation above, then an arbitrageur could borrow at the (low) quote currency rate, convert the funds to the base currency, invest at the (high) base currency rate, and enter into a forward contract to convert the (higher) amount back to quote currency funds, pay off the (lower) quote currency borrowings, and retain a riskless arbitrage profit.

EXAMPLE: Interest rate parity

Suppose you can invest in the New Zealand dollar (NZD) at 5.127%, or you can invest in Swiss francs at 5.5%. You are a resident of New Zealand, and the current spot rate is CHFNZD 0.79005. **Calculate** the 1-year forward rate expressed in CHFNZD.

Answer:

$$\text{forward} = \text{spot} \times \left[\frac{(1 + r_{YYY})}{(1 + r_{XXX})}\right] = 0.79005 \times \left(\frac{1.05127}{1.055}\right) = 0.78726$$

PROFESSOR'S NOTE

Notice here that the CHFNZD rate fell from 0.79005 to 0.78726. This implies that it now takes fewer NZD to buy one CHF. So, in other words, the New Zealand dollar is stronger in the forward market than in the spot market. Consequently, the Swiss franc is weaker in the forward market than in the spot market.

LO 35.l: Distinguish between covered and uncovered interest rate parity conditions.

Covered interest rate parity (CIRP) focuses on forward exchange rates as a function of spot rates and the risk-free domestic and foreign rates, all based on the concept of arbitrage as previously discussed. In contrast, **uncovered interest rate parity (UCIRP)** states that the same rate of return should be earned in any currency when all expected exchange rate movements are accounted for. For example, currency A and B have risk-free rates of 3% and 5%, respectively. According to UCIRP, the two currencies should earn the same rate of return. Therefore, UCIRP suggests that currency B should depreciate by 2% relative to currency A.

Assuming both CIRP and UCIRP hold, then the forward rate should be the same as the expected future spot rate.

MODULE QUIZ 35.2

1. The annual interest rate is 3% in the United States and 7% in Mexico. The spot rate for the Mexican peso is USDMXN 20. The six-month arbitrage-free forward rate is closest to:
   A. USDMXN 19.25.
   B. USDMXN 19.62.
   C. USDMXN 20.38.
   D. USDMXN 20.78.
2. The real interest rate in Country Z is 3% and expected inflation is 50%. The nominal interest rate is closest to:
   A. 47%.
   B. 53%.
   C. 55%.
   D. 95%.
3. Assume the CADJPY exchange rate has changed from CADJPY 82.4012 to 83.9912. The percentage change in the JPY price of a CAD is closest to:
   A. −1.89%.
   B. +1.89%.
   C. −1.93%.
   D. +1.93%.

KEY CONCEPTS

LO 35.a

Foreign exchange quotes have a base currency and a quote currency. They state how much of the quote currency is necessary to obtain one unit of the base currency.

Spot quotes are for an immediate currency trade and have four decimal places. The bid exchange rate is lower, and it is the rate the dealer would pay if buying the currency from the investor. The ask exchange rate is higher, and it is the rate the dealer would receive if selling the currency to the investor.

Forward quotes are for a future currency trade and are quoted as points that are multiplied by 0.0001 and then added to or subtracted from the spot quote.

FX futures use the USD as the quote currency (i.e., XXXUSD) which may be the inverse of the forward quote. For example, a forward quote for USDEUR of 0.8700 (USD as the base currency) would be quoted as a futures quote of 1.1494 (= 1 / 0.8700) USD per EUR.

LO 35.b

To illustrate spot bid-ask spreads, assume the CADUSD quote is bid 0.7535 and ask 0.7541. The spot bid-ask spread would be calculated as 0.0006.

To illustrate forward bid-ask spreads, assume the CADUSD three-month forward quote is bid 90.11 and ask 93.91. Therefore, the three-month forward bid quote is 0.7535 + 0.009011 = 0.762511 and the three-month forward ask quote is 0.7541 + 0.009391 = 0.763491. The forward bid-ask spread is 0.0006 + 0.00038 = 0.00098 or 0.763491 − 0.762511 = 0.00098.

Typically, the bid-ask spread will widen as the term of the forward contract increases, which accounts for greater risk/uncertainty compared with a spot transaction that would occur immediately with certainty.

LO 35.c

An outright (forward) FX transaction occurs when two parties agree to a transaction on a given date in the future. An FX swap transaction is more complex because it involves two steps: (1) FX buy/(sell) in spot market, and (2) FX sell/(buy) in forward market.

LO 35.d

Transaction risk occurs when cash flow of one currency must be exchanged for another at a future date to settle a specific transaction. For a firm, it can occur within the context of a receivable or a payable.

Translation risk occurs when the financial statements in a foreign currency must be converted to a (different) domestic currency, resulting in foreign exchange gains or losses. Although they will have an effect on reported earnings, they do not necessarily reflect real economic gain or loss (i.e., no effect on cash flows).

Economic risk is less directly measurable than the other two risks. Economic risk occurs when currency volatility affects the firm's cash flows or its competitive standing within the domestic market.

LO 35.e

Transaction risk can be hedged with outright forward transactions.

Hedging translation risk should be done on a specific future date. The best way to do it is to ensure that assets in a foreign country should be financed by borrowings in the same country.

Hedging economic risk may occur through operational production and/or sales decisions.

LO 35.f

The use of options versus forwards for hedging is because of the two-sided nature of options: (1) downside protection from undesirable changes in exchange rate, and (2) upward potential from desirable changes.

LO 35.g

Exchange rates are primarily driven by (1) balance of payments and trade flows, (2) monetary policy, and (3) inflation.

If Country X exports > Country Y exports, then Country X's currency will appreciate due to the increased demand for X's currency. If Country X imports > Country Y imports, then Country Y's currency will appreciate due to the increased demand for Y's currency.

All things being equal, an increase in a country's money supply will result in a depreciation of that country's currency.

LO 35.h

Regarding the effect of an appreciation/depreciation of a currency relative to a foreign currency, consider a EURUSD exchange rate that has changed from EURUSD 1.15 to 1.13. The percentage change in the USD price of a euro (depreciation) is simply:

$$1.13 / 1.15 - 1 = -0.0174 = -1.74\%$$

To calculate the percentage appreciation of the USD, we need to convert the quotes to USDEUR. Using the USDEUR quotes (1/1.13 and 1/1.15), we can calculate the change in the euro price of a USD as 0.8850 / 0.8696 − 1 = 0.0177 = +1.77%.

LO 35.i

Purchasing power parity (PPP) states that changes in exchange rates should exactly offset the price effects of any inflation differential between the two countries. The equation for PPP is as follows:

$$\%\Delta S(\text{domestic}) = \text{inflation(foreign)} - \text{inflation(domestic)}$$

LO 35.j

The real interest rate reflects a given currency's real demand and supply for its funds. The nominal interest rate is the compounded sum of the real interest rate and the expected rate of inflation over an estimation horizon.

LO 35.k

Interest rate parity (IRP) suggests that the discounted spread between domestic and foreign interest rates equals the percentage spread between forward and spot exchange rates. The IRP equation is as follows:

$$\text{forward} = \text{spot} \times \left[\frac{(1 + r_{YYY})}{(1 + r_{XXX})}\right]^T$$

where:
r_{YYY} = quote currency rate
r_{XXX} = base currency rate

LO 35.l

Covered interest rate parity (CIRP) focuses on forward exchange rates as a function of spot rates and the risk-free domestic and foreign rates, all based on the concept of arbitrage. In contrast, uncovered interest rate parity (UCIRP) states that the same rate of return should be earned in any currency when all expected exchange rate movements are accounted for.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 35.1

1. **A** Transaction risk can be hedged with forward transactions, and translation risk can be hedged by financing assets with liabilities in the same currency. Economic is much more challenging to determine than transaction or translation risk, therefore, it is the most challenging risk to hedge. (LO 35.e)

2. **C** The spot bid-ask spread is 0.0008 (= 1.2952 – 1.2944). The three-month forward bid quote is 1.2944 + 0.005634 = 1.300034, and the three-month forward ask quote is 1.2952 + 0.005885 = 1.301085. The forward bid-ask spread is 1.301085 – 1.300034 = 0.001051. (LO 35.b)

Module Quiz 35.2

1. **C** F(6-month) = USDMXN 20 × $(1.07 / 1.03)^{0.5}$ = MXNUSD 20.38

   (LO 35.k)

2. **C** Nominal interest rate = [(1.03)(1.50) − 1] = 54.5%

   (LO 35.j)

3. **D** The percentage change in the JPY price of a CAD is:

   (83.9912 / 82.4012) − 1 = +0.0193 = +1.93%

   Because the JPY price of a CAD has risen, the CAD has appreciated relative to the JPY, and a CAD now buys 1.93% more JPY. The CAD has appreciated by 1.93% relative to the JPY. (LO 35.h)

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Financial Markets and Products, Chapter 10.

READING 36

PRICING FINANCIAL FORWARDS AND FUTURES

Study Session 9

EXAM FOCUS

Both forward and futures contracts are obligations regarding a future transaction. Because the difference in pricing between these contract types is small, forward contract pricing and futures contract pricing are often presented interchangeably. The basic model for forward prices essentially connects the forward price to the cost incurred from purchasing and storing the underlying asset until the contract maturity date. Cash flows over the life of the contract are easily incorporated into the pricing model. It should be clear that interest rate parity is a key part of this reading.

MODULE 36.1: FORWARD AND FUTURES PRICES

Financial Assets

LO 36.a: Define and describe financial assets.

A **financial asset** derives its value from a specific claim. In this reading, we discuss three types of financial assets: (1) assets with no income, (2) assets with income that is fixed, and (3) assets with income that is a percentage of value. All financial assets are considered investment assets.

An **investment asset** is an asset that is held for investing. This type of asset is held by many different investors for the sake of investment. Examples of investment assets are stocks and bonds. A **consumption asset** is a noninvestment asset that is held for consumption. Examples of consumption assets are commodities such as oil and natural gas.

Short Selling

LO 36.b: Define short-selling and calculate the net profit of a short sale of a

dividend-paying stock.

Short selling (or "shorting") involves selling securities that the seller does not own, and it is possible with some investment assets. For a short sale, the short seller simultaneously borrows and sells securities through a broker and must return the securities at the request of the lender or when the short sale is closed out.

In terms of motivations to sell securities short, the seller thinks the current price is too high and that it will fall in the future, so the short seller hopes to sell high and then buy low. Ignoring all fees, if a short sale is made at $30 per share and the price falls to $20 per share, the short seller can buy shares at $20 to replace the shares borrowed and keep $10 per share as profit. Conversely, short selling will result in losses if the price of the security rises. Note that the short seller must pay all dividends due to the lender of the security.

EXAMPLE: Net profit of a short sale of a dividend-paying stock

Assume that a trader sold short XYZ stock in March by borrowing 200 shares and selling them for $50/share. In April, XYZ stock paid a dividend of $2/share. **Calculate** the net profit from the short sale assuming the trader bought back the shares in June for $40/share to replace the borrowed shares and close out his short position.

Answer:

The cash flows from the short sale on XYZ stock are as follows:

March: borrow 200 shares and sell them for $50/share	+$10,000
April: short seller dividend payment to lender of $2/share	−$400
June: buy back shares for $40/share to close short position	−$8,000
Total net profit =	+$1,600

LO 36.c: Describe the differences between forward and futures contracts and explain the relationship between forward and spot prices.

LO 36.d: Calculate the forward price given the underlying asset's spot price and describe an arbitrage argument between spot and forward prices.

LO 36.e: Distinguish between the forward price and the value of a forward contract.

LO 36.f: Calculate the value of a forward contract on a financial asset that does or does not provide income or yield.

Forward Price With No Income or Yield

The pricing model used to compute forward prices assumes no transaction costs or short-sale restrictions.

For the development of a forward pricing model, we will use the following notation:

- T = time to maturity (in years) of the forward contract
- S = underlying asset price today (t = 0)
- F = forward price today
- r = annually compounded risk-free rate

The forward price may be expressed as a function of the spot price, the risk-free rate, and the time to maturity:

Equation 1

$$F = S \times (1 + r)^T$$

The right-hand side of Equation 1 is the cost of borrowing funds to buy the underlying asset and carrying it forward to time *T*. Equation 1 states that this cost must equal the forward price.

If $F > S \times (1 + r)^T$, arbitrageurs will profit by selling the forward and buying the asset with borrowed funds.

If $F < S \times (1 + r)^T$, arbitrageurs will profit by selling the asset, lending out the proceeds, and buying the forward. Hence, the equality in Equation 1 must hold.

EXAMPLE: Computing a forward price with no interim cash flows

Suppose we have an asset currently priced at $1,000. The current annually compounded risk-free rate is 4%. **Compute** the price of a six-month forward contract on the asset.

Answer:

$$F = \$1{,}000 \times 1.04^{0.5} = \$1{,}019.80$$

Forward Price With Income or Yield

If the underlying pays a known amount of cash over the life of the forward contract, a simple adjustment is made to Equation 1. Because the owner of the forward contract does not receive any of the cash flows from the underlying asset between contract origination and delivery, the present value of those cash flows must be deducted from the spot price when calculating the forward price. That is most easily seen when the underlying asset makes a periodic payment. Therefore, we let *I* represent the *present value* of the cash flows over *T* years. Equation 1 then becomes:

Equation 2

$$F = (S - I) \times (1 + r)^T$$

The same arbitrage arguments used for Equation 1 are used here. The only modification is that the arbitrageur must account for the known cash flows.

EXAMPLE: Forward price when underlying asset has a cash flow

Compute the price of a six-month forward on a coupon bond worth $1,000 that pays a 5% coupon semiannually. A coupon is to be paid in three months. Assume the annual risk-free rate is 4%.

Answer:

The income in this case is computed as:

$$I = 25 / 1.04^{0.25} = \$24.756$$

Using Equation 2:

$$F = (\$1,000 - \$24.756) \times (1 + 0.04)^{0.5} = \$994.56$$

The Effect of a Known Dividend

When the underlying asset for a forward contract pays a dividend, we assume that the dividend is paid annually. Letting q represent the annually compounded dividend yield paid by the underlying asset, Equation 1 becomes:

Equation 3

$$F = S \times [(1 + r) / (1 + q)]^T$$

Once again, the same arbitrage arguments are used to prove that Equation 3 must be true.

EXAMPLE: Forward price when the underlying asset pays a dividend

Compute the price of a six-month forward contract for which the underlying asset is a stock index with a value of 1,000 and a continuous dividend yield of 1%. Assume the risk-free rate is 4%.

Answer:

Using Equation 3:

$$F = 1,000 \times (1.04 / 1.01)^{0.5} = 1,014.74$$

Value of a Forward Contract

The initial value of a forward contract is zero. After its inception, the contract can have a positive value to one counterparty (and a negative value to the other). Because the forward price at every moment in time is computed to prevent arbitrage, the value at inception of the contract must be zero. The forward contract can take on a non-zero value only after the contract is entered into and the obligation to buy or sell has been made. If we denote the obligated delivery price after inception as K, then the value of the long contract on an asset with no cash flows is computed as $S - [K / (1 + r)^T]$; with cash flows (with present value I) it is $S - I - [K / (1 + r)^T]$; and with an annual dividend yield of q, it is $[S / (1 + q)^T] - [K / (1 + r)^T]$.

EXAMPLE: Value of a stock index forward contract

Using the stock index forward in the previous example, **compute** the value of a long position if the index increases to 1,050 immediately after the contract is purchased.

Answer:

In this case, K = 1,014.74 and S = 1,050, so the value is:

$$(1{,}050 / 1.01^{0.5}) - (1{,}014.74 / 1.04^{0.5}) = 1{,}044.79 - 995.03 = 49.76$$

Forward Prices vs. Futures Prices

LO 36.g: Explain the relationship between forward and futures prices.

The most significant difference between forward contracts and futures contracts is the daily marking to market requirement on futures contracts. When interest rates are known (or change predictably) over the life of a contract, *T*, forward and futures prices can be shown to be the same. In practice, the changes are usually unpredictable, which results in price differences. Assume asset prices are positively correlated to interest rates. A gain from an asset price increase will be recognized immediately (due to daily settlement) and can be reinvested at a high rate of interest. That makes a long futures contract a bit more desirable than a long forward contract, so the former will be priced slightly higher. The opposite would hold true if asset prices are negatively correlated to interest rates—the forward would be priced slightly higher in that case.

Overall, the price differences are usually very small and can often be ignored. Therefore, futures and forward prices can be thought to be approximately equal and Equations 1 to 3 can be used to price both forward and futures contracts. Finally, because futures contracts have daily settlement and forward contracts may not settle for some time in the future (e.g., one year or more), a futures contract may recognize an immediate profit but the forward contract would only be able to recognize the present value of that profit.

Another difference between forwards and futures is that forwards have one delivery date while futures might have a choice of delivery dates. For example, with a financial asset, if the financing cost exceeds the investment income, then the short position would want to deliver earlier to minimize the loss. The opposite would be true if the investment income exceeds the financing cost.

Forward Exchange Rates

As discussed in the previous reading, **interest rate parity (IRP)** states that the forward exchange rate, *F* (using the quote format of XXXYYY [e.g., EURUSD]), must be related to the spot exchange rate, *S*, and to the interest rate differential between the domestic (currency YYY) and the foreign (currency XXX) country, $r_{YYY} - r_{XXX}$.

Equation 4

The IRP condition can be expressed as:

$$F = S \times [(1 + r_{YYY}) / (1 + r_{XXX})]^T$$

This equation is a no-arbitrage relationship.

Note that this is equivalent to Equation 3 with r_{XXX} replacing q. Just as the annual dividend yield q was used to adjust the income, we use the annual yield on a foreign currency deposit here.

EXAMPLE: Calculate forward foreign exchange rate

Suppose we wish to **compute** the forward foreign exchange rate of a 10-month futures contract on the Mexican peso. Each contract controls 500,000 pesos and is quoted in terms of MXNUSD. Assume that the annually compounded risk-free rate in Mexico is 14%, the annually compounded annual risk-free rate in the United States is 2%, and the current exchange rate is MXNUSD 0.12.

Answer:

Applying Equation 4:

$$F = 0.12 \times (1.02 / 1.14)^{10/12} = \$0.10938/\text{peso}$$

Stock Index Futures

LO 36.h: Calculate the value of a stock index futures contract and explain the concept of index arbitrage.

Stock index futures are valued similarly to forward contracts that pay dividends. If the average dividend yield for the contract term, q, is annually compounded, the futures price of the stock index will be computed using Equation 3. With stock index futures, arbitrage opportunities will be present if: $F > S \times [(1 + r) / (1 + q)]^T$ or $F < S \times [(1 + r) / (1 + q)]^T$.

If the futures price is greater than the theoretical value, an arbitrage profit is generated by shorting the futures contracts and going long stocks underlying the index at the spot price. This type of index arbitrage is typically performed by pension funds that hold a portfolio of index stocks.

Conversely, if the futures price is lower than the theoretical value, an arbitrage profit is generated by shorting stocks underlying the index and going long the futures contracts. This type of index arbitrage is typically performed by corporations or banks that hold shorter-term investments.

Index arbitrage is implemented through program trading; a computer sends out all the necessary trades for the stocks in the index for execution contemporaneously with the trading of the futures contract. The idea behind index arbitrage is to maintain the pricing relationship between the index and the futures on the index.

MODULE QUIZ 36.1

Use the following information to answer Questions 1 and 2.

An investor has an asset that is currently worth $500, and the annually compounded risk-free rate at all maturities is 3%.

1. Which of the following amounts is the closest to the no-arbitrage price of a three-month forward contract?
   A. $496.
   B. $500.
   C. $502.
   D. $504.

2. If the asset pays an annual dividend of 2%, which of the following amounts is the closest to the no-arbitrage price of a three-month forward contract?
   A. $495.
   B. $499.
   C. $501.
   D. $505.

3. A bond pays a semiannual coupon of $40 and has a current value of $1,109. The next payment on the bond is in four months, and the annual interest rate is 6.50%. Using an annual compounding assumption, the price of a six-month forward contract on this bond is closest to:
   A. $1,103.
   B. $1,104.
   C. $1,145.
   D. $1,185.

4. A trader is considering whether to invest in a forward or a futures contract on fixed-rate bonds as the underlying asset. Assuming interest rates rise, determine whether there is a gain or loss on the underlying asset and whether a forward or a futures contract is more desirable.
   A. There will be a gain on the underlying asset and a forward contract is more desirable.
   B. There will be a loss on the underlying asset and a forward contract is more desirable.
   C. There will be a gain on the underlying asset and a futures contract is more desirable.
   D. There will be a loss on the underlying asset and a futures contract is more desirable.

5. Which of the following statements regarding forward contracts and futures contracts with underlying fixed income assets is correct?
   A. Forward and futures prices often significantly diverge from each other.
   B. The path of interest rates is usually predictable over the life of a forward or futures contract.
   C. The most significant difference between forwards and futures is the number of delivery dates.
   D. When there are losses on the underlying asset, a forward contract on the underlying asset will be priced higher than a futures contract.

KEY CONCEPTS

LO 36.a

There are three types of financial assets: (1) assets with no income (or yield), (2) assets with income that is fixed, and (3) assets with income that is a percentage of value.

LO 36.b

Short sales are orders to sell securities that the seller does not own. The motivation for doing so arises because the seller thinks the current price is too high and that it will fall in the future. The short seller must pay all dividends due to the lender of the security.

LO 36.c

The most significant difference between forward contracts and futures contracts is the daily marking to market requirement on futures contracts. Another difference between forwards and futures is that forwards have one delivery date while futures might have a choice of delivery dates.

The basic relationship between forward and spot prices is as follows:

$$F = S \times (1 + r)^T$$

LO 36.d

If $F > S \times (1 + r)^T$, arbitrageurs will profit by selling the forward and buying the asset with borrowed funds.

If $F < S \times (1 + r)^T$, arbitrageurs will profit by selling the asset, lending out the proceeds, and buying the forward.

LO 36.e

The initial value of a forward contract is zero. After its inception, the contract can have a positive value to one counterparty (and a negative value to the other).

LO 36.f

The value of the long contract on an asset with no cash flows is computed as: $S - [K / (1 + r)^T]$; with cash flows (with present value *I*) it is $S - I - [K / (1 + r)^T]$; and with an annual dividend yield of *q*, it is $[S / (1 + q)^T] - [K / (1 + r)^T]$.

LO 36.g

When interest rates are known (or change predictably) over the life of a contract, *T*, forward and futures prices can be shown to be the same. In practice, the changes are usually unpredictable, which results in price differences. Overall, the price differences are usually very small and can often be ignored. Therefore, futures and forward prices can be thought to be approximately equal and forward pricing equations can be used to value both forward and futures contracts.

LO 36.h

Stock index futures are valued similarly to forward contracts that pay dividends. With stock index futures, arbitrage opportunities will be present if $F > S \times [(1 + r) / (1 + q)]^T$ or $F < S \times [(1 + r) / (1 + q)]^T$.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 36.1

1. **D** Using Equation 1:

   $\$500 \times 1.03^{0.25} = \$503.71$

   where S = 500, T = 0.25, r = 0.03

   (LO 36.f)

2. **C** Using Equation 3:

   $\$500 \times (1.03 / 1.02)^{0.25} = \$501.22$

   (LO 36.f)

3. **B** Use the formula $F = (S - I) \times (1 + r)^T$, where *I* is the present value of $40 to be received in 4 months, or 0.333 years. At a discount rate of 6.50%:

   $I = \$40 / 1.065^{0.333} = \$39.17$

   $F = (\$1{,}109 - 39.17) \times 1.065^{0.5} = \$1{,}104.05$

   (LO 36.f)

4. **B** If interest rates rise, then the fixed-rate bond assets will fall in value. With futures, a loss from an asset price decrease will be recognized immediately and must be financed at a high rate of interest. That makes a long futures contract less desirable than a long forward contract. Said another way, with forwards, a loss from an asset price decrease will be delayed and makes a long forward contract more desirable than a long futures contract. (LO 36.g)

5. **D** When there are losses on the underlying asset, a forward contract allows for a delay in recognizing the losses compared with a futures contract (due to daily settlement). In that regard, a forward contract is more desirable and would be priced higher.

   Forward and futures prices are often very close to each other. The path of interest rates is often unpredictable over the life of a forward or futures contract. The most significant difference between forwards and futures is the daily settlement feature of futures. (LO 36.c)

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Financial Markets and Products, Chapter 11.

READING 37

COMMODITY FORWARDS AND FUTURES

Study Session 9

EXAM FOCUS

This reading focuses on the pricing relationships that exist when commodities have characteristics such as lease rates, storage costs, and/or convenience yields. Before you begin this reading, recall the no-arbitrage pricing relationships for futures contracts that were discussed in the previous reading. For the exam, you should understand the basic futures pricing equation and how it is adjusted for lease rates, storage costs, and/or convenience yields.

MODULE 37.1: FUNDAMENTALS OF COMMODITIES

LO 37.a: Explain the key differences between commodities and financial assets.

The following four key differences explain why commodities futures price movements are different from those of financial assets.

1. There are significant storage costs associated with commodities, including costs for special care and insurance. In contrast, financial assets essentially have no storage costs.
2. Commodities prices often depend on their location, given the potential high cost to transport them. In contrast, financial assets essentially have no transport cost.
3. Shorting costs for commodities are expressed as a lease rate. Lease rates may be greater than the shorting fees for financial assets.
4. Financial assets usually provide price and/or income returns based on risk. In contrast, commodities only provide price returns and even at that, the price tends to be mean-reverting, so timing and holding period become relevant investment concerns.

LO 37.c: Identify factors that impact prices on agricultural commodities, metals, energy, and weather derivatives.

Agricultural Commodities

Agricultural commodities tend to fluctuate seasonally because the production of commodities is seasonal while their demand is relatively even throughout the year. This timing mismatch between production and consumption means that commodities must be stored. Thus, storage (and interest) costs are highly relevant in futures prices.

Good or bad harvest expectations will result in expectations of low and high commodity prices, respectively. Politics (e.g., import or export restrictions) can also have an impact on futures prices. Finally, weather is a key determinant of commodity futures prices; good or bad weather (and the impact on production) is likely to result in lower and higher prices, respectively.

Metals

Commodity metals differ significantly from agricultural commodities in that the pricing for the former is not impacted by the season or the weather. In addition, the storage costs for metals are not usually as high as for agricultural products. Futures prices for commodity metals are often easily found, so it would make sense for an investor to obtain exposure to metals via long forward contracts (synthetic) rather than physically buying and holding metals (and not earning a lease payment).

The pricing of metals is also influenced by the cost of production (e.g., extraction processes, recycling processes, environmental regulations), government policies, and inventory levels. Because metals are not always extracted and used in the same country, pricing is also impacted by foreign exchange rates.

Energy

Energy products could be subdivided into three classes: crude oil, natural gas, and electricity.

The physical characteristics of *crude oil* make it relatively easy to transport; therefore, the price of oil is comparable worldwide. Lower transportation costs and more constant worldwide demand cause the long-run futures price to be relatively stable.

Natural gas has dual functions of heating and producing electricity. It is an example of a commodity with constant production but seasonal demand (e.g., higher demand in winter for heating compared to summer for producing electricity for air-conditioning). Natural gas is expensive to store; therefore, futures prices vary geographically due to high transportation costs.

Electricity is not generally a storable commodity; once produced, it must be used or it will likely go to waste. The demand for electricity is not constant and will vary with time of day, day of the week, and with season (e.g., higher price in the summer due to increased demand for air-conditioning). Given the nonstorability characteristic of electricity, its price is set by demand and supply at a given point in time. Because arbitrage opportunities do not exist with electricity (e.g., the inability to buy electricity during one season and sell it in another season), futures prices on electricity can be much more volatile than financial futures.

Weather Derivatives

Energy companies use weather derivatives in their hedging activities. Pricing is impacted by temperature and measured using equations for heating degree days (HDD) and cooling degree days (CDD). Both HDD and CDD are influenced by the average of the highest and lowest temperatures for all days in a specified month.

MODULE QUIZ 37.1

1. Storage costs are most likely to be a significant consideration for which of the following commodity categories?
   A. Agricultural.
   B. Electricity.
   C. Metals.
   D. Natural gas.
2. Which of the following statements regarding commodities prices/returns is correct?
   A. Commodities prices tend to be mean-reverting.
   B. Commodities provide both price and income returns.
   C. Storage costs for commodities and financial assets are similar.
   D. Transport costs for commodities and financial assets are similar.

MODULE 37.2: PRICING COMMODITY FORWARDS

Forward Pricing

LO 37.d: Explain the formula for pricing commodity forwards.

LO 37.i: Explain how to create a synthetic commodity position and use it to explain the relationship between the forward price and the expected future spot price.

Assume that we do not know the forward price of the commodity and wish to derive it. $F_{0,T}$ is the forward price at time *0* for delivery at time *T* while S_t represents the spot price at time *t*.

A synthetic commodity forward price can be derived by combining a long position on a commodity forward, $F_{0,T}$, and a long zero-coupon bond that pays $F_{0,T}$ at time *T*.

The total cost at time *0* is equivalent to the cost of the bond, $e^{-rT}F_{0,T}$, where *r* represents the continuously compounded risk-free rate of return. The forward contract does not have any initial cash flows at time *0*. The payoff at time *T* will be the payoff from the forward contract ($S_T - F_{0,T}$) plus the payoff from the bond ($F_{0,T}$):

$S_T - F_{0,T} + F_{0,T} = S_T$ (same as payoff at time T from a long position in the commodity)

$F_{0,T}$ is the known (and certain) market price at time *0*. Given that the forward price is certain, we can discount the forward price at the risk-free rate. However, the *expected spot price* at time T is uncertain, and we cannot use the risk-free rate to discount it; we instead use a risk-adjusted discount rate (α). The higher the uncertainty in the future spot price, the higher the value of α.

The present value (at t = 0) of the expected spot price at time T is $E(S_T) / (1 + \alpha)^T$, where α represents the discount rate for the S_T cash flow at time T. This is equal to the present value of the forward price discounted at the risk-free rate:

$$F_{0,T} / (1 + r)^T = E(S_T) / (1 + \alpha)^T$$

Multiplying each side of the equation by $(1 + r)^T$ allows us to express the commodity forward price as follows:

$$F_{0,T} = E(S_T) \times [(1 + r) / (1 + \alpha)]^T$$

Thus, the forward price today is a biased estimate of the expected commodity spot price at time T. The bias is a function of the risk premium on the commodity, $r - \alpha$.

The net present value (NPV) of an investment in commodity would then be:

$$NPV = E(S_T) / (1 + \alpha)^T - S_0$$

Substituting $F_{0,T} / (1 + r)^T$ for $E(S_T) / (1 + \alpha)^T$ and setting NPV = 0, we can see that:

$$F_{0,T} / (1 + r)^T = S_0 \text{ or}$$
$$F_{0,T} = S_0(1 + r)^T$$

Since we set NPV = 0 for the preceding pricing relationship, we call it the no-arbitrage price. The no-arbitrage forward price is, therefore, just the future value of the spot price —or the value of the commodity adjusted for the cost of carrying it (interest only).

Commodity Arbitrage

LO 37.b: Define and apply commodity concepts such as storage costs, carry markets, lease rate, and convenience yield.

LO 37.e: Describe an arbitrage transaction in commodity forwards and compute the potential arbitrage profit.

The forward price of a commodity reflects the cost of carrying the commodity until the futures expiration date. The cost of carry includes interest cost (as already discussed), as well as any storage costs. **Storage costs** are the cost of storing a commodity and include incidental costs such as insurance and spoilage. The markets for those commodities that are storable are called as **carry markets**.

In carry markets, the forward price relationship is modified to include storage costs (U) as follows:

$$F_{0,T} = (S_0 + U) \times (1 + r)^T$$

However, if storage costs are negative, then *U* is replaced with –*I* (income). You can think of storage costs as negative income.

$$F_{0,T} = (S_0 - I) \times (1 + r)^T$$

A **cash-and-carry arbitrage** consists of buying the commodity, storing/holding the commodity, and selling the commodity at the futures price when the contract expires. The steps in a cash-and-carry arbitrage are as follows:

At the initiation of the contract:

1. Borrow money for the term of the contract at market interest rates.
2. Buy the underlying commodity at the spot price.
3. Sell a futures contract at the current futures price.

At contract expiration:

1. Deliver the commodity and receive the futures contract price.
2. Repay the loan plus interest.

If the futures contract is overpriced, this five-step transaction will generate a riskless profit. The futures contract is overpriced if the actual market price is greater than the no-arbitrage price.

EXAMPLE: Futures cash-and-carry arbitrage

Assume the spot price of gold is \$1,900/oz., that the one-year futures price is \$2,075/oz., and that an investor can borrow or lend funds at 5% annually. Storage costs are 2% annually. **Calculate** the arbitrage profit.

Answer:

The futures price, according to the no-arbitrage principle, should be:

$$U = \$1{,}900 \times 0.02 = \$38$$

$$F_{0,T} = (\$1{,}900 + \$38) \times (1.05)^1 = \$2{,}035$$

Instead, it is trading at \$2,075. That means that the futures contract is overpriced, so we should conduct cash-and-carry arbitrage by going short in the futures contract, buying gold in the spot market, and borrowing money to pay for the purchase. If we borrow \$1,938 to fund the purchase of gold (and the storage costs), we must repay \$2,035 after one year (at maturity of the futures contract).

Today		One Year From Today	
Transaction	Cash Flow	Transaction	Cash Flow
Short futures	$0	Settle short position by delivering gold	+$2,075
Buy gold in spot market	−$1,938		
Borrow at 5%	+$1,938	Repay loan	−$2,035
Total cash flow	$0	Total cash flow = arbitrage profit	+$40

The riskless profit is equal to the difference between the futures contract proceeds and the loan payoff, or $2,075 − $2,035 = $40. Notice that this profit is equal to the difference between the actual futures price of $2,075 and the no-arbitrage price of $2,035.

If the futures price is too low (which presents a profitable arbitrage opportunity), the opposite of each step should be executed to earn a riskless profit.

This is **reverse cash-and-carry arbitrage**. The steps in reverse cash-and-carry arbitrage are as follows.

At the initiation of the contract:

1. Sell commodity short.
2. Lend short sale proceeds at market interest rates.
3. Buy futures contract at market price.

At contract expiration:

1. Collect loan proceeds.
2. Take delivery of the commodity for the futures price and cover the short sale commitment.

EXAMPLE: Futures reverse cash-and-carry arbitrage

Assume again that the spot price of gold is $1,900/oz., and that the annual risk-free rate and storage costs are 5% and 2%, respectively, as before. Also, assume that the one-year futures price is now $2,000/oz. **Calculate** any profits from arbitrage.

Answer:

The futures price, according to the no-arbitrage principle, is again $2,035. Instead, it is trading at $2,000; the futures contract is underpriced. We should conduct reverse cash-and-carry arbitrage by going long the futures contract, shorting gold, and investing the short sale proceeds as follows:

Today		One Year From Today	
Transaction	Cash Flow	Transaction	Cash Flow
Long futures	$0	Settle long position by buying gold	–$2,000
Short gold	+$1,938	Deliver gold to close short position	$0
Invest short-sale proceeds at 5%	–$1,938	Receive investments proceeds	+$2,035
Total cash flow	$0	Total cash flow = arbitrage profit	+$35

The riskless profit is equal to the loan proceeds less the futures contract payment, or $2,035 – $2,000 = $35.

PROFESSOR'S NOTE

It may help to remember "buy low, sell high." If the futures price is "too high," sell the futures and buy the spot. If the futures price is "too low," buy the futures and sell the spot.

Lease Rates

LO 37.f: Define the lease rate and explain how it determines the no-arbitrage values for commodity forwards and futures.

A **lease rate** is the amount of interest a lender of a commodity requires. The lease rate is defined as the amount of return the investor requires to buy and then lend a commodity. From the commodity borrower's perspective, the lease rate represents the cost of borrowing the commodity. The lease rate and risk-free rate are important inputs to determine the commodity forward price.

The commodity forward price for time *T* with an active lease market is expressed as:

$$F_{0,T} = S_0 \times [(1 + r) / (1 + \delta)]^T$$

where:

S_0 = current spot price

r = risk-free rate

δ = lease rate

The lease rate, δ, is income earned only if the commodity is loaned out.

EXAMPLE: Pricing a commodity forward with a lease payment

Calculate the 12-month forward price for a bushel of corn that has a spot price of $5 and an annual lease rate of 7%. The appropriate annual risk-free rate for the commodity is equal to 9%.

Answer:

We can determine the 12-month forward price as follows:

$$F_{0,T} = S_0 \times [(1 + r) / (1 + \delta)]^T = \$5 \times [(1 + 0.09) / (1 + 0.07)]^1 = \$5.093$$

To further illustrate that this relationship must hold, consider the following no-arbitrage example.

EXAMPLE: No arbitrage for a commodity forward

Continuing with our previous example, suppose the one-year futures price of corn is $5.08/bushel. **Calculate** the arbitrage profit.

Answer:

Because the futures price of $5.08 is less than $5.093 calculated in the previous example, an arbitrage profit can be earned by reverse cash-and-carry arbitrage: Long futures contract and short in the spot market. Figure 37.1 shows the calculations.

Figure 37.1: No-Arbitrage Opportunity on Bushel of Corn

Transaction	Time = 0	Time = 1
Borrow corn at lease rate of 7%	_	$(0.35)
Sell borrowed corn	$5.00	_
Invest proceeds at 9%	$(5.00)	$5.45
Long futures contract	$0	$(5.08)
Return borrowed corn	$0	_
Total	$0	$0.02

Arbitrage profit at T = 1 is the same (except for the rounding difference) as the difference between the no-arbitrage futures price and the futures price in the market.

Storage Costs and Convenience Yields

LO 37.g: Describe the cost of carry model and determine the impact of storage costs and convenience yields on commodity forward prices and no-arbitrage bounds.

LO 37.h: Compute the forward price of a commodity with storage costs.

As discussed earlier, the existence of storage costs increases the forward price at least by the future value of the storage costs to compensate the seller for costs incurred while storing the commodity. If the owners of the commodity instead need the commodity for their business, holding physical inventory of the commodity creates value. For example, assume a manufacturer requires a specific commodity as a raw material. To reduce the risk of running out of inventory and slowing down production, excess inventory is held by the manufacturer. This reduces the risk of idle machines and workers. In the event that the excess inventory is not needed, it can always be sold. The nonmonetary benefit of holding excess inventory is called the **convenience yield**. The convenience yield is only relevant when a commodity is stored (i.e., in a carry market).

A convenience yield *cannot* be earned by the average investor who does not have a business reason for holding the commodity.

The forward price including a convenience yield is calculated as follows:

$$F_{0,T} = (S_0 + U) \times [(1 + r) / (1 + Y)]^T$$

where:

Y = annualized convenience yield

EXAMPLE: Impact of convenience yield on the no-arbitrage cash-and-carry commodity forward pricing range

Suppose the owner of a commodity decides to lend out the commodity. The commodity has an annualized convenience yield of *Y*, proportional to the value of the commodity. **Determine** the range of forward prices that must exist to prevent arbitrage.

Answer:

The minimum forward price that the owner of a commodity (i.e., the short) would accept is the current spot price plus interest and storage costs offset by any convenience yield (otherwise there would be the possibility of a reverse cash-and-carry arbitrage):

$$F_{0,T} \geq (S_0 + U) \times [(1 + r) / (1 + Y)]^T$$

The long party in a forward contract agrees to take delivery on some future date at a fixed forward price agreed on today. A long party can alternatively buy the commodity in the spot market and pay for storage until the delivery date. Therefore, the maximum forward price that the long party would be willing to pay is the spot price plus interest and storage cost:

$$F_{0,T} \leq (S_0 + U) \times (1 + r)^T$$

The two inequalities provide the range of arbitrage-free forward prices as follows:

$$(S_0 + U) \times [(1 + r) / (1 + Y)]^T \leq F_{0,T} \leq (S_0 + U)(1 + r)^T$$

The upper bound of the forward price depends on storage costs, but not on the convenience yield. The lower bound adjusts for the convenience yield and, therefore, explains why forward prices may appear lower than pure spot plus storage costs at times.

Futures Prices and Expected Spot Prices

LO 37.j: Explain the impact of systematic and nonsystematic risk on current futures prices and expected future spot prices.

LO 37.k: Define and interpret normal backwardation and contango.

The **expected future spot** price is the market's assessment of the future spot price. It is certain that the futures prices must converge to the spot price at maturity or else arbitrage opportunities would exist.

To examine the relationship mathematically, consider the following equation: $P = F / (1 + r)^T$, where: F = futures price, r = risk-free rate, T = time to maturity, and P = present value of futures price. If *P* is invested at the risk-free rate, then the payoff is *F* at time *T*. Therefore, the payoff can be replicated by investing *P* at the risk-free rate (initial cash flow = −P) and then taking a long futures position (cash flow at maturity = $+S_T$).

Consider the expected future spot price, $E(S_T) = P\ (1 + X)^T$, where X = expected return on futures position. By substituting for *P*, the result becomes $E(S_T) = F\ (1 + X)^T / (1 + r)^T$. The focus is now on the futures return (*X*), the risk-free return (*r*), and the level of systematic risk.

Keep in mind that systematic risk is a function of the correlation between the return on the market and the underlying asset. If the correlation is positive, then *X* exceeds *r*, which also means that $E(S_T)$ is above *F* (known as **normal backwardation**). This is an example of *positive systematic risk*. If the correlation is negative, then *r* exceeds *X*, which also means that $E(S_T)$ is below *F* (known as **contango**). This is an example of *negative systematic risk*. For zero correlation, there would be no systematic risk and $E(S_T) = F$ (i.e., *X* equals *r*). Note that normal backwardation (contango) could also refer to a situation where the futures price is below (above) the *current* spot price.

MODULE QUIZ 37.2

1. Which of the following statements regarding a lease rate is correct?
   A. It has a lower bound of zero.
   B. It is equal to the risk-free rate.
   C. It is very similar to the dividend yield in an equity forward contract.
   D. It will rise with the supply of the commodity available for borrowing.

2. Suppose there is an active lending market for a bushel of soybeans (which has a current spot price of $12/bushel). If the annual lease rate is equal to 7% and the effective annual risk-free rate is equal to 7%, how could an investor create an arbitrage opportunity (assuming the forward contract is overpriced)? An individual could:
   A. borrow money at 7% and purchase a bushel of soybeans and sell it forward.
   B. borrow a bushel of soybeans and sell at the spot price and long the forward.
   C. sell a bushel of soybeans at the forward price and lend the money at the risk-free rate.

D. go long in soybean forward contracts, short in soybean spot prices, and lend the excess proceeds at the risk-free rate.

3. Which of the following amounts is closest to the three-month forward price for a bushel of corn if the current spot price for corn is $3/bushel, the effective monthly interest rate is 1.5%, and the monthly storage costs are $0.03/bushel? Assume that the present value of storage costs over the next three months is $0.09/bushel.
   A. $3.18.
   B. $3.23.
   C. $3.29.
   D. $3.31.

KEY CONCEPTS

LO 37.a

Key differences between commodity futures and financial assets involve storage costs, transport costs, shorting costs, and sources of return.

LO 37.b

When holding a commodity requires storage costs, the forward price must be greater than the spot price to compensate for the storage costs.

The market in which a commodity is stored is called a carry market.

A lease rate is the amount of rent a lender of a commodity requires.

Convenience yield is the nonmonetary benefit earned from holding an excess inventory of a commodity.

LO 37.c

Agricultural commodities tend to fluctuate seasonally because the production of commodities is seasonal while their demand is relatively even throughout the year.

Commodity metals differ significantly from agricultural commodities in that the pricing for the former is not impacted by the season or the weather. In addition, the storage costs for metals are not usually as high as for agricultural products.

Energy products could be subdivided into three classes: crude oil, natural gas, and electricity.

- The physical characteristics of crude oil make it relatively easy to transport; therefore, the price of oil is comparable worldwide.
- Natural gas has constant production but seasonal demand. It is expensive to store; therefore, futures prices vary geographically due to high transportation costs.
- Electricity is not generally a storable commodity. Given the nonstorability characteristic of electricity, its price is set by demand and supply at a given point in time.

Energy companies use weather derivatives in their hedging activities. Pricing is impacted by the temperature.

LO 37.d

The commodity forward price today is defined as a biased estimate of the expected spot commodity price at time *T* as follows:

$$F_{0,T} = E(S_T) \times [(1 + r) / (1 + \alpha)]^T$$

LO 37.e

The steps in a cash-and-carry arbitrage are as follows:

At the initiation of the contract:

Step 1: Borrow money for the term of the contract at market interest rates.

Step 2: Buy the underlying commodity at the spot price.

Step 3: Sell a futures contract at the current futures price.

At contract expiration:

Step 1: Deliver the commodity and receive the futures contract price.

Step 2: Repay the loan plus interest.

LO 37.f

The lease rate is defined as the amount of return the investor requires to buy and then lend a commodity. If an active lease market exists for a commodity, a commodity lender can earn the lease rate by buying a commodity and immediately selling it forward.

The commodity forward price for time *T* with an active lease market is expressed as:

$$F_{0,T} = S_0 \times [(1 + r) / (1 + \delta)]^T$$

LO 37.g and LO 37.h

Convenience yield is the nonmonetary benefit enjoyed by producers who hold excess inventory of raw material inputs. This excess inventory precludes disruptions in the production process caused by temporary shortages of the input in the market. Accounting for the existence of a convenience yield, the forward price is calculated as:

$$F_{0,T} = (S_0 + U) \times [(1 + r) / (1 + Y)]^T$$

Accordingly, the arbitrage-free range of the forward price is:

$$(S_0 + U) \times [(1 + r) / (1 + Y)]^T \leq F_{0,T} \leq (S_0 + U) \times (1 + r)^T$$

LO 37.i

A synthetic commodity forward price can be derived by combining a long position on a commodity forward, $F_{0,T}$, and a long zero-coupon bond that pays $F_{0,T}$ at time *T*.

LO 37.j

$E(S_T) = F\,(1 + X)^T / (1 + r)^T$. Systematic risk is a function of the correlation between the return on the market and the underlying asset. If that correlation is positive, then *X* exceeds r, which also means that $E(S_T)$ is above F. The opposite where $E(S_T)$ is below F, would occur when there is negative correlation. For zero correlation, there would be no systematic risk and $E(S_T) = F$.

LO 37.k

Normal backwardation (contango) occurs when the futures price is below (above) the *expected future* spot price. However, the terms can also be used when dealing with the *current* spot price.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 37.1

1. **D** Storage costs are most significant for natural gas; storage is possible indefinitely but very costly. Storing agricultural products is costly but mitigated somewhat by the fact that some agricultural products can only be stored for a short amount of time. Metals are usually less costly to store than agricultural commodities. Electricity is generally non-storable. (LO 37.c)

2. **A** Commodities prices tend to be mean-reverting, which means they eventually revert back to an average value. Commodities do not provide periodic income but only provide income based on the price differential between purchase and sale price. Commodities may have significant storage and transportation costs, while financial assets have virtually neither of those costs. (LO 37.a)

Module Quiz 37.2

1. **C** A lease rate is the amount of interest a lender of a commodity requires. From the borrower's perspective, the lease rate represents the cost of borrowing the commodity. The lease rate in the pricing of a commodity future is very similar to the dividend payment in a financial forward.

   Lease rates can be negative. Lease rates involve some element of default risk; therefore, they are not risk free. As the supply of a commodity available for borrowing increases, the lease rate will generally fall. (LO 37.f)

2. **A** An individual could borrow money at the risk-free rate of 7% to purchase a bushel of soybeans and sell it forward. The individual immediately lends the bushel of soybeans out at a lease rate of 7%. At the end of the lease period, T_1, the individual would pay back the loan with interest at \$12.84, sell the soybeans at forward price (> \$12.00), and receive the lease payment of \$0.84. In order for a no-arbitrage position to exist, the forward price, $F_{0,1}$, must be equal to \$12.00.

   No-Arbitrage Opportunity on Bushel of Soybeans

Transaction	Time = T_0	Time = T_1
Borrow @ 7%	\$12.00	(\$12.84)
Buy a bushel of soybeans	(\$12.00)	–
Lend bushel of soybeans	\$0	\$0.84
Short forward	\$0	$F_{0,1}$
Total	\$0	$F_{0,1}$ – \$12.00

(LO 37.e)

3. **B** The storage cost (*U*) is \$0.03 per month, so that is \$0.09 in total for the three-month period.

$$F_{0,T} = (S_0 + U) \times (1 + r)^T = (\$3.00 + \$0.09) \times (1.015)^3 = \$3.23$$

(LO 37.g)

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Financial Markets and Products, Chapter 12.

READING 38

OPTIONS MARKETS

Study Session 10

EXAM FOCUS

Stock options give the owner the right, but not the obligation, to buy or sell a stock at a specific price on or before a specific date. Call options give the owner the right to buy the stock, and put options give the owner the right to sell the stock. An option is exercised when the owner executes the right to buy or sell the stock. This reading covers the basic mechanics of option trading. For the exam, understand the different kinds of options and the system by which exchange-traded options are bought and sold.

MODULE 38.1: OPTION TYPES, POSITIONS, AND UNDERLYING ASSETS

LO 38.a: Describe the various types and uses of options; define moneyness.

LO 38.b: Explain the payoff function and calculate the profit and loss from an options position.

Option contracts have asymmetric payoffs. The buyer of an option has the right to exercise the option but is not obligated to exercise. Therefore, the maximum loss for the buyer of an option contract is the loss of the price (premium) paid to acquire the position, while the potential gains in some cases are theoretically infinite. Because option contracts are a zero-sum game, the seller of the option contract could incur substantial losses, but the maximum potential gain is the amount of the premium received for writing the option. **American options** (which comprise most exchange-traded options) may be exercised at any time up to and including the contract's expiration date (or maturity date), while **European options** (which comprise many over-the-counter options) can be exercised only on the contract's expiration date.

From an analytical perspective, American options are more challenging and can only be valued using binomial tress, for example. European options are less challenging and can be valued using the Black-Scholes-Merton model. Note that American options must be worth the same or more than an otherwise identical European option.

To understand the potential returns, we need to introduce the standard symbols used to represent the relevant factors:

X = strike price or exercise price specified in the option contract (a fixed value)
S_t = price of the underlying asset at time *t*
C_t = market value of a call option at time *t*
P_t = market value of a put option at time *t*
t = time subscript, which can take any value between 0 and T, where *T* is the maturity or expiration date of the option

Call Options

A **call option** gives the *owner* the right, but not the obligation, to buy the stock from the seller of the option. The owner is also called the *buyer* or the holder of the *long position*. The buyer benefits, at the expense of the option *seller*, if the underlying stock price is greater than the exercise price. The option *seller* is also called the *writer* or holder of the *short position*.

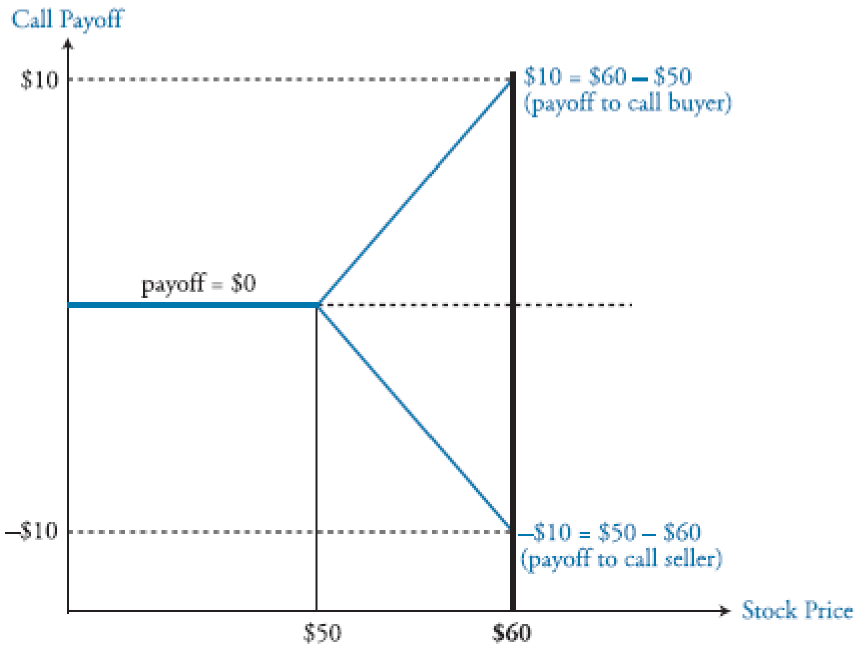
At maturity time *T*, if the price of the underlying stock is less than (out-of-the-money) or equal (at-the-money) to the exercise (or strike) price of a call option (i.e., $S_T \leq X$), the payoff is zero, so the option owner would not exercise the option. On the other hand, if the stock price is higher than (in-the-money) the exercise price (i.e., $S_T > X$) at maturity, then the payoff of the call option is equal to the difference between the market price and the strike price ($S_T - X$). The "payoff" (at the option's maturity) to the call option seller is the mirror image (opposite sign) of the payoff to the buyer. The *intrinsic value* is the maximum of zero or the difference between the underlying asset and the strike price. Therefore, the intrinsic value of a call option = max(0, S − X).

Note that whether the option is in-the-money, at-the-money, or out-of-the-money is known as the option's **moneyness**. Because of the linear relationships between the value of the option and the price of the underlying asset, simple graphs can clearly illustrate the possible value of option contracts at the expiration date. Figure 38.1 illustrates the payoff of a call with an exercise price equal to 50.

PROFESSOR'S NOTE

An option payoff graph ignores the initial cost of the option.

Figure 38.1: Payoff of Call With Exercise Price Equal to $50



> **EXAMPLE: Payoff of a call option**
>
> An investor writes an at-the-money call option on a stock with an exercise price of 50 (X = 50). If the stock price rises to $60, what will be the *payoff* to the owner and seller of the call option?
>
> **Answer:**
>
> The call option may be exercised with the holder of the long position buying the stock from the writer at 50 for a $10 gain. The payoff to the option buyer is $10, and the payoff to the option writer is *negative* $10. This is illustrated in Figure 38.1 and, as mentioned, does not include the premium paid for the option.
>
> This example shows just how easy it is to determine option payoffs. At expiration time *T* (the option's maturity), the payoff to the option owner, represented by C_T, is:
>
> $$C_T = S_T - X \quad \text{if} \quad S_T > X$$
> $$C_T = 0 \quad \text{if} \quad S_T \leq X$$

Another popular way of writing this is with the "max(0, variable)" notation. If the variable in this expression is greater than zero, then max(0, variable) = variable; if the variable's value is less than zero, then max(0, variable) = 0. Thus, letting the variable be the quantity $S_0 - X$, we can write:

$$C_T = \max(0, S_T - X)$$

The payoff to the option seller is the negative value of these numbers. In what follows, we will always talk about payoff in terms of the option owner, unless otherwise stated. We should note that $\max(0, S_t - X)$, where $0 < t < T$, is also the payoff if the owner decides to exercise the call option early. In this reading, we will only consider time *T* in our analysis.

Although our focus here is not to calculate C_t, we should clearly define it as the initial cost of the call when the investor purchases at time *0*, which is *T* units of time before *T*. C_0 is also called the premium. Thus, we can write that the profit to the owner at t = T is:

$$\text{profit} = C_T - C_0$$

This says that at time *T*, the owner's profit is the option payoff minus the premium paid at time *0*. Incorporating C_0 into Figure 38.1 gives us the profit diagram for a call at expiration, and this is Figure 38.2.

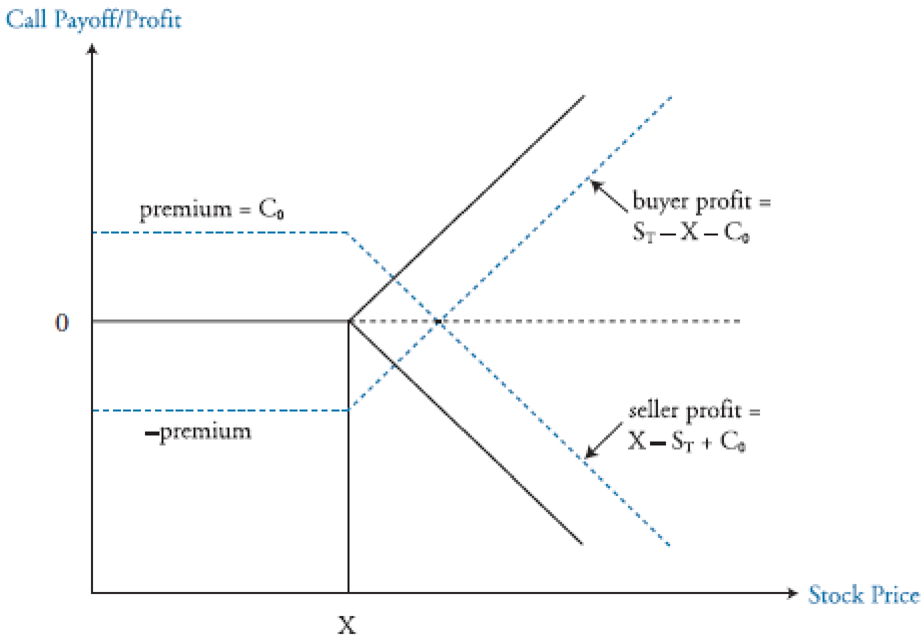
Figure 38.2 illustrates an important point, which is that the profit to the owner is negative when the stock price is less than the exercise price plus the premium. At expiration, we can say that:

if $S_T < X + C_0$ then: call buyer profit $< 0 <$ call seller profit

if $S_T = X + C_0$ then: call buyer profit $= 0 =$ call seller profit

if $S_T > X + C_0$ then: call buyer profit $> 0 >$ call seller profit

Figure 38.2: Profit Diagram for a Call at Expiration



Put Options

If you understand the properties of a call, the properties of a put should come to you fairly easily. A **put option** gives the owner the right to sell a stock to the seller of the put at a specific price. At expiration, the buyer benefits if the price of the underlying is less than the exercise price *X*:

$$P_T = X - S_T \quad \text{if} \quad S_T < X$$
$$P_T = 0 \quad \text{if} \quad X \leq S_T$$

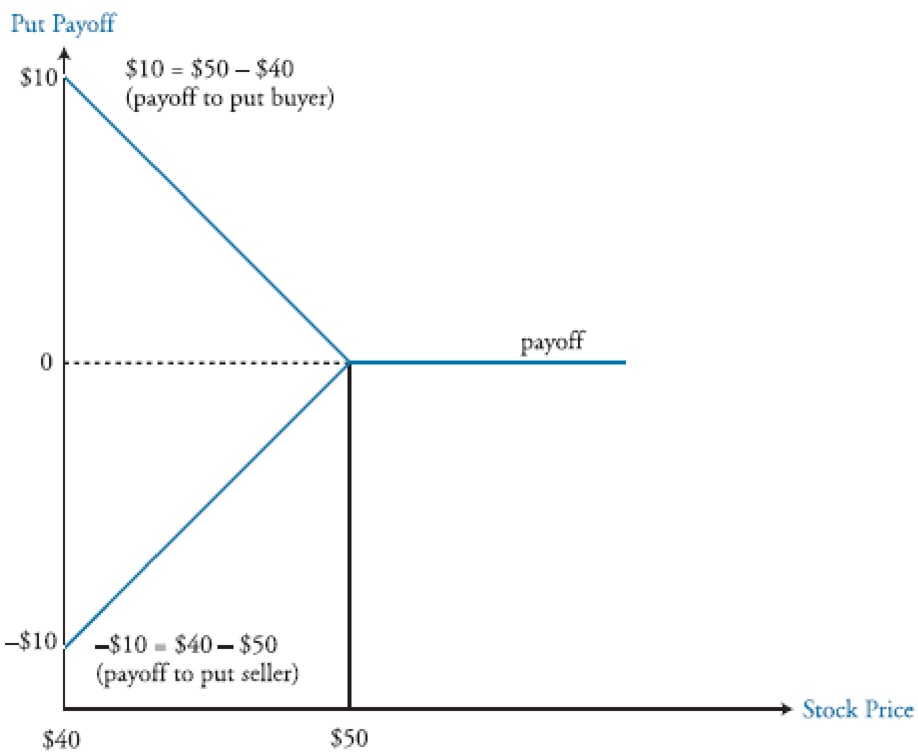
or:

$$P_T = \max(0, X - S_T)$$

The *intrinsic value* is the maximum of zero or the difference between the underlying asset and the strike price. Therefore, the intrinsic value of a put option = max(0, X − S).

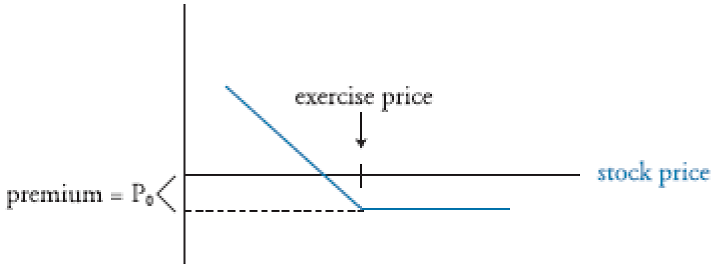
For example, an investor writes a put option on a stock with a strike price of X = 50. If the stock stays at $50 or above, the payoff of the put option is zero (because the holder may receive the same or better price by selling the underlying asset on the market rather than exercising the option). But if the stock price falls below $50, say to $40, the put option may be exercised with the option holder buying the stock from the market at $40 and selling it to the put writer at $50 for a $10 gain. The writer of the put option must pay the put price of $50 when it can be sold in the market at only $40, resulting in a $10 loss. The gain to the option holder is the same magnitude as the loss to the option writer. Figure 38.3 illustrates this example, excluding the initial cost of the put and transaction costs. Figure 38.4 includes the cost of the put (but not transaction costs) and illustrates the profit to the put owner.

Figure 38.3: Put Payoff to Buyer and Seller



Given the "mirror image quality" that results from the "zero-sum game" nature of options, we often just draw the profit to the buyer as shown in Figure 38.4. Then, we can simply remember that each positive (negative) value is a negative (positive) value for the seller.

Figure 38.4: Put Profit to Buyer



Underlying Assets

Exchange-traded options trade on assets, including individual stocks, stock indices, and exchange-traded funds (ETFs):

- **Stock options.** Stock options are typically exchange-traded, American-style options. Each option contract is normally for 100 shares of stock. For example, if the last trade on a call option occurred at $3.60, the option contract would cost $360. After issuance, stock option contracts are adjusted for stock splits but not cash dividends. The primary U.S. exchanges for stock options are the Chicago Board Options Exchange (CBOE), Boston Options Exchange, NYSE Euronext, and the International Securities Exchange.
- **Index options.** Options on stock indices are typically European-style options and are cash settled. Index options can be found on both the over-the-counter (OTC) markets and the exchange-traded markets. The payoff on an index call is the amount (if any) by which the index level at expiration exceeds the index level specified in the option (the strike price), multiplied by the contract multiplier (typically 100).
- **ETF options.** While similar to index options, ETF options are typically American-style options and utilize delivery of shares rather than cash at settlement.

EXAMPLE: Index options

Assume you own a call option on an index with an exercise price equal to 950. The multiplier for this contract is 100. **Compute** the payoff on this option assuming that the index is 956 at expiration.

Answer:

The payoff on an index call (long) is the amount (if any) by which the index level at expiration exceeds the index level specified in the option (the exercise price), multiplied by the contract multiplier. An equal amount will be deducted from the account of the index call option writer. In this example, the expiration date payoff is (956 – 950) × $100 = $600.

MODULE QUIZ 38.1

1. If an option is quoted at $2.75, the cost of one contract to the potential buyer is closest to:
   A. $0.275.
   B. $2.75.
   C. $275.00.

D. $2,750.00.

2. Which of the following statements regarding options is correct?
   A. Stock options are typically American-style options.
   B. All options expire on the third Wednesday of the expiration month.
   C. American-style options are less valuable than European-style options.
   D. Index options are typically American-style options and are cash settled.

MODULE 38.2: OPTION SPECIFICATION AND TRADING

Option Expiration

On the CBOE, an option will be included in one of three maturity cycles:

- *January cycle*: January, April, July, October
- *February cycle*: February, May, August, November
- *March cycle*: March, June, September, December

The actual day of expiration is the third Friday of the expiration month. Before the third Friday of the current month, options trade with maturities in the current month, the following month, and the next two months in the cycle. Following the third Friday of the current month, options trade with maturities in the next month, the month after that, and the next two months in the cycle.

For example, assume Intel is on a March cycle. That means that at the beginning of September, options trade with maturities in September, October, December, and March. After the third Friday in September, options trade with maturities in October, November, December, and March.

Short-term options called **weeklys** are available. Weeklys mature on Fridays (but not on the third Friday of the month). **Long-term equity anticipation securities (LEAPS)** are simply long-dated options with expirations greater than one year and up to three years. LEAPS expire on the third Friday of January.

Strike Prices

Strike prices are dictated by the value of the stock. Low-value stocks have smaller strike increments than higher-value stocks. Typically, stocks that are priced between $5 and $25 have increments of $2.50, stocks that are priced between $25 and $200 have increments of $5.00, and stocks that are priced above $200 have increments of $10.00. The strike price is usually denoted as *X* and the underlying stock as *S*.

At first, the three strike prices nearest to the current stock price are provided and additional strike prices are provided as the stock price changes. In the end, there could end up being a large number of available options contracts (e.g. multiple strike prices for each expiration date). All options of the same type (e.g., puts, calls) are called a *class*, and all options in a class with a given expiration and strike price are called an *option series* (e.g., put options on Intel maturing in September 2019).

Nonstandard Products

Nonstandard option products include flexible exchange (FLEX) options, Asian options, and cliquet options:

- **FLEX options.** FLEX options are exchange-traded options on equity indices and equities that allow some alteration of the options contract specifications. The nonstandard terms include alteration of the strike price, different expiration dates, or European-style (rather than the standard American-style). FLEX options were developed in order for the exchanges to better compete with the nonstandard options that trade over the counter. The minimum size for FLEX trades is typically 100 contracts.
- **Asian options.** Asian options generate payoffs calculated on the average price of the underlying for the life of the option.
- **Cliquet options.** Cliquet options generate payoffs calculated as the sum of the monthly capped returns earned by the asset.

The Effect of Dividends and Stock Splits

LO 38.c: Explain how dividends and stock splits can impact the terms of a stock option.

In general, options are not adjusted for cash dividends. This will have option pricing consequences that will need to be incorporated into a valuation model.

Options are adjusted for *stock splits*. For example, if a stock has a 2-for-1 split, then the strike price will be reduced by half and the number of shares underlying the option will double. In general, if a stock experiences a b-for-a split, the strike price becomes (a/b) of its previous value and the number of options owned by the trader is increased by multiples of (b/a).

Stock dividends are dealt with in the same manner. For example, if a stock pays a 25% stock dividend, this is treated in the same manner as a 5-for-4 stock split in that the strike price becomes 4/5 of its previous level.

Bottom line, all investors' positions should be relatively unchanged due to stock splits or stock dividends.

LO 38.d: Describe the application of commissions, margin requirements, and exercise procedures to exchange-traded options, and explain the trading characteristics of these options.

Trading

As mentioned previously, options are quoted relative to one underlying stock. To compute the actual option cost, the quote needs to be multiplied by 100. This is because an options contract represents an option on 100 shares of the underlying stock. The quotes will also include the strike, expiration month, volume, and the option class.

Market makers will quote bid and offer (or ask) prices whenever necessary. They profit on the bid-offer spread and add liquidity to the market. Floor brokers represent a particular firm and execute trades for the general public. The order book official enters limit orders relayed from the floor broker. An offsetting trade takes place when a long (short) option position is offset with a sale (purchase) of the same option, which is often done when a trader is trying to exit a position. If a trade is not an offsetting trade, then open interest increases by one contract.

The number of options a trader can have on one stock is limited by the exchange. This is called a **position limit**. Additionally, short calls and long puts are considered to be part of the same position. The exercise limit equals the position limit and specifies the maximum number of option contracts that can be exercised by an individual over any five consecutive business days. Traders are subject to position limits and exercise limits to discourage them from potentially manipulating the market.

Commissions

Option investors must consider the commission costs associated with their trading activity. Commission costs often vary based on trade size and broker type (discount vs. full service). Brokers typically structure commission rates as a fixed amount plus a percentage of the trade amount. The following example provides an illustration on how commission costs affect an option trade's profitability.

EXAMPLE: Commission costs

An investor buys a call contract with a strike price of $260. The current price of the underlying stock is $245. Assume the option price is $10 and the contract is settled with shares rather than cash. Using the commission schedule for a discount broker, **calculate** (1) the commission costs incurred by the investor based on the initial trade and (2) the investor's net profit if the stock price increases to $280 prior to expiration. Assume the cost to exercise the option is 1% of the trade amount and the cost to sell stock is also 1% of the trade amount.

Figure 38.5: Commission Schedule

Trade Amount	Commission Rate
≤ $3,000	$30 + 0.8% of trade amount
$3,001 to $14,999	$30 + 0.6% of trade amount
≥ $15,000	$30 + 0.4% of trade amount
Other details:	
Minimum charge per contract: $4	
Maximum charge per contract: $35	

Answer:

1. Contract cost = $10 × 100 = $1,000

   Initial commission costs = $30 + ($1,000 × 0.8%) = $38. Because this exceeds the maximum contract charge, $35 is charged (i.e., the maximum contract charge).
2. Gross profit: $280 − $260 = $20 per share. $20 × 100 shares = $2,000

Additional commission costs = 1% × 2 × $280 × 100 = $560

Total commission costs = $35 + $560 = $595

Net profit = $2,000 – $1,000 – $595 = $405

Due to the costs associated with exercising the option and then selling the stock, some retail investors may find it more efficient to simply sell the option to another investor.

One final note on option commission costs is that they fail to account for the cost embedded in the bid-offer spread. The cost associated with this spread for options can be calculated by multiplying the spread by 50%. For example, if the bid price is $12 and the offer price is $12.20, the associated cost for both the option buyer and the option seller would be $0.10 per contract [= ($12.20 − $12.00) × 50%]. This cost is also present in stock transactions.

Margin Requirements

Options with maturities of nine months or fewer cannot be purchased on margin. This is because the leverage would become too high. For options with longer maturities, investors can borrow a maximum of 25% of the option value.

Investors who engage in writing options must have a margin account due to the high potential losses and potential default. The required margin for option writers is dependent on the amount and position of option contracts written.

Uncovered calls are those in which the writer does not also own a position in the underlying asset. The size of the initial and maintenance margin for uncovered call writing is equal to the option premium plus a percentage of the underlying share price.

Writing *covered calls* (selling a call option on a stock that is owned by the seller of the option) is far less risky than uncovered call writing and, therefore, requires no margin.

The Options Clearing Corporation

Similar to a clearinghouse for futures, the **Options Clearing Corporation (OCC)** guarantees that buyers and sellers in the exchange-traded options market will honor their obligations and records all option positions. Exchange-traded options have no default risk because of the OCC, while OTC options possess default risk. The OCC requires that all trades are cleared by one of its clearing members. OCC members must meet net capital requirements and help finance an emergency fund that is utilized in the event of a member default. Nonmember brokers must contact a clearing member to clear their option trades. The OCC guarantees contract performance and therefore requires option writers to post margin as a means of supporting their obligation and option buyers to deposit required funds by the morning of the business day immediately following the day the option is purchased.

Exercising an Option

When an investor decides to exercise an option before contract expiration, her broker contacts the assigned OCC member responsible for clearing that broker's trades. This OCC member then submits an exercise order to the OCC, which matches it with a clearing member who identifies an investor who has written a stock option. This assigned investor then must sell (if a call option) or buy (if a put option) the underlying at the specified strike price on the third business day after the order to exercise is received. Exercising an option results in the open interest being reduced by one. At contract expiration, unexercised options that are in-the-money after accounting for transaction costs will be exercised by brokers.

Other Option-Like Securities

LO 38.e: Define and describe warrants, convertible bonds, and employee stock options.

Warrants are often issued by a company to make a bond issue more attractive (e.g., equity upside) and will typically trade separately from the bond at some point. Warrants are like call options except that, upon exercise, the company may issue new shares and the warrant holders can purchase the shares at the exercise price.

The same distinction applies to **employee stock options**, which are issued as an incentive to company employees and provide a benefit if the stock price rises above the exercise price. A vesting period often applies before the options may be exercised, so the employee generally must still be employed by the company to receive the options or else the options are forfeited. Employee stock options are not transferrable to a third party.

Convertible bonds contain a provision that gives the bondholder the option of exchanging the bond for a prespecified number of shares of the company's common stock. At exercise, the newly issued shares increase the number of shares outstanding and debt is retired based on the amount of bonds exchanged for the shares.

MODULE QUIZ 38.2

Use the following information to answer Questions 1 and 2.

An investor owns a stock option that currently has a strike price of $100.

1. If the stock experiences a 4-to-1 split, the strike price becomes:
   A. $20.
   B. $25.
   C. $50.
   D. $100.

2. The number of options owned by the investor is:
   A. 1.
   B. 2.
   C. 3.
   D. 4.

3. Which of the following statements regarding options is correct?

A. Position limits usually exceed exercise limits.
B. Writing covered calls does not require any margin.
C. Options are adjusted for cash dividends and stock dividends.
D. LEAPS are long-term (over one year) options that expire in December of each year.

KEY CONCEPTS

LO 38.a

American options may be exercised at any time up to and including the contract's expiration date, while European options can be exercised only on the contract's expiration date. Exchange-traded options are typically American options. Exchange-traded options include those on individual stocks, stock indices, and ETFs.

LO 38.b

A call (put) option gives the owner the right to purchase (sell) the underlying asset at a strike price. When the owner executes this right, the option is said to be exercised. Because long (buy, purchase) option positions give the owner the right to exercise, the seller (short, writer) of the option has the obligation to meet the terms of the option.

For a call (put), when the underlying asset price is less (greater) than the strike price, the option is said to be out-of-the-money. For both a call and a put, when the underlying asset price is equal to the strike price, the option is said to be at-the-money. For a call (put), when the underlying asset price is greater (less) than the strike price, the option is said to be in-the-money.

LO 38.c

Options are not adjusted for cash dividends, but they are adjusted for stock splits and stock dividends. In the end, all investors' positions should be relatively unchanged due to stock splits or stock dividends.

LO 38.d

Traders are subject to position limits and exercise limits to discourage them from potentially manipulating the market.

Options with a maturity of nine months or fewer cannot be purchased on margin and must be paid in full due to the leverage effect of options. For options with longer maturities, investors can borrow up to 25% of the option value. Writers of options are required to have margin accounts with a broker.

Investors must account for commission costs when utilizing option. Commissions vary based on trade size and broker type. Commission rates typically are structured as a fixed dollar amount plus a percentage of the trade amount. In some instances, investors can earn higher profits by selling in-the-money options rather than exercising the options.

The Options Clearing Corporation (OCC) guarantees that buyers and sellers in the options market will honor their obligations and records all option positions. This minimizes default risk.

LO 38.e

Warrants, employee stock options, and convertible bonds are option-like securities that are often used as incentives. Unlike options, these securities are issued by financial institutions or companies.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 38.1

1. **C** Multiply the quote by 100 because each option contract is for 100 shares: $2.75 × 100 = $275.00.

   (LO 38.b)

2. **A** Stock options are typically exchange-traded, American-style options. Options expire after the third Friday of the month. American-style options are at least as valuable as European-style options. Index options are typically European-style (not American-style) options and are cash settled. (LO 38.a)

Module Quiz 38.2

1. **B** $\frac{a}{b} = \frac{1}{4} \times \$100 = \$25$

   (LO 38.c)

2. **D** $\frac{b}{a} = \frac{4}{1} \times \$100 = \$400$ (Each option contract is originally for 100 shares.)

   (LO 38.c)

3. **B** Writing covered calls is much less risky than writing uncovered calls because with covered calls, the seller owns the underlying asset so there would presumably be no default risk. Position limits are usually the same as exercise limits. Options are adjusted for stock dividends but not cash dividends. LEAPS expire in January, not December. (LO 38.d)

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Financial Markets and Products, Chapter 13.

READING 39

PROPERTIES OF OPTIONS

Study Session 10

EXAM FOCUS

Stock options have several properties relating both to their value and to the factors that affect their price. Six factors affect option prices: the current value of the stock, the strike price, the time to expiration, the volatility of the stock price, the risk-free rate, and dividends. The values of stock options have upper and lower pricing bounds. For the exam, be familiar with these pricing bounds, as well as the relationships that exist between the value of European and American options.

MODULE 39.1: OPTION PRICING FACTORS

LO 39.a: Identify the six factors that affect an option's price.

The following six factors will impact the value of an option:

1. S_0 = current stock price
2. X = strike price of the option
3. T = time to expiration of the option
4. r = short-term risk-free interest rate over T
5. D = present value of the dividend of the underlying stock
6. σ = expected volatility of stock prices over T

When evaluating a change in any one of the factors, hold the other factors constant.

Current Price of the Stock

For call options, as *S* increases (decreases), the value of the call increases (decreases). For put options, as *S* increases (decreases), the value of the put decreases (increases). This simply states that as an option becomes closer to or more in-the-money, its value increases.

Strike Price of the Option

The effect of strike prices on option values will be exactly the opposite of the effect of the current price of the stock. For call options, as X increases (decreases), the value of the call decreases (increases). For put options, as X increases (decreases), the value of the put increases (decreases). This is the same as the logic for the current price of the stock: the option's value will increase as it becomes closer to or more in-the-money.

The Time to Expiration

For American-style options, increasing time to expiration will increase the option value. With more time, the likelihood of being in-the-money increases. A general statement cannot be made for European-style options. Suppose we have a one-month and three-month call option on the same underlying with the same exercise price. Also suppose a large dividend is expected to be paid in two months. Because the stock price and three-month option price will fall when the dividend is paid in two months, the one-month option may be worth more than the three-month option.

The Risk-Free Rate Over the Life of the Option

As the risk-free rate increases, the value of the call (put) will increase (decrease). The intuition behind this property involves arbitrage arguments that require the use of synthetic securities.

Dividends

The option owner does not have access to the cash flows of the underlying stock, and the stock price decreases when a dividend is paid. Thus, as the dividend increases, the value of the call (put) will decrease (increase).

Volatility of the Stock Price Over the Life of the Option

Volatility is the friend of all options. As volatility increases, option values increase. This is due to the asymmetric payoff of options. Because long option positions have a maximum loss equal to the premium paid, increased volatility only increases the chances that the option will expire in-the-money. Many consider volatility to be the most important factor for option valuation.

Figure 39.1 summarizes the factors' effects on option prices: "+" indicates a positive effect on option price from an increase in the factor, and "–" is a negative effect on option price.

Figure 39.1: Summary of Effects of Increasing a Factor on the Price of an Option

Factor	European Call	European Put	American Call	American Put
S	+	_	+	_
X	_	+	_	+
T	?	?	+	+
σ	+	+	+	+
r	+	_	+	_
D	_	+	_	+

MODULE QUIZ 39.1

1. Which of the following will not cause a decrease in the value of a European call option position on XYZ stock?
   A. XYZ declares a 3-for-1 stock split.
   B. XYZ raises its quarterly dividend from $0.15 per share to $0.17 per share.
   C. The Federal Reserve lowers interest rates by 0.25% to stimulate the economy.
   D. Investors believe that the volatility of XYZ stock has declined.

MODULE 39.2: UPPER AND LOWER OPTION PRICING BOUNDS

LO 39.b: Identify and compute upper and lower bounds for option prices on non-dividend and dividend paying stocks.

In addition to those previously introduced, consider the following variables:

c = value of a European call option
C = value of an American call option
p = value of a European put option
P = value of an American put option
S_T = value of the stock at expiration

Also, assume in the following examples that there are no transaction costs, that all profits are taxed at the same rate, and that borrowing and lending can be done at the risk-free rate.

Upper Pricing Bounds for European and American Options

A call option gives the right to purchase one share of stock at a certain price. Under no circumstance can the option be worth more than the stock. If it were, everyone would sell the option and buy the stock and realize an arbitrage profit. We express this as:

$$c \leq S_0 \text{ and } C \leq S_0$$

Similarly, a put option gives the right to sell one share of stock at a certain price. Under no circumstance can the put be worth more than the sale or strike price. If it were, everyone would sell the option and invest the proceeds at the risk-free rate over the life of the option. We express this as:

$$p \leq X \text{ and } P \leq X$$

For a European put option, we can further reduce the upper bound. Because it cannot be exercised early, it can never be worth more than the present value of the strike price:

$$p \leq PV(X)$$

Lower Pricing Bounds for European Calls on Non-Dividend-Paying Stocks

Consider the following two portfolios:

- Portfolio P_1: one European call, *c*, with exercise price *X* plus a zero-coupon risk-free bond that pays *X* at *T*.
- Portfolio P_2: one share of the underlying stock, *S*.

At expiration, *T*, Portfolio P_1 will always be the greater of *X* (when the option expires out-of-the-money) or S_T (when the option expires in-the-money). Portfolio P_2, on the other hand, will always be worth S_T. Therefore, P_1 is always worth at least as much as P_2 at expiration. If we know that at *T*, $P_1 \geq P_2$, then it always has to be true because if it were not, arbitrage would be possible. Therefore, we can state the following:

$$c + PV(X) \geq S_0$$

Because the value of a call option cannot be negative (if the option expires out-of-the-money, its value will be zero), the lower bound for a European call on a non-dividend-paying stock is:

$$c \geq \max(S_0 - PV(X), 0)$$

Lower Pricing Bounds for European Puts on Non-Dividend-Paying Stocks

Consider the following two portfolios:

- Portfolio P_3: one European put, *p*, plus one share of the underlying stock, *S*.
- Portfolio P_4: zero-coupon risk-free bond that pays *X* at *T*.

At expiration, *T*, Portfolio P_3 will always be the greater of *X* (when the option expires in-the-money) or S_T (when the option expires out-of-the-money). Portfolio P_4, on the other hand, will always be worth *X*. Therefore, P_3 is always worth at least as much as P_4 at expiration. If we know that at *T*, $P_3 \geq P_4$, it has to be true always because if it were not, arbitrage would be possible. Therefore, we can state the following:

$$p + S_0 \geq PV(X)$$

Because the value of a put option cannot be negative (if the option expires out-of-the-money, its value will be zero), the lower bound for a European put on a non-dividend-paying stock is:

$$p \geq \max(PV(X) - S_0, 0)$$

Computing Option Values Using Put-Call Parity

LO 39.c: Explain put-call parity and apply it to the valuation of European and American stock options, with dividends and without dividends, and express it in terms of forward prices.

The derivation of **put-call parity** is based on the payoffs of two portfolio combinations, a fiduciary call and a protective put.

A *fiduciary call* is a combination of a pure-discount (i.e., zero coupon), riskless bond that pays *X* at maturity and a call with exercise price *X*. The payoff for a fiduciary call at expiration is *X* when the call is out-of-the-money, and X + (S − X) = S when the call is in-the-money.

A *protective put* is a share of stock together with a put option on the stock. The expiration date payoff for a protective put is (X − S) + S = X when the put is in-the-money, and *S* when the put is out-of-the-money.

PROFESSOR'S NOTE

When working with put-call parity, it is important to note that the exercise prices on the put and the call and the face value of the riskless bond are all equal to X.

When the put is in-the-money, the call is out-of-the-money, and both portfolios pay *X* at expiration.

Similarly, when the put is out-of-the-money and the call is in-the-money, both portfolios pay *S* at expiration.

Put-call parity holds that portfolios with identical payoffs must sell for the same price to prevent arbitrage. We can express the put-call parity relationship as:

$$c + PV(X) = S + p$$

Equivalencies for each of the individual securities in the put-call parity relationship can be expressed as:

$$S = c - p + PV(X)$$
$$p = c - S + PV(X)$$
$$c = S + p - PV(X)$$
$$PV(X) = S + p - c$$

The single securities on the left-hand side of the equations all have the same payoffs as the portfolios on the right-hand side. The portfolios on the right-hand side are the "synthetic" equivalents of the securities on the left. Note that the options must be European style and the puts and calls must have the same exercise price for these relations to hold.

For example, to synthetically produce the payoff for a long position in a share of stock, you use the relationship:

$$S = c - p + PV(X)$$

This means that the payoff on a long stock can be synthetically created with a long call, a short put, and a long position in a risk-free discount bond.

The other securities in the put-call parity relationship can be constructed in a similar manner.

PROFESSOR'S NOTE

After expressing the put-call parity relationship in terms of the security you want to synthetically create, the sign on the individual securities will indicate whether you need a long position (+ sign) or a short position (− sign) in the respective securities.

EXAMPLE: Call option valuation using put-call parity

Suppose that the current stock price is $52 and the risk-free rate is 5%. You have found a quote for a three-month put option with an exercise price of $50. The put price is $1.50, but due to light trading in the call options, there was not a listed quote for the three-month, $50 call. **Estimate** the price of the three-month call option.

Answer:

Rearranging put-call parity, we find that the call price is:

$$\text{call} = \text{put} + \text{stock} - PV(X)$$

$$\text{call} = \$1.50 + \$52 - (\$50 / 1.05^{0.25}) = \$4.11$$

This means that if a three-month, $50 call is available, it should be priced at $4.11 per share.

Put-call parity is expressed as:

$$p + S = c + PV(X)$$

It can also be expressed in terms of forward prices by substituting PV(F) for *S* as follows:

$$p + PV(F) = c + PV(X)$$

The left side of the equation represents a forward contract to purchase asset *F* at option expiration (at an agreed upon forward price of F) plus cash in the amount of PV(F) plus a European put option; recall that it does not cost anything to enter into a forward contract, so there is a zero value for the forward contract on the right side.

Lower Pricing Bounds for an American Call Option on a Non-Dividend-Paying Stock

LO 39.d: Explain and assess potential rationales for using the early exercise features of American call and put options.

Recall the following equation from our earlier discussion of the lower pricing bounds for a *European* call option:

$$c \geq \max(S_0 - PV(X), 0)$$

Because the only difference between an American option and a European option is that the American option can be exercised early, American options can always be used to replicate their corresponding European options simply by choosing not to exercise them until expiration. Therefore, it follows that:

$$C \geq c \geq \max(S_0 - PV(X), 0)$$

Note that when an American call is exercised, it is only worth S_0 – X. Because this value is never larger than S_0 – PV(X) for any *r* and T > 0, it is never optimal to exercise early. In other words, the investor can keep the cash equal to *X*, which would be used to exercise the option early, and invest that cash to earn interest until expiration. Because exercising the American call early means that the investor would have to forgo this interest, it is never optimal to exercise an American call on a non-dividend-paying stock before the expiration date (i.e., c = C).

Lower Pricing Bounds for an American Put Option on a Non-Dividend-Paying Stock

While it is never optimal to exercise an American call on a non-dividend-paying stock, American puts are optimally exercised early if they are sufficiently in-the-money. If an option is sufficiently in-the-money, it can be exercised, and the payoff (X – S_0) can be invested to earn interest. In the extreme case when S_0 is close to zero, the future value of the exercised cash value, PV(X), is always worth more than a later exercise, *X*. We know that:

$$P \geq p \geq \max(PV(X) - S_0, 0) \text{ for the same reasons that } C \geq c$$

However, we can place an even stronger bound on an American put because it can always be exercised early:

$$P \geq \max(X - S_0, 0)$$

Figure 39.2 summarizes what we now know regarding the boundary prices for American and European options.

Figure 39.2: Lower and Upper Bounds for Options

Option	Minimum Value	Maximum Value
European call	$c \geq \max(0, S_0 - PV(X))$	S_0
American call	$C \geq \max(0, S_0 - PV(X))$	S_0
European put	$p \geq \max(0, PV(X) - S_0)$	$PV(X)$
American put	$P \geq \max(0, X - S_0)$	X

PROFESSOR'S NOTE

For the exam, know the price limits in Figure 39.2. You will not be asked to derive them, but you may be expected to use them.

EXAMPLE: Minimum prices for American vs. European puts

Compute the lowest possible price for four-month American and European 65 puts on a stock that is trading at 63 when the risk-free rate is 5%.

Answer:

$P \geq \max(0, X - S_0) = \max(0, 2) = \$2$

$p \geq \max(0, PV(X) - S_0) = \max(0, (65 / 1.0167) - 63) = \$0.93$

EXAMPLE: Minimum prices for American vs. European calls

Compute the lowest possible price for three-month American and European 65 calls on a stock that is trading at 68 when the risk-free rate is 5%.

Answer:

$C \geq \max(0, S_0 - PV(X)) = \max(0, 68 - (65 / 1.0125)) = \$3.80$

$c \geq \max(0, S_0 - PV(X)) = \max(0, 68 - (65 / 1.0125)) = \$3.80$

Relationship Between American Call Options and Put Options

Put-call parity only holds for European options. For American options, we have an inequality. This inequality places upper and lower bounds on the difference between the American call and put options.

$$S_0 - X \leq C - P \leq S_0 - PV(X)$$

EXAMPLE: American put option bounds

Consider an American call and put option on Stock XYZ. Both options have the same one-year expiration and a strike price of $20. The stock is currently priced at $22, and the annual interest rate is 6%. What are the upper and lower bounds on the American put option if the American call option is priced at $4?

Answer:

The upper and lower bounds on the difference between the American call and American put options are:

$$S_0 - X \leq C - P \leq S_0 - PV(X)$$
$$S_0 - X = 22 - 20 = \$2$$
$$S_0 - PV(X) = 22 - (20 / 1.06) = 22 - 18.87 = \$3.13$$
$$\$2 \leq C - P \leq \$3.13$$

or

$$-\$2 \geq P - C \geq -\$3.13$$

Therefore, when the American call is valued at \$4, the upper and lower bounds on the American put option will be:

$$\$2 \geq P \geq \$0.87$$

The Impact of Dividends on Option Pricing Bounds

Because most stock options have an expiration of less than a year, dividends can be estimated fairly accurately. Recall that to prevent arbitrage, when a stock pays a dividend, its value must decrease by the amount of the dividend. This increases the value of a put option and decreases the value of a call option.

Consider the following portfolios:

- Portfolio P_6: one European call option, *c*, plus cash equal to D + PV(X).
- Portfolio P_7: one share of the underlying stock, *S*.

Similar to the development of the $c \geq \max(S_0 - PV(X), 0)$ equation, Portfolio P_6 is always at least as large as P_7, or:

$$c \geq S_0 - D - PV(X)$$

All else equal, the payment of a dividend will reduce the lower pricing bound for a call option.

For put options:

- Portfolio P_8: one European put, *p*, plus one share of the underlying stock, *S*.
- Portfolio P_9: cash equal to D + PV(X).

Using the same development as the $p \geq \max(PV(X) - S_0, 0)$ equation:

$$p \geq D + PV(X) - S_0$$

All else being equal, the payment of a dividend will increase the lower pricing bound for a put option.

Impact of Dividends on Early Exercise for American Calls and Put-Call Parity

When the dividend is large enough, American calls might be optimally exercised early. This will be the case if the amount of the dividend exceeds the amount of interest that is forgone as a result of the early exercise. Note that if a large dividend makes early exercise optimal, exercise should take place immediately before the ex-dividend date. Put-call parity is adjusted for dividends in the following manner:

$$p + S_0 = c + D + PV(X)$$

This equation is verified using the same development as that used to derive the $p + S_0 = c + PV(X)$ equation. The $S_0 - X \leq C - P \leq S_0 - PV(X)$ equation that we used to show the relationship between American call and put options is modified as follows:

$$S_0 - X - D \leq C - P \leq S_0 - PV(X)$$

MODULE QUIZ 39.2

1. Consider a European put option on a stock trading at \$50. The put option has an expiration of six months, a strike price of \$40, and a risk-free rate of 5%. The lower bound and upper bound on the put are closest to:
   A. \$10, \$40.00.
   B. \$10, \$39.00.
   C. \$0, \$40.00.
   D. \$0, \$39.00.

2. Consider a one-year European put option that is currently valued at \$5 on a \$25 stock and a strike of \$27.50. The one-year risk-free rate is 6%. Which of the following amounts is closest to the value of the corresponding call option?
   A. \$0.00.
   B. \$3.89.
   C. \$4.06.
   D. \$5.00.

3. Consider an American call and put option on the same stock. Both options have the same one-year expiration and a strike price of \$45. The stock is currently priced at \$50, and the annual interest rate is 10%. Which of the following amounts could be the difference in the two option values?
   A. \$4.95.
   B. \$7.95.
   C. \$9.35.
   D. \$12.50.

4. According to put-call parity for European options, purchasing a put option on ABC stock would be equivalent to:
   A. buying a call, buying ABC stock, and buying a zero-coupon bond.
   B. buying a call, selling ABC stock, and buying a zero-coupon bond.
   C. selling a call, selling ABC stock, and buying a zero-coupon bond.
   D. buying a call, selling ABC stock, and selling a zero-coupon bond.

KEY CONCEPTS

LO 39.a

Six factors influence the value of an option: current value of the underlying asset (stock), the strike price, the time to expiration of the option, the volatility of the stock price, the risk-free rate, and dividends.

With the exception of time to expiration, all these factors affect European- and American-style options in the same way.

LO 39.b

Call options cannot be worth more than the underlying security, and put options cannot be worth more than the strike price.

When the stock does not pay a dividend, European call options cannot be worth less than the difference between the current stock price and the present value of the strike price. European put options cannot be worth less than the difference between the present value of the strike price and the current stock price.

LO 39.c

Put-call parity is a no-arbitrage relationship for European-style options with the same characteristics. It states that a portfolio consisting of a call option and a zero-coupon bond with a face value equal to the strike must have the same value as a portfolio consisting of the corresponding put option and the stock:

$$p + S_0 = c + PV(X)$$

Put-call parity can be expressed in terms of forward prices as follows:

$$p + PV(F) = c + PV(X)$$

LO 39.d

It is never optimal to exercise an American call option on non-dividend-paying stock before expiration.

American put options on non-dividend-paying stocks can be optimally exercised before expiration if the put is sufficiently in-the-money.

Call options are always worth more than corresponding put options before expiration when both are at-the-money.

The difference between prices of an American call and corresponding put is bounded below by the difference between the current stock price and the strike price, and above by the difference between the current stock price and the present value of the strike price.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 39.1

1. **A** After a stock split, both the price of the stock and the strike price of the option will be adjusted, so the value of the option position will be the same. An increase in the dividend, a lower risk-free interest rate, and a lower volatility of the price

of the underlying stock will all decrease the value of a European call option. (LO 39.a)

Module Quiz 39.2

1. **D** The upper bound is the present value of the exercise price: $\$40 / 1.05^{0.5} = \$39.04$. Because the put is out-of-the-money, the lower bound is zero.
   (LO 39.b)

2. **C** $c = p - PV(X) + S_0 = \$5 - (\$27.50 / 1.06) + \$25 = \$4.06$
   (LO 39.c)

3. **B** The upper and lower bounds are: $S_0 - X \leq C - P \leq S_0 - PV(X)$ or $\$5 \leq C - P \leq \$9.09$. Only \$7.95 falls within the bounds. (LO 39.d)

4. **B** The formula for put-call parity is $p + S_0 = c + PV(X)$. Rearranging to solve for the price of a put, we have $p = c - S_0 + PV(X)$. (LO 39.c)

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Financial Markets and Products, Chapter 14.

READING 40

TRADING STRATEGIES

Study Session 10

EXAM FOCUS

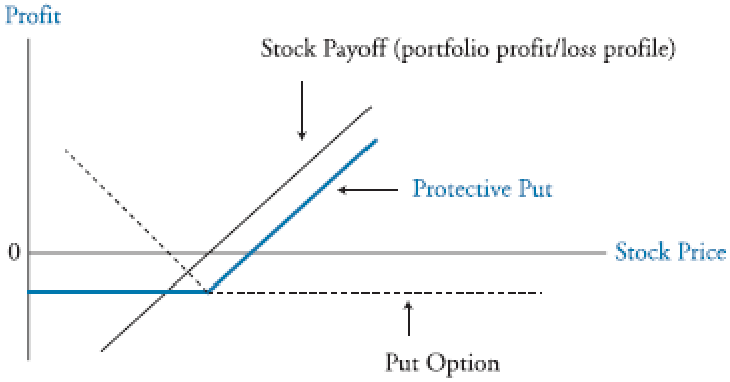
Traders and investors use option-based trading strategies to create an extraordinary spectrum of payoff profiles. This enables investors to take positions based on almost any possible expectation of the underlying stock over the life of the options. This reading describes the common option trading strategies and implementation. For the exam, know the general payoff graphs for each strategy discussed. In addition, know how to calculate profits for some of the more popular strategies including protective put, covered call, bull call spread, butterfly spread, and straddle.

MODULE 40.1: PROTECTIVE PUTS, COVERED CALLS, AND PRINCIPAL PROTECTED NOTES

LO 40.a: Explain the motivation to initiate a covered call or a protective put strategy.

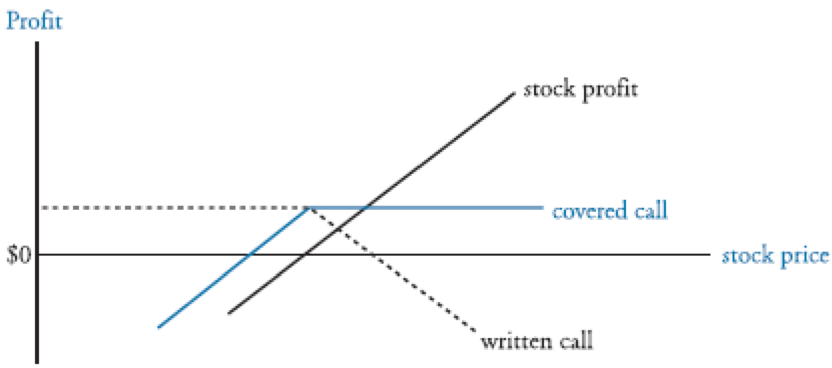
When an at-the-money long put position is combined with the underlying stock, we have created a **protective put** strategy. A protective put (also called *portfolio insurance* or a *hedged portfolio*) is constructed by holding a long position in the underlying security and buying a put option. You can use a protective put to limit the downside risk at the cost of the put premium, P_0. You will see by the diagram in Figure 40.1 that the investor will still be able to benefit from increases in the stock's price, but it will be lower by the amount paid for the put, P_0. Notice that the combined strategy looks very much like a call option. This should not be surprising because put-call parity requires that $p + S_0$ be the same as $c + PV(X)$. Figure 40.1 illustrates this property.

Figure 40.1: Protective Put Strategy



Another common strategy is to sell a call option on a stock that is owned by the option writer. This is called a **covered call** position. By writing an out-of-the-money call option, the combined position caps the upside potential at the strike price. In return for giving up any potential gain beyond the strike price, the writer receives the option premium. This strategy is used to generate cash on a stock that is not expected to increase above the exercise price over the life of the option.

Figure 40.2: Profit Profile for a Covered Call



LO 40.b: Describe principal protected notes (PPNs) and explain necessary conditions to create them.

Principal protected notes (PPNs) are securities that are generated from one option. Investors may participate in gains on a portfolio but do not suffer from any losses.

To illustrate PPNs, consider the following example. Investment X consists of two items:

- A five-year zero-coupon bond with a face value of $100,000 (annually compounded discount rate of 4%)
- A five-year call option on Investment Y, with a current market value and strike price of $100,000

An investor in Investment X stands to gain if Investment Y goes up in value (call option exercised) and will not lose anything if Investment Y goes down in value (call option not exercised). The zero-coupon bond costs about $82,193 [= $100,000 / $(1.04)^5$], so if the call option is priced at below $17,807, then Investment X can be sold as a PPN.

Note that principal protection does come with a cost. An investor in the preceding PPN forfeits interest income for five years on $100,000, as well as any income that might have been earned on Investment Y.

PPNs can only be generated from investments that have an income stream. To illustrate, the call is currently at-the-money (S = X = $100,000) and using put-call parity:

$$c = p + S - PV(S) = \$0 + \$100{,}000 - \$82{,}193 = \$17{,}807$$

Because the income from Investment Y is not earned by the investor, that lowers the value of the call. Therefore, if the income from Investment Y is high enough, then it allows the opportunity to issue a PPN.

MODULE QUIZ 40.1

1. A covered call position is the:
   A. simultaneous purchase of a call and the underlying asset.
   B. purchase of a share of stock with a simultaneous sale of a call on that stock.
   C. purchase of a share of stock with a simultaneous sale of a put on that stock.
   D. short sale of a stock with a simultaneous sale of a call on that stock.

MODULE 40.2: OPTION SPREAD STRATEGIES

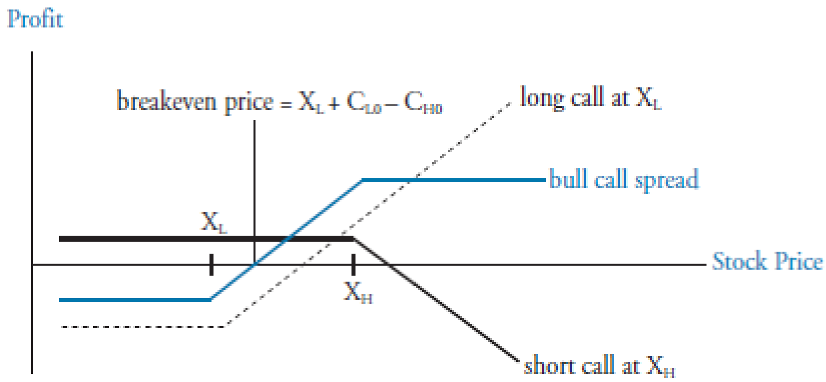
LO 40.c: Describe the use and calculate the payoffs of various spread strategies.

Several spread strategies exist. These strategies combine options positions to create a desired payoff profile. The differences between the options are either the strike prices and/or the time to expiration. We will discuss bull and bear spreads, butterfly spreads, calendar spreads, and diagonal spreads.

Bull and Bear Spreads

In a *bull call spread* (Figure 40.3), the buyer of the spread purchases a European call option with a low exercise price, X_L, and subsidizes the purchase price of the call by selling a European call with a higher exercise price, X_H. The expiration dates are identical for both options. The buyer of a bull call spread expects the stock price to rise and the purchased call to finish in-the-money. However, the buyer does not believe that the price of the stock will rise above the exercise price for the out-of-the-money written call. Therefore, in exchange for a lower net cost (compared with purchasing the call with exercise price X_L only), there is the loss of upside potential beyond X_H. Additionally, if the bull call spread is set up with exercise prices that make the options out-of-the-money (in-the-money), then the net cost will be relatively low (high).

Figure 40.3: Bull Call Spread



EXAMPLE: Bull call spread

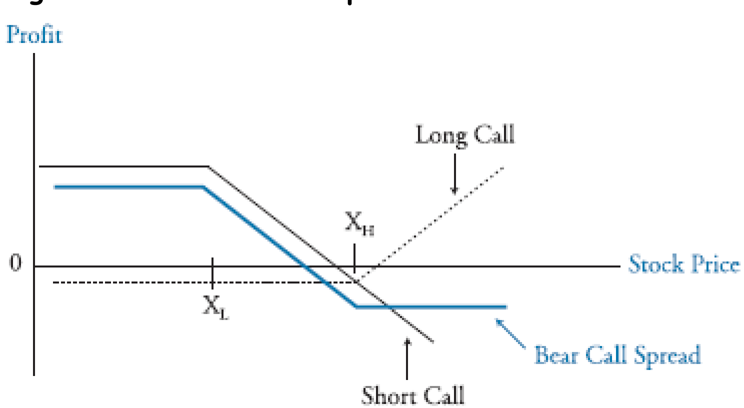
An investor purchases a call for C_{L0} = \$3.00 with a strike of X = \$40 and sells a call for C_{H0} = \$1.00 with a strike price of \$50. **Compute** the profit of a bull call spread strategy when the price of the stock is at \$45.

Answer:

$$\text{profit} = \max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{LO} + C_{HO}$$
$$\text{profit} = \max(0, 45 - 40) - \max(0, 45 - 50) - 3 + 1 = \$3.00$$

A *bear call spread* is the sale of a bull spread. That is, the bear spread trader will buy the call with the higher exercise price and sell the call with the lower exercise price. This strategy is designed to profit from falling stock prices (i.e., a "bear" strategy). As stock prices fall, the investor keeps the premium from the written call, net of the long call's cost. The purpose of the long call is to protect from sharp increases in stock prices. The payoff is the opposite (mirror image) of the bull call spread and is shown in Figure 40.4.

Figure 40.4: Bear Call Spread



Puts can also be used to replicate the payoffs for both a bull call spread and a bear call spread. In a *bear put spread*, the investor buys a put with a higher exercise price and sells a put with a lower exercise price.

EXAMPLE: Bear put spread

An investor sells a put for P_{L0} = \$3.00 with a strike of X = \$20 and purchases a put for P_{H0} = \$4.50 with a strike price of \$40. **Compute** the profit of a bear put spread strategy when the price of the stock is at \$35.

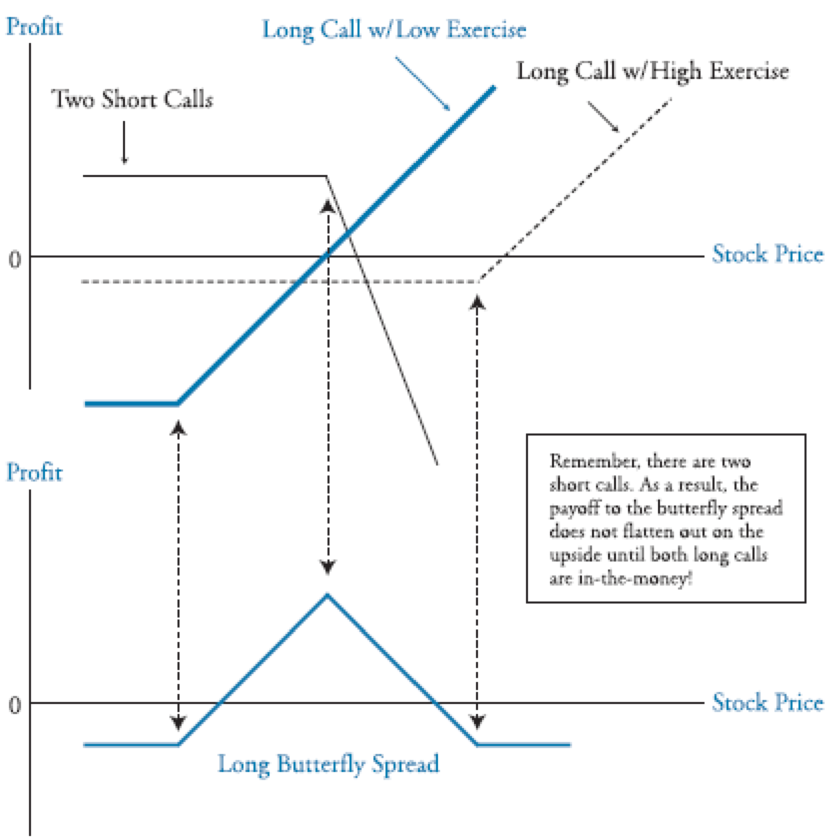
Answer:

$$\text{profit} = \max(0, X_H - S_T) - \max(0, X_L - S_T) - P_{H0} + P_{L0}$$
$$\text{profit} = \max(0, 40 - 35) - \max(0, 20 - 35) - 4.50 + 3 = \$3.50$$

Butterfly Spreads

A *butterfly spread* involves the purchase or sale of *three* different call options. Here, the investor buys one European call with a low exercise price, buys another European call with a high exercise price, and sells *two* European calls with an exercise price in between (usually near the current stock price). The expiration dates are identical for all the options. The net cost of the butterfly spread is always positive because the payoff is always zero or more; it will be zero for large moves in either direction. The buyer of a butterfly spread is essentially betting that the stock price will stay near the exercise price of the written calls. However, the loss that the butterfly spread buyer sustains if the stock price strays from this level is limited. The two graphs in Figure 40.5 illustrate the construction and payoffs of a butterfly spread.

Figure 40.5: Butterfly Spread Construction and Behavior



EXAMPLE: Butterfly spread with calls

An investor makes the following transactions in calls on a stock:

- Buys one call defined by C_{L0} = \$7.00 and X_L = \$55.
- Buys one call defined by C_{H0} = \$2.00 and X_H = \$65.
- Sells two calls defined by C_{M0} = \$4.00 and X_M = \$60.

Compute the profit of a butterfly spread strategy with calls when the stock is at \$60.

Answer:

$$\begin{aligned}\text{profit} &= \max(0, S_T - X_L) - 2\max(0, S_T - X_M) + \max(0, S_T - X_H) \\ &\quad - C_{L0} + 2C_{M0} - C_{H0}\end{aligned}$$

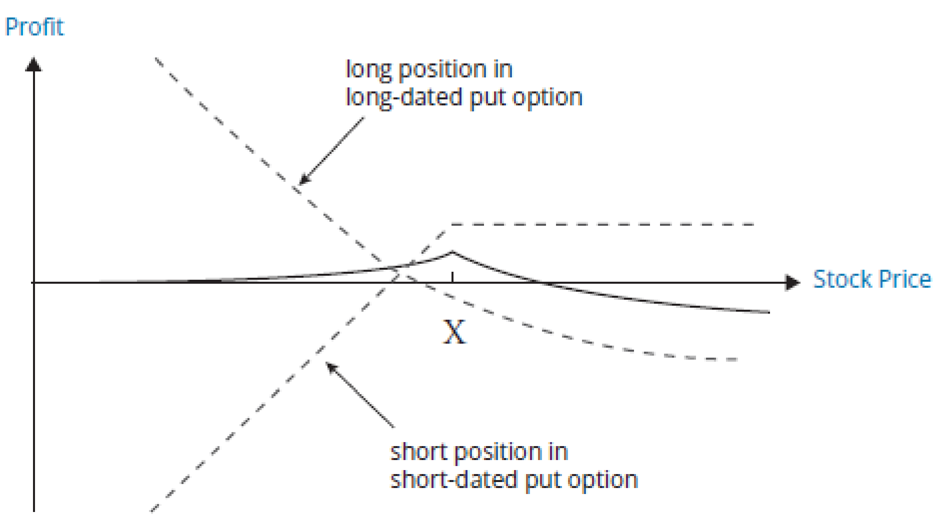
$$\begin{aligned}\text{profit} &= \max(0, 60 - 55) - 2\max(0, 60 - 60) + \max(0, 60 - 65) - 7 \\ &\quad + 2(4) - 2 = \$4.00\end{aligned}$$

To create a butterfly spread with put options, the investor would buy a low and a high strike put option and sell two puts with an intermediate strike price. Again, the combined position is constructed by summing the payoffs of the individual options at each stock price.

Calendar Spreads

A *calendar spread* is created by transacting in two options that have the same strike price but different expirations. Figure 40.6 shows a calendar spread using put options. The strategy sells the short-dated option and buys the long-dated option. Notice that the payoff here is similar to the butterfly spread. The investor profits slightly only if the stock remains in a narrow range (e.g., close to strike price), but losses are limited to about the net option premium cost. In this case, the losses are not symmetrical as they are in the butterfly spread. A calendar spread based on calls is created in similar fashion.

Figure 40.6: Calendar Spread (Using Two Put Options)



Diagonal Spreads

A *diagonal spread* is similar to a calendar spread (e.g., long call/put and short call/put), except that instead of using options with the same strike price and different expirations, the options in a diagonal spread can have different strike prices and different expirations.

Box Spreads

A *box spread* is a combination of a bull call spread and a bear put spread on the same asset. This strategy will produce a constant payoff that is equal to the high exercise price (X_H) minus the low exercise price (X_L). Under a no arbitrage assumption, the present value of the payoff will equal the net premium paid (i.e., profit will equal zero).

When the profit from this strategy is different than zero, an investor can capitalize on the arbitrage opportunity by either buying or selling the box. If the profit is positive, the investor will create a long box spread by buying a call at X_L, selling a call at X_H, buying a put at X_H, and selling a put at X_L. If the profit is negative, the investor will create a short box spread by buying a call at X_H, selling a call at X_L, buying a put at X_L, and selling a put at X_H. Note that box spread arbitrage is only successful with European options.

MODULE QUIZ 40.2

1. An investor is very confident that a stock will change significantly over the next few months; however, the direction of the price change is unknown. Which strategies will most likely produce a profit if the stock price moves as expected?

   I. Short butterfly spread.
   II. Bearish calendar spread.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

2. Which of the following will create a bear spread?

   A. Buy a call with a strike price of X = 45 and sell a call with a strike price of X = 50.
   B. Buy a call with a strike price of X = 50 and buy a put with a strike price of X = 55.
   C. Buy a put with a strike price of X = 45 and sell a put with a strike price of X = 50.
   D. Buy a call with a strike price of X = 50 and sell a call with a strike price of X = 45.

3. Consider an option strategy where an investor buys one call option with an exercise price of \$55 for \$7, sells two call options with an exercise price of \$60 for \$4, and buys one call option with an exercise price of \$65 for \$2. If the stock price declines to \$25, what will be the profit or loss on the strategy?

   A. –\$3.
   B. –\$1.
   C. \$1.

D. $2.

MODULE 40.3: OPTION COMBINATION STRATEGIES

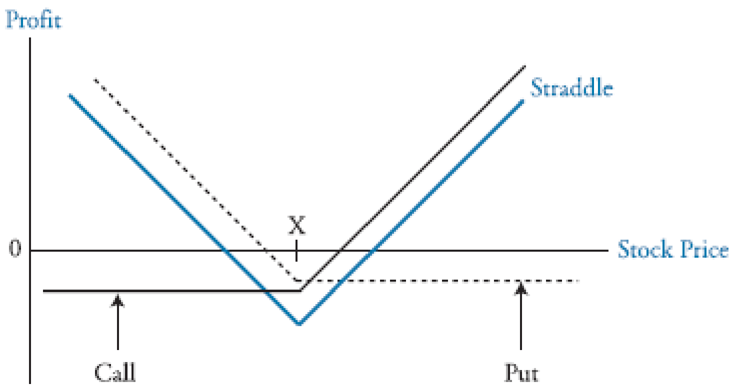
LO 40.d: Describe the use and explain the payoff functions of combination strategies.

Combinations are option strategies involving both puts and calls. We will discuss straddles, strangles, strips, and straps.

Straddle

A (long) *straddle* is created by purchasing a call and a put with the same strike price (often near current stock price) and expiration. Figure 40.7 illustrates the payoff for a long straddle position. Given the need to pay for two option premiums, this strategy is only profitable when the stock price moves significantly in either direction; it is a bet on volatility but without certainty on the direction. Straddle payoffs are symmetric around the strike price.

Figure 40.7: Long Straddle Profit/Loss



EXAMPLE: Straddle

An investor purchases a call on a stock, with an exercise price of $45 and a premium of $3, and purchases a put option with the same maturity that has an exercise price of $45 and a premium of $2. **Compute** the profit of a straddle strategy if the stock is at $35.

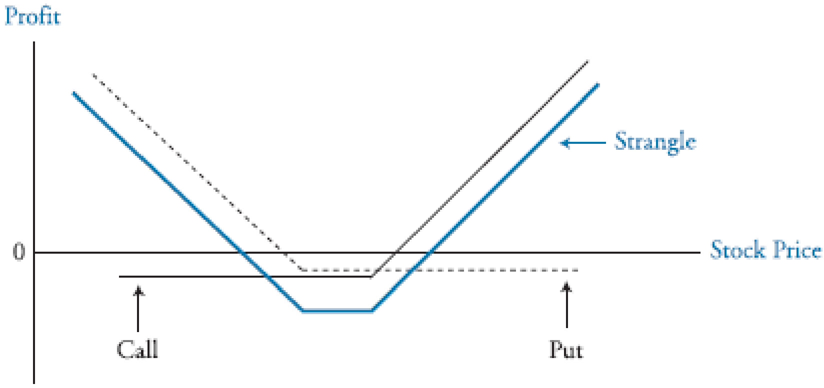
Answer:

$$\text{profit} = \max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0$$
$$\text{profit} = \max(0, 35 - 45) + \max(0, 45 - 35) - 3 - 2 = \$5$$

Strangle

A *strangle* is similar to a straddle, except that the options purchased are slightly out-of-the-money (e.g., call strike price > put strike price), so it is cheaper to implement than the straddle. The payoff is similar to the straddle except for a flat section between the strike prices, as shown in Figure 40.8. Because it is cheaper, the stock will have to move even further relative to the straddle before the strangle is profitable. Strangle payoffs are also symmetric around the strike prices.

Figure 40.8: Long Strangle Profit/Loss

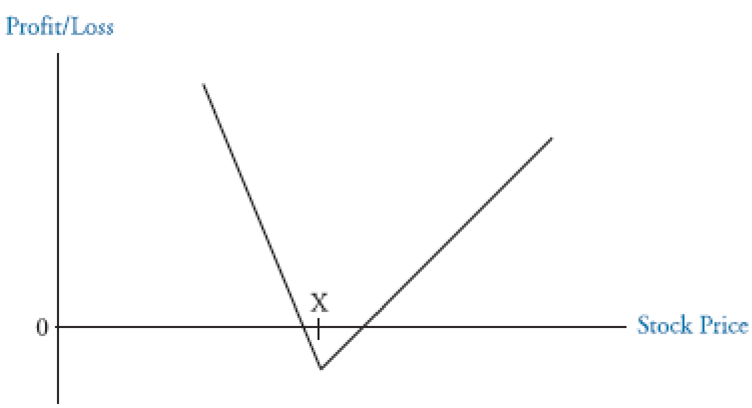


> **EXAMPLE: Strangle**
>
> An investor purchases a call on a stock, with an exercise price of $50 and a premium of $1.50, and purchases a put option with the same maturity that has an exercise price of $45 and a premium of $2. **Compute** the profit of a strangle strategy if the stock is at $40.
>
> **Answer:**
>
> $$\text{profit} = \max(0, S_T - X_H) + \max(0, X_L - S_T) - C_0 - P_0$$
> $$\text{profit} = \max(0, 40 - \$50) + \max(0, 45 - 40) - 1.50 - 2 = \$1.50$$

Strips and Straps

A *strip* involves purchasing two puts and one call with the same strike price and expiration, so it is similar to a straddle. Figure 40.9 illustrates a strip. Notice the asymmetry of the payoff. A strip is betting on volatility but is more bearish because it pays off more on the downside.

Figure 40.9: Strip Profit/Loss



A *strap* involves purchasing two calls and one put with the same strike price and expiration, so again, it is similar to a straddle. A strap is betting on volatility but is more bullish since it pays off more on the upside.

MODULE QUIZ 40.3

1. An investor believes that a stock will either increase or decrease greatly in value over the next few months but believes a down move is more likely. Which of the following strategies will be most appropriate for this investor?
   A. A protective put.
   B. An at-the-money strip.
   C. An at-the-money strap.
   D. A straddle.
2. An investor constructs a straddle by buying an April $30 call for $4 and buying an April $30 put for $3. If the price of the underlying shares is $27 at expiration, what is the profit on the position?
   A. –$4.
   B. –$2.
   C. $2.
   D. $3.

KEY CONCEPTS

LO 40.a

Stock options can be combined with their underlying stock to generate various payoff profiles. A protective put combines an at-the-money long put position with the underlying stock. A covered call involves selling a call option on a stock that is owned by the option writer.

LO 40.b

Principal protected notes (PPNs) are securities that are generated from one option. Investors may participate in gains on a portfolio but do not suffer from any losses.

Note that principal protection does come with a cost. An investor in the PPN forfeits interest income for the term of the investment, as well as any interest income that would have been earned on the underlying investment. In that regard, PPNs can only be generated from investments that have an income stream.

LO 40.c

Spread strategies combine options in the same option class to generate various payoff profiles.

The buyer of a bull call spread expects the stock price to rise and the purchased call to finish in-the-money. However, the buyer does not believe that the price of the stock will rise above the exercise price for the out-of-the-money written call.

The bear call spread trader will purchase the call with the higher exercise price and sell the call with the lower exercise price. This strategy is designed to profit from falling stock prices (i.e., a "bear" strategy). As stock prices fall, the investor keeps the premium from the written call, net of the long call's cost.

A box spread is an extreme method of locking in value. The dollar return for a box spread is fixed. It is a combination of a bull call spread and a bear put spread.

A calendar spread is created by transacting in two options that have the same strike price but different expirations.

The buyer of a butterfly spread is essentially betting that the stock price will stay near the strike price of the written calls. However, the loss that the butterfly spread buyer sustains if the stock price strays from this level is not large.

In a diagonal spread, options can have different strike prices and different expirations.

Bull call spread:

$$\text{profit} = \max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0}$$

Bear put spread:

$$\text{profit} = \max(0, X_H - S_T) - \max(0, X_L - S_T) - P_{H0} + P_{L0}$$

Butterfly spread:

$$\text{profit} = \max(0, S_T - X_L) - 2\max(0, S_T - X_M) + \max(0, S_T - X_H) - C_{L0} + 2C_{M0} - C_{H0}$$

LO 40.d

Combination strategies combine puts and calls to generate various payoff strategies.

A (long) straddle is created by purchasing a call and a put with the same strike price and expiration. Note that this strategy only pays off when the stock moves in either direction.

A strangle is similar to a straddle except that the option purchased is slightly out-of-the-money, so it is cheaper to implement than the straddle.

A strip is betting on volatility but is more bearish because it pays off more on the downside.

A strap is betting on volatility but is more bullish because it pays off more on the upside.

Straddle:

$$\text{profit} = \max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0$$

Strangle:

$$\text{profit} = \max(0, S_T - X_H) + \max(0, X_L - S_T) - C_0 - P_0$$

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 40.1

1. **B** The covered call is a stock plus a short call. The term *covered* means that the stock covers the inherent obligation assumed in writing the call. Why would you write a covered call? You feel the stock's price will not go up anytime soon, and you want to increase your income by collecting some call option premiums. To add some insurance that the stock won't get called away, the call writer can write

out-of-the money calls. You should know that this strategy for enhancing one's income is not without risk. The call writer is trading the stock's upside potential for the call premium. The desirability of writing a covered call to enhance income depends upon the chance that the stock price will exceed the exercise price at which the trader writes the call. (LO 40.a)

Module Quiz 40.2

1. **A** A short butterfly spread will produce a modest profit if there is a large amount of volatility in the price of the stock. A bearish calendar spread is a play using options with different expiration dates. (LO 40.c)

2. **D** Spread strategies involve purchasing and selling an option of the same type. A bear spread with calls involves buying a call with a high strike price and selling a call with a low strike price. The investor profits if stock prices fall by keeping the premium from the written call, net of the premium from the purchased call. Note that a bear spread can also be constructed with put options by buying a put with a high strike price and selling a put with a low strike price. With a bear put spread, if the stock price declines and both puts are exercised, the investor receives the difference between the strike prices less the net premium paid. (LO 40.b)

3. **B** The strategy described is a butterfly spread where the investor buys a call with a low exercise price, buys another call with a high exercise price, and sells two calls with a price in between. In this case, if the option moves to \$25, none of the call options will be in-the-money, so the profit is equal to the net premium paid, which is $-\$7 + (2 \times \$4) - \$2 = -\$1$. (LO 40.b)

Module Quiz 40.3

1. **B** An at-the-money strip bets on volatility but is more bearish because it pays off more on the downside. A straddle is possible, but a strip is even more appropriate. (LO 40.d)

2. **A** The sum of the premiums paid for the position is \$7. With the underlying stock at \$27, the put will be worth \$3, while the call option will be worthless. The value of the position is $(-\$7 + \$3) = -\$4$. (LO 40.d)

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Financial Markets and Products, Chapter 15.

READING 41

EXOTIC OPTIONS

Study Session 10

EXAM FOCUS

In this reading, we define and discuss the important characteristics of a variety of exotic options. The difference between exotic options and more traditional exchange-traded instruments is also highlighted. For the exam, be familiar with the payoff structures for the various exotic options discussed.

MODULE 41.1: EXOTIC OPTION DEVELOPMENT

LO 41.a: Define and contrast exotic derivatives and plain vanilla derivatives.

LO 41.b: Describe some of the reasons that drive the development of exotic derivative products.

Plain vanilla options are generally traded on exchanges in fairly liquid markets. In contrast, exotic options are customized (e.g., traded in the OTC markets) to fit a specific firm need for hedging that cannot be met by plain vanilla options. With plain vanilla options, there is little uncertainty about the cost, the current market value, when they will pay, how much they will pay, and the cost of exiting the position. With exotic derivatives, some or all of these may be in question.

Exotic options were developed for several reasons. The main purpose is to provide a unique hedge for a firm's underlying assets as mentioned previously. Other reasons include addressing tax and regulatory concerns as well as speculating on the expected future direction of market factors, including interest rates and exchange rates, for example.

Using Packages to Formulate a Zero-Cost Product

LO 41.c: Explain how any derivative can be converted into a zero-cost product.

A package is defined as some combination of standard European options on an underlying asset. Bull, bear, and calendar spreads, as well as straddles and strangles, are examples of packages. Packages usually consist of selling one instrument with certain characteristics and buying another with somewhat different characteristics. Because packages often consist of a long position and a short position, they can be constructed so that the initial cost to the investor is zero.

For example, consider a zero-cost short collar. A short collar combines a long standard put option with an exercise price X_L and a short standard call option with exercise price X_H (where $X_L < X_H$). If the premium the investor pays for the put option is exactly offset by the premium the investor receives for the short call position, the investor's net cost for implementing the short collar strategy is zero. In any case where the investor's cash outflows from long positions are offset by cash inflows from short positions, the investor can use a package to create a zero-cost product.

Additionally, an option can be structured as zero-cost if the otherwise up-front premium can be deferred to the expiration date of the option. The option buyer will pay interest on the option premium (c), so the total payment (A) is expressed as $A = c(1 + r)^T$. The payoff on a European call, therefore, would be $\max(S_T - X - A, -A)$. The deferral is the key difference between a futures-style option and a regular equity-style option, with the equity-style option requiring the payment of an up-front premium.

Transforming Standard American Options Into Nonstandard American Options

LO 41.d: Describe how standard American options can be transformed into nonstandard American options.

Recall that standard exchange-traded American options can be exercised at any time before expiration. If some of the available expiration periods are restricted, or changes are made to other standard features, standard options become what we call **nonstandard options**. Nonstandard options are common in the over-the-counter (OTC) market.

There are three common features that transform standard American options into nonstandard options:

- The most common transformation can be made to restrict early exercise to certain dates (e.g., a three-month call option may only be exercised on the last day of each month). This type of transformation results in a **Bermudan option**.
- Early exercise can be limited to a certain portion of the life of the option (e.g., there is a "lockout" period that does not allow a six-month call option to be exercised in the first three months of the call's life).
- The option's strike price may change (e.g., the strike price of a three-year call option with a strike price of 40 at initiation may rise to 44 in Year 2 and 48 in Year 3).

MODULE QUIZ 41.1

1. A Bermudan option is one where the:
   A. volatility is assumed to increase.

B. exercise is restricted to certain dates.
C. strike price is changed to half the initial stock price.
D. strike price is chosen to be the average between the maximum and minimum stock price over the life of the option.

MODULE 41.2: TYPES OF EXOTIC OPTIONS

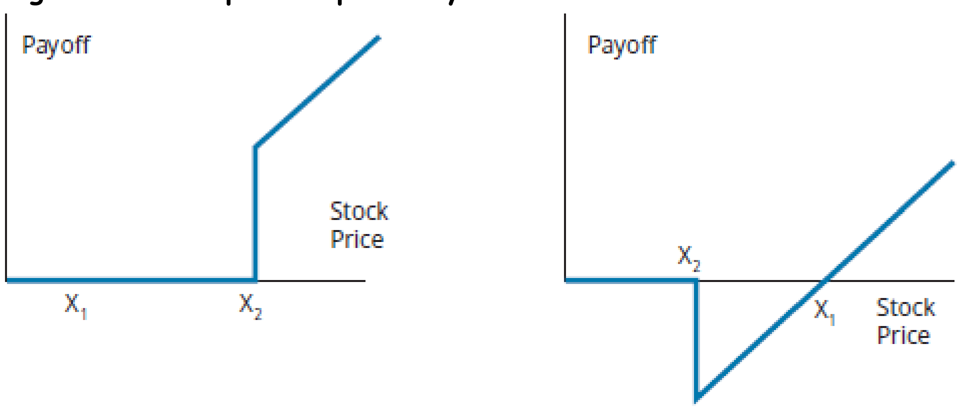
LO 41.e: Identify and describe the characteristics and payoff structures of the following exotic options: gap, forward start, compound, chooser, barrier, binary, lookback, Asian, exchange, and basket options.

Gap Options

A gap option is a European option with two strike prices, X_1 and X_2. (X_2 is sometimes called the trigger price.) If these two strike prices are equal, the gap option payoff will be the same as an ordinary option. If the two strike prices differ and the payoff for a gap option is non-zero, there will be a gap in the payoff graph that is either increased or decreased by the difference between the strike prices.

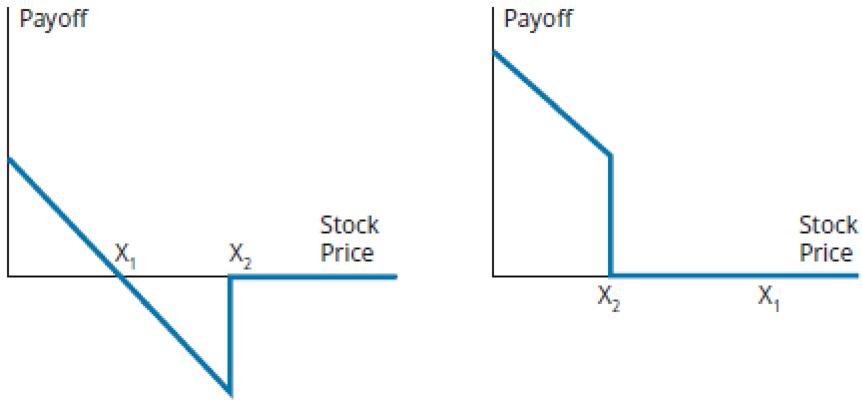
For a *gap call option*, if X_2 is greater than X_1, and the stock price at maturity, S_T, is greater than the trigger price, X_2, then the payoff for the call option will be equal to $S_T - X_1$. If the stock price is less than or equal to X_2, the payoff will be zero. Note that a negative payoff can occur if the stock price is greater than X_2 and X_2 is less than X_1. In this case, the payoff will be reduced by $X_2 - X_1$.

Figure 41.1: Gap Call Option Payoffs



For a *gap put option*, if X_2 is less than X_1, and the stock price at maturity, S_T, is less than the trigger price, X_2, then the payoff for the put option will be equal to $X_1 - S_T$. If the stock price is greater than or equal to X_2, the payoff will be zero. A negative payoff can occur if the stock price is less than X_2 and X_2 is greater than X_1. As with a gap call option, if this is the case, the payoff will be reduced by $X_2 - X_1$.

Figure 41.2: Gap Put Option Payoffs



Forward Start Options

Forward start options are options that begin their existence at some time in the future. For example, today an investor may purchase a three-month call option that will not come into existence until six months from today. Employee incentive plans commonly incorporate forward start options in which at-the-money options will be created after some period of employment has passed. Note that when the underlying asset is a non-dividend-paying stock, the value of a forward start option will be identical to the value of a European at-the-money option with the same time to expiration as the forward start option.

Cliquet options are structured as a set of forward start options that have specific guidelines on computing the exercise prices. For example, it could be structured as three put options: a one-year option, a one-year option beginning in one year, and a one-year option beginning in two years. In other words, it is a one-year option plus two forward start options.

Compound Options

Compound options are options on options. There are four key types of compound options:

- A *call on a call* gives the investor the right to buy a call option at a set price for a set period of time.
- A *call on a put* gives the investor the right to buy a put option at a set price for a set period of time.
- A *put on a call* gives the investor the right to sell a call option at a set price for a set period of time.
- A *put on a put* gives the investor the right to sell a put option at a set price for a set period of time.

Compound options have two levels of the underlying that determine their value—the value of the underlying option, which in turn is determined by the value of the underlying asset.

Compound options consist of two strike prices and two exercise dates. The first strike price and exercise date are used by the holder to evaluate whether to exercise the first option to receive the second option, where the second option is an option on the underlying asset, or just let the compound option expire. For example, a call on a call would be exercised if the price of the call on the underlying for the second call option were greater than the strike price of the initial option. The strike price and exercise date on the second call, however, are related to the value of the underlying asset.

Compared with standard options, compound options provide more leverage potential and are more sensitive to price volatility fluctuations.

Chooser Options

This option allows the buyer, after a certain amount of time has elapsed (but before expiration), to choose whether the option is a call or a put. The option with the greater value after the requisite time has elapsed will determine whether the owner will choose the option to be a put or a call.

Assuming T_1 = the time when the option choice is made and T_2 = expiration date, a chooser option can be thought of as a package of two European options: (1) call option with exercise price X expiring at T_2, and (2) put option with exercise price $PV(X)$ expiring at T_1.

Barrier Options

Barrier options are European options whose payoffs (and existence) depend on whether the underlying's asset price reaches a certain barrier level over the life of the option. These options are usually less expensive than standard options and essentially come in either *knock-out* or *knock-in* flavors. Specific types of barrier options are as follows:

- *Down-and-out call (put)*. A standard call (put) option that ceases to exist if the underlying asset price hits the barrier level, which is set below the current stock value.
- *Down-and-in call (put)*. A standard call (put) option that only comes into existence if the underlying asset price hits the barrier level, which is set below the current stock value.
- *Up-and-out call (put)*. A standard call (put) option that ceases to exist if the underlying asset price hits a barrier level, which is set above the current stock value.
- *Up-and-in call (put)*. A standard call (put) option that only comes into existence if the underlying asset price hits the above-current-stock-price barrier level.

Barrier options have characteristics that can be very different from those of standard options. For example, vega, the sensitivity of an option's price to changes in volatility, is always positive for a standard option but may be negative for a barrier option. Increased volatility on a down-and-out option and an up-and-out option does not increase value because the closer the underlying gets to the barrier price, the greater the chance the option will expire. In addition, just like binary and gap options, there is discontinuity in the payoff profile.

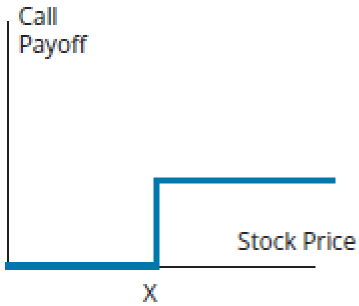
Note that the value of a down-and-out call combined with the value of a down-and-in call is equal to the value of a standard call option. In other words, by knowing the value of two of these three options you can calculate the value of the remaining option (e.g., down-and-out call = standard call – down-and-in call). Similarly, the value of a standard put option is equal to the value of an up-and-out put plus the value of an up-and-in put.

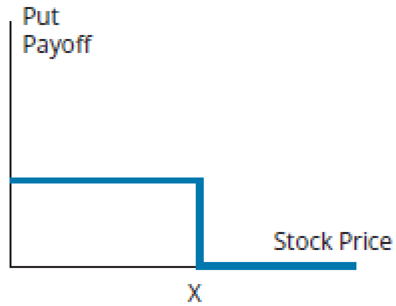
Binary Options

Binary options generate discontinuous payoff profiles because they pay only one price at expiration if the asset value is above the strike price. The term *binary* means that the option payoff has one of two states: the option pays a set dollar amount at expiration if the option is above the strike price, or the option pays nothing if the price is below the strike price. Hence, a payoff discontinuity results from the fact that the payoff is only one value—it does not increase continuously with the price of the underlying asset as in the case of a traditional option.

In the case of a **cash-or-nothing call (put)**, a fixed amount, Q, is paid if the asset ends up above (below) the strike price. Otherwise, no payment is made.

Figure 41.3: Cash-or-Nothing Call and Put Payoffs

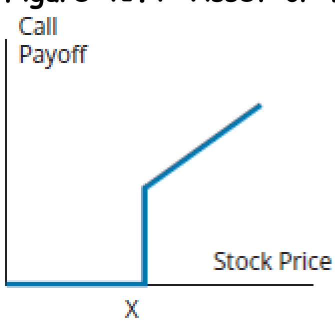


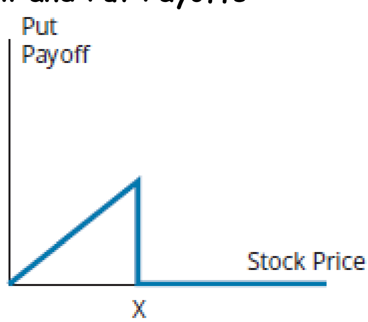


An **asset-or-nothing call (put)** pays the value of the stock if the stock price ends up above (below) the strike price at expiration. Otherwise, no payment is made.

Structurally, the buyer of a European call option is (1) long an asset-or-nothing call and (2) short a cash-or-nothing call that has the exercise price as the payoff.

Figure 41.4: Asset-or-Nothing Call and Put Payoffs





Lookback Options

Lookback options are options whose payoffs depend on the maximum or minimum price of the underlying asset during the life of the option. A **floating lookback call** pays the difference between the expiration price and the minimum price of the stock over the horizon of the option. This essentially allows the owner to purchase the security at its lowest price over the option's life. On the other hand, a **floating**

lookback put pays the difference between the expiration and maximum price of the stock over the time period of the option. This translates into allowing the owner of the option to sell the security at its highest price over the life of the option.

Lookback options can also be fixed when an exercise price is specified. A **fixed lookback call** has a payoff function that is identical to a European call option. However, for this exotic option, the final stock price (or expiration price) in the European call option payoff is replaced by the maximum price during the option's life. Similarly, a **fixed lookback put** has a payoff like a European put option but replaces the final stock price with the minimum price during the option's life.

Given the potential benefits that could accrue to the owner of the option, all things being equal, lookback options will command a higher premium than standard options. Lookback options are more valuable if the underlying asset is looked at more often.

Asian Options

Asian options have payoff profiles based on the average price of the security over the life of the option. *Average price* calls and puts pay off the difference between the average stock price and the strike price. Note that the average price will be much less volatile than the actual price. This means that the price for an Asian average price option will be lower than the price of a comparable standard option. *Average strike* calls and average strike puts pay off the difference between the stock expiration price and average price, which essentially represents the strike price in a typical intrinsic value calculation.

All other things being equal, Asian options are cheaper than regular options and could be more effectively used for hedging. For example, if a firm needs to hedge its weekly exposure to foreign exchange rate fluctuations for the next six months, it would be cheaper to purchase an Asian option for the six-month period rather than 26 regular weekly options.

Because most Asian options base their average calculations on arithmetic averages, it complicates the pricing process and a lognormal distribution of prices is assumed to provide an adequate approximation.

Exchange Options

A common use of an option to exchange one asset for another, often called an exchange option, is to exchange one currency with another. For example, consider a U.S. investor who holds an option to purchase euros with yen at a specified exchange rate. In this particular case, the option will be exercised if euros are more valuable than yen to the U.S. investor. The option can be thought of as a long position in yen plus an option to exchange yen for euros (if euros are more valuable) or a long position in euros plus an option to exchange euros for yen (if yen are more valuable).

Basket Options

Basket options are simply options to purchase or sell multiple securities. These baskets may be defined specifically for the individual investor and may be composed of specific stocks, indices, or currencies. From a hedging perspective, they may be cheaper because only one trade is required to cover the exposure rather than multiple trades. From a pricing perspective, basket options are more (less) expensive when there is more (less) correlation of returns between the underlying securities.

Volatility and Variance Swaps

LO 41.f: Describe and contrast volatility swaps and variance swaps.

A **volatility swap** involves the exchange of volatility based on a notional principal. One side of the swap pays based on a prespecified fixed volatility, while the other side pays based on realized volatility; so like a forward contract, it is zero-sum game between the two sides. Unlike the exotic options we have discussed thus far, volatility swaps are a bet on volatility alone as opposed to a bet on volatility and the price of the underlying asset. In calculating the realized volatility (in the process of calculating the payoff), daily volatility must be multiplied by the square root of 252 (estimate of the number of annual trading days) to arrive at annual volatility.

Much like a volatility swap, a **variance swap** involves exchanging a prespecified fixed variance rate for a realized variance rate. The variance rate being exchanged is simply the square of the volatility rate. However, unlike volatility swaps, variance swaps are easier to price and hedge because they can be replicated using a collection of call and put options.

Static Option Replication

LO 41.g: Explain the basic premise of static option replication and how it can be applied to hedging exotic options.

The typical dynamic option-hedging situation uses option Greeks to measure sensitivity of the option value to changes in underlying asset characteristics (i.e., creating a delta-neutral portfolio). Hedging is simpler with some exotic options than it is with plain vanilla options. Asian options, for instance, depend on the average price of the underlying. Through time, the uncertainty of the average value gets smaller. Hence, the option begins to become less sensitive to changes in the value of the security because the payoff can be estimated more accurately.

Hedging positions in barrier and other exotic options are not so straightforward. This type of hedging requires the replication of a portfolio that is exactly opposite to the option position, so when the underlying asset price moves toward the barrier price, it is more difficult to estimate the payoff accurately.

To address the problem, a **static options replication** approach may be used to hedge positions in exotic options. In this case, a short portfolio of actively traded options that approximates the option position to be hedged is constructed. This short replication options portfolio is created once without any changes until the relevant barrier is reached (for a barrier option, for example). Then the existing hedging portfolio must be unwound and a new hedging portfolio is established.

MODULE QUIZ 41.2

1. A down-and-in call option is an option that comes into existence only when the underlying asset price:
   A. rises to a set barrier level.
   B. falls to a set barrier level.
   C. falls to a set average barrier level.
   D. rises to a set average barrier level.

2. A cash-or-nothing put option has a payout profile equivalent to zero or:
   A. the underlying asset price if the value of the asset ends below the strike price.
   B. the underlying asset price if the value of the asset ends above the strike price.
   C. a set amount if the value of the asset ends below the strike price.
   D. a set amount if the value of the asset ends above the strike price.

3. An Asian option can be hedged dynamically because the:
   A. average value of the underlying asset price decreases uncertainty the closer the option gets to expiration.
   B. average value of the underlying asset price increases uncertainty the closer the option gets to expiration.
   C. maximum value of the underlying asset price decreases uncertainty the closer the option gets to expiration.
   D. minimum value of the underlying asset price increases uncertainty the closer the option gets to expiration.

4. Which of the following options is most likely to have a negative vega?
   A. A chooser option close to expiration.
   B. A forward start put option before the start date.
   C. An Asian put option close to the beginning of the option's life.
   D. An up-and-out put when the stock price is close to the barrier.

5. Under which of the following circumstances would the value of an up-and-out call option be zero?
   A. The strike price is above the barrier price.
   B. The stock price is below the barrier price.
   C. The stock price is above the strike price.
   D. The stock price is below the strike price.

KEY CONCEPTS

LO 41.a

Plain vanilla derivatives include listed futures contracts and commonly used forwards and other OTC derivatives that are traded in fairly liquid markets. Exotic derivatives are customized to fit a specific firm need.

LO 41.b

The main purpose for the development of exotic derivatives is to provide a unique hedge for a firm's underlying assets. Additional reasons include addressing tax and regulatory concerns, as well as speculating on the expected future direction of market prices.

LO 41.c

Packages are portfolios of European options. Given that packages often consist of a long position and a short position, they can be constructed so that the initial cost to the investor is zero. Additionally, an option can be structured as zero-cost if the otherwise up-front premium can be deferred to the expiration date of the option.

LO 41.d

Restricting exercise dates and changing strike prices can transform standard options into nonstandard options.

LO 41.e

A gap option has two strike prices. If the two strike prices differ and the payoff is non-zero, there will be a gap in the payoff graph that is either increased or decreased by the difference between the strike prices.

Forward start options (including cliquet options) are options that commence in the future.

A compound option is defined as an option on another option.

Chooser options allow the owner to choose whether the option is a call or a put, after option initiation.

Barrier options are options whose payoffs (and existence) depend on whether the underlying's asset price reaches a certain barrier level over the life of the option.

Binary options either pay nothing (if price is below strike price) or a fixed amount at expiration.

Lookback options depend on the maximum or minimum value of the underlying asset during the life of the option.

Asian options have payoff profiles that depend on the average underlying asset price over the life of the option.

An exchange option is an option to exchange one asset for another.

Basket options allow the owner to buy or sell portfolios of assets, thereby reducing hedging costs.

LO 41.f

A volatility swap involves the exchange of volatility based on a notional principal. A variance swap involves exchanging a prespecified fixed variance rate for a realized variance rate.

LO 41.g

Exotic options can be hedged in a static context, which involves shorting a portfolio of options that has about the same value as an exotic option on a boundary.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 41.1

1. **B** Bermudan options are options that restrict exercise to certain dates, not any time over the life of the option. (LO 41.d)

Module Quiz 41.2

1. **B** Down-and-in call options are standard options that come into existence only if the asset price falls to a set barrier price level, which is set below the current stock price. (LO 41.e)

2. **C** Cash-or-nothing put options pay only a set amount if the stock price ends below the strike price. These options differ from standard put options because the payment is a set amount that does not continuously increase with the decrease in stock price. (LO 41.e)

3. **A** Dynamic hedging can be used to hedge Asian options because uncertainty in the expiration value is decreased the closer one gets to expiration. This occurs because the intrinsic value becomes "set" due to the averaging effect over the life of the option. (LO 41.e)

4. **D** Vega is the sensitivity of the price of an option to changes in volatility of the underlying stock. For most options, vega is always positive—as volatility of the underlying stock increases, the price of the option also increases. An exception would be a knock-out barrier option when the stock price is close to the barrier. Higher volatility means the barrier is more likely to be reached and the option will cease to exist. (LO 41.e)

5. **A** With an up-and-out call, if the stock price rises beyond the barrier price, the option ceases to exist. It therefore follows that if the strike price is above the barrier price, the option will never come into the money because the option will cease to exist before the option will ever come into the money. (LO 41.e)

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Financial Markets and Products, Chapter 16.

READING 42

PROPERTIES OF INTEREST RATES

Study Session 11

EXAM FOCUS

Spot, or zero, rates are computed from coupon bonds using a method known as bootstrapping. Forward rates can then be computed from the spot or zero curve. For the exam, understand how to use the bootstrapping method and how to compute forward rates from spot rates. Also, be familiar with discrete and continuous compounding methods. In addition, the concepts of duration and convexity and how they are used to determine the change in a bond's price are crucial.

MODULE 42.1: TYPES OF INTEREST RATES

LO 42.a: Describe Treasury rates, LIBOR, Secured Overnight Financing Rate (SOFR), and repo rates, and explain what is meant by the "risk-free" rate.

Interest rates that play a key role in interest rate derivatives include Treasury rates, the Secured Overnight Financing Rate (SOFR), repo rates, and overnight indexed swap (OIS) rates. Keep in mind that interest rates increase as the credit risk of the underlying instrument increases.

- **Treasury rate.** Treasury rates are the rates that correspond to government borrowing in its own currency. They are considered risk-free rates.
- **LIBOR.** The London Interbank Offered Rate (LIBOR) was the rate at which large international banks funded their activities. However, LIBOR was based on estimates, which subjected it to potential manipulation. This was a contributing reason why LIBOR was phased out in mid-2023.
- **SOFR.** The Secured Overnight Financing Rate (SOFR) is a one-day, repo-based rate that is derived from actual transactions. It is one of the proposed replacements for LIBOR.
- **Repo rate.** The "repo" or repurchase agreement rate is the implied rate on a repurchase agreement. In a repo agreement, one party agrees to sell a security to another with the understanding that the selling party will buy it back later at a specified higher price. The interest rate implied by the price differential is the repo

rate. The most common repo is the overnight repurchase agreement. Longer-term agreements are called term repos. Depending on the parties and structure involved, there is some credit risk with repurchase agreements.

The overnight rate is the rate at which large financial institutions borrow from each other in the overnight market. In the United States, this rate is called the federal funds rate and is monitored and influenced by the central bank. If a financial institution borrows (lends) funds at the overnight rate, the rate it pays (earns) during the period is the weighted average of the overnight rates.

An **overnight indexed swap (OIS)** is an interest rate swap where a fixed rate is exchanged for a floating rate and where the floating rate is the geometric average of the overnight federal funds rates during the period. The fixed rate in the OIS is known as the **OIS rate**. If the fixed rate is greater than the geometric average of the overnight rates, the fixed side makes a payment; otherwise, the floating side makes a payment.

As mentioned, Treasury rates (such as T-bill and T-bond rates) are often considered the benchmark for nominal risk-free rates. However, derivative traders view these rates as being too low to be considered risk free (because part of the demand for these bonds comes from fulfilling regulatory requirements, which drives prices up and rates down). As a result, traders instead use OIS rates for short-term risk-free rates, because OIS better reflects a trader's opportunity cost of capital.

Compounding Frequencies

LO 42.b: Calculate the value of an investment using different compounding frequencies.

LO 42.c: Convert interest rates based on different compounding frequencies.

Derivative pricing often uses a framework called continuous time mathematics. In this framework, it is assumed that returns are continuously compounded. This is a theoretical construct only because returns cannot literally be compounded continuously. Fortunately, converting discrete compounding to continuous compounding is straightforward.

If we have an initial investment of A that earns an annual rate R, compounded m times a year for n years, then it has a future value of:

$$FV_1 = A\left(1 + \frac{R}{m}\right)^{m \times n}$$

If our same investment is continuously compounded over that period, it has a future value of:

$$FV_2 = Ae^{R \times n}$$

For any rate, R, FV_2 will always be greater than FV_1. The difference will decrease as m increases. In fact, as m becomes infinitely large, the difference goes to zero.

In most circumstances, rates are discretely compounded, so we need to find the continuously compounded rate that gives the same future value. Using the previous two equations, the goal is to solve the following:

$$A\left(1+\frac{R}{m}\right)^{m\times n} = Ae^{R_c n}$$

where:
R_c = continuous rate

We can solve for R_c as:

$$R_c = m \times \ln\left(1+\frac{R}{m}\right)$$

We can also solve for R as:

$$R = m\left(e^{R_c/m} - 1\right)$$

PROFESSOR'S NOTE

In order to algebraically solve for R or R_c, given one of the previous equations, it is helpful to understand that *e* is the base of the natural log (ln). In other words, the natural log is the inverse function of the exponential function:

$$e^{\ln(x)} = \ln(e^x) = x$$

So if you are given an equation such that $R = e^x$, *x* will be equal to ln(R).

EXAMPLE: Computing continuous rates

Suppose we have a 5% rate that is compounded semiannually. **Compute** the corresponding continuous rate. Repeat this for quarterly, monthly, weekly, and daily compounding.

Answer:

$$R_c = 2\ln\left(1+\frac{0.05}{2}\right) = 0.049385$$

The results for other compounding frequencies are shown in Figure 42.1.

Figure 42.1: Compounding Frequencies and Returns

M	R_c
4	0.049690
12	0.049896
52	0.049976
250	0.049995

Notice that as *m* increases, the difference between the rates decreases.

EXAMPLE: Discrete compounding rate

A loan is quoted at 12% annually with continuous compounding. Interest is paid monthly. **Calculate** the equivalent rate with monthly compounding.

Answer:

$$R = 12(e^{0.12/12} - 1) = 12.06\%$$

MODULE QUIZ 42.1

1. What is the continuously compounded rate of return for an investment that has a value today of $86.50 and will have a future value of $100 in one year?
   A. 13.62%.
   B. 14.50%.
   C. 15.61%.
   D. 16.38%.

MODULE 42.2: SPOT RATES, FORWARD RATES, AND FORWARD RATE AGREEMENTS

Spot (Zero) Rates

LO 42.d: Calculate the theoretical price of a bond using spot rates.

LO 42.j: Calculate zero-coupon rates using the bootstrap method.

Spot rates are the rates that correspond to zero-coupon bond yields. They are the appropriate discount rates for a single cash flow at a particular future time or maturity. Spot rates are also often called *zero rates*. Most interest rates that are observed in the market, such as coupon bond yields, are not spot rates.

Bond Pricing

A coupon bond makes a series of cash flows. Each cash flow considered in isolation is equivalent to a zero-coupon bond. Using this interpretation, a coupon bond is a series of zero-coupon bonds. Its value assuming semiannual coupons is:

$$B = \left[\frac{c}{2} \times \left(1 + \frac{z_j}{2}\right)^{-2\times j}\right] + \left[FV \times \left(1 + \frac{z_N}{2}\right)^{-N}\right]$$

where:

c = annual coupon
N = number of semiannual payment periods
z_j = bond equivalent spot rate that corresponds to *j* periods
FV = face value of the bond

Don't let this formula intimidate you. It simply says that the value of a bond is the present value of its cash flows, where each cash flow is discounted at the appropriate spot rate for its maturity. Note that the negative sign in the exponent just means that the coupon and principal payments are being discounted back to the present.

Alternately, the semiannual cash flows could be divided by (1 + the periodic rate) taken to the power of the number of semiannual periods:

$$\frac{c/2}{(1 + z_j/2)^{2 \times j}}$$

The following example is a good illustration of this discounting process.

EXAMPLE: Calculating bond price

Compute the price of a $100 face value, two-year, 4% semiannual coupon bond using the annualized spot rates in Figure 42.2.

Figure 42.2: Spot Rates

Maturity (Years)	Spot Rate (%)
0.5	2.5
1.0	2.6
1.5	2.7
2.0	2.9

Answer:

$$B = \frac{\$2}{(1 + 0.025/2)^1} + \frac{\$2}{(1 + 0.026/2)^2} + \frac{\$2}{(1 + 0.027/2)^3} + \frac{\$102}{(1 + 0.029/2)^4} = \$102.14$$

Note that this formula uses semiannual compounding (i.e., discrete compounding). If continuous compounding were used instead, the bond price would be computed as follows:

$$B = \left(2 \times e^{-\frac{0.025}{2} \times 1}\right) + \left(2 \times e^{-\frac{0.026}{2} \times 2}\right) + \left(2 \times e^{-\frac{0.027}{2} \times 3}\right) + \left(102 \times e^{-\frac{0.029}{2} \times 4}\right) = \$102.10$$

PROFESSOR'S NOTE

Notice that the two discounting approaches will produce a similar result. On the exam, using either method will most likely lead to the same answer choice given that you will need to choose a response that is closest to one listed in the question.

Bond Yield

The yield of a bond is the single discount rate that equates the present value of a bond to its market price. You can use a financial calculator to compute bond yield, as in the following example.

EXAMPLE: Calculating bond yield

Compute the yield for the bond in the previous example (using discrete compounding).

Answer:

$$PMT = 2; N = 4; PV = -102.14; FV = 100; CPT \rightarrow I/Y = 1.446;$$
$$Y = 1.446\% \times 2 \approx 2.89\%$$

The bond's **par yield** is the rate that makes the price of a bond equal to its par value. When the bond is trading at par, the coupon will be equal to the bond's yield.

Bootstrapping Spot Rates

The theoretical spot curve is derived by interpreting each Treasury bond (T-bond) as a package of zero-coupon bonds. Using the prices for each bond, the spot curve is computed using the bootstrapping methodology.

For example, suppose there is a T-bond maturing on a coupon date in exactly six months. Further assume that the bond is priced at 102.2969% of par and has a semiannual coupon of 6.125% How is the corresponding spot rate computed? In this case, this is truly a zero-coupon bond, since there is only one cash flow, which occurs in six months. Simply solve for z_1 in the bond valuation equation, given the price, as follows:

$$\$102.2969 = \frac{\$100 + (\$6.125/2)}{1 + z_1/2}$$

Solving this for z_1 produces:

$$z_1 = 2 \times \left(\frac{\$100 + (\$6.125/2)}{\$102.2969} - 1 \right) = 1.497\%$$

The six-month spot rate on a bond equivalent basis is 1.497%. Also note that the yield to maturity (YTM) did not need to be computed in this case because the YTM and the spot rate are the same.

How is the spot rate that corresponds to one year found? Suppose a T-bond that matures in one year is priced at 104.0469% of par and has a semiannual coupon of 6.25%. From the previous computation, the six-month spot rate is known, so the bond valuation equation can be written as:

$$\$104.0469 = \frac{\$6.25/2}{(1 + 0.01497/2)^1} + \frac{\$100 + (\$6.25/2)}{(1 + z_2/2)^2}$$
$$z_2 = 0.02148 = 2.148\%$$

Thus, the one-year spot rate with discrete compounding is 2.148%.

EXAMPLE: Bootstrapping spot rates

Compute the corresponding spot rate curve using the information in Figure 42.3. Note that we've already computed the first two spot rates.

Figure 42.3: Input Information to Bootstrap Spot Rates

Price as a Percentage of Par	Coupon	Semiannual Period	Maturity (Years)
102.2969	6.125	1	0.5
104.0469	6.250	2	1.0
104.0000	5.250	3	1.5
103.5469	4.750	4	2.0

Answer:

The spot rates derived by bootstrapping are shown in Figure 42.4.

Figure 42.4: Bootstrapped Spot Rate Curve

Price as a Percentage of Par	Coupon	Semiannual Period	Maturity (Years)	Spot Rates
102.2969	6.125	1	0.5	1.497%
104.0469	6.250	2	1.0	2.148%
104.0000	5.250	3	1.5	2.531%
103.5469	4.750	4	2.0	2.936%

An alternative verification is to use the spot rates to check if they result in the same prices using the bond valuation equation. For example, using the spot rates will ensure computation of the same price for the two-year bond:

$$B = \frac{\$4.75/2}{(1+0.01497/2)^1} + \frac{\$4.75/2}{(1+0.02148/2)^2}$$

$$+ \frac{\$4.75/2}{(1+0.02531/2)^3} + \frac{\$100+(\$4.75/2)}{(1+0.02936/2)^4} = \$103.5469$$

This results in a bond price of $103.5469. Notice that this is exactly the price of the two-year bond.

Forward Rates

LO 42.h: Derive forward interest rates from a set of spot rates.

Forward rates are interest rates implied by the spot curve for a specified future period. Recall that spot rates are the appropriate rates that an investor should expect to realize for various maturities. Suppose an investor is faced with the following two investments, which are based on the spot curve.

1. Invest for two years at 2.915%.

2. Invest for a year at 2.136%, and then roll over that investment for another year at the forward rate.

It does not matter which investment is chosen if they both offer the same return at the end of two years. This is the same as stating that both strategies give the same future value at the end of two years. Equating the two future values:

$$e^{\frac{0.02915}{2}\times 4} = e^{\frac{0.02136}{2}\times 2} \times e^{\frac{R_{Forward}}{2}\times 2}$$

where:

$R_{Forward}$ = one-year forward rate one year from now

For the two strategies to be equal, $R_{Forward}$ must be 3.694%.

We can simplify this calculation by using the following equation (which assumes continuously compounded rates):

$$R_{Forward} = \frac{R_2T_2 - R_1T_1}{T_2 - T_1} = R_2 + (R_2 - R_1) \times \left(\frac{T_1}{T_2 - T_1}\right)$$

where:

R_i = spot rate corresponding with T_i periods

$R_{Forward}$ = forward rate between T_1 and T_2

For example, if the one-year rate is 2.136% and the two-year rate is 2.915%, the one-year forward rate one year from now is:

$$R_{Forward} = 0.02915 + (0.02915 - 0.02136) \times \left(\frac{1}{2-1}\right) = 0.03694 = 3.694\%$$

This is the same forward rate that was calculated before.

As a further example, consider the problem of finding the one-year forward rate three years from now, given a three-year spot rate of 7.424% and a four-year spot rate of 8.216% (both continuously compounded annual rates). Based on the previous formula, the continuously compounded one-year rate three years from now is:

$$0.08216 + (0.08216 - 0.07424) \times \frac{3}{4-3} = 0.10592$$

Forward Rate Agreements

LO 42.i: Derive the value of the cash flows from a forward rate agreement (FRA).

A **forward rate agreement (FRA)** is a forward contract obligating two parties to agree that a certain interest rate will apply to a principal amount during a specified future time. Obviously, forward rates play a crucial role in the valuation of FRAs. The T_2 cash flow of an FRA that promises the receipt or payment of R_K is:

$$\text{cash flow (if receiving } R_K) = L \times (R_K - R) \times (T_2 - T_1)$$

$$\text{cash flow (if paying } R_K) = L \times (R - R_K) \times (T_2 - T_1)$$

where:
L = principal
R_K = annualized fixed rate, expressed with compounding period $T_2 - T_1$
R = annualized floating rate, expressed with compounding period $T_2 - T_1$
T_i = time i, expressed in years

The value of an FRA if receiving or paying the fixed interest rate is:

$$\text{value (if receiving } R_K) = PV\left(\frac{L \times (R_K - R_{Forward}) \times (T_2 - T_1)}{1 + R_{Forward} \times (T_2 - T_1)}\right)$$

$$\text{value (if paying } R_K) = PV\left(\frac{L \times (R_{Forward} - R_K) \times (T_2 - T_1)}{1 + R_{Forward} \times (T_2 - T_1)}\right)$$

where:
$R_{Forward}$ = forward rate between T_1 and T_2

Any interest should be due at the end of the FRA period (e.g., six months in the example that follows); however, common practice is for the FRA to be settled at the beginning of the FRA period. Therefore, any payoffs must be discounted at the relevant discount rate (i.e., the risk-free rate from the beginning of the FRA period) to arrive at the value of the FRA.

EXAMPLE: Computing the payoff from an FRA

Suppose an investor has entered into an FRA where he has contracted to pay a fixed rate of 3% on $1 million based on the quarterly rate in three months. Assume that rates are compounded quarterly. **Compute** the payoff from the FRA if the quarterly rate is 1% in three months.

Answer:

For this FRA, the payoff will take place in six months. The net payoff will be the difference between the fixed-rate payment and the floating rate receipt. If the floating rate is 1% in three months, the payoff at the end of the sixth month will be:

$$\$1{,}000{,}000(0.01 - 0.03)(0.25) = -\$5{,}000$$

EXAMPLE: Computing the payoff from an FRA

Suppose the three-month and six-month floating rates are 4% and 5%, respectively (continuously compounded rates). An investor enters into an FRA in which she will receive 8% (assuming quarterly compounding) on a principal of $5,000,000 between Months 3 and 6. **Calculate** the payoff from the FRA.

Answer:

$$R_{Forward} = 0.05 + (0.05 - 0.04) \times \left(\frac{1}{2-1}\right) = 0.06 = 6\%$$

$$R_{Forward} \text{ (with quarterly compounding)} = 4 \times \left(e^{0.06/4} - 1\right)$$
$$= 0.060452 = 6.05\%$$

$$\text{payoff} = \frac{\$5{,}000{,}000 \times (0.08 - 0.0605) \times (0.50 - 0.25)}{1 + 0.05 \times (0.50 - 0.25)}$$
$$= \$24{,}074$$

Term Structure Theories

LO 42.k: Compare and contrast the major theories of the term structure of interest rates.

The **market segmentation theory** states that the bond market is segmented into different maturity sectors and that supply and demand for bonds in each maturity range dictate rates in that maturity range. The market segmentation theory does not fully make sense because many investors are more likely to move between the maturity sector based on the attractiveness of the available yields.

The **expectations theory** suggests that forward rates correspond to expected future spot rates. That is, forward rates are good predictors of expected future spot rates. An expectation of rising (falling) interest rates would suggest an upward-sloping (downward-sloping) yield curve. In reality, the expectations theory may be in doubt because upward-sloping yield curves occur far more frequently than downward-sloping and a logical expectation would be for upward- and downward-sloping curves to occur with equal frequency.

The **liquidity preference theory** attempts to clear up the doubt with the expectations theory. Liquidity preference suggests that most depositors prefer short-term liquid deposits to meet current needs. In order to coax them to lend/invest longer term, the intermediary will raise longer-term rates by adding a liquidity premium.

MODULE QUIZ 42.2

1. What is the bond price of a $100 face value, 2.5-year, 3% semiannual coupon bond using the following annual spot rates: z_1 = 3%, z_2 = 3.1%, z_3 = 3.2%, z_4 = 3.3%, and z_5 = 3.4%?
   A. $97.27.
   B. $97.83.
   C. $98.15.
   D. $99.07.

2. Assume that the continuously compounded 10-year spot rate is 5% and the 9-year spot rate is 4.9%. Which of the following amounts is closest to the 1-year forward rate 9 years from now?
   A. 4.1%.
   B. 5.1%.

C. 5.9%.
D. 6.0%.

3. An investor enters into a one-year forward rate agreement (FRA) where she will receive the contracted rate on a principal of $1 million. The contracted rate is a one-year rate at 5%. Which of the following amounts is closest to the cash flow if the actual rate is 6% at maturity of the underlying asset (loan)?
   A. –$10,000.
   B. –$1,000.
   C. +$1,000.
   D. +$10,000.

MODULE 42.3: DURATION AND CONVEXITY

LO 42.e: Calculate the Macaulay duration, modified duration, and dollar duration of a bond.

The duration of a bond is the average time until the cash flows on the bond are received. For a zero-coupon bond, this is simply the time to maturity. For a coupon bond, its duration will be necessarily shorter than its maturity. The weights on the time in years until each cash flow is to be received are the proportion of the bond's value represented by each of the coupon payments and the maturity payment. The formula for duration using continuously compounded discounting of the cash flows is:

$$\text{duration} = \sum_{i=1}^{n} t_i \left[\frac{c_i e^{-yt_i}}{B}\right]$$

where:
t_i = time (in years) until cash flow c_i is to be received
y = continuously compounded yield (discount rate) based on a bond price of B

The usefulness of the duration measure lies in the fact that the approximate change in a bond's price, *B*, for a parallel shift in the yield curve of Δ*y* is:

$$\frac{\Delta B}{B} = -\text{duration} \times \Delta y$$

The change in yield is often expressed as a **basis point** change. One basis point is equivalent to 0.01%. So a 100 basis point change is a change of 1% in the yield. When yields are continuously compounded, the provided duration measure is known as **Macaulay duration**.

Modified duration is used when the yield given is something other than a continuously compounded rate. When the yield is expressed as a semiannually compounded rate, for example, modified duration = duration / (1 + y/2).

In general we can express this relation as:

$$\text{modified duration} = \frac{\text{duration}}{1 + {}^{y}\!/_{m}}$$

where *m* is the number of compounding periods per year.

Note that as *m* goes to infinity (continuous compounding), the two measures are equal and there is no difference between the two.

On the exam, you may also see a reference to **dollar duration**. Dollar duration is simply modified duration multiplied by the price of the bond.

LO 42.f: Evaluate the limitations of duration and explain how convexity addresses some of them.

Duration is a good approximation of price changes for an option-free bond, but it's only good for relatively small changes in interest rates. As rate changes grow larger, the curvature of the bond price/yield relationship becomes more important, meaning that a linear estimate of price changes, such as duration, will contain errors.

In fact, the relationship between bond price and yield is not linear (as assumed by duration) but convex. This convexity shows that the difference between actual and estimated prices widens as the yield swings grow. That is, the widening error in the estimated price is due to the curvature of the actual price path. This is known as the **degree of convexity**.

Fortunately, the amount of convexity in a bond can be measured and used to supplement duration in order to achieve a more accurate estimate of the change in price. It's important to note that all convexity does is account for the amount of error in the estimated price change based on duration. In other words, it picks up where duration leaves off and converts the straight (estimated price) line into a curved line that more closely resembles the convex (actual price) line.

In order to obtain an estimate of the percentage change in price due to convexity, or the amount of price change that is not explained by duration, the following calculation will need to be made:

$$\text{convexity effect} = 1/2 \times \text{convexity} \times \Delta y^2$$

The convexity effect is typically quite small. However, remember that convexity is simply correcting for the error embedded in the duration, so you would expect convexity to have a much smaller effect than duration. Also note that for an option-free bond, the convexity effect is always positive, no matter which direction interest rates move. Thus, for option-free bonds, convexity is always added to duration to modify the price volatility errors embedded in duration. This decreases the drop in price (due to an increase in yields) and adds to the rise in price (due to a fall in yields).

Analogous to modified duration and used together with it, we can express modified convexity as follows: modified convexity = convexity / $(1 + y/m)^2$.

LO 42.g: Calculate the change in a bond's price given its duration, its convexity, and a change in interest rates.

By combining duration and convexity, we can obtain a far more accurate estimate of the percentage change in the price of a bond, especially for large swings in yield. That is, you can account for the amount of convexity embedded in a bond by adding the convexity effect to the duration effect.

EXAMPLE: Estimating price changes with the duration/convexity approach

Estimate the effect of a 100 basis point increase and decrease on a 10-year, 5%, option-free bond currently trading at par, using the duration/convexity approach. The bond has a duration of 7 and a convexity of 90.

Answer:

Using the duration/convexity approach:

$$\text{percentage bond price change} \approx \text{duration effect} + \text{convexity effect}$$

$$\Delta B_{+\Delta y} \approx [-7 \times 0.01] + [(1/2) \times 90 \times (0.01\ 2)]$$

$$\approx -0.07 + 0.0045 = -0.0655 = -6.55\%$$

$$\Delta B_{-\Delta y} \approx [-7 \times -0.01] + \left[(1/2) \times 90 \times \left(-0.01^2\right)\right]$$

$$\approx 0.07 + 0.0045 = 0.0745 = 7.45\%$$

MODULE QUIZ 42.3

1. A 12-year, 8% semiannual coupon bond with $100 par value currently trades at $78.75 and has a duration of 9.8 years and a convexity of 130. What is the price of the bond if the yield falls by 150 basis points?
   A. $67.17.
   B. $86.47.
   C. $91.48.
   D. $95.43.

KEY CONCEPTS

LO 42.a

Interest rates that are particularly relevant in the interest rate derivative markets include Treasury rates, the Secured Overnight Financing Rate (SOFR), and repo rates. Treasury rates (such as T-bill and T-bond rates) are often considered the benchmark for nominal risk-free rates.

An overnight indexed swap (OIS) is an interest rate swap where a fixed rate is exchanged for a floating rate and where the floating rate is the geometric average of the overnight federal funds rates during the period. The fixed rate in the OIS is known as the OIS rate.

LO 42.b

If we have an initial investment of *A* that earns an annual rate *R*, compounded *m* times a year for *n* years, then it has a future value of:

$$FV = A\left(1 + \frac{R}{m}\right)^{m \times n}$$

LO 42.c

In most circumstances, rates are discretely compounded, so we need to find the continuously compounded rate that gives the same future value. The continuous rate can be solved as follows:

$$R_c = m \times \ln\left(1 + \frac{R}{m}\right)$$

LO 42.d

Zero (spot) rates correspond to the interest earned on a single cash flow at a single point in time. Bond prices are computed using the spot curve by discounting each cash flow at the appropriate spot rate.

The yield of a bond is the single discount rate that equates the present value of a bond to its market price.

LO 42.e

Duration and modified duration are the same when continuously compounded yields are used, and they both estimate the percentage price change of a bond from an absolute change in yield. Dollar duration is modified duration multiplied by the price of the bond.

LO 42.f

Duration is only good for relatively small changes in interest rates. As rate changes grow larger, the curvature of the bond price/yield relationship becomes more important, meaning that a linear estimate of price changes, such as duration, will contain errors. The amount of convexity in a bond can be measured and used to supplement duration in order to achieve a more accurate estimate of the change in price.

LO 42.g

The approximate change in a bond's price, B, for a parallel shift in the yield curve of Δy is:

$$\frac{\Delta B}{B} = -\text{duration} \times \Delta y$$

In order to obtain an estimate of the percentage change in price due to convexity, the following calculation will need to be made:

$$\text{convexity effect} = \frac{1}{2} \times \text{convexity} \times \Delta y^2$$

Combining duration and convexity creates a more accurate estimate of the percentage change in the price of a bond:

$$\text{percentage bond price change} \approx \text{duration effect} + \text{convexity effect}$$

LO 42.h

Forward rates are computed from spot rates. When the spot curve is upward-sloping, the corresponding forward rate curve is upward-sloping and above the spot curve.

When the spot curve is downward-sloping, the corresponding forward rate curve is downward-sloping and below the spot curve.

LO 42.i

A forward rate agreement (FRA) is a contract between two parties that an interest rate will apply to a specific principal during some future time period.

LO 42.j

The theoretical spot curve is derived by interpreting each Treasury bond (T-bond) as a package of zero-coupon bonds. Using the prices for each bond, the spot curve is computed using the bootstrapping methodology.

LO 42.k

The market segmentation theory states that bonds are segmented into different maturity sectors and that supply and demand dictate rates in the segmented maturity sectors. The expectations theory suggests that forward rates correspond to expected future spot rates. The liquidity preference theory expands on the expectations theory and suggests that longer-term rates incorporate a liquidity premium.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 42.1

1. **B** The formula to solve this problem is:

$$R_c = m \times \ln\left(1 + \frac{R}{m}\right)$$

First, we need to compute R as the rate earned on the \$86.50 investment:

$$R = \frac{\$100 - \$85.50}{\$86.50} = 0.15607$$

This is essentially the effective rate earned over one year with annual compounding. So, m = 1, and $R_c = 1 \times \ln(1.15607) = 0.1450$. Alternatively, since m = 1:

$$\ln\left(\frac{100}{86.50}\right) = 0.1450 = 14.50\%$$

(LO 42.c)

Module Quiz 42.2

1. **D** $B = 1.5/(1 + 0.03/2) + 1.5/(1 + 0.031/2)^2 + 1.5/(1 + 0.032/2)^3$
$+ 1.5/(1 + 0.033/2)^4 + 101.5/(1 + 0.034/2)^5$
$= 1.478 + 1.455 + 1.430 + 1.405 + 93.30 = \$99.07$

(LO 42.d)

2. **C** $R_{Forward} = R_2 + (R_2 - R_1) \times [T_1 / (T_2 - T_1)]$
$= 0.05 + (0.05 - 0.049) \times [9 / (10 - 9)] = 5.9\%$

(LO 42.h)

3. **A** \$1,000,000(0.05 − 0.06)(1) = −\$10,000

(LO 42.i)

Module Quiz 42.3

1. **C** percentage price change =
$[(-)(\text{duration})(\Delta y)] + [(1/2)(\text{convexity})(\Delta y)^2]$
$= [(-)(9.80)(-0.015)(100)] + [(0.5)(130)(-0.015)^2(100)]$
$= 16.16$

estimated price = 78.75(1 + 0.1616) = $91.48

(LO 42.g)

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Financial Markets and Products, Chapter 17.

READING 43

CORPORATE BONDS

Study Session 11

EXAM FOCUS

The term “bond” refers to a variety of assets that offer a wide range of interest rate payments from fixed cash payments, to accruals without cash, to payments in the form of additional securities. In this reading, we will provide an overview of major fixed-income instruments and their payment structures. We will also address the impact of credit risk and event risk on bond ratings and features. For the exam, be familiar with the types of bonds discussed and the methods for retiring bonds. Also, know the terminology associated with high-yield issues.

MODULE 43.1: CORPORATE BOND FUNDAMENTALS AND TYPES

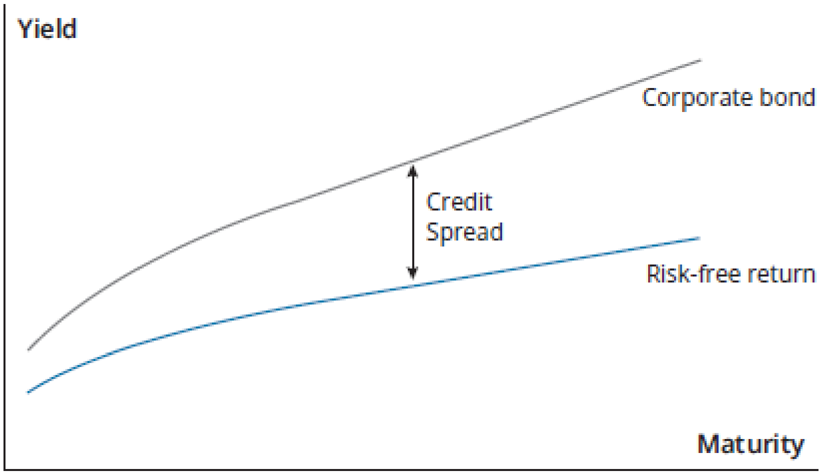
Bond Trading

LO 43.a: Describe features of bond trading and explain the behavior of bond yield.

Publicly traded bonds are usually traded in the over-the-counter (OTC) market as opposed to exchanges. Dealers exist in the bond market to buy and sell bonds and earn profit through the bid-ask spread (e.g., buy low, sell high). Bond pricing is based on the laws of supply and demand. If demand > supply, then price rises, and if demand < supply, then price falls.

A corporate **bond yield** is a function of the risk-free return plus a credit spread to reflect the risk of default. The longer the bond maturity, the greater the risk of default as illustrated in Figure 43.1.

Figure 43.1: Corporate Bond Yield



The fundamental bond pricing rule is that an increase (decrease) in yield leads to a price decrease (increase). Also note that with higher (lower) bond liquidity comes lower (higher) yield demanded by investors.

Bond Indenture and Corporate Trustee

LO 43.b: Describe a bond indenture and explain the role of the corporate trustee in a bond indenture.

The **bond indenture** is a document that sets forth the obligation of the issuer and the rights of the investors in the bonds (i.e., the bondholders). It is usually a detailed document filled with legal language. One of the roles of the **corporate trustee** is to interpret this language and represent the interests of the bondholders. Banks or trust companies most often serve as corporate trustees, and the position requires that they act in a fiduciary capacity on behalf of the bondholders. The trustee would authenticate the issue, which includes keeping track of the amount of bonds issued and making sure the number does not exceed the limit specified in the indenture.

The trustee would monitor the corporation's activities to make sure the issuer abides by the indenture's covenants. Such covenants include *negative* or *restrictive* (e.g., limited additional debt financing, dividend declarations), *positive* (e.g., to produce financial statements, maintain insurance), and *financial* (e.g., maintaining key ratios above/below a given number).

The corporate trustee's requirements are explicitly stated in the indenture, and the trustee only needs to meet those requirements and no more. The indenture would specify how and the frequency with which the trustee would make reports to bondholders and what to do if the issuer fails to pay interest or principal. As mentioned earlier, the basic goal of the trustee is to protect the rights of bondholders.

Bond Classifications

LO 43.f: Describe different characteristics of bonds such as issuer, maturity,

Bond Issuers and Maturities

There are five general groups of bond issuers:

- Utilities
- Transportation companies
- Industrials
- Financial institutions
- Internationals (e.g., World Bank, International Monetary Fund)

Short-term notes have maturities from 1 to 5 years. Medium-term notes have maturities from 5 to 12 years. Long-term bonds have maturities greater than 12 years.

Interest Payment Classifications

Fixed-rate bonds have a fixed interest rate set for the entire life of the issue. In some cases, interest might be paid in a foreign currency.

Floating-rate bonds are also known as variable rate bonds. The interest paid is generally linked to the market reference rate such as SOFR plus a fixed spread. The interest rate (coupon) to be paid is determined at the beginning of the period and the interest is paid at the end of the period.

Zero-coupon bonds pay the face value or principal at maturity. There is not a cash interest payment; instead, the bondholder earns a return by purchasing the bond at a discount to face value and receiving the face value at maturity.

The value of the bond grows each year and thus pays implicit interest, which is a function of the discount and the time to maturity. If the issuer goes into bankruptcy before the maturity of a zero-coupon bond, the bondholders are only entitled to the issue price plus accrued interest up to that date and not the face value of the bond.

One advantage of zero-coupon bonds is zero **reinvestment risk**. Unlike coupon bonds, the zero-coupon bondholder does not have to make an effort to reinvest cash interest payments or worry about the available rates at which to reinvest them. Another advantage is that in some tax jurisdictions, it is possible to convert the bond interest income to a capital gain. If capital gains are taxed more favorably than interest income in such jurisdictions, then the investor will have higher after-tax proceeds.

Bond Types

Corporate bonds can have collateral, such as real property, underlying the issue. The collateral may be useful if a defaulting firm will be liquidated because the sale proceeds from the collateral will be paid first to the bondholders who have a collateral position. If the defaulting firm is reorganized, then the bondholders with collateral will have better negotiating powers.

Mortgage bonds have supporting collateral that can be sold to pay off the bondholders if there is a default. Mortgage bondholders may want to protect themselves by restricting future bond issues. Additionally, an *after-acquired clause* could be used to restrict any assets acquired after the bond issuance to be used as collateral only for the existing bonds (and not new bond issues).

Collateral trust bonds are backed by stocks, notes, bonds, or other similar obligations that the company owns. The underlying assets are called the collateral or personal property. The issuers are holding companies, and the collateral consists of claims on their subsidiaries.

The trustee holds the collateral and acts for the benefit of the bondholders (and not the shareholders). The issuer retains voting rights for stock used as collateral if there has not been a default, so the issuer retains control over their subsidiaries. The indenture may have provisions covering what to do if the value of the collateral falls below the value of the loan. For example, the issuer may have to contribute additional securities to back the bonds.

Equipment trust certificates (ETCs) are a variation of a mortgage bond where a particular piece of equipment underlies the bond. The usual arrangement is that the borrower does not actually purchase the equipment. Instead, the trustee purchases the equipment and leases it to the user of the equipment (the effective borrower), who pays rent on the equipment, and that rent is passed through to the holders of the ETCs. Upon full payment of the debt, the title is transferred to the borrower. It is especially attractive if the equipment is standardized, as in the case of aircraft, which provides for easy lease of the equipment to another borrower in case the original borrower defaults.

Debentures are unsecured bonds (i.e., no collateral). Most corporate bonds are debentures and usually pay a higher interest rate for that reason; they rank below mortgage bonds and collateral trust bonds. Typically, the issuer is restricted to one issue of debentures if there is already secured debt. If there is no secured debt, and the company issues debentures, there is often a negative-pledge clause that says that the debentures will be secured equally with any secured bonds that may be issued in the future.

Subordinated debenture bonds have a claim that is at the bottom of the list of creditors if the issuer goes into default. They are bonds that are unsecured and have another unsecured bond with a higher claim above them. This means that the issuer has to offer a higher interest rate on the subordinated debentures.

Bonds issued by one company may also be guaranteed by other companies. A guarantee does not ensure that the issue will be free of default risk because the risk will depend on the ability of the guarantor(s) to satisfy all obligations. If the correlation between issuer profitability and the guarantor increases (decreases), then the value of the guarantee decreases (increases); in other words, the lower (or more negative) the correlation, the higher the value.

Methods for Retiring Bonds

LO 43.g: Describe the mechanisms by which corporate bonds can be retired before maturity.

There are various methods for retiring debt, and some are included in the bond's indenture while others are not included. The indenture would include the call provisions, sinking funds, maintenance and replacement funds, and redemption through sale of assets. The indenture would not include fixed-spread tender offers.

Call provisions are call options on the bonds that the issuer owns and give the issuer the right to purchase at a fixed price either in whole or in part before maturity. These provisions allow a firm to call back debt that has a high coupon and reissue debt with a lower coupon. Other reasons for exercising these options are to eliminate restrictive covenants, alter capital structure, increase shareholder value, or improve financial/managerial flexibility. A call provision can either be a fixed-price call or a make-whole call.

- **Fixed-price call.** The firm can call back the bonds at specific prices that can vary over the life of the bonds as specified in the indenture. They generally start out high and decline toward par. Also, for most bonds, the bonds are not callable during the first few years of the issue's life. An alternate form for bond retirement is to allow the bonds to be converted to common shares at a predetermined rate. The fact that a call option (potential benefit to the issuer) is often combined with the conversion option (potential benefit to investor) may incentivize the investor to exercise the conversion option earlier (e.g., before the stock price has risen too much). Otherwise, the issuer may have already called the bonds; the issuer is likely to call the bonds if the stock price rises significantly to avoid issuing stock at the predetermined price, which would now be considered relatively low.
- **Make-whole call.** In this case, market rates determine the call price, which is the present value of the bond's remaining cash flows subject to a floor price equal to par value. A discount rate based on the yield of comparable-maturity Treasury securities (usually the rate plus a premium) determines the present value and the bond's price.

A **sinking-fund provision** generally means that the issuing firm retires a specified portion of the debt each year as outlined in the indenture. The bonds can either be retired by use of a lottery where the owners of the selected bonds must redeem them, or the bonds are purchased in the open market. The indenture may give the issuer some flexibility as to how much of the bonds to call back each period, which would give the firm some latitude to call back more bonds when the market conditions are favorable to do so. One example is an accelerated sinking-fund provision, which allows the firm to call back more bonds in early years, which the firm would do if interest rates fall in those early years. Sinking-fund provisions also make sense when the value of the collateral goes down with time; therefore, the provisions would reduce the borrowings at the same time. Alternatively, if it is desired not to reduce the borrowing levels, then additional collateral can be provided to offset the potential decline in the value of existing collateral.

A **maintenance and replacement fund** has the same goal as a sinking-fund provision, which is to maintain the credibility of the property backing the bonds. The provisions differ in that the maintenance and replacement fund provision is more complex because it requires valuation formulas for the underlying assets. The main point is that the provision specifies that the fund must keep up the value of the underlying assets much like a home mortgage specifies that the homebuyer must keep up the value of the home. One way to satisfy the provision is to acquire sufficient cash to maintain the health of the firm. That cash can then be used to retire debt.

Selling the collateral is another possibility because it is usually on the condition that the sale proceeds are applied toward retiring the bonds early.

Tender offers are usually a means for retiring debt for most firms. The firm openly indicates an interest in buying back a certain dollar amount of bonds or, more often, all of the bonds at a set price. Firms can also announce that they will buy back bonds at an amount calculated as the present value of future cash flows based on a specific discount rate (e.g., the yield to maturity on a comparable-maturity Treasury plus a spread).

MODULE QUIZ 43.1

1. A fixed-income security has a maturity of three years. What is the most appropriate maturity classification of the security?
   A. Money market.
   B. Short-term note.
   C. Medium-term note.
   D. Long-term bond.

2. The holder of a zero-coupon bond obligation of a bankrupt corporation would have a claim equal to:
   A. the face value of the bond only.
   B. the issuing price of the bond only.
   C. the issuing price plus accrued interest.
   D. nothing, because zero-coupon bonds are unsecured.

3. All other things being equal, which of the following types of bond instruments would have the lowest interest rate?
   A. Equipment trust certificates.
   B. Mortgage bonds.
   C. Junior debentures.
   D. Senior debentures.

4. Which of the following methods for retiring bonds before maturity is generally considered the most detrimental for the bondholders?
   A. Tender offers.
   B. Call provision.
   C. Sinking-fund provision.

D. Maintenance and replacement funds.

MODULE 43.2: CREDIT RISK, EVENT RISK, AND HIGH-YIELD BONDS

Credit Risk

LO 43.d: Differentiate between credit default risk and credit spread risk.

Credit risk includes credit default risk and credit spread risk.

Credit default risk is the uncertainty concerning the issuer's making timely payments of interest and principal as prescribed by the bond's indenture. The most widely used indicators of this risk are bond ratings that major rating agencies assign when those agencies perform credit analysis of a firm. Fitch Ratings, Moody's, and Standard & Poor's are the main rating agencies in the United States. The agencies assign a symbol associated with the rating (e.g., AAA or Aaa for the corporate debt with the least credit default risk). The rating can be interpreted as a probability of default within some time period, as well as the probability of a change in a rating within some time period.

Credit spread risk focuses on the difference between a corporate bond's yield and the yield on a comparable-maturity benchmark Treasury security. This difference is known as the **credit spread**. It should be noted that other factors such as embedded options and liquidity factors can affect this spread; therefore, it is not only a function of credit risk. Credit spread risk increases when the economy deteriorates (e.g., moves through the business cycle).

A method commonly used to evaluate credit spread risk is **spread duration**. The duration of the spread is the approximate percentage change in a bond's price for a 100-basis-point change in the credit spread assuming that the Treasury rate is constant. If a bond has a spread duration of 4, for example, a 50-basis-point change in the spread will change the value of the bond by 2%.

Event Risk

LO 43.e: Describe event risk and explain what may cause it to manifest in corporate bonds.

Event risk addresses the adverse consequences from possible events involving significant increases in leverage, such as mergers, recapitalizations, restructurings, acquisitions, leveraged buyouts, and share repurchases, which may escape being included in the indenture. Such events can drastically change the firm's capital structure and reduce the creditworthiness of the bonds and their value.

In order to protect bondholders, a company may include in the indenture a **maintenance of net worth** clause that can require the company to maintain a minimum equity level. If that level is breached, then it must repurchase a sufficient amount of its debt at par value to reach the minimum equity level.

High-Yield Bonds

LO 43.c: Define high-yield bonds and describe types of high-yield bond issuers and some of the payment features unique to high-yield bonds.

High-yield bonds (a.k.a. junk bonds) are those bonds rated below investment grade by ratings agencies. This includes a broad range of ratings below the cutoff, (e.g., Ba1/BB+ down to default). Over long periods of time, high-yield bonds should offer higher average returns. However, over shorter periods, the returns will be volatile where large losses are possible.

There are many types of high-yield bonds. One type includes companies who issue bonds with a non-investment-grade rating. Such issuers include young and growing companies that do not have strong financial statements but have promising prospects. A company that otherwise has consistent cash flows may issue high-yield debt and use those proceeds to pay high dividends to shareholders.

Fallen angels are another type of high-yield bond. They are bonds that were issued with an investment-grade rating, but then events led to the ratings agencies lowering the rating to below investment grade.

High-yield bonds can have several types of coupon structures. There are **deferred-coupon bonds**, which would sell at a discount and not pay any interest for an initial period and then pay the stated coupon afterward. **Step-up bonds** pay a low coupon in the early years and then a higher coupon in later years. **Payment-in-kind bonds** allow the issuer to pay interest in the form of additional bonds over the initial period. **Extendable reset bonds** allow the issuer to reset the coupon as frequently as needed to keep the bond price at a specified level.

Recovery Rate and Default Rate

LO 43.h: Define recovery rate and default rate, and differentiate between an issue default rate and a dollar default rate.

A default occurs if there are any missed or delayed disbursements of interest and/or principal. It has been proven that lower credit ratings indicate a higher probability of default, but there are two ways to measure default: by the raw number of issuers that defaulted or the dollar amount of issues that defaulted. For each approach in measuring default rates, there are different formulas, which can lead to researchers reporting different default rates for the same data set.

The **issuer default rate** is the number of issuers that defaulted over a year divided by the total number of issuers at the beginning of the year. It is only a proportion of the number of issuers who do fulfill their obligations and does not include a measure of the dollar amount involved.

The **dollar default rate** is the par value of all bonds that defaulted in a given calendar year divided by the total par value of all bonds outstanding during the year. Over a multiyear period, often-used measures are ratios of cumulative dollar value of all defaulted bonds divided by some weighted-average measure of all bonds issued. One such measure attempts to weight the bonds outstanding by the number of years they are in the market:

$$\frac{\text{cumulative dollar value of all defaulted bonds}}{\begin{pmatrix}\text{cumulative dollar value}\\ \text{of all issuance}\end{pmatrix} \times \begin{pmatrix}\text{weighted average \# of}\\ \text{years outstanding}\end{pmatrix}}$$

Another measure simply takes a raw total as shown in the following equation:

$$\frac{\text{cumulative dollar value of all defaulted bonds}}{\text{cumulative dollar value of all issuance}}$$

The recovery rate is the amount received as a proportion of the total obligation after a bond defaults. Measuring this can be complicated because the value of the total obligation requires computing the present value of the remaining cash flows at the time of the default. Furthermore, some of the amount that the investor recovers may be in the form of securities (e.g., stock in the company). A study by Moody's estimated that the recovery rate for bonds has been about 38%. Bonds with higher seniority will obviously have higher recovery rates.

Expected Return

LO 43.i: Evaluate the expected return from a bond investment and identify the components of the bond's expected return.

A bond's expected return is calculated as: risk-free rate + credit spread − expected loss rate. The expected loss rate here is equal to: probability of default × (1 − expected recovery rate). Also, the credit spread is greater than the expected loss rate, which provides a return for investors. One study has noted that the excess of the credit spread over the expected loss rate is lower (higher) when the credit quality of the issuer is higher (lower).

Although the Treasury rate is often used as a risk-free measure, it may not be appropriate for corporate bonds and a higher rate such as the interbank borrowing rate may be appropriate. Regardless of the risk-free measure, in calculating expected return, investors in corporate bonds expect to earn more than the risk-free rate.

MODULE QUIZ 43.2

1. Corporate Bond X has a credit spread of 120 bps over Treasuries and the probability of default is 2% with an expected recovery rate of 25%. The risk-free rate is 3%. What is the expected return of Corporate Bond X?
   A. 2.7%.
   B. 3.7%.
   C. 4.7%.
   D. 5.7%.

KEY CONCEPTS

LO 43.a

Publicly traded bonds are usually traded in the over-the-counter (OTC) market. Dealers earn profit through the bid-ask spread. A corporate bond yield is a function of the risk-free return plus a credit spread to reflect the risk of default.

LO 43.b

A bond indenture sets forth the obligations of the issuer. The trustee interprets the legal language of the indenture and works to make sure the issuer fulfills obligations to bondholders.

LO 43.c

High-yield bonds can either be issued by growing, risky firms or established firms with senior debt outstanding. High-yield bonds may also be fallen angels (i.e., one-time investment grade bonds).

High-yield bonds may have coupon structures that allow the firm to conserve cash in early years, such as (1) deferred-interest bonds, (2) step-up bonds, and (3) payment-in-kind bonds.

LO 43.d

Credit default risk is the possibility that the issuer does not make the payments specified in the indenture. Credit spread risk is the price risk from changes in the spread of a bond's interest rate over the corresponding Treasury rate.

LO 43.e

Event risk is the possibility that a merger, restructuring, acquisition, et cetera increases the risk of the bond by changing the ability of the firm to pay off the bonds (e.g., significantly increasing leverage). The indenture can try to address some of these events, but some can be omitted.

LO 43.f

The main types of interest payment classifications are fixed-rate bonds, zero-coupon bonds, and floating-rate bonds. Fixed-rate bonds pay a fixed cash coupon periodically. Floating-rate bonds pay a cash amount that varies with market rates. Zero-coupon bonds increase in value over the life of the issue.

Mortgage bonds have supporting collateral that can be sold to pay off the bondholders if there is a default.

Collateral trust bonds are backed by stocks and bonds that the company owns.

Equipment trust certificates are a form of mortgage bond where the trustee actually owns the property and rents it to the bond issuer. The property is often in the form of standardized equipment (e.g., aircraft) that is easily leased.

Debentures are unsecured bonds (i.e., no collateral). Most corporate bonds are debentures and usually pay a higher interest rate for that reason.

LO 43.g

Call provisions allow the firm to retire debt early at a given price. Sinking-fund provisions require the firm to buy back portions of debt each year. Call provisions are generally considered detrimental to bondholders, but sinking-fund provisions may be beneficial to bondholders.

A maintenance and replacement fund helps maintain the financial health of the firm. Cash in the fund can be used to retire debt.

Bond issuers can retire debt through a tender offer. The offer price may either be a fixed price or a price that varies with a market rate such as that on comparable Treasury securities.

LO 43.h

The issuer default rate is a proportion based on the number of issues that default as a proportion of all issues. The dollar default rate estimates the dollar amount of defaulted bonds compared with the dollar amount of the corresponding population of bonds outstanding.

In the event of default, the recovery rate refers to the amount a bondholder receives as a proportion of the amount owed. Bonds with higher seniority usually have higher recovery rates.

LO 43.i

A bond's expected return is calculated as: risk-free rate + credit spread − expected loss rate.

- Expected loss rate = probability of default × (1 − expected recovery rate).
- The credit spread > expected loss rate, which provides a return for investors.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 43.1

1. **B** Short-term notes have maturities from 1 to 5 years, therefore, a 3-year security would be considered a short-term note. Any fixed-income security with a maturity less than 1 year is considered money market. Medium-term notes have maturities from 5 to 12 years. Long-term bonds have maturities greater than 12 years. (LO 43.f)

2. **C** The claim equals the value at that point in time as implied by the issuing price, the discount, and accrued interest. (LO 43.f)

3. **A** Equipment trust certificates (ETCs) are generally the most secure because the underlying assets are actually owned by the trustee and rented to the borrower. Also, the assets are usually standardized for easy lease to other potential borrowers if required. (LO 43.f)

4. **B** The call provision gives the issuer the right to purchase the bonds at a given price, which the issuer would not likely do unless that price was below the market

price.

Sinking-fund provisions can benefit bondholders because the issuer is obligated to purchase bonds, which improves the creditworthiness of the issue, and the issuer may have to do so at a price higher than the market price. There are no features in M&R funds or tender offers that would be detrimental to bondholders. (LO 43.g)

Module Quiz 43.2

1. **A** Expected return = risk-free rate + credit spread − expected loss rate = 0.03 + 0.012 − [0.02 × (1 − 25%)] = 0.027

   (LO 43.i)

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Financial Markets and Products, Chapter 18.

READING 44

MORTGAGES AND MORTGAGE-BACKED SECURITIES

Study Session 11

EXAM FOCUS

Mortgage-backed securities (MBSs) are debt securities backed by a pool of residential loans, which serve to transform mortgages from an illiquid asset into a liquid asset. Because the underlying mortgages can be prepaid, prepayment risk is a major concern for MBS investors. Monte Carlo simulation is the most common methodology used for valuing MBSs because it is able to account for prepayment risk. Alternate interest rate paths are assumed in the model to generate an option-adjusted spread (OAS). For the exam, be able to calculate the payments for a fixed-rate, level-paying mortgage. Also, be familiar with the factors that affect prepayment rates and how to measure prepayment speeds with a conditional prepayment rate (CPR). Finally, be prepared to discuss the steps involved in valuing an MBS using the Monte Carlo methodology and understand the advantages and disadvantages of using OAS.

MODULE 44.1: MORTGAGE LOANS

LO 44.a: Describe the various types of residential mortgage products.

Conventional Mortgage

A **mortgage** is a loan that is collateralized with a specific piece of real property, usually for 15 to 30 years. Before the 1970s, mortgages existed solely in the **primary market** where banks that issued the mortgage loans collected all interest and principal payments from the borrower. Within the past few decades, it is more common for mortgage lenders to sell the loans in the **secondary market** through a process known as **securitization** (discussed later in this reading). The secondary market has allowed more banks to issue mortgage loans.

Fixed-rate mortgages have a set rate of interest for the term of the mortgage. Payments are constant for the term and consist of blended amounts of interest and principal.

Adjustable-rate mortgages (ARMs) have rate changes throughout the term of the mortgage. The rate is initially fixed for a few years and then adjusts based on an interest rate index (e.g., Treasury rates). Rates can usually change on a monthly, semiannual, or annual basis. The risk of default is high, especially if there are large rate increases after the first year, thereby significantly increasing the total payment amount (due to the increase in interest).

Mortgage-Backed Securities

In the secondary market, mortgages are pooled together and packaged to investors in the form of a **mortgage-backed security (MBS)**. The payments of an MBS can follow a **pass-through structure** where the interest and principal collected from the borrower pass through the banks and ultimately end up with the MBS investor. Because default risk is present in mortgage lending, banks will often guarantee the borrower's payments when mortgages are securitized. Therefore, the ability to create mortgage-backed securities requires loans that have credit guarantees.

Government loans are those that are backed by federal government agencies (e.g., Government National Mortgage Association or GNMA).

Conventional loans could be securitized by either of two government-sponsored enterprises (GSEs): Federal Home Loan Mortgage Corporation (FHLMC) or Federal National Mortgage Association (FNMA). For a guarantee fee, these GSEs will guarantee payment of principal and interest to the investors.

Agency (or conforming) MBSs are those that are guaranteed by any of three government-sponsored entities (GSEs): GNMA, FNMA, and FHLMC. Most of the MBSs are issued by these GSEs. The GSEs have restrictions on what mortgages they can guarantee/securitize (e.g., dollar value limit, loan-to-value [LTV] ratio limit). With agency MBSs, the investor bears no credit risk because the GSEs have been paid a fee to guarantee the underlying mortgages. If there is a default with a mortgage, the GSE will pay the outstanding balance to the investors. However, there is no coverage for prepayments, so the investor bears such risk especially when interest rates are low and there is more incentive to prepay mortgages. The investor receives the funds back earlier than expected and can only reinvest them at lower rates.

Nonagency (or nonconforming) MBSs are issued privately (e.g., financial institutions) with no guarantee from the GSEs.

Fixed-Rate, Level-Payment Mortgages

LO 44.b: Calculate a fixed-rate mortgage payment and its principal and interest components.

There are a wide variety of mortgage designs that specify the rates, terms, amortization, and repayment methods. All the concepts associated with risk analysis and valuation, however, can be understood through an examination of *fixed-rate, level-payment, fully amortized mortgage loans*. This common type of mortgage loan requires equal payments (usually monthly) over the life of the mortgage. Each of these payments consists of an interest component and a principal component.

EXAMPLE: Calculating a mortgage payment

Consider a 30-year, $500,000 level-payment, fully amortized mortgage with a fixed annual rate of 12% (monthly compounding). **Calculate** the monthly payment and **prepare** an amortization schedule for the first three months.

Answer:

The monthly payment is $5,143.06:

N = 360; I/Y = 1.0 (= 12/12); PV = –500,000; FV = 0;
CPT → PMT = 5,143.06

With reference to the partial amortization schedule in the figure that follows, the portion of the first payment that represents interest is $5,000.00 (= 0.01 × $500,000). The remainder of the payment, $143.06 (= $5,143.06 – $5,000.00), goes toward the reduction of principal. The portion of the second payment that represents interest is $4,998.57 (= 0.01 × $499,856.94). The remaining $144.49 (= $5,143.06 – $4,998.57) goes toward the further reduction of principal.

Monthly Amortization Schedule for a 30-Year, $500,000 Mortgage at 12%

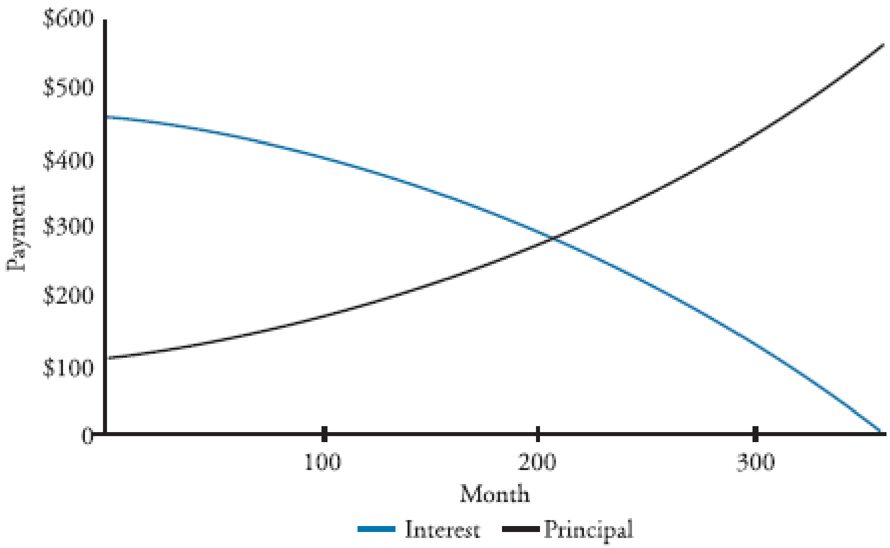
Payment Number	Initial Principal	Monthly Payment	Interest Component	Reduction of Principal	Outstanding Principal
1	$500,000.00	$5,143.06	$5,000.00	$143.06	$499,856.94
2	499,856.94	5,143.06	4,998.57	144.49	499,712.45
3	499,712.45	5,143.06	4,997.12	145.94	499,566.51

Notice that the monthly interest charge is based on the beginning-of-period outstanding principal. As time passes, the proportion of the monthly payment that represents interest decreases, and because the payment is level, the proportion that goes toward the repayment of principal increases. This process continues until the outstanding principal reaches zero and the loan is paid in full.

The incremental reduction of outstanding principal is called scheduled amortization (or scheduled principal repayment). The previous figure is a portion of what is commonly called an **amortization schedule**. Amortization schedules are easily constructed using an electronic spreadsheet.

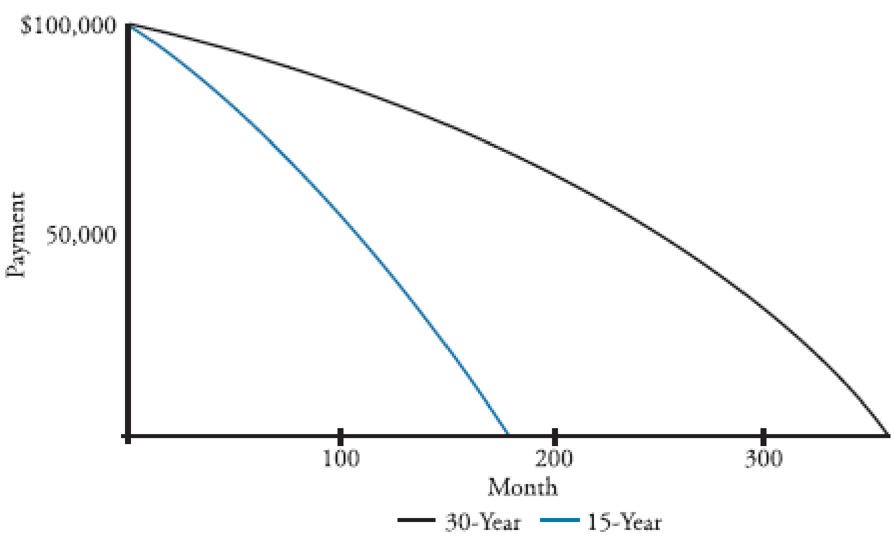
Figure 44.1 illustrates the relationship of interest and principal over the term of the loan.

Figure 44.1: Interest and Principal Over Time



As indicated previously, principal payments increase over time is because of the reduction in the outstanding loan balance. Figure 44.2 illustrates the relationship between loan balance and time for a $100,000 loan.

Figure 44.2: Loan Balance Over Time



MODULE QUIZ 44.1

1. Consider a 30-year, $750,000 level-payment, fully amortized mortgage with a fixed annual rate of 5% (monthly compounding). Which of the following amounts is closest to the monthly payment?
   A. $3,125.
   B. $4,010.
   C. $4,025.
   D. $4,065.

MODULE 44.2: MORTGAGE-BACKED SECURITIES

LO 44.c: Summarize the securitization process of mortgage-backed securities (MBS), particularly the formation of mortgage pools, including specific pools and

to-be-announced (TBA).

LO 44.d: Calculate the weighted average coupon, weighted average maturity, single monthly mortality rate (SMM), and conditional prepayment rate (CPR) for a mortgage pool.

To reduce the risk from holding a potentially undiversified portfolio of mortgage loans, a number of financial institutions (i.e., originators) will work together to pool residential mortgage loans with similar characteristics (e.g., interest rate, origination date) into a more diversified portfolio. They will then sell the loans to a separate entity, called a special purpose vehicle (SPV), in exchange for cash. An issuer will purchase those mortgage assets in the SPV and then use the SPV to issue MBSs to investors; the securities are backed by the mortgage loans as collateral.

Pass-Through Securities

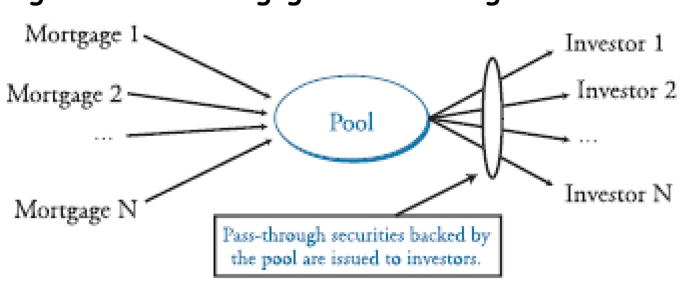
A **mortgage pass-through security** represents a claim against a pool of mortgages. Any number of mortgages may be used to form the pool, and any mortgage included in the pool is called a **securitized mortgage**.

The **weighted average maturity (WAM)** of the pool is equal to the weighted average of all mortgage ages in the pool, each weighted by the relative outstanding mortgage balance to the value of the entire pool. For example, if a mortgage pool consists of only two mortgages: $300,000 for 180 months and $500,000 for 240 months, then the WAM would be [(300,000 / 800,000) × 180] + [(500,000 / 800,000) × 240] = 217.5 months.

The **weighted average coupon (WAC)** of the pool is the weighted average of the mortgage rates in the pool. For example, if a mortgage pool consists of only two mortgages: $300,000 at a 3% interest rate and $500,000 at a 4% interest rate, then the WAC would be [(300,000 / 800,000) × 3%] + [(500,000 / 800,000) × 4%] = 3.625%.

As illustrated in Figure 44.3, pass-through security investors receive the monthly cash flows generated by the underlying pool of mortgages, less any servicing and guarantee/insurance fees.

Figure 44.3: Mortgage Pass-Through Cash Flow



The most important characteristic of pass-through securities is their prepayment risk; because the mortgages used as collateral for the pass-through can be prepaid, the pass-throughs themselves have significant prepayment risk.

Measuring Prepayment Speeds

Prepayments cause the timing and amount of cash flows from mortgage loans and MBSs to be uncertain; they speed up principal repayments and reduce the amount of interest paid over the life of the mortgage. Thus, it is necessary to make specific assumptions about the rate at which prepayment of the pooled mortgages occurs when valuing pass-through securities. One industry convention that has been adopted as a benchmark for prepayment rates is the **conditional prepayment rate (CPR)**.

The *CPR* is the annual rate at which a mortgage pool balance is assumed to be prepaid during the life of the pool. A mortgage pool's CPR is a function of past prepayment rates and expected future economic conditions.

We can convert the CPR into a monthly prepayment rate called the **single monthly mortality rate (SMM)** (also called constant maturity mortality) using the following formula:

$$\text{SMM} = 1 - (1 - \text{CPR})^{1/12}$$

If given the SMM rate, you can annualize the rate to solve for the CPR using the following formula:

$$\text{CPR} = 1 - (1 - \text{SMM})^{12}$$

An SMM of 10% implies that 10% of a pool's beginning-of-month outstanding balance, less scheduled payments, will be prepaid during the month.

The *PSA prepayment benchmark* assumes that the monthly prepayment rate for a mortgage pool increases as it ages or becomes seasoned. The PSA benchmark is expressed as a monthly series of CPRs.

The PSA standard benchmark is called 100% PSA (or just 100 PSA). A 100 PSA assumes the following graduated CPRs for 30-year mortgages:

- CPR = 0.2% for the first month after origination, increasing by 0.2% per month up to 30 months. For example, the CPR in month 14 is 14(0.2%) = 2.8%.
- CPR = 6% for months 30 to 360.

Remember that the CPRs are expressed as annual rates.

A particular pool of mortgages may exhibit prepayment rates faster or slower than 100% PSA, depending on the current level of interest rates and the coupon rate of the issue. A 50% PSA refers to half of the CPR prescribed by 100% PSA, and 200% PSA refers to two times the CPR called for by 100% PSA.

EXAMPLE: Computing the SMM

Assume the CPR is 0.2% for the first month after origination, increasing by 0.2% per month up to 30 months. **Compute** the CPR and SMM for the 5th and 25th months, assuming 100 PSA and 150 PSA.

Answer:

Assuming 100 PSA:

CPR (month 5) = 5 × 0.2% = 1%

100 PSA = 1 × 0.01 = 0.01

$$\text{SMM} = 1 - (1 - 0.01)^{1/12} = 0.000837$$

CPR (month 25) = 25 × 0.2% = 5%

100 PSA = 1 × 0.05 = 0.05

$$\text{SMM} = 1 - (1 - 0.05)^{1/12} = 0.004265$$

Assuming 150 PSA:

CPR (month 5) = 5 × 0.2% = 1%

150 PSA = 1.5 × 0.01 = 0.015

$$\text{SMM} = 1 - (1 - 0.015)^{1/12} = 0.001259$$

CPR (month 25) = 25 × 0.2% = 5%

150 PSA = 1.5 × 0.05 = 0.075

$$\text{SMM} = 1 - (1 - 0.075)^{1/12} = 0.006476$$

Trading Pass-Through Agency MBS

LO 44.e: Describe the process of trading pass-through agency MBS.

Pass-through MBS state the issuer, coupon, and maturity, where the maturity refers to the original (not current) mortgage maturities. Pass-through MBSs are subject to prepayment risk.

Trading is done in one of the following ways:

- The specified pools market
- The to-be-announced (TBA) market

The **specified pools** market identifies the number and balances of the pools prior to a trade. As a result, the characteristics of a given pool will influence the price of a trade. For example, high loan-balance pools, which make more use of prepayment options, trade for relatively lower prices.

The **to-be-announced (TBA)** market, which is more liquid than specified pools, involves identifying the security and establishing the price in a forward market. However, there is a pool allocation process whereby the actual pools are not revealed to the seller until two days before settlement. The characteristics of the pools that can be used for TBA trades are regulated to ensure reasonable consistency. For example, consider where the seller must provide mortgages from a GNMA 25-year 4% pool with a par value of $250 million for proceeds of $260 million. The seller may provide mortgages from any GNMA 25-year pool with a 4% coupon (e.g., cheapest-to-deliver option). The remaining maturity of the mortgages provided would likely be under 25 years, but there may be a stipulated suitable minimum.

Dollar Roll Transaction

LO 44.g: Describe a dollar roll transaction and how to value a dollar roll.

MBS trading requires the same securities to be priced for different settlement dates. A **dollar roll transaction** occurs when an MBS market maker sells TBA positions for one settlement month and, at the same time, buys TBA positions for settlement in the next month. Therefore, it has a superficial resemblance to a repurchase agreement but with two key differences. First, the securities bought in the next month may or may not be the same as in the first month. Second, there is no interest added onto the repurchase price in the second month so the initiating trader loses a month of interest while the other trader gains a month of interest.

How to Value a Dollar Roll

To value a dollar roll transaction, the following four components are needed:

value of a dollar roll = A − B + C − D

A = price at which pool is sold in Month 1, with accrued interest
B = price at which pool is bought in Month 2, with accrued interest
C = interest earned on funds from the sale for one month
D = coupon and principal payment that was foregone on the pool sold in Month 1

For example, assume $2 million par value of a 4% pool is sold for 104.00 in August and repurchased in September for 103.25. Note that 104 and 103.25 are a percentage of par value (i.e., 2,000,000 × 1.04 = $2,080,000). The payment date is the 15^{th} day of each month so accrued interest is calculated for each month as: 15/30 × 0.04/12 × $2 million = $3,333. Therefore, A is $2,083,333 and B = $2,068,333.

The funds received from the sale in Month 1 will earn interest at 0.15%. Therefore, C = $2,083,333 × 0.0015 = $3,125.

For D, assume the coupon and principal payments foregone were 0.6% of par value. Therefore, D = $2 million × 0.006 = $12,000.

Therefore, the value of the dollar roll = $2,083,333 − $2,068,333 + $3,125 − $12,000 = $6,125.

Agency MBS Products

LO 44.f: Explain the mechanics of different types of agency MBS products, including collateralized mortgage obligations (CMOs), interest-only securities

(IOs), and principal-only securities (POs).

Collateralized Mortgage Obligations

All investors have varying degrees of concern about exposure to prepayment risk. Some are primarily concerned with extension risk (the increase in the expected life of a mortgage pool due to rising interest rates and lower prepayment rates), while others want to minimize exposure to contraction risk (the decrease in the expected life of a mortgage pool due to falling interest rates and higher prepayment rates). Fortunately, all the pass-through securities issued on a pool of mortgages do not have to be the same. The ability to partition and distribute the cash flows generated by a mortgage pool into different risk packages has led to the creation of **collateralized mortgage obligations (CMOs)**.

CMOs are securities issued against pass-through securities (securities secured by other securities) for which the cash flows have been reallocated to different bond classes called *tranches*. Each tranche has a different claim against the cash flows of the mortgage pass-throughs or pool from which it was derived. Each CMO tranche represents a different mixture of contraction and extension risk. Hence, CMO securities can be more closely matched to the unique asset/liability needs of institutional investors and investment managers.

Planned Amortization Class Tranches

The most common type of CMO today is the **planned amortization class (PAC)**. A PAC is a tranche that is amortized based on a sinking fund schedule that is established within a range of prepayment speeds called the *initial PAC collar* or *initial PAC bond*.

What makes a PAC bond work is that it is packaged with a *support*, or *companion*, tranche created from the original mortgage pool. Support tranches are included in a structure with PAC tranches specifically to provide prepayment protection for the PAC tranches (each tranche is, of course, priced according to the timing risk of the cash flows). If prepayment rates are faster than the upper repayment rate, the PAC tranche receives principal according to the PAC schedule, and the support tranche absorbs (i.e., receives) the excess. If prepayment speeds are below the lower repayment rate, the funds needed to keep the PAC on schedule come from the cash flows scheduled for the support tranche(s). It should be pointed out that the extent of prepayment risk protection provided by a support tranche increases as its par value increases relative to its associated PAC tranche.

There is an *inverse* relationship between the prepayment risk of PAC tranches and the prepayment risk associated with the support tranches. In other words, *the certainty of PAC bond cash flow comes at the expense of increased risk to the support tranches*.

To understand the relatively high prepayment risk for support tranches, consider the situation in which prepayments are slower than planned. Because the PAC tranches have priority claim against the cash flows, principal payments to the support tranches must be deferred until the PAC repayment schedule is satisfied. Thus, the average life of the support tranche is extended. Similarly, when actual prepayments come at a rate that is faster than expected, the support tranches must absorb the amount that is in excess of that required to maintain the repayment schedule for the PAC. In this case, the average life of the support tranche is contracted. If these excesses continue to occur, the support tranches will eventually be paid off and the principal will then go to the PAC holders. When this happens, the PAC is called a *broken* or *busted* PAC, and any further prepayments go directly to the PAC tranche. Essentially, the PAC tranche becomes an ordinary sequential-pay structure.

Notice that the prepayment risk protection provided by the support tranches causes their average lives to extend and contract. This relationship is such that as the prepayment risk protection for a PAC tranche increases, its average life variability decreases and the average life variability of the support tranche increases.

Principal-Only and Interest-Only Securities

The two most common types of stripped MBSs are **principal-only securities** (PO securities) and **interest-only securities** (IO securities). PO securities are a class that receives only the principal payment portion of each mortgage payment, while IO securities are a class that receives only the interest component of each payment.

The PO cash flow stream starts small and increases with the passage of time as the principal component of the mortgage payments grows. The investment performance of a PO is extremely sensitive to prepayment rates. Higher prepayment rates result in a faster-than-expected return of principal and, thus, a higher yield. Because prepayment rates increase as mortgage rates decline, PO prices increase when interest rates fall. The entire par value of a PO is ultimately paid to the PO investor. The only question is whether realized prepayment rates will cause it to be paid sooner or later than expected.

In contrast to PO securities, an IO cash flow starts big and gets smaller over time. Thus, IOs have shorter effective lives than POs.

The major risk associated with IO securities is that the value of the cash flow investors receive over the life of the mortgage pool may be less than initially expected and possibly less than the amount originally invested. Why? The amount of interest produced by the pool depends on its beginning-of-month balance. If market rates fall, the mortgage pool will be paid off sooner than expected, leaving IO investors with no interest cash flow. Therefore, IO investors want prepayments to be slow.

In general, the uncertainty surrounding interest rates and prepayment rates cause IOs to be lower in value and POs to be greater in value.

MODULE QUIZ 44.2

1. If the conditional prepayment rate (CPR) for a pool of mortgages is assumed to be 5% on an annual basis and the weighted average maturity of the underlying mortgages is 15 years, which of the following amounts is closest to the constant maturity mortality?
   A. 0.333%.
   B. 0.405%.
   C. 0.427%.

D. 0.500%.

2. Which of the following statements regarding principal-only (PO) and interest-only (IO) securities is correct?
   A. Only IO securities are at risk of incurring losses.
   B. PO securities cash flows decline over time.
   C. Higher prepayment rates result in a higher yield for IO securities.
   D. The uncertainty surrounding interest rates and prepayment rates causes POs and IOs to be lower in value.

MODULE 44.3: PREPAYMENT MODELING

LO 44.h: Describe the mortgage prepayment option and factors that affect it; explain prepayment modeling and its four components: refinancing, turnover, defaults, and curtailments.

In many cases, a borrower pays the exact amount of the monthly payment on a loan, and the interest and principal follow an amortization schedule. However, it is possible for a borrower to pay an amount in excess of the required payment or even to pay off the loan entirely. The option to prepay a mortgage is essentially a call option for the borrower. The borrower is in a position that is very similar to the issuer of a callable bond. In the absence of any prepayment penalties, the prepayment option may be highly beneficial to the borrower.

Mortgage prepayments come in three general forms: (1) increasing the frequency or amount of payments (where permitted), (2) refinancing the outstanding balance, and (3) repaying the outstanding balance because the property is sold. Prepayments are much more likely to occur when market interest rates fall and borrowers wish to refinance their existing mortgages at a new and lower rate. Note that prepayments could also occur if the borrower has defaulted on the mortgage and the lender is forced to sell the property to cover the mortgage.

For the lender, prepayments represent a loss for two reasons: (1) they stop receiving interest income at the high rate and (2) they have to reinvest the proceeds received from prepayment at the prevailing lower market rates. Therefore, the pricing of the initial mortgage rate should be somewhat higher to take into account the possibility of prepayment.

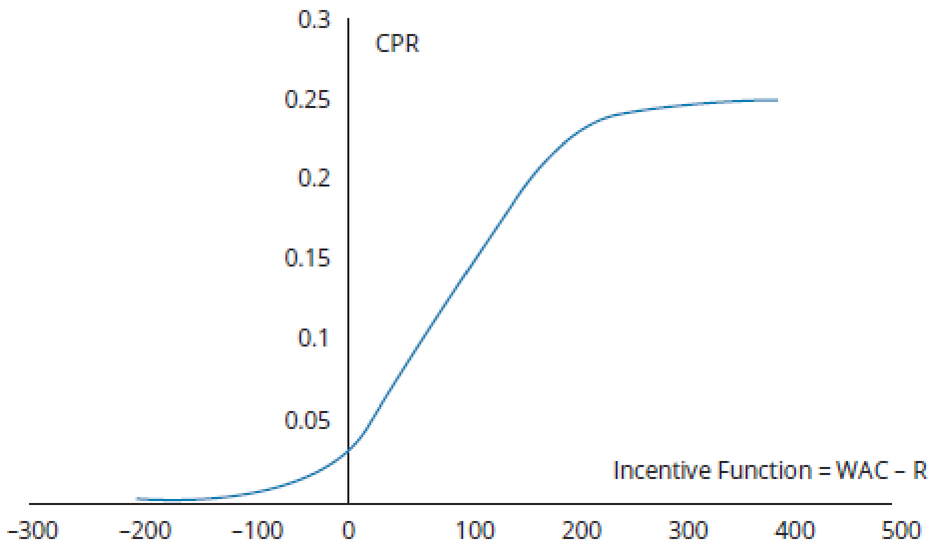
With agency MBSs, prepayments and defaults have the same impact on investors. Prepayments result in the investors actually receiving cash from the borrowers, whereas with defaults, the borrower does not pay the outstanding mortgage balances, but the GSE does, thereby causing a prepayment.

Refinancing a mortgage involves using the proceeds of a new mortgage to pay off the principal from an existing mortgage. If a borrower is holding a high interest rate mortgage and the current mortgage rates fall, there is incentive to refinance to benefit from lower mortgage payments (given that rates decline enough to cover the transaction costs of refinancing). Alternatively, a borrower might have an improved credit rating that would qualify for a lower rate (even if rates remain unchanged).

Extracting home equity is another motive for refinancing a mortgage. Given a substantial increase in property value, a borrower may take out a new mortgage with a higher balance that not only pays off the existing mortgage but also has extra cash for other purposes. Extracting home equity is also known as *cash-out refinancing*.

Incentive functions are used to model refinancing activity and are based on the term structure of mortgage rates. Other items such as past rates and average loan size can be included in the model to help explain refinancing behavior. Incentive functions essentially forecast the present value of any dollar gains, given that a borrower will refinance. An example of an incentive function for a pool could be I = WAC − R, where R = current mortgage rate. The incentive function essentially considers how much interest the borrowers will save by refinancing. Figure 44.4 outlines roughly how prepayment occurs.

Figure 44.4: Incentive Function



At the bottom of the curve when interest rates are high (when WAC − R is negative), there is much less prepayment activity occurring. However, as interest rates fall (and WAC − R becomes positive), more prepayment activity will occur.

The path that mortgage rates follow on their way to the current level affects prepayments through *burnout*. To better understand this phenomenon, consider a mortgage pool that was formed when rates were 6%, then interest rates dropped to 3%, rose to 6%, and then dropped again to 3%. Many homeowners will have refinanced when interest rates dipped the first time. On the second occurrence of 3% interest rates, most homeowners in the pool who were able to refinance would have already done so.

It is typically the case that the mortgage is due once the property is sold. Because most borrowers sell their homes without regard for the path of mortgage rates, MBS investors will be subjected to a degree of **turnover** that does not correlate with the behavior of rates. One factor that slows the degree of housing turnover is known as the *lock-in effect*. This essentially means that borrowers may wish to avoid the costs of a new mortgage, which likely consists of a higher mortgage rate.

Modeling turnover typically starts with a base rate and then adjusts for seasonality (turnover is higher in the summer and lower in the winter). The turnover model may also include a factor based on improvements to creditworthiness over time and, thus, the homeowner's increased ability to prepay the mortgage. The rate of turnover is dependent on the stage of the mortgage (e.g., less likely to move shortly after getting a mortgage), the age of the mortgagees (e.g., younger individuals are less likely to sell until they have built up sufficient savings to "upsize" to a better home), and the property location.

When a borrower **defaults**, mortgage guarantors pay the interest and principal outstanding. These payments act as a source of prepayment. Modeling prepayments from default requires an analysis of loan-to-value (LTV) ratios and FICO scores, as well as an overall analysis of the housing market.

Partial payments by the borrower are called **curtailments**. These partial payments tend to occur when a mortgage is older or has a relatively low balance. Thus, prepayment modeling due to curtailment typically takes into account the age of the mortgage.

Valuation Using Monte Carlo Simulation

LO 44.i: Describe the steps in valuing an MBS using Monte Carlo simulation.

As discussed earlier, mortgage borrowers have an option to prepay the underlying securities. The value of MBSs with embedded options to prepay cannot be determined using traditional option valuation techniques. Therefore, **Monte Carlo simulation** is used to value MBSs and other fixed-income securities with embedded options.

Prepayments on mortgage pass-through securities are interest rate path-dependent. This means that a given month's prepayment rate depends on whether there were prior opportunities to refinance since the origination of the underlying mortgages. For example, if mortgage rates trend downward over a period of time, prepayment rates will increase at the beginning of the trend as homeowners refinance their mortgages, but prepayments will slow as the trend continues because many of the homeowners who can refinance will have already done so. As mentioned earlier, this prepayment pattern is called refinancing burnout. Prepayments also depend on the path of housing prices after mortgage origination. Significant drops in housing prices may lead to defaults, which increases prepayments. Significant rises in housing prices may lead to more cash-out refinancing, which increases prepayments. Monte Carlo simulation can easily take into account path dependence, which is not as easy to do with other tools such as binomial trees.

Certain aspects of the mortgage pool affect prepayments. For example, the larger the average mortgage size, the greater the prepayment rate. Or the lower the FICO score and the higher the loan-to-value ratio, the greater the likelihood of defaults.

The Monte Carlo approach is a process of steps rather than a specific model. It is extremely useful when there are numerous variables with multiple outcomes. Monte Carlo simulation is used to provide a probability distribution of the value of an MBS.

The Monte Carlo approach provides a range of possible outcomes with a probability distribution for the value of an MBS. The mean or average value of this range of outcomes is then taken as the estimated value of the MBS.

The following steps are required to value an MBS using the Monte Carlo approach:

Step 1: Simulate a monthly path for risk-free rates and housing prices using samples from the probability distributions.

Step 2: Determine prepayment rates for each month based on the prepayment model, interest rate path, housing prices, and mortgage pool characteristics.

Step 3: Project monthly cash flows of MBS based on prepayment rates.

Step 4: Compute the present value of cash flows using the risk-free rate for the month.

Step 5: Repeat Steps 1 to 4 many times.

Step 6: Compute the value of the MBS pool (average of the present values).

Steps 1 and 2:

The first step in applying the Monte Carlo approach is to estimate monthly interest rates for the entire life of the mortgage security. For example, a 30-year mortgage security would theoretically require 360 monthly interest rates, although some simplifying assumptions could be made about interest rates not changing for specific amounts of time. Random interest rate paths are generated using the term structure of interest rates and a volatility assumption. The term structure of interest rates is created using the theoretical spot rate (zero-coupon) curve for the market on the pricing date. The simulations are adjusted to ensure that the average simulated price of a zero-coupon Treasury bond is equal to the actual price corresponding to the pricing date.

The derivatives market is used to construct an arbitrage-free term structure of future interest rates. Short-term interest rate paths are used to discount the cash flows in Step 4 of the Monte Carlo process. These interest rate paths are also used to create the prepayment paths, which are cash flows for each interest rate path. The prepayment path is computed based on refinancing rates that are available each month. The mortgagor has an incentive to refinance if the refinancing rate is low relative to the mortgagor's original coupon rate. The relationship between refinancing rates and short-term interest rates is an important assumption of the model.

Step 3:

Cash flows for each month on each interest rate path are equal to the scheduled principal for the mortgage pool, the net interest, and prepayments. Scheduled principal payments are simply calculated based on the projected mortgage balance from the prior month. A prepayment model is used rather than a simple prepayment rate. A prepayment rate is specified for each month on a given interest rate path, and rates for a given month across all interest rate paths are not the same.

CMO deal structures dictate how principal and interest is to be paid. Therefore, it is necessary to reverse engineer the deal to determine the cash flows for a senior CMO.

The cash flows for each month on an interest rate path are calculated using the scheduled principal, net interest, and prepayments for the collateral. The tranche's cash flows for each path are determined by the total principal and interest paid to the tranche, the interaction of the cash flow rules, and the prepayment model.

Step 4:

The present values of cash flows for each interest rate path are calculated by discounting the cash flows for each path by a discount rate. The discount rate is estimated using the simulated spot rates for each month on the interest rate path plus an appropriate spread. The simulated spot rates are determined from the simulated future monthly rates.

The interest rate paths for the simulated future one-month rates are converted to the interest rate paths for the simulated monthly spot rates.

The present value for a given path is determined as the sum of the present values of the cash flows for each month on the path.

Step 6:

The theoretical value for a specific interest rate path is thought of as the present value of all cash flows in that path, assuming that path was actually realized. The theoretical value of the MBS is calculated as the average present value of all theoretical values for each interest rate path.

Option-Adjusted Spread

LO 44.j: Define Option-Adjusted Spread (OAS) and explain its uses and challenges.

The **option-adjusted spread (OAS)** can be interpreted as a measure of MBS returns that indicates the potential compensation after adjusting for prepayment risk. In other words, the OAS is *option adjusted* because the cash flows on the interest rate paths take into account the borrowers' option to prepay. It can be expressed as the excess of the expected MBS return over the return on Treasuries.

The OAS can be determined using the following steps:

Step 1: Perform a preliminary OAS estimate.

Step 2: Perform a Monte Carlo simulation using a discount rate equal to the sum of the Treasury rate and an OAS estimate.

Step 3: Compare the computed price in Step 2 to the market price.

Step 4: If the market price is higher (lower) than the simulated price, decrease (increase) the OAS estimate.

Step 5: Continue with the iterative process by adjusting OAS estimate so that the simulated price and the market price are identical.

The OAS is defined as the spread that when added to all the spot rates of all interest rate paths will make the average present value of the paths equal to the actual observed market price (plus accrued interest). In the simulation process, the average of the high and low prices is used. For example, in Step 3, assume that the value of the MBS is determined to be 98.55 but the market price is 97.70. In that instance, the OAS is the incremental spread to be added to the previously determined discount rates that would force the price down from 98.55 to 97.70, which is what is done in Steps 4 and 5.

An investor could estimate the value of a security using the OAS for comparable MBSs to determine whether or not to invest in the pool. All other things being equal, a higher OAS (greater return over Treasuries) is preferred over lower OAS. From a due diligence perspective, one should try to determine why a pool has a relatively high or low OAS (e.g., underlying business/economic reasons versus model assumptions).

The OAS is generated through Monte Carlo simulations. Therefore, the OAS is subject to all prepayment modeling risks associated with the simulation. Interest rate paths must be adjusted to ensure that securities or rates making up the benchmark curve are properly valued when using Monte Carlo methods. This process of adjusting interest rate paths is subject to modeling error. If there is a term structure to the OAS, then this is not reflected in the Monte Carlo process because the OAS methodology assumes a constant OAS.

The prepayment model is very complex, given the amount of uncertainty regarding important variables. The behavior of both borrowers and lenders changes over time. Thus, the greatest weakness of using OAS valuation estimates generated from the Monte Carlo simulation is the dependence on the prepayment model. Assuming the model is sound, then in theory, it should allow for interest rate hedging (using Treasury futures). However, it would not be a perfect hedge because there is not a perfect correlation between mortgage rates and Treasury rates.

MODULE QUIZ 44.3

1. When estimating the value of mortgage-backed securities (MBSs), which of the following statements is correct?
   A. Prepayment rates tend to be higher on smaller mortgages.
   B. Both binomial trees and Monte Carlo simulation can easily account for path dependence.
   C. Greater attention should be paid to significant decreases in housing prices than significant increases.
   D. The Monte Carlo approach provides a range of outcomes of which the mean is taken to determine the MBS value.

2. Assume \$1 million par value of a 5% mortgage pool is sold for 103.50 in March and repurchased in April for 102.60. The coupon and principal payments for the month are approximately 0.5% of par value. The payment date is the 10th day of each month, and funds can be invested at a monthly rate of 0.2%. Which of the following amounts is closest to the value of the dollar roll?
   A. \$1,927.
   B. \$4,684.
   C. \$6,073.

D. $7,462.

KEY CONCEPTS

LO 44.a

A mortgage is a loan that is collateralized with a specific piece of real property, usually for 15 to 30 years. Fixed-rate mortgages have a set rate of interest for the term of the mortgage with constant payments of blended interest and principal. Adjustable-rate mortgages have rate changes throughout the term of the mortgage.

Mortgage-based securities (MBSs) are mortgages that are pooled together and packaged to investors in the secondary market. Agency MBSs are those that are guaranteed by government-sponsored enterprises (GSEs). Most of the MBSs are issued by these GSEs. The GSEs have restrictions on which mortgages they can guarantee/securitize, which opened up the market for nonagency MBSs for those participants willing to take on the risks inherent in nonconventional loans.

LO 44.b

A mortgage is a loan that is collateralized with a specific piece of real property, either residential or commercial. A level-payment, fixed-rate conventional mortgage has a fixed term, a fixed interest rate, and a fixed monthly payment. Even though the term, rate, and payment are fixed, the cash flows are not known with certainty because the borrower has the right to repay all or any part of the mortgage balance at any time.

LO 44.c

To reduce the risk from holding a potentially undiversified portfolio of mortgage loans, a number of financial institutions (originators) will work together to pool residential mortgage loans with similar characteristics into a more diversified portfolio. They will then sell the loans to a separate entity, called a special purpose vehicle (SPV), in exchange for cash. An issuer will purchase those mortgage assets in the SPV and then use the SPV to issue mortgage-backed securities (MBSs) to investors; the securities are backed by the mortgage loans as collateral.

LO 44.d

The value of an MBS is a function of the following:

- Weighted average maturity (WAM)
- Weighted average coupon (WAC)
- Speed of prepayments

Regarding prepayment speeds, the single monthly mortality (SMM) rate is derived from the conditional prepayment rate and is used to estimate monthly prepayments for a mortgage pool:

$$SMM = 1 - (1 - CPR)^{1/12}$$

LO 44.e

Fixed-rate, pass-through securities trade in one of the following ways:

- The specified pools market
- The to-be-announced (TBA) market

The specified pools market identifies the number and balances of the pools before a trade. As a result, the characteristics of a given pool will influence the price of a trade.

The TBA market, which is more liquid than specified pools, involves identifying the security and establishing the price in a forward market. However, there is a pool allocation process whereby the actual pools are not revealed to the seller until two days before settlement. The characteristics of the pools that can be used for TBA trades are regulated to ensure reasonable consistency.

LO 44.f

The ability to partition and distribute the cash flows generated by a mortgage pool into different risk packages has led to the creation of collateralized mortgage obligations (CMOs). CMOs are securities issued against pass-through securities for which the cash flows have been reallocated to different bond classes called tranches. Each tranche has a different claim against the cash flows of the mortgage pass-throughs or pool from which it was derived. Each CMO tranche represents a different mixture of contraction risk and extension risk. Hence, CMO securities can be more closely matched to the unique asset/liability needs of institutional investors and investment managers.

The two most common types of stripped MBSs are PO securities and IO securities. PO securities are a class that receives only the principal payment portion of each mortgage payment, while IO securities are a class that receives only the interest component of each payment. The PO cash flow stream starts small and increases with the passage of time as the principal component of the mortgage payments grows. The investment performance of a PO is extremely sensitive to prepayment rates. In contrast to PO securities, an IO cash flow starts big and gets smaller over time. Thus, IOs have shorter effective lives than POs.

LO 44.g

A dollar roll transaction occurs when an MBS market maker sells TBA positions for one settlement month and, at the same time, buys TBA positions for settlement in the next month.

value of a dollar roll = A − B + C − D

A = price at which pool is sold in Month 1, with accrued interest
B = price at which pool is bought in Month 2, with accrued interest
C = interest earned on funds from the sale for one month
D = coupon and principal payment that was foregone on the pool sold in Month 1

LO 44.h

Mortgage prepayments come in three general forms: (1) increasing the frequency or amount of payments (where permitted), (2) refinancing the outstanding balance, and

(3) repaying the outstanding balance because the property is sold. Prepayments are much more likely to occur when market interest rates fall and borrowers wish to refinance their existing mortgages at a new and lower rate.

LO 44.i

The Monte Carlo simulation approach is used for valuing MBSs. Monte Carlo is a process of steps rather than a specific model.

The Monte Carlo approach provides a range of possible outcomes with a probability distribution for the value of a mortgage security. The mean or average value of this range of outcomes is then taken as the estimated value of the MBS.

The following steps are required to value an MBS using the Monte Carlo approach:

Step 1: Simulate a monthly path for risk-free rates and housing prices using samples from the probability distributions.

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Step 3: Project monthly cash flows of MBS based on the prepayment rates.

Step 4: Compute the present value of cash flows using the risk-free rate for the month.

Step 5: Repeat Steps 1 to 4 many times.

Step 6: Compute the value of the MBS pool (average of the present values).

LO 44.j

The option-adjusted spread (OAS) can be interpreted as a measure of MBS returns that indicates the potential compensation after adjusting for prepayment risk. It can be expressed as the excess of the expected MBS return over the return on Treasuries.

The OAS can be determined using the following steps:

Step 1: Perform a preliminary OAS estimate.

Step 2: Perform a Monte Carlo simulation using a discount rate equal to the sum of the Treasury rate and an OAS estimate.

Step 3: Compare the computed price in Step 2 to the market price.

Step 4: If the market price is higher (lower) than the simulated price, decrease (increase) the OAS estimate.

Step 5: Continue with the iterative process by adjusting OAS estimate so that the simulated price and the market price are identical.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 44.1

1. **C** The monthly payment is $4,026:

N = 360, I/Y = 0.4167 (= 5 / 12), PV = −750,000, FV = 0; CPT → PMT = 4,026
(LO 44.b)

Module Quiz 44.2

1. **C** The constant maturity mortality (or single monthly mortality rate) is a monthly measure. Its relationship to CPR is as follows:

   $SMM = 1 - (1 - CPR)^{1/12} = 1 - (1 - 0.05)^{1/12} = 1 - 0.95^{1/12} = 0.43\%$

   (LO 44.d)

2. **A** The entire par value of a PO is ultimately paid to the investor. The only question is whether realized prepayment rates will cause it to be paid sooner or later than expected. In contrast, with IO securities, the value of the cash flows that investors receive over the life of the mortgage pool may be less than initially expected and possibly less than the amount originally invested.

   IO securities cash flows decline over time as the mortgages are paid off and the balances decrease. Higher prepayment rates result in a faster-than-expected return of principal and, thus, a higher yield for PO securities. The uncertainty surrounding interest rates and prepayment rates causes POs to be greater in value. (LO 44.f)

Module Quiz 44.3

1. **D** The Monte Carlo approach provides a range of outcomes of which the mean is taken to determine the MBS value. Prepayment rates tend to be higher as the average loan size increases. Although Monte Carlo simulation can easily account for path dependence, binomial trees cannot do it so easily. Significant increases in housing prices may lead mortgage holders to utilize cash-out refinancing, which has an effect on prepayments; therefore, significant increases should be paid no less attention than significant decreases. (LO 44.i)

2. **C** Value of a dollar roll = A − B + C − D

   Accrued interest is calculated for each month as: 10/30 × 0.05/12 × \$1 million = \$1,389. Therefore, A is \$1,036,389 and B = \$1,027,389.

   The funds received from the sale in month 1 will earn interest at 0.2%. Therefore, C = \$1,036,389 × 0.002 = \$2,073.

   For D, assume that the coupon and principal payments foregone were 0.5% of par value. Therefore, D = \$1 million × 0.005 = \$5,000.

   The value of the dollar roll = \$1,036,389 − \$1,027,389 + \$2,073 − \$5,000 = \$6,073. (LO 44.g)

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Financial Markets and Products, Chapter 19.

READING 45

INTEREST RATE FUTURES

Study Session 11

EXAM FOCUS

In this reading, we examine Treasury bonds (T-bonds) and Eurodollar futures contracts. These instruments are two of the most popular interest rate futures contracts that trade in the United States. For the exam, be able to define the cheapest-to-deliver bond for T-bonds, and know how to use the convexity adjustment for Eurodollar futures. Duration-based hedging using interest rate futures is also discussed. In addition, be familiar with the equation to calculate the number of contracts needed to conduct a duration-based hedge.

MODULE 45.1: DAY COUNT CONVENTIONS AND QUOTATIONS

LO 45.a: Identify the most commonly used day count conventions, describe the markets that each one is typically used in, and apply each to an interest calculation.

Day count conventions play a role when computing the interest that accrues on a fixed-income security. When a bond is purchased, the buyer must pay any **accrued interest** earned through the settlement date.

$$\text{accrued interest} = \text{coupon} \times \frac{\text{\# of days from last coupon to the settlement date}}{\text{\# of days in coupon period}}$$

In the United States, there are three commonly used day count conventions.

1. U.S. Treasury bonds use **actual/actual**
2. U.S. corporate and municipal bonds use **30/360**
3. U.S. money-market instruments (Treasury bills) use **actual/360**

The following examples demonstrate the use of day count conventions when computing accrued interest.

EXAMPLE: Day count conventions

Suppose there is a semiannual-pay bond with a $100 par value. Further assume that coupons are paid on March 1 and September 1 of each year. The annual coupon is 6%, and it is currently July 13. **Compute** the accrued interest of this bond as a T-bond and a U.S. corporate bond.

Answer:

The T-bond uses actual/actual (in period), and the reference (March 1 to September 1) period has 184 days. There are 134 actual days from March 1 to July 13, so the accrued interest is:

$$\frac{134}{184} \times \$3 = \$2.1848$$

The corporate bond uses 30/360, so the reference period now has 180 days. Using this convention, there are 132 (= 30 × 4 + 12) days from March 1 to July 13, so the accrued interest is:

$$\frac{132}{180} \times \$3 = \$2.20$$

Quotations for T-Bonds

LO 45.c: Differentiate between the clean and dirty price for a U.S. Treasury bond; calculate the accrued interest and dirty price on a U.S. Treasury bond.

T-bond prices are quoted relative to a $100 par amount in dollars and 32nds. So a 95–05 is 95 5/32, or 95.15625. The quoted price of a T-bond is not the same as the cash price that is actually paid to the owner of the bond. In general:

cash price = quoted price + accrued interest

The cash price (a.k.a. **invoice price** or **dirty price**) is the price that the seller of the bond must be paid to give up ownership. It includes the present value of the bond (a.k.a. **quoted price** or **clean price**) plus the accrued interest. This relationship is shown in the preceding equation. Conversely, the clean price is the cash price less accrued interest:

quoted price = cash price − accrued interest

This relationship can also be expressed as:

clean price = dirty price − accrued interest

EXAMPLE: Calculate the cash price of a bond

Assume that the bond in the previous example is a T-bond currently quoted at 102–11. **Compute** the cash price.

Answer:

cash price = $102.34375 + $2.1848 = $104.52855

For a $100,000 par amount, this is $104,528.55.

Quotations for T-Bills

LO 45.b: Calculate the conversion of a discount rate to a price for a U.S. Treasury bill.

T-bills and other money-market instruments use a discount rate basis and an actual/360 day count. A T-bill with a $100 face value with *n* days to maturity and a cash price of *Y* is quoted as:

$$\text{T-bill discount rate} = \frac{360}{n}(100 - Y)$$

This is called the discount rate in annual terms. However, this discount rate is not the actual rate earned on the T-bill. The following example shows the calculation of the annualized yield on a T-bill, given its price.

EXAMPLE: Calculating the cash price on a T-bill

Suppose you have a 180-day T-bill with a discount rate, or quoted price, of five (i.e., the annualized rate of interest earned is 5% of face value). If face value is $100, what is the true rate of interest and the cash price?

Answer:

Interest is equal to $2.5 (= $100 × 0.05 × 180 / 360) for a 180-day period. The true rate of interest and cash price are therefore:

true rate of interest: 2.5 / (100 − 2.5) = 2.564%

cash price: 5 = (360 / 180) × (100 − Y); Y = $97.5

MODULE QUIZ 45.1

1. Consider day count convention and, specifically, the following example: A semiannual bond with $100 face value has a 4% coupon. Today is August 3. Assume coupon dates of March 1 and September 1. Which of the following statements is correct?
   A. Corporate bonds accrue more interest in July than T-bonds.
   B. Corporate bonds accrue more interest from March 1 to September 1 than from September 1 to March 1.
   C. Corporate bonds accrue more interest than T-bonds for this period (March 1 to August 3).
   D. The T-bond accrued interest is $1.76 for this period (March 1 to August 3).

2. Assume that the cash price on a 90-day T-bill is quoted as 98.75. The discount rate is closest to:
   A. 4%.
   B. 5%.
   C. 6%.

D. 7%.

MODULE 45.2: TREASURY BOND AND EURODOLLAR FUTURES

Treasury Bond Futures

LO 45.d: Explain and calculate a U.S. Treasury bond futures contract conversion factor.

LO 45.e: Calculate the cost of delivering a bond into a Treasury bond futures contract.

LO 45.f: Describe the impact of the level and shape of the yield curve on the cheapest-to-deliver Treasury bond decision.

Because deliverable bonds have very different market values, the Chicago Board of Trade (CBOT) has created **conversion factors**. The conversion factor defines the price received by the short position of the contract (i.e., the short position is delivering the contract to the long). Specifically, the cash received by the short position is computed as follows:

$$\text{cash received} = (\text{QFP} \times \text{CF}) + \text{AI}$$

where:
QFP = quoted futures price (most recent settlement price)
CF = conversion factor for the bond delivered
AI = accrued interest since the last coupon date on the bond delivered

Conversion factors are supplied by the CBOT on a daily basis. Conversion factors for 10-year Treasury notes or longer are calculated as the time to maturity from Day 1 of the delivery month to the maturity of the bond. That amount is then rounded down to the nearest three months. Finally, the rounded-down time to maturity is used to calculate the clean price per dollar of face value on the assumption of a 6% annual yield, compounded semiannually. For shorter periods, such as for 2-year or 5-year Treasury notes, the rounding down is to the nearest month (not three months).

EXAMPLE: Calculating conversion factor

Suppose you have a November 2021 Treasury bond futures contract on a 4% coupon bond maturing on June 15, 2033. Assume the valuation is done immediately after the last coupon was paid. **Calculate** the conversion factor.

Answer:

The time to maturity on the first day of the delivery month (November 1, 2021) is 11 years and 7.5 months, which is rounded down to 11 years and 6 months. Using a calculator, the price of the bond is calculated as N = 23, I/Y = 3, PMT = 2, FV = 100. CPT → PV = 83.55.

Because there is no accrued interest, 83.55 is both the dirty and clean price. Therefore, the conversion factor is 0.8355.

Cheapest-to-Deliver Bond

The conversion factor system is not perfect and often results in one bond that is the cheapest (or most profitable) to deliver. The procedure to determine which bond is the cheapest to deliver (CTD) is as follows:

cash received by the short = (QFP × CF) + AI

cost to purchase bond = (quoted bond price + AI)

The CTD bond minimizes the following: quoted bond price − (QFP × CF). This expression calculates the cost of delivering the bond.

EXAMPLE: The cheapest-to-deliver bond

Assume an investor with a short position is about to deliver a bond and has four bonds to choose from, which are listed in the following table. The last settlement price is $95.75 (this is the quoted futures price). **Determine** which bond is the cheapest to deliver.

Bond	Quoted Bond Price	Conversion Factor
1	99	1.01
2	125	1.24
3	103	1.06
4	115	1.14

Answer:

Cost of delivery:

Bond 1: 99 − (95.75 × 1.01) = $2.29

Bond 2: 125 − (95.75 × 1.24) = $6.27

Bond 3: 103 − (95.75 × 1.06) = $1.51

Bond 4: 115 − (95.75 × 1.14) = $5.85

Bond 3 is the cheapest to deliver with a cost of delivery of $1.51.

Finding the cheapest-to-deliver bond does not require any arcane procedures but could involve searching among a large number of bonds. The following guidelines give an indication of what type of bonds tend to be the cheapest to deliver under different circumstances:

- When yields > 6%, CTD bonds tend to be low-coupon, long-maturity bonds.
- When yields < 6%, CTD bonds tend to be high-coupon, short-maturity bonds.
- When the yield curve is upward sloping, CTD bonds tend to have longer maturities.
- When the yield curve is downward sloping, CTD bonds tend to have shorter maturities.

The settlement price is the trading price of the futures contract at 2:00 pm Chicago time (Central). Because the investor with the short position gets to decide when to deliver, there is the possibility to deliver after 2:00 pm if bond prices fall after that time and that may reduce the delivery cost. That benefit is known as the **wild card play**. Combined with the short position's choice to deliver on any day of the delivery month, the impact is to lower the futures price. In other words, because the futures contract has so many benefits that accrue to the short position, the short position should be willing to accept a lower delivery price.

Treasury Bond Futures Price

LO 45.g: Calculate the theoretical futures price for a Treasury bond futures contract.

Recall the cost-of-carry relationship, where the underlying asset pays a known cash flow, as was presented in the previous reading. The futures price is calculated in the following fashion:

$$F_0 = (S_0 - I) \times (1 + r)^T$$

where:

I = present value of cash flow

On the assumption of continuous compounding, the equation becomes:

$$F_0 = (S_0 - I)e^{rT}$$

We can use this equation to calculate the theoretical futures price when accounting for the CTD bond's accrued interest and its conversion factor.

EXAMPLE: Theoretical futures price

Suppose that the CTD bond for a Treasury bond futures contract pays 10% semiannual coupons. This CTD bond has a conversion factor of 1.1 and a quoted bond price of 100. Assume that there are 180 days between coupons and the last coupon was paid 90 days ago. Also assume that the Treasury bond futures contract is to be delivered 180 days from today, and the risk-free rate of interest is 3%. **Calculate** the theoretical price for this T-bond futures contract.

Answer:

The cash price of the CTD bond is equal to the quoted bond price plus accrued interest. Accrued interest is computed as follows:

$$AI = \text{coupon} \times \left(\frac{\text{number of days from last coupon to settlement date}}{\text{number of days in coupon period}}\right)$$

$$AI = 5 \times \frac{90}{180} = 2.5$$

$$\text{cash price} = 100 + 2.5 = 102.5$$

Because the next coupon will be received 90 days from today, that cash flow should be discounted back to the present using the familiar present value equation, which discounts the cash flow using the risk-free rate:

$$5e^{-0.03\times(90/365)} = \$4.96$$

Using the cost-of-carry model, the cash futures price (which expires 180 days from today) is then calculated as follows:

$$F_0 = (102.5 - 4.96)e^{(0.03)(180/365)} = 98.99$$

We are not done, however, because the futures contract expires 90 days after the last coupon payment. The quoted futures price at delivery is calculated after subtracting the amount of accrued interest (recall: QFP = cash futures price − AI).

$$98.99 - \left(5 \times \frac{90}{180}\right) = \$96.49$$

Finally, the conversion factor is utilized, producing a theoretical price for this T-bond futures contract of:

$$\text{QFP} = \frac{96.49}{1.1} = \$87.72$$

Eurodollar Futures

LO 45.h: Calculate the final contract price on a Eurodollar futures contract and compare Eurodollar futures to FRAs.

LO 45.i: Describe and compute the Eurodollar futures contract convexity adjustment.

The three-month **Eurodollar futures** contract trades on the Chicago Mercantile Exchange (CME) and is the most popular interest rate futures in the United States. This contract settles in cash, and the minimum price change is one “tick,” which is a price change of one basis point, or $25 per $1 million contract. Eurodollar futures are based on a Eurodollar deposit (a Eurodollar is a U.S. dollar deposited outside the United States) with a face amount of $1 million. The interest rate underlying this contract is essentially the three-month (90-day) SOFR futures contract. If *Z* is the quoted price for a Eurodollar futures contract, the contract price is:

Eurodollar futures price = $10,000[100 − (0.25)(100 − Z)]

For example, if the quoted price, *Z*, is 97.8:

contract price = $10,000[100 − (0.25)(100.0 − 97.8)] = $994,500

There are two key differences between Eurodollar futures and forward-rate agreements (FRAs). First, assuming a three-month contract, the interest on the FRA is paid at the end of the three months, while for Eurodollar futures the interest is paid at the beginning. Second, FRAs are settled only at the very end, while Eurodollar futures have daily settlement.

Convexity Adjustment

The corresponding 90-day SOFR futures (on an annual basis) for each contract is 100 – Z. For example, assume that the previous Eurodollar contract was for a futures contract that matured in six months. Then the 90-day SOFR futures six months from now is approximately 2.2% (= 100 – 97.8). However, the daily marking to market aspect of the futures contract can result in differences between actual forward rates and those implied by futures contracts. This difference is reduced by using the convexity adjustment. In general, long-dated Eurodollar futures contracts result in implied forward rates larger than actual forward rates. The two are related as follows:

$$\text{forward rate} = \text{futures rate} - (\tfrac{1}{2} \times \sigma^2 \times T_1 \times T_2)$$

where:

T_1 = maturity (in years) of the futures contract

T_2 = time to the maturity of the rate underlying the contract (T_1 + 90 days)

σ = standard deviation of the change in the short-term rate underlying the futures contract over one year

futures rate = 100 – futures quote

Notice that as T_1 increases, the convexity adjustment will need to increase. So as the maturity of the futures contract increases, the necessary convexity adjustment increases. Also, note that the σ and the T_2 are largely dictated by the specifications of the futures contract.

EXAMPLE: Calculating the forward rate

Suppose a Eurodollar futures quote is 97.300, the standard deviation of the change in the monthly rate over one year is 1.7%, and the time to maturity is three years. Ignoring compounding effects, **calculate** the forward rate.

Answer:

Based on the information provided, the futures rate is 2.7% (= 100 – 97.3), σ = 0.017, and T_1 = 3. The forward rate is calculated as: $0.027 - (0.5 \times 0.017^2 \times 3 \times 3.25) = 0.02559$, or about 2.56%.

MODULE QUIZ 45.2

1. Assume that an investor is about to deliver a short bond position and has four options to choose from, which are listed in the following table. The settlement price is $92.50 (i.e., the quoted futures price). Determine which bond is the cheapest to deliver.

Bond	Quoted Bond Price	Conversion Factor
1	98	1.02
2	122	1.27
3	105	1.08
4	112	1.15

A. Bond 1.

B. Bond 2.
C. Bond 3.
D. Bond 4.

MODULE 45.3: DURATION-BASED HEDGING

LO 45.j: Calculate the duration-based hedge ratio and create a duration-based hedging strategy using interest rate futures.

The objective of a **duration-based hedge** is to create a combined position that does not change in value when yields change by a small amount. In other words, a position that has a duration of zero needs to be produced. The combined position consists of our portfolio with a hedge horizon value of P and a futures position with a contract value of F. Denote the duration of the portfolio at the hedging horizon as D_P and the corresponding duration of the futures contract as D_F. Using this notation, the duration-based hedge ratio can be expressed as follows:

$$\text{number of contracts} = -\frac{\text{portfolio value} \times \text{duration}_{\text{portfolio}}}{\text{futures value} \times \text{duration}_{\text{futures}}}$$

The minus sign suggests that the futures position is the opposite of the original position. In other words, if the investor is long the portfolio, he must short N contracts to produce a position with a zero duration.

EXAMPLE: Duration-based hedge

Assume there is a 6-month hedging horizon and a portfolio value of $100 million. Further assume that the 6-month T-bond contract is quoted (as a % of par) at 105, with a contract size of $100,000. The duration of the portfolio is 10, and the duration of the futures contract is 12. **Calculate** the appropriate hedge for small changes in yield.

Answer:

$$N = -\frac{100{,}000{,}000 \times 10}{105{,}000 \times 12} = -793.65$$

Rounding up to the nearest whole number means the manager should short 794 contracts.

Limitations of Duration

LO 45.k: Explain the limitations of using a duration-based hedging strategy.

The price/yield relationship of a bond is convex, meaning that it is nonlinear in shape. Duration measures are linear approximations of this relationship. Therefore, as the change in yield increases, the duration measures become progressively less accurate. Moreover, duration implies that all yields are perfectly correlated. Both of these assumptions place limitations on the use of duration as a single risk measurement tool. When changes in interest rates are both large and nonparallel (i.e., not perfectly correlated), duration-based hedge strategies will perform poorly.

MODULE QUIZ 45.3

1. Assume a 6-month hedging horizon and a portfolio value of $30 million. Further assume that the 6-month Treasury bond (T-bond) contract is quoted at 100.41, with a contract size of $100,000. The duration of the portfolio is 8, and the duration of the futures contract is 12. Which of the following is closest to the appropriate hedge for small changes in yield?
   A. Long 298 contracts.
   B. Short 298 contracts.
   C. Long 199 contracts.
   D. Short 199 contracts.

2. Which of the following statements regarding the use of duration as a risk metric is correct?
   A. It can consider yield changes of any size.
   B. It assumes that interest rate volatility is constant.
   C. It assumes that the price/yield relationship is linear.
   D. It can consider parallel and nonparallel yield curve changes.

KEY CONCEPTS

LO 45.a

Day count conventions play a role when computing the interest that accrues on a fixed-income security. When a bond is purchased, the buyer must pay any accrued interest earned through the settlement date. The most common day count conventions are actual/actual, 30/360, and actual/360.

LO 45.b

T-bills are quoted on a discount-rate basis. A T-bill with a $100 face value with *n* days to maturity and a cash price of *Y* is quoted as:

$$\text{T-bill discount rate} = \frac{360}{n}(100 - Y)$$

LO 45.c

For a U.S. Treasury bond, the dirty price is the price that the seller of the bond must be paid to give up ownership. It includes the present value of the bond plus the accrued interest. Conversely, the clean price is the dirty price less accrued interest.

LO 45.d

Because deliverable bonds have very different market values, the Chicago Board of Trade (CBOT) has created conversion factors.

Conversion factors for 10-year Treasury notes or longer are calculated as the time to maturity from Day 1 of the delivery month to the maturity of the bond. That amount is then rounded down to the nearest three months. Finally, the rounded-down time to maturity is used to calculate the clean price per dollar of face value on the assumption of a 6% annual yield, compounded semiannually. For shorter periods, such as for 2-year or 5-year Treasury notes, the rounding down is to the nearest month (not three months).

LO 45.e

The conversion factor system is not perfect and often results in one bond that is the cheapest (or most profitable) to deliver. The cheapest-to-deliver (CTD) bond is the bond that minimizes the following:

quoted bond price − (quoted futures price × conversion factor)

LO 45.f

When the yield curve is not flat, there is a single bond that is the CTD. When the yield curve is upward sloping, CTD bonds tend to have longer maturities. When the yield curve is downward sloping, CTD bonds tend to have shorter maturities.

LO 45.g

The theoretical price for a T-bond futures contract is calculated as:

(cash futures price − accrued interest) / conversion factor

LO 45.h

Eurodollar contracts are based on SOFR futures contracts and are quoted on a discount rate basis. If *Z* is the quoted price for a Eurodollar futures contract, the contract price is:

Eurodollar futures price = \$10,000 × [100 − (0.25) × (100 − Z)]

LO 45.i

Long-dated Eurodollar contracts must be adjusted for convexity before being used to estimate the corresponding forward rates. As the maturity of the futures contract increases, the necessary convexity adjustment increases.

LO 45.j

Duration can be used to compute the number of futures contracts needed to implement a duration-based hedging strategy. The duration-based hedge ratio can be expressed as follows:

$$\text{number of contracts} = -\frac{\text{portfolio value} \times \text{duration}_{\text{portfolio}}}{\text{futures value} \times \text{duration}_{\text{futures}}}$$

LO 45.k

The effectiveness of duration-based hedging strategies is limited when there are large changes in yield or nonparallel shifts in the yield curve.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 45.1

1. **C** July accrued T-bond interest is 31/184 = 0.1685; July accrued corporate bond interest is 30/180 = 0.1667. T-bonds accrue 155/184 = 0.8424 × \$2 = \$1.6848; C-bonds accrue 152/180 = 0.8444 × \$2 = \$1.6889. (LO 45.a)

2. **B** The discount rate on a U.S. T-bill is calculated using the following equation:

$$\text{discount rate} = \frac{360}{n} \times (100 - \text{cash price})$$

$$\text{discount rate} = \frac{360}{90} \times (100 - 98.75) = 5\%$$

(LO 45.b)

Module Quiz 45.2

1. **A** Cost of delivery:

Bond 1: 98 − (92.50 × 1.02) = \$3.65

Bond 2: 122 − (92.50 × 1.27) = \$4.53

Bond 3: 105 − (92.50 × 1.08) = \$5.10

Bond 4: 112 − (92.50 × 1.15) = \$5.63

Bond 1 is the cheapest to deliver with a cost of delivery of \$3.65. (LO 45.e)

Module Quiz 45.3

1. **D** $N = -\dfrac{(\$30{,}000{,}000 \times 8)}{(\$100{,}410 \times 12)} = -199$

The appropriate hedge is to short 199 contracts. (LO 45.j)

2. **C** The limitations of duration include: (1) that it is valid for only *small changes in yield*, (2) that it assumes that the price/yield relationship is linear, and (3) that it assumes that changes in yield are the same across all maturities and risk levels (i.e., they're perfectly correlated). (LO 45.k)

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. Cross-reference to GARP FRM Part I Financial Markets and Products, Chapter 20.

READING 46

SWAPS

Study Session 11

EXAM FOCUS

An interest rate swap is an agreement between two parties to exchange interest payments based on a specified principal over a period of time. In a plain vanilla interest rate swap, one of the interest rates is floating, and the other is fixed. Swaps can be used to efficiently alter the interest rate risk of existing assets and liabilities. A currency swap exchanges interest rate payments in two different currencies. For valuation purposes, swaps can be thought of as a long and short position in two different bonds or as a package of forward rate agreements. Credit risk in swaps cannot be ignored.

MODULE 46.1: MECHANICS OF INTEREST RATE SWAPS

LO 46.a: Explain the mechanics of a plain vanilla interest rate swap and compute its cash flows.

The most common interest rate swap is a **plain vanilla interest rate swap**. In this swap arrangement, Company X agrees to pay Company Y a periodic fixed rate on a notional principal over the tenor of the swap. In return, Company Y agrees to pay Company X a periodic floating rate on the same notional principal. Both payments are in the same currency. Therefore, only the net payment is exchanged. Most interest rate swaps use overnight interest rates (e.g., the Secured Overnight Financing Rate [SOFR]) as the reference rate for the floating leg of the swap. Finally, because the payments are based in the same currency, there is no need for the exchange of principal at the inception of the swap. That is why it is called notional principal; the notional principal is used only to determine the respective interest rates.

For example, Companies X and Y enter into a two-year plain vanilla interest rate swap. The swap cash flows are exchanged semiannually, and the reference rate is six-month SOFR. The SOFR rates are shown in Figure 46.1. The fixed rate of the swap is 3.784%, and the notional principal is $100 million. We will compute the cash flows for Company X, the fixed payer of this swap.

Figure 46.1: Six-Month SOFR

Beginning of Period	SOFR
1	3.00%
2	3.50%
3	4.00%
4	4.50%
5	5.00%

The first cash flow takes place at the end of Period 1 and uses the SOFR at the beginning of that same period. In other words, at the beginning of each period, both payments for the end of the period are known. The gross cash flows for the end of the first period for both parties are calculated in the following manner:

$$\text{floating} = \$100 \text{ million} \times 0.03 \times 0.5 = \$1.5 \text{ million}$$

$$\text{fixed} = \$100 \text{ million} \times 0.03784 \times 0.5 = \$1.892 \text{ million}$$

Note that 0.5 is the semiannual day count. The net payment for Company X is an outflow of $0.392 million. Note that for simplicity to focus on the key issues, we are ignoring the less crucial day-count and business-day conventions associated with swaps. Figure 46.2 shows the other cash flows.

Figure 46.2: Swap Cash Flows

End of Period	SOFR at Beginning of Period	Floating Cash Flow	Fixed Cash Flow	Net X Cash Flow
1	3.00%	$1,500,000	$1,892,000	−$392,000
2	3.50%	$1,750,000	$1,892,000	−$142,000
3	4.00%	$2,000,000	$1,892,000	$108,000
4	4.50%	$2,250,000	$1,892,000	$358,000

LO 46.b: Explain how a plain vanilla interest rate swap can be used to transform an asset or a liability and calculate the resulting cash flows.

Let's continue with Companies X and Y. Suppose that X has a two-year floating-rate liability, and Y has a two-year fixed-rate liability. After they enter into the swap, interest rate risk exposure from their liabilities has completely changed for each party. X has transformed the floating-rate liability into a fixed-rate liability, and Y has transformed the fixed-rate liability to a floating-rate liability. Note that X pays fixed and receives floating, so X's liability becomes fixed.

Similarly, assume that X has a fixed-rate asset and Y has a floating-rate asset tied to SOFR. After entering into the swap, X has transformed the fixed-rate asset into a floating-rate asset, and Y has transformed the floating-rate asset into a fixed-rate asset.

Financial Intermediaries

LO 46.c: Explain the role of financial intermediaries in the swaps market.

LO 46.d: Describe the role of the confirmation in a swap transaction.

There are swap intermediaries who bring together parties with needs for the opposite side of a swap. Dealers, large banks, and brokerage firms act as principals or market makers in trades. In many cases, a swap party will not be aware of the other party on the offsetting side of the swap because both parties will likely only transact with the intermediary. Financial intermediaries, such as banks, will typically earn a spread for bringing two nonfinancial companies together in a swap agreement. This fee is charged to compensate the intermediary for the risk involved. If one of the parties defaults on its swap payments, the intermediary is responsible for making the other party whole.

Confirmations, as drafted by the International Swaps and Derivatives Association (ISDA), outline the details of each swap agreement. A representative of each party signs the confirmation, ensuring that they agree with all swap details (such as tenor, fixed/floating rates, and payment dates) and the steps taken in the event of default.

Comparative Advantage

LO 46.e: Describe the comparative advantage argument for the existence of interest rate swaps and evaluate some of the criticisms of this argument.

Let's return to Companies X and Y and assume that they have access to borrowing for two years as specified in Figure 46.3.

Figure 46.3: Borrowing Rates for X and Y

Company	Fixed Borrowing	Floating Borrowing
X	6.5%	SOFR + 100 bps
Y	5.0%	SOFR + 10 bps

With the lowest borrowing rates in both markets, Company Y has an absolute advantage in both markets but a comparative advantage in the fixed market. Notice that the differential between X and Y in the fixed market is 1.5%, or 150 basis points (bps), and the corresponding differential in the floating market is only 90 basis points. When this is the case, Y has a comparative advantage in the fixed market, and X has a comparative advantage in the floating market. When a comparative advantage exists, a swap arrangement will reduce the costs of both parties. In this example, the net potential borrowing savings by entering into a swap is the difference between the differences, or 60 bps. In other words, by entering into a swap, the total savings shared between X and Y is 60 bps.

To better understand where the 60 bps comes from, suppose Y borrows fixed at 5% for two years, X borrows floating for two years at SOFR + 1%, and then X and Y enter into a swap to transform their liabilities. Specifically, X pays Y fixed and Y pays X floating based on SOFR. If we assume that the net savings is split evenly, the net borrowing costs for X are then 6.2% and SOFR − 20 bps for Y. Each has saved 30 bps for a total of 60 bps. If an intermediary were used, part of the 60 bps would be used to pay the bid-ask spread.

A problem with the comparative advantage argument is that it assumes X can borrow at SOFR + 1% over the life of the swap, which is not correct because the 1% spread would likely vary as X's credit rating changes. It also ignores the credit risk taken on by Y by entering into the swap. If X were to raise funds by borrowing directly in the capital markets, no credit risk is taken, so perhaps the savings is compensation for that risk. The same criticisms exist when an intermediary is involved.

MODULE QUIZ 46.1

1. Two companies, C and D, have the borrowing rates shown in the following table.

   Borrowing Rates for C and D

Company	Fixed Borrowing	Floating Borrowing
C	10%	SOFR + 50 bps
D	12%	SOFR + 100 bps

   According to the comparative advantage argument, what is the total potential savings for C and D if they enter into an interest rate swap?

   A. 0.5%.
   B. 1.0%.
   C. 1.5%.
   D. 2.0%.

2. Which of the following swaps would properly transform a floating-rate liability to a fixed-rate liability?

   A. Entering into a pay foreign currency swap.
   B. Entering into a pay fixed interest rate swap.
   C. Entering into a pay domestic currency swap.
   D. Entering into a pay floating interest rate swap.

MODULE 46.2: VALUATION OF INTEREST RATE SWAPS

The Discount Rate

LO 46.f: Explain how the discount rates in a plain vanilla interest rate swap are computed.

Because a swap is nothing more than a sequence of cash flows, its value is determined by discounting each cash flow back to the valuation date. The question is, what is the appropriate *discount rate* to use? It turns out that the forward rates implied by either forward rate agreements (FRAs) or the convexity-adjusted Eurodollar futures are used to produce an SOFR spot curve. The swap cash flows are then discounted using the corresponding spot rate from this curve. The following connection between forward rates and spot rates exists:

$$R_{Forward} = R_2 + (R_2 - R_1)\frac{T_1}{T_2 - T_1}$$

where:

R_i = spot rate corresponding with T_i years

$R_{Forward}$ = forward rate between T_1 and T_2

We will utilize this equation later when we value an interest rate swap using a sequence of forward rate agreements.

Valuing an Interest Rate Swap With Bonds

LO 46.g: Calculate the value of a plain vanilla interest rate swap based on two simultaneous bond positions.

Let's return to our two companies, X and Y, in our two-year swap arrangement. From X's perspective, there are two series of cash flows—one fixed going out and one floating coming in. Essentially, X has a long position in a floating-rate note (because it is an inflow) and a short position in a fixed-rate note (because it is an outflow). From Y's perspective, it is exactly the opposite—Y has a short position in a floating-rate note (because it is an outflow) and a long position in a fixed-rate note (because it is an inflow).

If we denote the present value of the fixed-leg payments as B_{fix} and the present value of the floating-leg payments as B_{flt}, the value of the swap can be written for both X and Y as:

$$V_{swap}(X) = B_{flt} - B_{fix}$$
$$V_{swap}(Y) = B_{fix} - B_{flt}$$

Note that $V_{swap}(X) + V_{swap}(Y) = 0$. This is by design because the two positions are mirror images of each other. At inception of the swap, it is convention to select the fixed payment so that $V_{swap}(X) = V_{swap}(Y) = 0$. As expected floating rates in the future change, the swap value for each party is no longer zero.

Valuing an interest rate swap in terms of bond positions involves understanding that the value of a floating-rate bond will be equal to the notional amount at any of its periodic settlement dates when the next payment is set to the market (floating) rate. Because $V_{swap} = \text{Bond}_{fixed} - \text{Bond}_{floating}$, we can value the fixed-rate bond using the spot rate curve and then discount the next (known) floating-rate payment plus the notional amount at the current discount rate. The following example illustrates this method.

EXAMPLE: Valuing an interest rate swap

Consider a $1 million notional swap that pays a floating rate based on 6-month SOFR and receives a 6% fixed rate semiannually. The swap has a remaining life of 15 months with pay dates at 3, 9, and 15 months. Spot SOFR rates are as follows: 3 months at

5.4%; 9 months at 5.6%; and 15 months at 5.8%. The SOFR at the last payment date was 5.0%. **Calculate** the value of the swap to the fixed-rate receiver using the bond methodology.

Answer:

$$B_{fixed} = (\$30{,}000 / 1.054^{0.25}) + (\$30{,}000 / 1.056^{0.75}) + (\$1{,}030{,}000 / 1.058^{1.25})$$
$$= \$29{,}608 + \$28{,}799 + \$959{,}909$$
$$= \$1{,}018{,}316$$

$$B_{floating} = \{\$1{,}000{,}000 + [\$1{,}000{,}000 \times (0.05 / 2)]\} / 1.054^{0.25}$$
$$= \$1{,}011{,}611$$

$$V_{swap} = B_{fixed} - B_{floating} = \$1{,}018{,}316 - \$1{,}011{,}611 = \$6{,}705$$

Figure 46.4: Valuing an Interest Rate Swap With Two Bond Positions

Time	Fixed Cash Flow	Floating Cash Flow	Present Value Factor	PV Fixed CF	PV Floating CF
0.25 (3 months)	30,000	1,025,000	0.987*	29,608	1,011,611
0.75 (9 months)	30,000		0.960*	28,799	
1.25 (15 months)	1,030,000		0.932*	959,909	
Total				1,018,316	1,011,611

*Note that some rounding has occurred.

Again we see that the value of the swap = 1,018,316 − 1,011,611 = $6,705.

Valuing an Interest Rate Swap With FRAs

LO 46.h: Calculate the value of a plain vanilla interest rate swap from a sequence of FRAs.

At settlement, the payment made on a forward rate agreement is the notional amount multiplied by the difference between a market (floating) rate such as SOFR and the contract (fixed) rate specified in the FRA. This is identical to a periodic payment on an interest rate swap when the reference floating rates and notional principal amounts are the same and the swap fixed rate is equal to the contract rate specified in the FRA. Viewed this way, we can see that an interest rate swap is equivalent to a series of FRAs. One way to value a swap would be to use expected forward rates to forecast the expected net cash flows and then discount these expected cash flows at the corresponding spot rates, consistent with forward rate expectations.

EXAMPLE: Valuing an interest rate swap with FRAs

Consider the previous example on valuing an interest rate swap with two bond positions. An investor has a $1 million notional swap that pays a floating rate based on 6-month SOFR and receives a 6% fixed rate semiannually. The swap has a remaining life of 15 months with pay dates at 3, 9, and 15 months. Spot SOFR rates are as follows: 3 months at 5.4%; 9 months at 5.6%; and 15 months at 5.8%. The SOFR at the last payment date was 5.0%. **Calculate** the value of the swap to the fixed-rate receiver using the FRA methodology.

Answer:

To calculate the value of the swap, we'll need to find the floating-rate cash flows by calculating the expected forward rates via the SOFR based spot curve.

The first floating-rate cash flow is calculated in a similar fashion to the previous example.

SOFR rate (last payment date): 5%

Floating-rate cash flow in 3 months: 1,000,000 × (0.05 / 2) = $25,000

The second floating-rate cash flow is calculated by finding the forward rate that corresponds to the period between 3 months and 9 months. To calculate forward rate for the period between 3 and 9 months, use the previously mentioned (continuously compounded) forward rate formula:

$$R_{forward} = R_2 + (R_2 - R_1)\frac{T_1}{T_2 - T_1}$$

$$R_{forward} = 0.056 + (0.056 - 0.054)\frac{0.25}{0.75 - 0.25} = 0.057 = 5.7\%$$

Floating-rate cash flow in 9 months: 1,000,000 × (0.057 / 2) = $28,500

The third floating-rate cash flow is calculated by finding the forward rate that corresponds to the period between 9 months and 15 months.

$$R_{forward} = 0.058 + (0.058 - 0.056)\frac{0.75}{1.25 - 0.75} = 0.061 = 6.1\%$$

Floating-rate cash flow in 15 months: 1,000,000 × (0.061 / 2) = $30,500

Figure 46.5: Valuing an Interest Rate Swap Based on a Sequence of FRAs

Time	Fixed Cash Flow	Floating Cash Flow	Present Value Factor	PV Fixed CF	PV Floating CF
0.25 (3 months)	30,000	25,000	0.987*	29,610	24,675
0.75 (9 months)	30,000	28,500	0.960*	28,800	27,360
1.25 (15 months)	30,000	30,500	0.932*	27,960	28,426
Total				86,370	80,461

* Note that some rounding has occurred.

The value of the swap based on a sequence of FRAs = $86,370 − $80,461 = $5,909.

As you can see from the previous two examples, valuing a swap based on a sequence of forward rate agreements produces a similar result as valuing a swap based on two simultaneous bond positions. However, valuation based on FRAs assumes that forward rates are continuously compounded, while valuation based on bond positions uses discrete compounding.

MODULE QUIZ 46.2

1. Consider the following information:
   - \$1 million notional value, semiannual, 18-month maturity.
   - Spot SOFR rates: 6 months, 2.6%; 12 months, 2.65%; 18 months, 2.75%.
   - The fixed rate is 2.8%, with semiannual payments.

   Which of the following amounts is closest to the value of the swap to the floating-rate payer, assuming that it is currently the floating-rate reset date?

   A. –\$1,026.
   B. \$1,026.
   C. –\$12,416.
   D. \$12,416.

MODULE 46.3: OTHER TYPES OF SWAPS

Currency Swaps

LO 46.j: Calculate the value of a currency swap based on two simultaneous bond positions.

A **currency swap** exchanges both principal and interest rate payments with payments in different currencies. The exchange rate used in currency swaps is the spot exchange rate. The valuation and application of currency swaps is similar to the interest rate swap. However, because the principals in a currency swap are not the same currency, they are exchanged at the inception of the currency swap so that they have equal value using the spot exchange rate. Also, the periodic cash flows throughout the swap are not netted as they are in the interest rate swap.

Suppose we have two companies, A and B, that enter into a fixed-for-fixed currency swap with periodic payments annually. Company A pays 6% in Great Britain pounds (GBP) to Company B and receives 5% in U.S. dollars (USD) from Company B. Company A pays a principal amount to B of USD 175 million, and B pays GBP 100 million to A at the outset of the swap. Notice that A has effectively borrowed GBP from B, and so it must pay interest on that loan. Similarly, B has borrowed USD from A. The cash flows in this swap are actually more easily computed than in an interest rate swap because both legs of the swap are fixed. Every period (12 months), A will pay GBP 6 million to B, and B will pay USD 8.75 million to A. At the end of the swap, the principal amounts are re-exchanged.

From Company A's perspective, there are two series of cash flows: one fixed GBP cash flow stream going out and one fixed USD cash flow stream coming in. Essentially, A has a long position in a USD-denominated note (because it's an inflow) and a short position in a GBP-denominated note (because it's an outflow).

If we denote the present value of the GBP-denominated payments as B_{GBP} and the present value of the USD payments as B_{USD}, the value of the swap in USD to Company A is:

$$V_{swap}(USD) = B_{USD} - (S_0 \times B_{GBP})$$

where:

S_0 = spot rate in USD per GBP

EXAMPLE: Calculate the value of a currency swap

Suppose the yield curves in the United States and Great Britain are flat at 2% and 4%, respectively, and the current spot exchange rate is USD 1.50 = GBP 1. **Value** the currency swap just discussed assuming that the swap will last for three more years.

Answer:

$$B_{USD} = 8.75 / 1.02 + 8.75 / 1.02^2 + 183.75 / 1.02^3 = \text{USD } 190.14 \text{ million}$$

$$B_{GBP} = 6 / 1.04 + 6 / 1.04^2 + 106 / 1.04^3 = \text{GBP } 105.55 \text{ million}$$

$$V_{swap} \text{ (to A in USD)} = 190.14 - (1.50 \times 105.55) = \text{USD } 31.82 \text{ million}$$

LO 46.k: Calculate the value of a currency swap based on a sequence of forward exchange rates.

The value of a currency swap can also be calculated based on a sequence of forward exchange rates.

EXAMPLE: Value of a currency swap with forward exchange rates

Suppose the yield curves in the United States and Great Britain are flat at 2% and 4%, respectively, and the current spot exchange rate is USD 1.50 = GBP 1. Note that in this case, GBP is the base currency and USD is the quote currency.

Compute the value of the currency swap discussed previously using a sequence of forward exchange rates to Company A. Assume the swap will last for three more years.

The corresponding forward rates are as follows:

Figure 46.6: Forward Rates

Year 1: $1.47/£
Year 2: $1.44/£
Year 3: $1.41/£

PROFESSOR'S NOTE

The Year 1 forward rate is calculated as follows: $F_1 = 1.50 \times 1.02 / 1.04 =$ $1.47/£. Interest rate parity suggests that the dollar will appreciate relative to the pound, so the $/£ forward rate will decline (i.e., it will take fewer USD to buy 1 GBP).

Answer:

Figure 46.7 denotes cash flows and forward rates for this currency swap.

Figure 46.7: Valuing a Currency Swap Based on a Sequence of Forward Exchange Rates

Time	USD Cash Flow	GBP Cash Flow	Forward Rate	$ Value of £	Net Cash Flows	PV of Net CF
1	8.75	6	1.47	8.82	–0.07	–0.069
2	8.75	6	1.44	8.64	0.11	0.106
3	8.75	6	1.41	8.46	0.29	0.273
3	175	100	1.41	141	34	32.02
Total						32.33*

* Note that some rounding has occurred.

Ignoring the rounding differences, we see that the value of the currency swap to Company A is about $32 million using both the two simultaneous bond positions and the forward rate agreements.

LO 46.i: Explain how a currency swap can be used to transform an asset or liability and calculate the resulting cash flows.

Currency swaps can be combined with existing positions to completely alter the risk of a liability or an asset. For example, suppose that Company A has a dollar-based liability. By entering into a currency swap, the liability has become a pound-based liability at the GBP fixed (or floating) rate. This is analogous to an interest rate swap where a floating-rate liability has become a fixed-rate liability (or vice versa).

Comparative advantage is also used to explain the success of currency swaps for multinational companies in terms of differing tax rates. For example, assume a multinational has operations in Canada and Great Britain and that Canada has the relatively higher tax rate. If the multinational needs to borrow in pounds (GBP), one strategy would be to borrow in Canadian dollars (CAD) and benefit from a large tax deduction for interest (due to the higher Canadian tax rate). Then it could swap the CAD borrowings for GBP.

Swap Credit Risk

LO 46.m: Describe the credit risk exposure in a swap position.

Because $V_{swap}(A) + V_{swap}(B) = 0$, whenever one side of a swap has a positive value, the other side must be negative. For example, if $V_{swap}(A) > 0$, $V_{swap}(B) < 0$. As $V_{swap}(A)$ increases in value, $V_{swap}(B)$ must become more negative. This results in increased credit risk to A because the likelihood of default increases as B has larger and larger payments to make to A. However, the potential losses in swaps are generally much smaller than the potential losses from defaults on debt with the same principal. This is because the value of swaps is generally much smaller than the value of the debt.

Additional Swaps

LO 46.l: Identify and describe other types of swaps, including commodity, volatility, credit default, and exotic swaps.

In an **equity swap**, the return on a stock, a portfolio, or a stock index is paid each period by one party in return for a fixed-rate or floating-rate payment. The return can be the capital appreciation or the total return including dividends on the stock, portfolio, or index.

A **credit default swap (CDS)** provides the holder with protection against default of a subject company (or reference entity). The holder of the CDS makes periodic payments to the seller for a specific term, and if the reference entity defaults during that time, the seller will make a prespecified payment to the buyer. An index CDS is the same as a regular CDS, except instead of one reference entity, there are multiple reference entities.

Firms may enter into **commodity swap** agreements where they agree to pay a fixed rate for the multiperiod delivery of a commodity and receive a corresponding floating rate based on the average commodity spot rates at the time of delivery. Although many commodity swaps exist, the most common use is to manage the costs of purchasing energy resources such as oil and electricity.

A **volatility swap** involves the exchanging of volatility based on a notional principal. One side of the swap pays based on a prespecified volatility, while the other side pays based on historical volatility.

Swaps are also sometimes created for exotic structures. An example of an **exotic swap** was between Procter and Gamble (P&G) and Banker's Trust where P&G's payments were based on a complicated combination of the commercial paper rate, a medium-term Treasury, and a long-term Treasury.

MODULE QUIZ 46.3

1. Which type of swap is most likely to be considered the most complicated in terms of structure?
   A. Commodity swap.
   B. Credit default swap.
   C. Exotic swap.
   D. Volatility swap.

2. Suppose Company X pays 5% annually (in euros) to Company Y and receives 4% annually (in dollars). Company X pays a principal amount of \$150 million to Y, and Y pays a €100 million to X at the inception of the swap. Assume the yield curve is flat in the United States and in Germany (Europe). The U.S. rate is 3%, and the German rate is 5%. The current spot exchange rate is \$1.45/€. What is the value of the currency swap to Company X using the bond methodology if it is expected to last for

two more years?

A. –$3.34 million.
B. $3.34 million.
C. –$7.86 million.
D. $7.86 million.

KEY CONCEPTS

LO 46.a

A plain vanilla interest rate swap exchanges floating-rate payments (overnight interest rates) for fixed-rate payments over the life of the swap. The floating-rate payments at time t in a plain vanilla interest rate swap are computed using the floating rate at time $t - 1$.

LO 46.b

Interest rate swaps can be combined with existing asset and liability positions to drastically change the interest rate risk (e.g., floating to fixed for more certainty, fixed to floating for less certainty).

LO 46.c

There are swap intermediaries who bring together parties with needs for the opposite side of a swap. Dealers, large banks, and brokerage firms act as principals or market makers in trades.

LO 46.d

Confirmations outline the details of each swap agreement. A representative of each party signs the confirmation, ensuring that they agree with all swap details and the steps taken in the event of default.

LO 46.e

The comparative advantage argument suggests that when one of two borrowers has a comparative advantage in either the fixed- or floating-rate market, both borrowers will be better off by entering into a swap to exploit the advantage. The comparative advantage argument is flawed in that it assumes rates can be borrowed for the life of the swap. It also ignores the credit risk associated with the swap that does not exist if funds were raised directly in the capital markets.

LO 46.f

Because a swap is nothing more than a sequence of cash flows, its value is determined by discounting each cash flow back to the valuation date. The cash flows are discounted using the corresponding spot rate from the SOFR spot curve.

LO 46.g

The value of a swap to the fixed-rate receiver at a point in time is the difference between the present value of the remaining fixed-rate payments and the present value of the remaining floating-rate payments.

LO 46.h

Valuing a swap based on a sequence of forward rate agreements (FRAs) produces the same result as valuing a swap based on two simultaneous bond positions.

LO 46.i

A currency swap exchanges interest rate payments in two different currencies. The exchange rate used in currency swaps is the spot exchange rate. Currency swaps can be combined with existing positions to completely alter the risk of a liability or an asset.

LO 46.j

Because the principals in a currency swap are not the same currency, they are exchanged at the inception of the currency swap so that they have equal value using the spot exchange rate. Also, the periodic cash flows throughout the swap are not netted as they are in an interest rate swap.

LO 46.k

In addition to valuing a currency swap based on two simultaneous bond positions, the value of a currency swap can also be calculated based on a sequence of FRAs.

LO 46.l

Many different types of swaps exist. Examples of swaps, in addition to interest rate swaps and currency swaps, include equity swaps, commodity swaps, credit default swaps, and volatility swaps.

LO 46.m

Credit risk is an important factor in existing swap positions, although potential losses are usually smaller than that with debt agreements.

ANSWER KEY FOR MODULE QUIZZES

Module Quiz 46.1

1. **C** The difference of the differences is (12% − 10%) − [(SOFR + 1%) − (SOFR + 0.5%)] = 1.5%. (LO 46.e)

2. **B** The fixed interest rate swap will allow for the conversion of a floating-rate liability to a fixed-rate liability. (LO 46.b)

Module Quiz 46.2

1. **B** B_{fixed} = (\$14,000 / $1.026^{0.5}$) + (\$14,000 / 1.0265^{1}) + [(\$1,000,000 + \$14,000) / $1.0275^{1.5}$] = \$13,821 + \$13,639 + \$973,566 = \$1,001,026

   Note that we are at a (semiannual) reset date, so the floating-rate portion has a value equal to the notional amount.

   V_{swap} = (B_{fixed} − $B_{floating}$) = \$1,001,026 − \$1,000,000 = \$1,026

   (LO 46.g)

Module Quiz 46.3

1. **C** Exotic swaps are probably the most complicated and can involve complex calculations of payments whereby the risks of the swaps are not likely fully understood by clients. The other three swaps (commodity, credit default, and volatility) are more common and, therefore, much less likely to be overly complicated. (LO 46.l)

2. **D** $B_{\$} = 6 / 1.03 + 156 / 1.03^2 = \$5.82 + \$147.04 = \$152.86$

   $B_{€} = 5 / 1.05 + 105 / 1.05^2 = €4.76 + €95.24 = €100.00$

   V_{swap} (to X) $= 152.86 - (1.45 \times 100.00) = \$7.86$ million

   (LO 46.j)

FORMULAS

Reading 28

loss ratio + expense ratio = combined ratio

combined ratio + dividends = combined ratio after dividends

combined ratio after dividends − investment income = operating ratio

Reading 29

$$\text{net asset value: NAV} = \frac{\text{fund assets} - \text{fund liabilities}}{\text{total shares outstanding}}$$

Reading 30

call option payoff:

$C_T = \max(0, S_T - X)$

where:
C_T = payoff on call option
S_T = stock price at maturity
X = strike price of option

put option payoff:

$P_T = \max(0, X - S_T)$

where:
P_T = payoff on put option
S_T = stock price at maturity
X = strike price of option

forward contract payoff:

$\text{payoff} = S_T - K$

where:
S_T = spot price at maturity
K = delivery price

Reading 33

basis = spot price − futures price

Reading 34

$$\text{optimal hedge ratio: HR} = \rho_{S,F}\frac{\sigma_S}{\sigma_F}$$

$$\text{correlation: } \rho = \frac{\text{Cov}_{S,F}}{\sigma_S \sigma_F}$$

hedging with stock index futures:

$$\text{number of contracts} = \beta_{\text{portfolio}} \times \left(\frac{\text{portfolio value}}{\text{value of futures contract}}\right)$$
$$= \beta_{\text{portfolio}} \times \left(\frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}}\right)$$

adjusting portfolio beta: $\text{number of contracts} = (\beta^* - \beta)\dfrac{P}{A}$

Reading 35

purchasing power parity:

$\%\Delta S = \text{inflation(foreign)} - \text{inflation(domestic)}$

where:
$\%\Delta S$ = change in domestic spot rate

nominal interest rate:

exact methodology: $(1 + r) = (1 + \text{real } r) \times [1 + E(i)]$

linear approximation: $r \approx \text{real} + E(i)$

interest rate parity:

$$\text{forward} = \text{spot} \times \left[\frac{(1 + r_{YYY})}{(1 + r_{XXX})}\right]^T$$

where:
r_{YYY} = quote currency rate
r_{XXX} = base currency rate

Reading 36

forward price: $F = S \times (1 + r)^T$

forward price with income: $F = (S - I) \times (1 + r)^T$

forward price with dividends: $F = S \times [(1 + r) / (1 + q)]^T$

Reading 37

forward price with storage costs: $F_{0,T} = (S_0 + U) \times (1 + r)^T$

forward price with lease rate:

$F_{0,T} = S_0 \times [(1 + r) / (1 + \delta)]^T$

where:
S_0 = current spot price
r = risk-free rate
δ = lease rate

forward price with convenience yield:

$F_{0,T} = (S_0 + U) \times [(1 + r) / (1 + Y)]^T$
where:
Y = annualized convenience yield

Reading 39

put-call parity equations:

$S = c - p + PV(X)$
$p = c - S + PV(X)$
$c = S + p - PV(X)$
$PV(X) = S + p - c$

lower and upper bounds for options:

Option	Minimum Value	Maximum Value
European call	$c \geq \max(0, S_0 - PV(X))$	S_0
American call	$C \geq \max(0, S_0 - PV(X))$	S_0
European put	$p \geq \max(0, PV(X) - S_0)$	$PV(X)$
American put	$P \geq \max(0, X - S_0)$	X

Reading 40

bull call spread: profit = $\max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0}$

bear put spread: profit = $\max(0, X_H - S_T) - \max(0, X_L - S_T) - P_{H0} + P_{L0}$

butterfly spread with calls:

$$\text{profit} = \max(0, S_T - X_L) - 2\max(0, S_T - X_M) + \max(0, S_T - X_H) - C_{L0} + 2C_{M0} - C_{H0}$$

straddle: profit = $\max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0$

strangle: profit = $\max(0, S_T - X_H) + \max(0, X_L - S_T) - C_0 - P_0$

Reading 42

discrete compounding: $FV = A\left(1 + \frac{R}{m}\right)^{m \times n}$

continuous compounding: $FV = Ae^{R \times n}$

$$B = \left[\frac{c}{2} \times \left(1 + \frac{z_j}{2}\right)^{-2 \times j}\right] + \left[FV \times \left(1 + \frac{z_N}{2}\right)^{-N}\right]$$

where:

c = annual coupon
N = number of semiannual payment periods
z_j = bond equivalent spot rate that corresponds to *j* periods
FV = face value of the bond

forward rate between T_1 and T_2:

$$R_{Forward} = \frac{R_2T_2 - R_1T_1}{T_2 - T_1} = R_2 + (R_2 - R_1) \times \left(\frac{T_1}{T_2 - T_1}\right)$$

where:
R_i = spot rate corresponding with T_i periods
$R_{Forward}$ = forward rate between T_1 and T_2

forward rate agreement:

cash flow (if receiving R_K) = $L \times (R_K - R) \times (T_2 - T_1)$

cash flow (if paying R_K) = $L \times (R - R_K) \times (T_2 - T_1)$

where:
L = principal
R_K = annualized fixed rate, expressed with compounding period $T_2 - T_1$
R = annualized floating rate, expressed with compounding period $T_2 - T_1$
T_i = time *i*, expressed in years

percentage bond price change ≈ duration effect + convexity effect

Reading 43

dollar default rate:

$$\frac{\text{cumulative dollar value of all defaulted bonds}}{\left(\begin{array}{c}\text{cumulative dollar value}\\ \text{of all issuance}\end{array}\right) \times \left(\begin{array}{c}\text{weighted average \# of}\\ \text{years outstanding}\end{array}\right)}$$

expected loss rate = probability of default × (1 − expected recovery rate)

Reading 44

single monthly mortality rate: $SMM = 1 - (1 - CPR)^{1/12}$

value of a dollar roll = A − B + C − D

A = price at which pool is sold in Month 1, with accrued interest
B = price at which pool is bought in Month 2, with accrued interest
C = interest earned on funds from the sale for one month
D = coupon and principal payment that was foregone on the pool sold in Month 1

Reading 45

$$\text{accrued interest} = \text{coupon} \times \frac{\text{\# of days from last coupon to the settlement date}}{\text{\# of days in coupon period}}$$

cash price of a bond: cash price = quoted price + accrued interest

annual rate on a T-bill: $\text{T-bill discount rate} = \frac{360}{n}(100 - Y)$

cash received by the short position:

$$\text{cash received} = (\text{QFP} \times \text{CF}) + \text{AI}$$

where:
QFP = quoted futures price (most recent settlement price)
CF = conversion factor for the bond delivered
AI = accrued interest since the last coupon date on the bond delivered

cheapest-to-deliver bond: quoted bond price − (QFP × CF)

Eurodollar futures price = $10,000[100 − (0.25)(100 − Z)]

duration-based hedge ratio:

$$\text{number of contracts} = -\frac{\text{portfolio value} \times \text{duration}_{\text{portfolio}}}{\text{futures value} \times \text{duration}_{\text{futures}}}$$

Reading 46

interest rate swap value:

$$V_{swap}(X) = B_{flt} - B_{fix}$$
$$V_{swap}(Y) = B_{fix} - B_{flt}$$

currency swap value:

$$V_{swap}(USD) = B_{USD} - (S_0 \times B_{GBP})$$

where:
S_0 = spot rate in USD per GBP

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