

233 Project 1 Overview

Hybrid Digital and Analog Beamforming Design for Large-Scale Antenna Arrays

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Slides adapted from Benjamin Domae's presentation on 2021

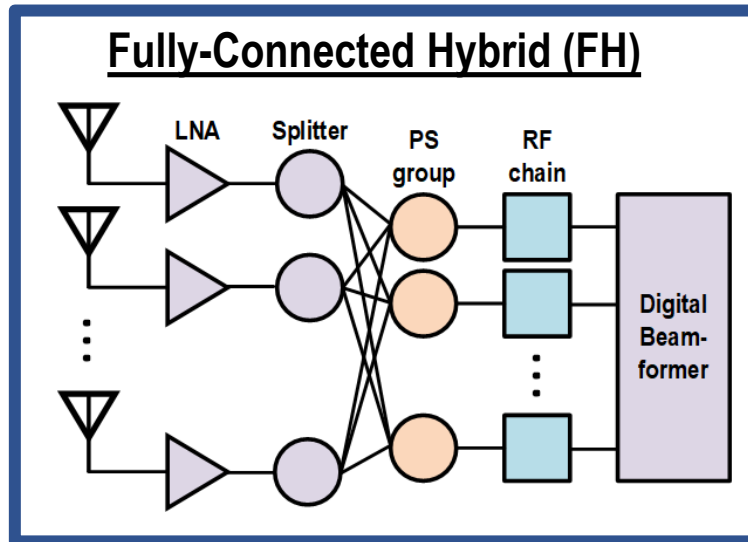
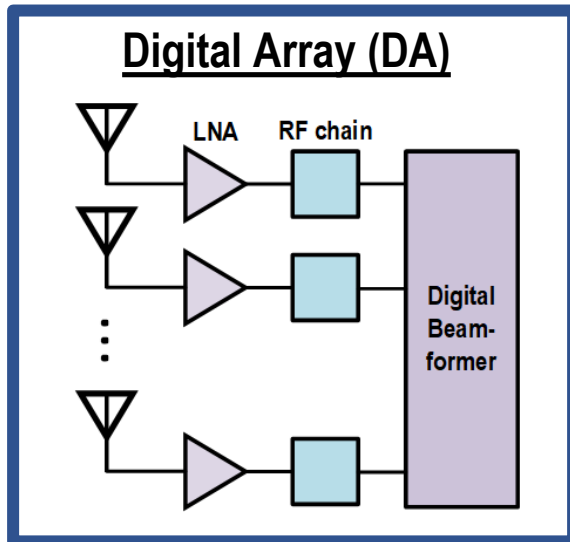
What the paper discussed



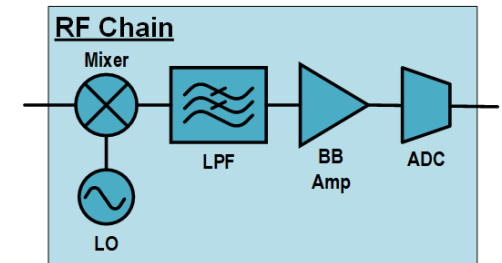
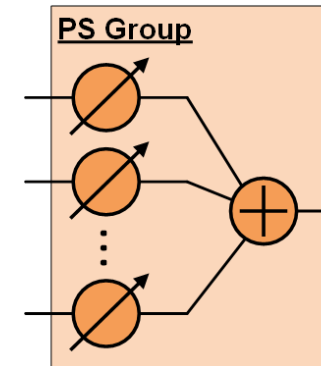
- **Authors present algorithms for hybrid MIMO precoding/combining**
 - Iterative algorithms for optimization
 - Aim for optimal precoding/combining (fully digital array solution)
 - Simple adjustment to handle PS quantization
- **Three scenarios depicted:**
 1. Large number of streams ($N^{RF} \geq 2d$) – Proof of optimal performance
 2. Point-to-point SU-MIMO ($N^{RF} < 2d$) – Simulated near-optimal sum rate
 3. MU-MISO ($N^{RF} < 2d$) – Simulated near-optimal sum rate
 - Simulated performance with PS quantization close to optimal (exhaustive search)

Motivation

- Hybrid antenna arrays can reduce power consumption and cost
- Analog processing limits MIMO precoding/combining
 - Phase-only weights for the analog stages = sub-optimal solutions
 - Traditional MIMO algorithms designed for fully digital arrays

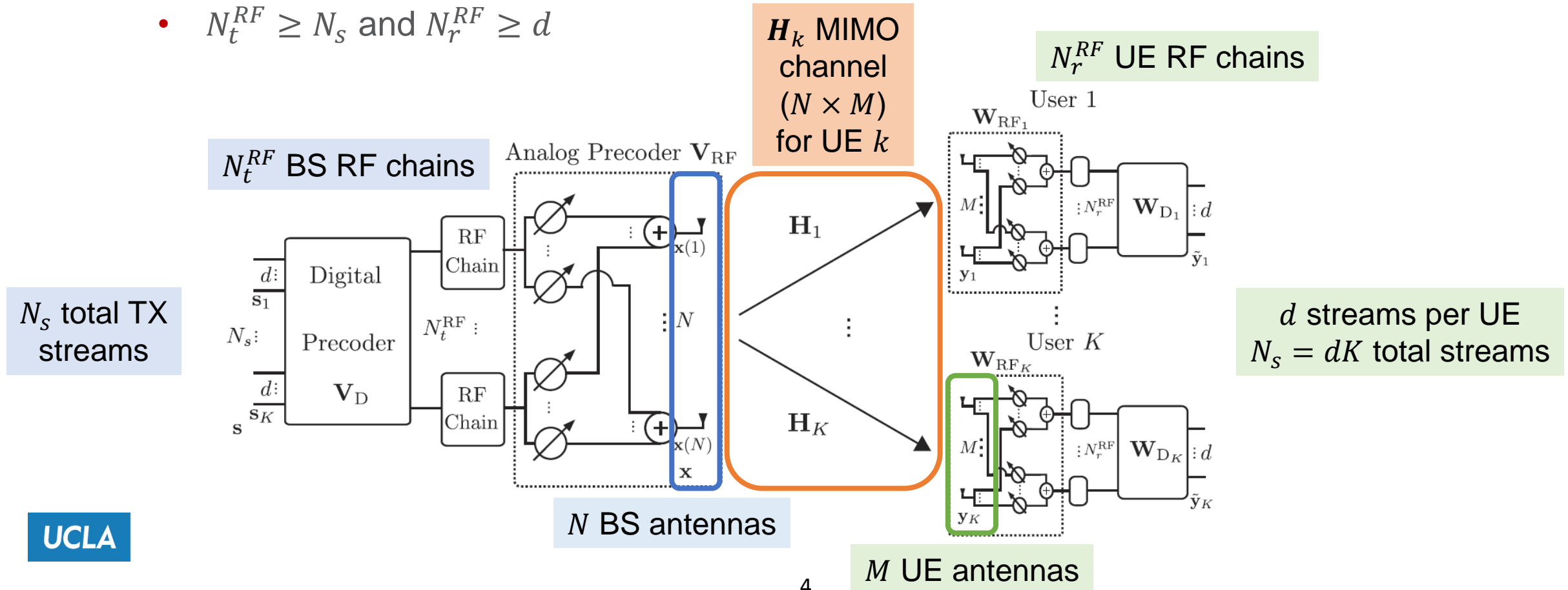


Example RX components



System Model Block Diagram

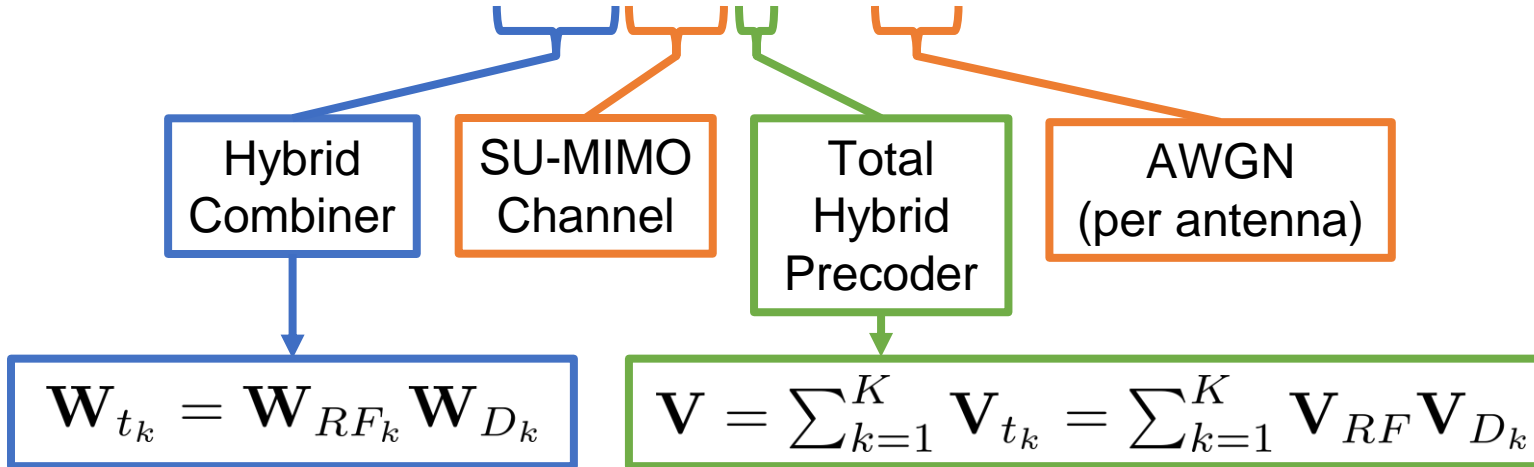
- **Multi-user (MU) MIMO downlink:**
 - Base station (BS) transmitting to K users (UEs)
 - Both BS and UEs have hybrid arrays
 - $N_t^{RF} \geq N_s$ and $N_r^{RF} \geq d$



System Model Equations

General MU-MIMO downlink received symbols

$$\tilde{\mathbf{y}}_k = \mathbf{W}_{t_k}^H (\mathbf{H}_k \mathbf{V} \mathbf{s} + \mathbf{z}_k)$$



$$\tilde{\mathbf{y}}_k = \underbrace{\mathbf{W}_{t_k}^H \mathbf{H}_k \mathbf{V}_{t_k} \mathbf{s}_k}_{\text{Desired Signals}} + \underbrace{\mathbf{W}_{t_k}^H \mathbf{H}_k \sum_{\ell \neq k} \mathbf{V}_{t_\ell} \mathbf{s}_\ell}_{\text{Inter-user Interference}} + \underbrace{\mathbf{W}_{t_k}^H \mathbf{z}_k}_{\text{Spatially-correlated Noise}}$$

Transmitted symbols

$$\mathbf{x} = \mathbf{V} \mathbf{s} = \mathbf{V}_{t_k} \mathbf{s}_k + \sum_{\ell \neq k} \mathbf{V}_{t_\ell} \mathbf{s}_\ell$$

Variable	Dimension
$\tilde{\mathbf{y}}_k$	N_s
\mathbf{W}_{RF_k}	$N_s \times N_t^{RF}$
\mathbf{W}_{D_k}	$N_t^{RF} \times N$
\mathbf{H}_k	$N \times M$
\mathbf{V}_{RF}	$M \times N_r^{RF}$
\mathbf{V}_{D_k}	$N_t^{RF} \times d$
\mathbf{s}_k	d
\mathbf{z}_k	N

Mathematical Objective

- Rate (spectral efficiency) for user k :

$$R_k = \log_2 \left| \mathbf{I}_M + \mathbf{W}_{t_k} \mathbf{C}_k^{-1} \mathbf{W}_{t_k}^H \mathbf{H}_k \mathbf{V}_{t_k} \mathbf{V}_{t_k}^H \mathbf{H}_k^H \right|$$

Inter-user interference
+ noise covariance

$$\mathbf{C}_k = \mathbf{W}_{t_k}^H \mathbf{H}_k \left(\sum_{\ell \neq k} \mathbf{V}_{t_\ell} \mathbf{V}_{t_\ell}^H \right) \mathbf{H}_k^H \mathbf{W}_{t_k} + \sigma^2 \mathbf{W}_{t_k}^H \mathbf{W}_{t_k}$$

- Optimization problem:

maximize
 $\mathbf{V}_{\text{RF}}, \mathbf{V}_{\text{D}}, \mathbf{W}_{\text{RF}}, \mathbf{W}_{\text{D}}$

subject to

$$\sum_{k=1}^K \beta_k R_k$$

$$\text{Tr}(\mathbf{V}_{\text{RF}} \mathbf{V}_{\text{D}} \mathbf{V}_{\text{D}}^H \mathbf{V}_{\text{RF}}^H) \leq P$$

$$|\mathbf{V}_{\text{RF}}(i, j)|^2 = 1, \forall i, j$$

$$|\mathbf{W}_{\text{RF}_k}(i, j)|^2 = 1, \forall i, j, k,$$

Sum-rate
weighted by user
priority β_k

Total power
constraint P

Analog hardware
constraints

Project Requirement



- **Solve the problem for single-user case:**

- Implement hybrid precoding and combining algorithms (SU case only)
- Implement quantized design algorithm of analog precoder (SU case only)
- Explore the case when you have more RF chains than the number of stream

- **Optimization problem:**

- Spectral efficiency for single user case:

$$R = \log_2 \left| \mathbf{I}_M + \frac{1}{\sigma^2} \mathbf{W}_t (\mathbf{W}_t^H \mathbf{W}_t)^{-1} \mathbf{W}_t^H \mathbf{H} \mathbf{V}_t \mathbf{V}_t^H \mathbf{H}^H \right|. \quad (8)$$

where $\mathbf{V}_t = \mathbf{V}_{\text{RF}} \mathbf{V}_D$ and $\mathbf{W}_t = \mathbf{W}_{\text{RF}} \mathbf{W}_D$.

- Geometric channel model:

$$\mathbf{H}_k = \sqrt{\frac{NM}{L}} \sum_{\ell=1}^L \alpha_k^\ell \mathbf{a}_r(\phi_{r_k}^\ell) \mathbf{a}_t(\phi_{t_k}^\ell)^H, \quad (33)$$

Point-to-Point SU-MIMO ($N^{RF} < 2d$)



- **Nested joint optimization problem:**
 - Optimizing 4 total matrices (V_{RF}, V_D, W_{RF}, W_D)
- **Two stage procedure overall:**
 - Initially assume a fixed optimal combiner structure
 1. Design the precoders
 2. Design the combiners using the computed precoders
- **Precoders and combiners are each designed in two stages as well:**
 - Initially assume a fixed digital precoder/combiner structure
 1. Iteratively solve for the RF precoder/combiner
 2. Solve for the optimal digital precoder/combiner

Iterative Analog Precoder Design

- Iteratively maximize the contribution of the j^{th} column to the rate
 - Note: $\bar{\mathbf{V}}_{RF}^j$ is \mathbf{V}_{RF} with the j^{th} column removed
- Combiner design similar: $\gamma^2 \rightarrow \frac{1}{M}$ and $\mathbf{F}_1 \rightarrow \mathbf{F}_2 = \mathbf{H}\mathbf{V}_t\mathbf{V}_t^H\mathbf{H}^H$

Parameters

$\mathbf{F}_1 = \mathbf{H}^H\mathbf{H}$ = Channel covariance matrix

γ^2 = Average power per antenna

σ^2 = Noise variance (per antenna)

Solve simplified optimization problem:

$$\max_{\mathbf{V}_{RF}} \log_2 \left| \mathbf{I} + \frac{\gamma^2}{\sigma^2} \mathbf{V}_{RF}^H \mathbf{F}_1 \mathbf{V}_{RF} \right| \quad (12a)$$

$$\text{s.t. } |\mathbf{V}_{RF}(i, j)|^2 = 1, \forall i, j, \quad (12b)$$

Algorithm 1. Design of \mathbf{V}_{RF} by solving (12)

Given: $\mathbf{F}_1, \gamma^2, \sigma^2$

- 1: Initialize $\mathbf{V}_{RF} = \mathbf{1}_{N \times N_{RF}}$.
- 2: **for** $j = 1 \rightarrow N_{RF}$ **do**
- 3: Calculate $\mathbf{C}_j = \mathbf{I} + \frac{\gamma^2}{\sigma^2} (\bar{\mathbf{V}}_{RF}^j)^H \mathbf{F}_1 \bar{\mathbf{V}}_{RF}^j$.
- 4: Calculate $\mathbf{G}_j = \frac{\gamma^2}{\sigma^2} \mathbf{F}_1 - \frac{\gamma^4}{\sigma^4} \mathbf{F}_1 \bar{\mathbf{V}}_{RF}^j \mathbf{C}_j^{-1} (\bar{\mathbf{V}}_{RF}^j)^H \mathbf{F}_1$.
- 5: **for** $i = 1 \rightarrow N$
- 6: Find $\eta_{ij} = \sum_{\ell \neq i} \mathbf{G}_j(i, \ell) \mathbf{V}_{RF}(\ell, j)$.
- 7: $\mathbf{V}_{RF}(i, j) = \begin{cases} 1, & \text{if } \eta_{ij} = 0, \\ \frac{\eta_{ij}}{|\eta_{ij}|}, & \text{otherwise.} \end{cases}$
- 8: **end for**
- 9: **end for**
- 10: Check convergence. If yes, stop; if not go to Step 2.

Coordinate descent (greedy updates based on [34] [41])

Point-to-Point SU-MIMO Algorithm

1. RF precoder (V_{RF}): Iterative optimization
2. Digital precoder (V_D): Water-filling solution
3. RF combiner (W_{RF}): Iterative optimization
4. Digital combiner (W_D): MMSE solution

Algorithm 2. Design of Hybrid Beamformers for Point-to-Point MIMO systems

Given: σ^2, P

- 1: Assuming $V_D V_D^H = \gamma^2 \mathbf{I}$ where $\gamma = \sqrt{P/(N N^{RF})}$, find V_{RF} by solving the problem in (12) using Algorithm 1.
 - 2: Calculate $V_D = (V_{RF}^H V_{RF})^{-1/2} U_e \Gamma_e$ where U_e and Γ_e are defined as following (11).
 - 3: Find W_{RF} by solving the problem in (16) using Algorithm 1.
 - 4: Calculate $W_D = J^{-1} W_{RF}^H H V_{RF} V_D$ where $J = W_{RF}^H H V_{RF} V_D V_D^H V_{RF}^H H^H W_{RF} + \sigma^2 W_{RF}^H W_{RF}$.
-

Phase Shifter (PS) Quantization



- Simple integrations into the algorithms
 - n_{PS} possible phases (steps of $\omega = e^{j2\pi/n_{PS}}$) for V_{RF} and $W_{RF} \rightarrow \mathcal{F} = \{1, \omega, \omega^2, \dots, \omega^{n_{PS}-1}\}$

Algorithm 1. Design of V_{RF} by solving (12)

Given: F_1, γ^2, σ^2

- 1: Initialize $V_{RF} = \mathbf{1}_{N \times N_{RF}}$.
 - 2: **for** $j = 1 \rightarrow N_{RF}$ **do**
 - 3: Calculate $C_j = \mathbf{I} + \frac{\gamma^2}{\sigma^2} (\bar{V}_{RF}^j)^H F_1 \bar{V}_{RF}^j$.
 - 4: Calculate $G_j = \frac{\gamma^2}{\sigma^2} F_1 - \frac{\gamma^4}{\sigma^4} F_1 \bar{V}_{RF}^j C_j^{-1} (\bar{V}_{RF}^j)^H F_1$.
 - 5: **for** $i = 1 \rightarrow N$
 - 6: Find $\eta_{ij} = \sum_{\ell \neq i} G_j(i, \ell) V_{RF}(\ell, j)$.
 - 7: $V_{RF}(i, j) = \begin{cases} 1, & \text{if } \eta_{ij} = 0, \\ \frac{\eta_{ij}}{|\eta_{ij}|}, & \text{otherwise.} \end{cases}$
 - 8: **end for**
 - 9: **end for**
 - 10: Check convergence. If yes, stop; if not go to Step 2.
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Quantized Version:

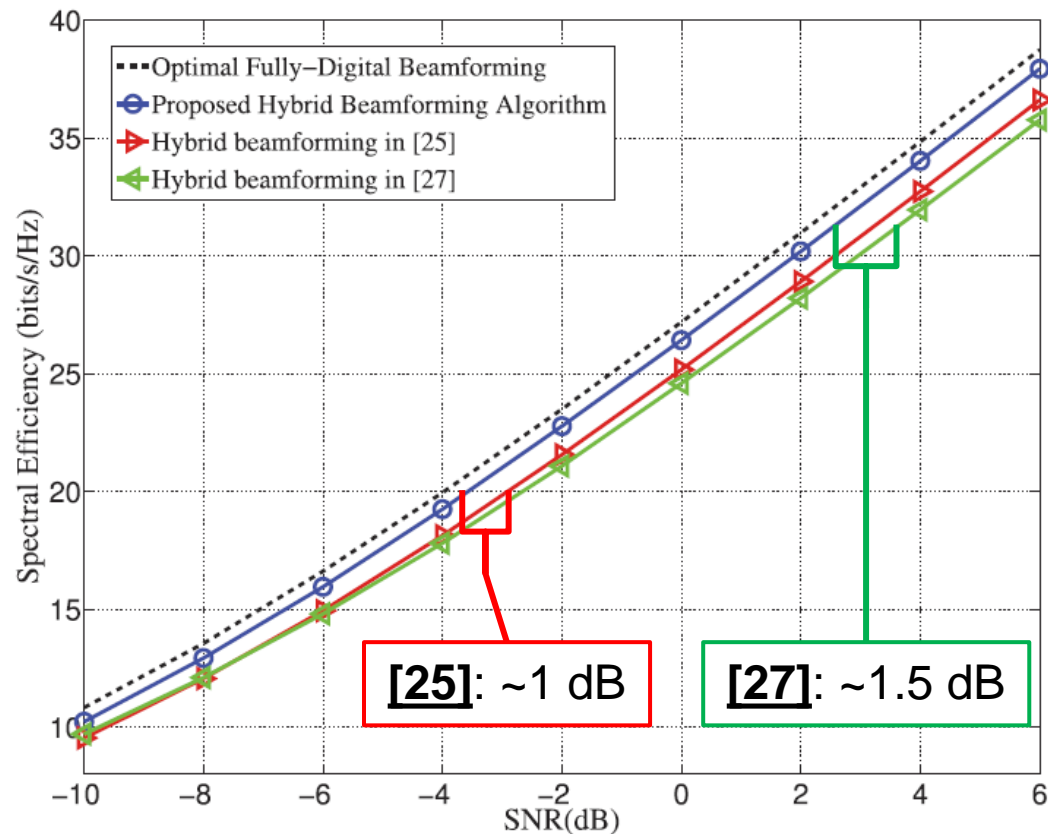
$$V_{RF}(i, j) = Q \left(\frac{\eta_{ij}}{|\eta_{ij}|} \right)$$

$$= \arg \min_{V_{RF}(i, j) \in \mathcal{F}} \left| V_{RF}(i, j) - \frac{\eta_{ij}}{|\eta_{ij}|} \right|^2$$

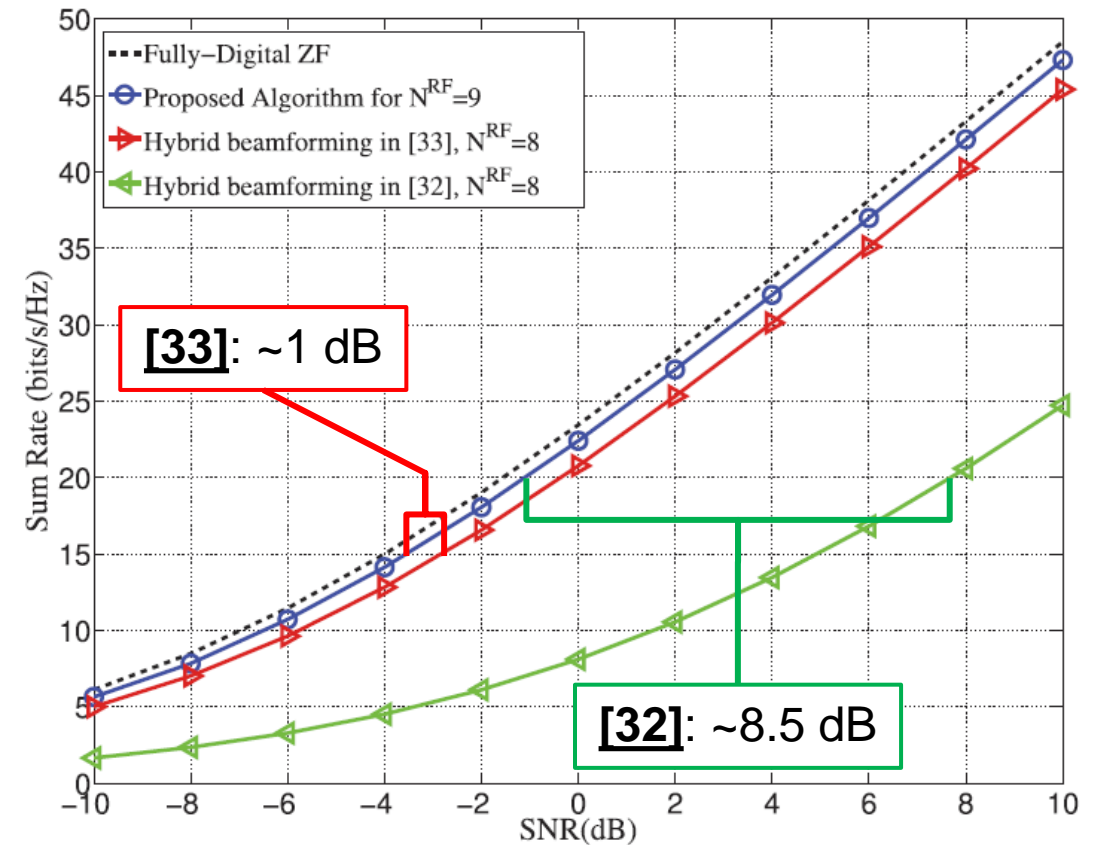
Simulation Results (Perfect PS)

- Random geometric channels – 15 paths per link
 - Average spectral efficiency from 100 trials

Case 2: 64 x 16 MIMO, $N_{RF} = N_S = 6$



Case 3: 64 x 8, 8-user MISO, $N_{RF} = 8$ or 9



Projects on True-Time-Delay (TTD) Arrays

Project 4: Beam Training with Analog TTD Arrays

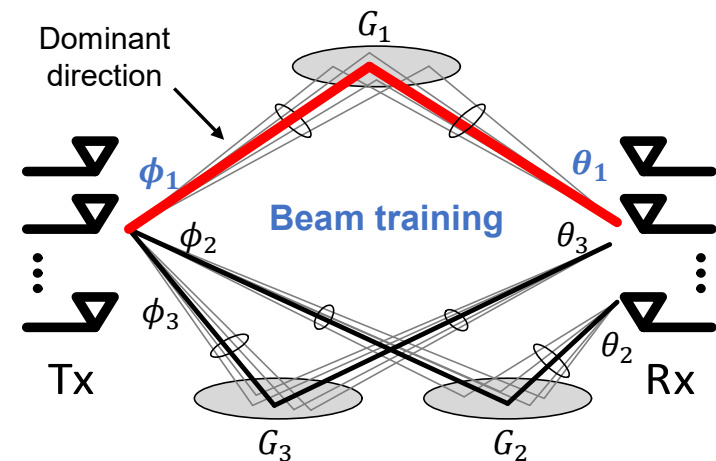
Project 2: Flexible Frequency-Dependent beamforming with Analog TTD arrays

Project 4: Beam Training w/ Analog TTD



Beam Training: Link establishment procedure to estimate the AoA of the strongest path

- **Partial channel knowledge** –angles (AoA) corresponding to the *dominant propagation path*
- Enables beam-steering in data communication
- **Low training overhead** – *compared to channel estimation*



Spatially clustered multipath: few spatial clusters reach the receiver

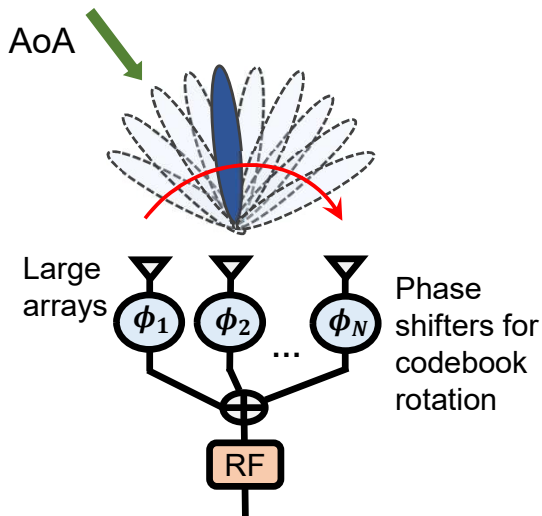
Project 4: Beam Training w/ Analog TTD



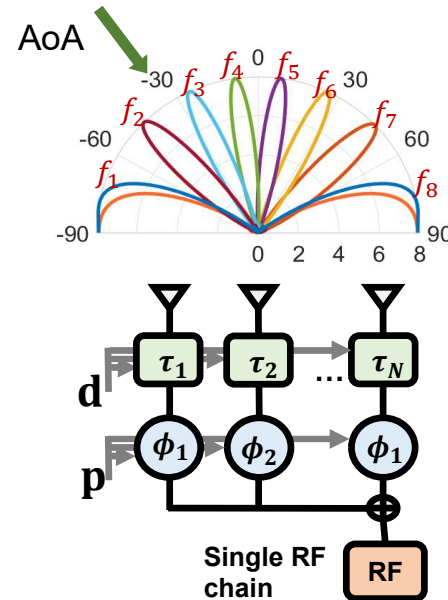
Beam Training with Analog PS vs TTD array:

Analog PS-based

Exhaustive beam sweep to estimate AoA



Analog TTD



Single shot AoA estimation



f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8

Upto M_{tot} beams using 1 OFDM symbol

$$\Delta\tau = \frac{R}{BW}$$

Project 4: Beam Training w/ Analog TTD

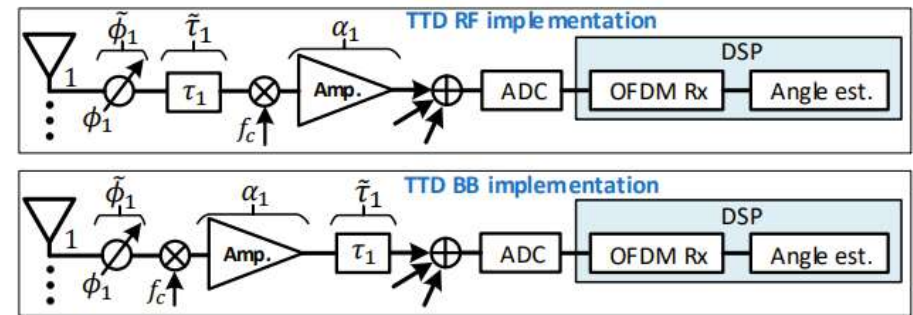


System Model for Rx Beam Training:

- $N_R \times 1$ Analog TTD array
- Freq. dependent TTD combiner

RF $[\mathbf{w}_{\text{RF}}[m]]_n = \alpha_n \exp \left[-j \left(2\pi f_m \tilde{\tau}_n + \tilde{\phi}_n \right) \right]$

BB $[\mathbf{w}_{\text{BB}}[m]]_n = \alpha_n \exp \left[-j \left(2\pi (f_m - f_c) \tilde{\tau}_n + \tilde{\phi}_n \right) \right]$

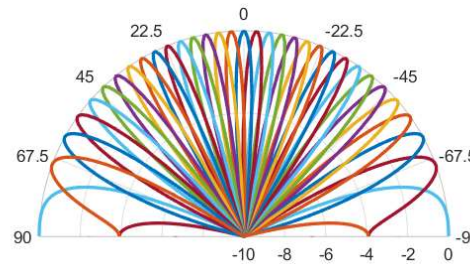


Hardware impairments

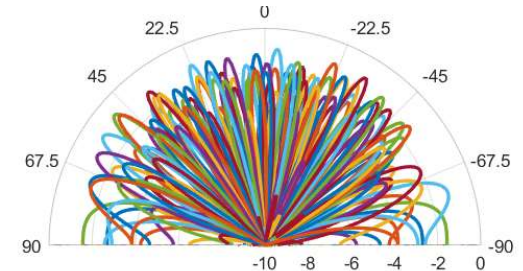
Cause distortions in TTD Rx beam patterns

- Delay error: $\tilde{\tau}_n \sim \mathcal{N}(\tau_n, \sigma_T^2)$
- Phase error: $\tilde{\phi}_n \sim \mathcal{N}(\phi_n, \sigma_P^2)$
- Gain error: $10 \log_{10} \alpha_n \sim \mathcal{N}(0, \sigma_A^2)$

Rainbow beams w/o error



Rainbow beams w/ phase error



Project 4: Beam Training w/ Analog TTD



Channel Model: wideband frequency selective [Sec II]

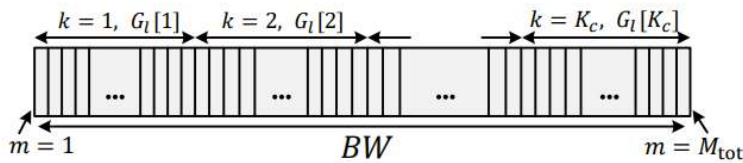


Fig. 1. The illustration of sub-bands and their corresponding channel gains for the l -th cluster.

$$\mathbf{H}[k] = \sum_{l=1}^L G_l[k] \mathbf{a}_R(\theta_l^{(R)}) \mathbf{a}_T^H(\theta_l^{(T)}),$$

$$\mathbb{E} (G_{l_1}[k_1] G_{l_2}^*[k_2]) = \begin{cases} \sigma_{l_1}^2, & \text{if } l_1 = l_2, k_1 = k_2 \\ 0, & \text{otherwise.} \end{cases}$$

TTD codebook design [Sec III]

$$[\mathbf{w}[m]]_n = \exp[-j(2\pi f_m \tau_n + \phi_n)],$$

$$\tau_n = (n-1)R/BW, \quad \phi_n = (n-1)[\text{sgn}(\psi)\pi - \psi], \quad (9)$$

where $\psi = \text{mod}(2\pi R(f_c - BW/2)/BW + \pi, 2\pi) - \pi$.

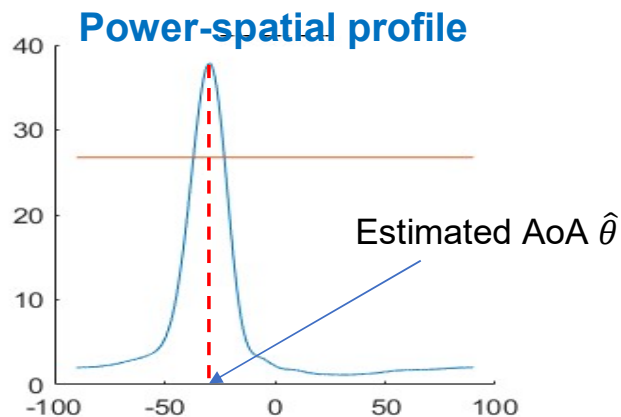
Project 4: Beam Training w/ Analog TTD



Beam Training Algorithm [Sec IV]

Received signal: $Y[m] = M^{-1/2} \mathbf{w}^H[m] \mathbf{H}[k] \mathbf{v} + \mathbf{w}^H[m] \mathbf{n}[m], m \in \mathcal{M}.$

Projection onto Dictionary matrix: $\mathbf{B} \in \mathbb{R}^{D \times Q}, [\mathbf{B}]_{d,q} = |\mathbf{f}_d^H \mathbf{a}_R(\xi_q)|^2.$



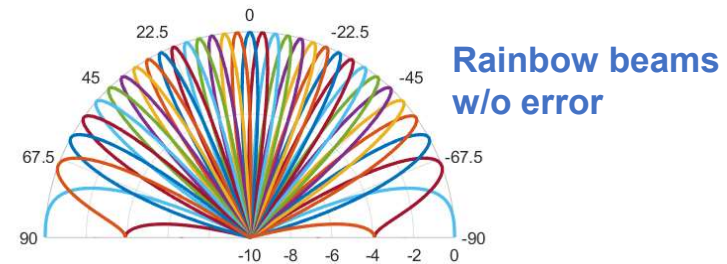
Find the RMSE

Project 4: Beam Training w/ Analog TTD



Problem statements:

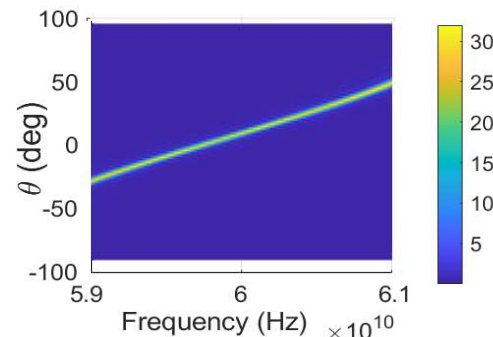
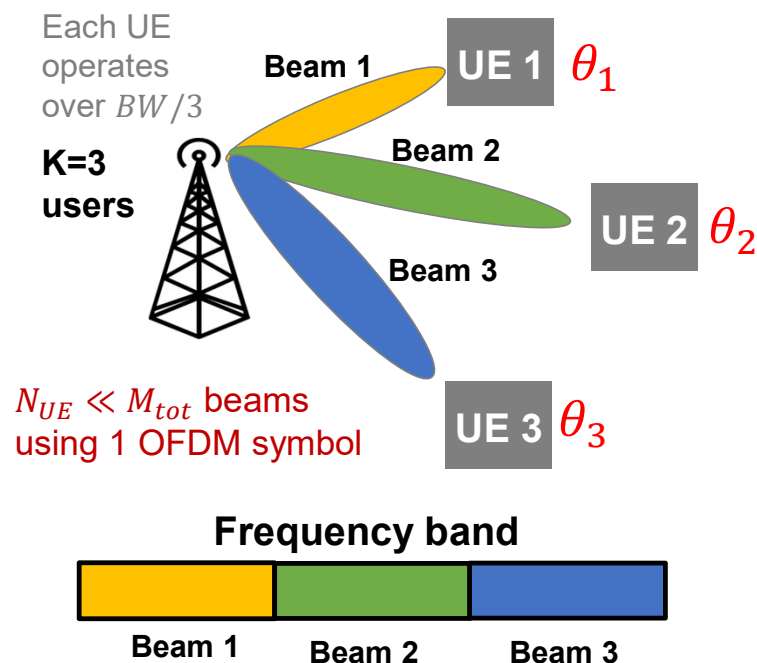
1. Plot the beam patterns of designed codebook for analog TTD arrays.
2. Plot the RMSE of angle estimation vs SNR (-20 dB to 20 dB)
 - In absence of hardware impairments
 - For $R = 1, 2, 4$
3. RMSE of angle estimation vs standard deviation of gain error σ_A (0 dB to 4.5 dB)
4. **[Bonus]:** RMSE of angle estimation vs standard deviation of phase error σ_P (0° to 50°)



Project 2: Flexible beamforming w/ TTD

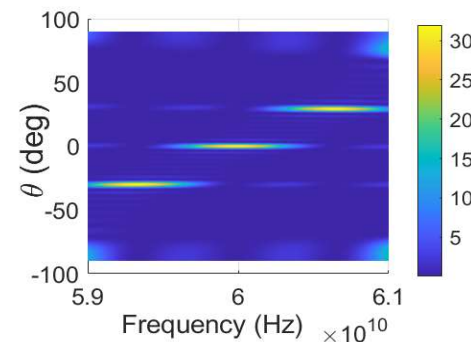
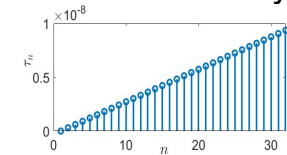


Sub-band-specific beams for simultaneous multi-user data communication



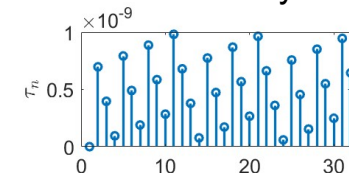
Rainbow beams: complete frequency-angle dispersion

- Fast beam Training
- Uniform TTD arrays



Quantized sub-band specific dispersion^[1]

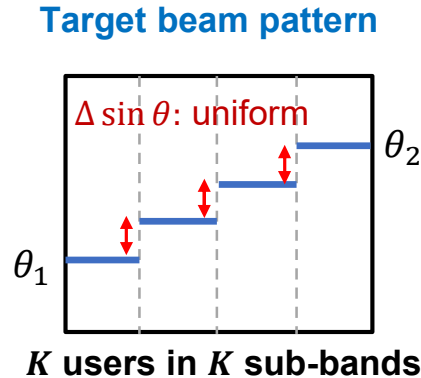
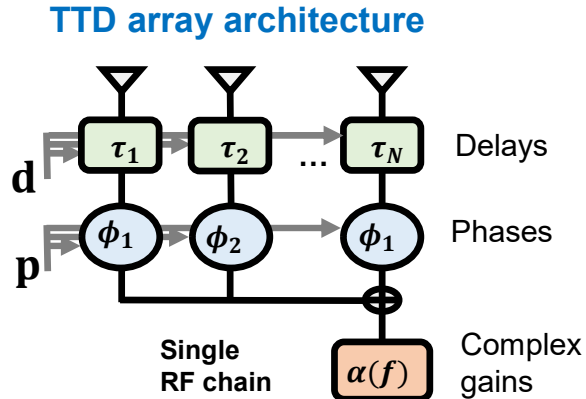
- Multi-user data communication
- Non-uniform delays and phases



Project 2: Flexible beamforming w/ TTD



Problem statement: How to design TTD codebook to implement sub-band beams with single RF chain?



Given:

Target freq. angle map: $\Theta(f)$

Target beam:

$$\mathbf{b}_f = \frac{1}{\sqrt{N_T}} \exp(-j\pi \frac{f_m}{f_c} \sin \Theta(f))$$

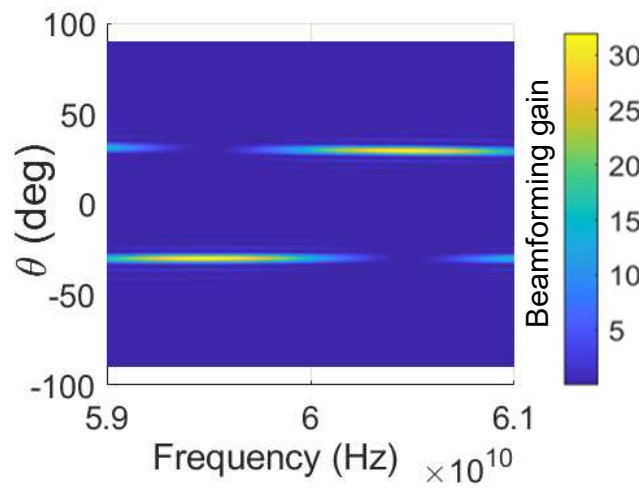
TTD combiner: $\mathbf{w}(f) = \frac{1}{\sqrt{N_T}} \alpha(f) \cdot \exp(-j(2\pi f_m \tau - \Phi)) \quad \forall f = 1, \dots, M_{tot}$

How to design $\tau_n, \phi_n \quad \forall n \in \{1, \dots, N_R\}, \quad \alpha(f)$

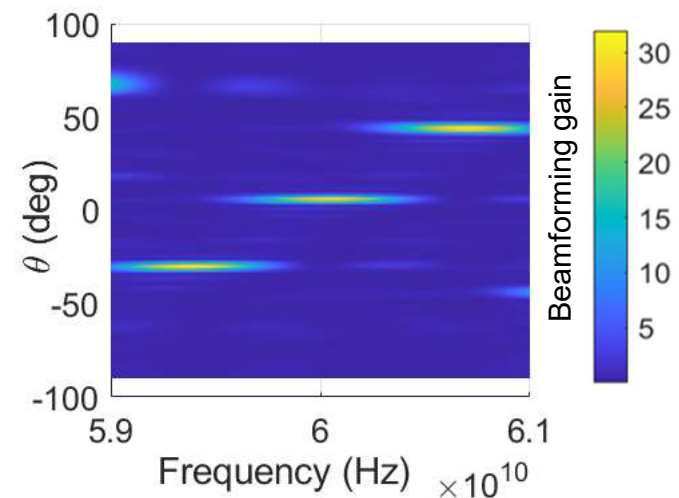
Project 2: Flexible beamforming w/ TTD



Examples of sub-band angle maps



$$K = 2; \Theta(f) = \begin{cases} -30^\circ & ; f < f_c \\ 30^\circ & ; f \geq f_c \end{cases}$$



$$K = 3; \Theta(f) = \begin{cases} -30^\circ & ; f < f_c - BW/6 \\ 5^\circ & ; f_c - \frac{BW}{6} \leq f < f_c + BW/6 \\ 45^\circ & ; f \geq f_c + BW/6 \end{cases}$$

Project 2: Flexible beamforming w/ TTD



Algorithms: to design TTD codebook to implement required sub-band-angle mapping

Algo 1: Alternating minimization [2, Sec V, Algo 1]

Given: Target beam:

$$\mathbf{b}_f = \frac{1}{\sqrt{N_T}} \exp(-j\pi \frac{f_m}{f_c} \sin \Theta(f))$$

Determine: $\tau_n, \phi_n \forall n \in \{1, \dots, N_R\}, \alpha(f)$

- Iterative adaptation of τ_n : [2, eqn 9 – *simplify to obtain closed form expression*]
- Iterative adaptation of ϕ_n : [2, eqn 8b]
- Iterative adaptation of $\alpha(f)$: [2, eqn 11]

Algo 2: mmFlexible [3, Appendix A, Theorem 2]

Given: Target angle mapping: $\Theta(f)$

Determine: $\tau_n, \phi_n \forall n \in \{1, \dots, N_R\}$

Project 2: Flexible beamforming w/ TTD



Goodness of fit: measure of how well the designed beams achieve the target angle mapping

Goodness of fit: The goodness of fit of combiner $\mathbf{w}(f) \in \mathbb{C}^{N_T \times 1}$ with respect to the target beam \mathbf{b}_f is defined as $\frac{1}{M_{tot}} \sum_f |\mathbf{w}^H(f) \mathbf{b}_f|$.

Project 2: Flexible beamforming w/ TTD



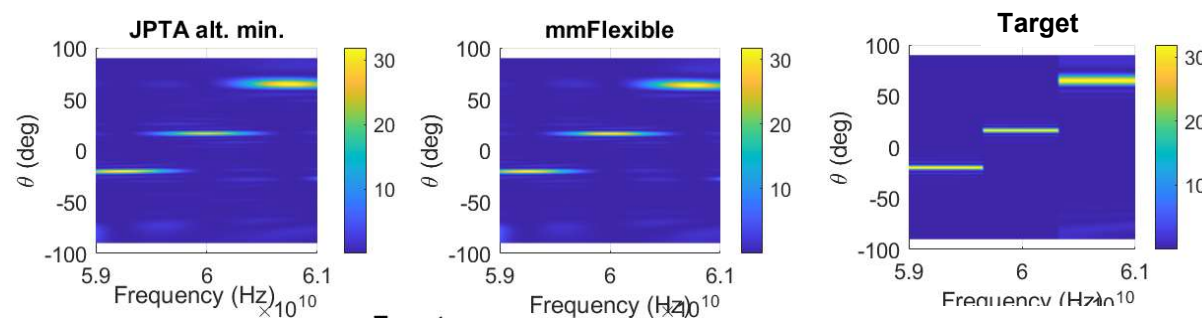
Required results:

1. You are given three target frequency-angle mappings. For each case, show the beamforming gain as a function of frequency-angle obtained via

- i. 20 iterations of alternating minimization
- ii. mmFlexible closed form solution
- iii. Target beam pattern

In each case, compute and report the goodness of fit of resulting beams

Example:



Project 2: Flexible beamforming w/ TTD



Required results:

2. For $K=2$ users, vary $\theta^{(1)}, \theta^{(2)}$ across 25 angles in the range of $[-60^\circ, 60^\circ]$. Compute goodness of fit for each of the resulting 625 angle-mappings, and plot its CDF for
 - i. 20 iterations of alternating minimization & mmFlexible closed form solution
 - ii. Plot the CDFs for $BW = 1\%$ of f_c and 10% of f_c for each algorithm (4 CDF curves totally)
3. For $K=4$ users, vary $\theta^{(i)} \forall i = 1, \dots, 4$ across 10 angles in the range of $[-60^\circ, 60^\circ]$. Repeat steps in (2) for each of the resulting 10000 angle-mappings.
4. List three main observations about the proposed algorithms that you make based on the results in parts 1 to 3. Provide adequate justification backed by evidence to substantiate your inferences.

ECE 233: Spring 2023

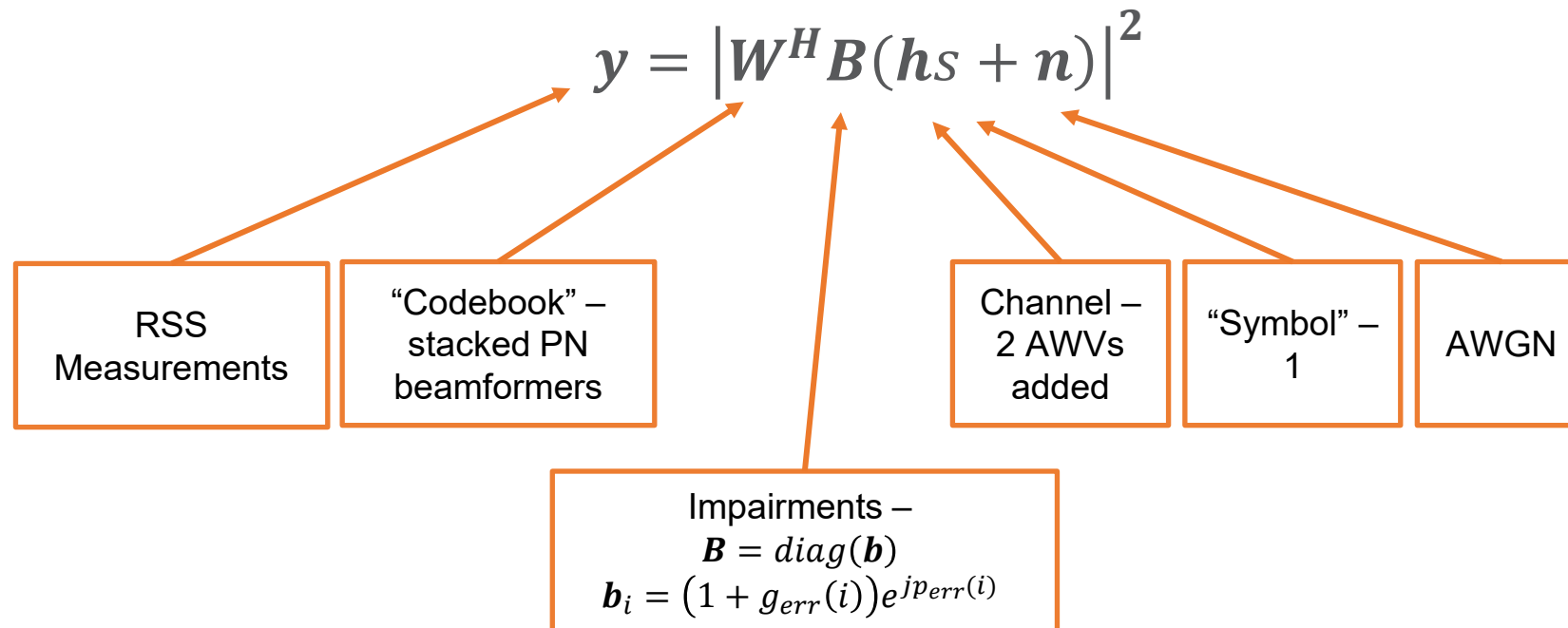
Project 5: Beam Training with Compressive Sensing and Machine Learning

Benjamin Domae

May 25, 2023

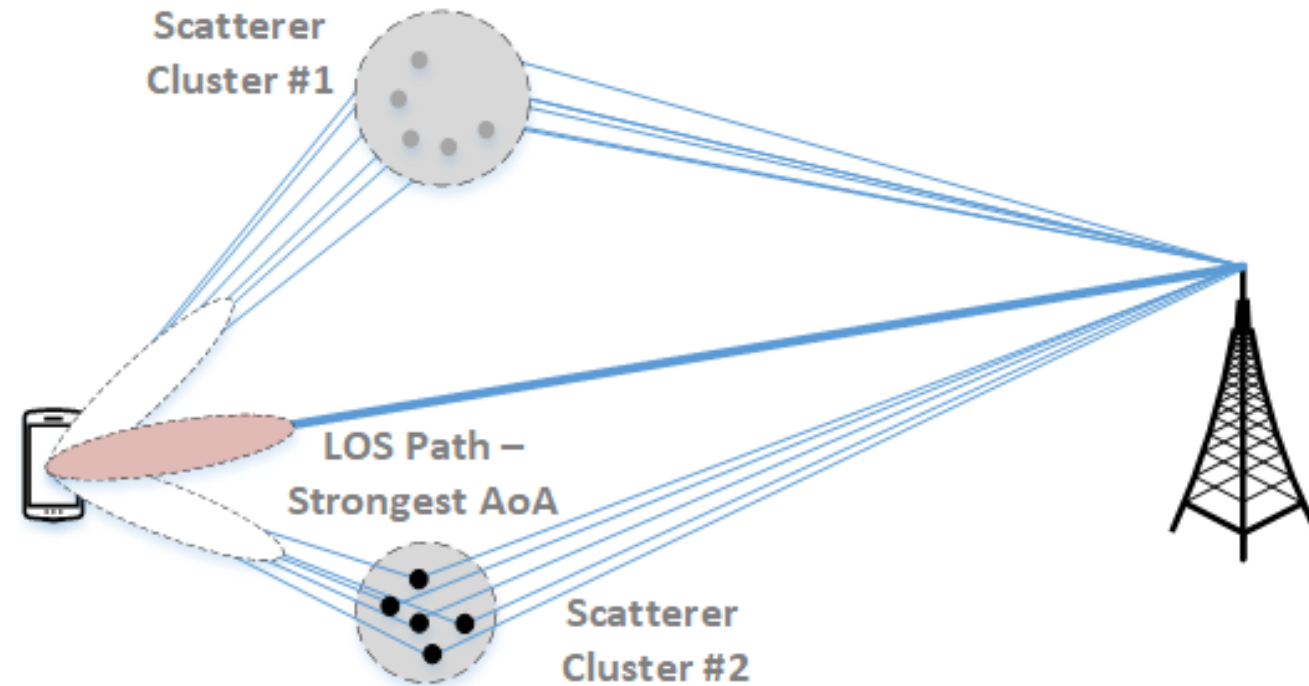
System Model

- 32 element array (uniform linear, 0.5λ spacing)
- Channels: 2 paths (-15 dB relative gain)



Algorithm Objectives

- **Objective: Predict the strongest path**
 - Analog phased array -> only 1 probing/data stream at the time
- **mmW channels are sparse**
 - Typically, only a few dominant paths
- **Phase-less measurements:**
 - Only using received signal strength (RSS)
 - No raw IQ samples or phase measurements

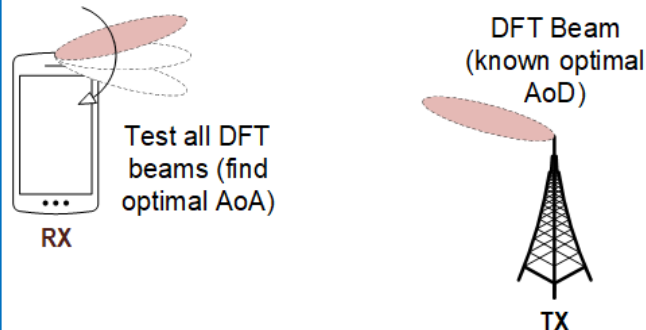


mmW Beam Alignment Codebooks

- Codebook: set of antenna phases/weights
 - Beam training: whole measurement/sounding codebook \mathbf{W}_S (size M)
 - Data comm: one beam from the directional codebook \mathbf{W}_D (size K)

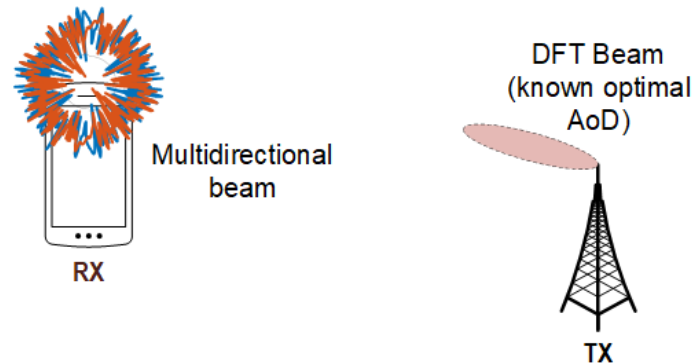
Traditional BA

- Exhaustive search
- Codebook: $\mathbf{W}_S = \mathbf{W}_D$
- Overhead $M = K \propto$ number of elements (N)



Compressed Sensing (CS) BA

- Pseudorandom noise (PN) beams as probing beams \mathbf{W}_S
- Overhead $M \propto \log(N)$
- Sensitive to array impairment

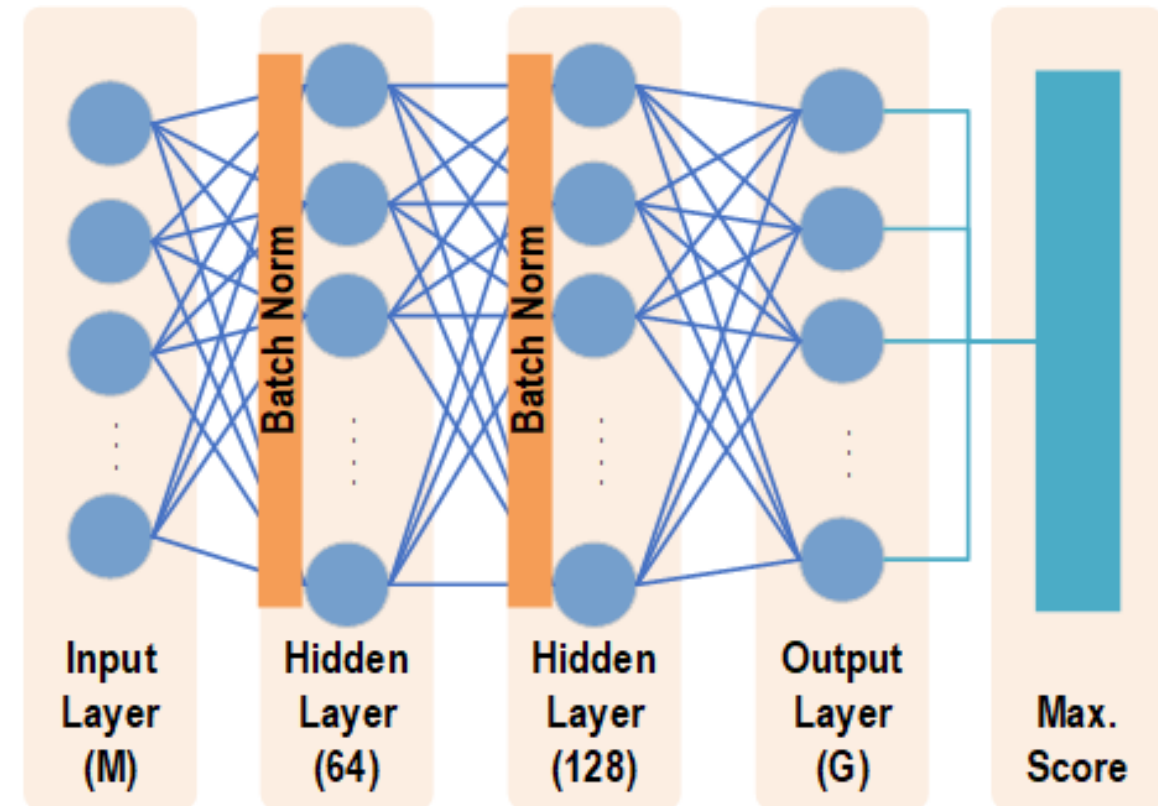


Combine CS and Machine Learning (ML)

- RSS from PN beams
-> *Reduced measurement complexity*
- Neural Network for signal processing
-> *Learns inherent hardware impairment*

mmRAPID BA Algorithm

- ***K*-level classification problem**
 - Features: RSS values from M PN Rx beams
 - Labels: Best DFT Rx beam in codebook \mathbf{W}_D
- **Two stage usage:**
 - Training: Rx measures PN and DFT beams and feed into neural network (NN)
 - Operation: Use NN to predict DFT beam from just PN sounding
 - **Retrain for every number of meas. and impairments instance**
 $M = \{4, 8, 12\}$



Plots Required



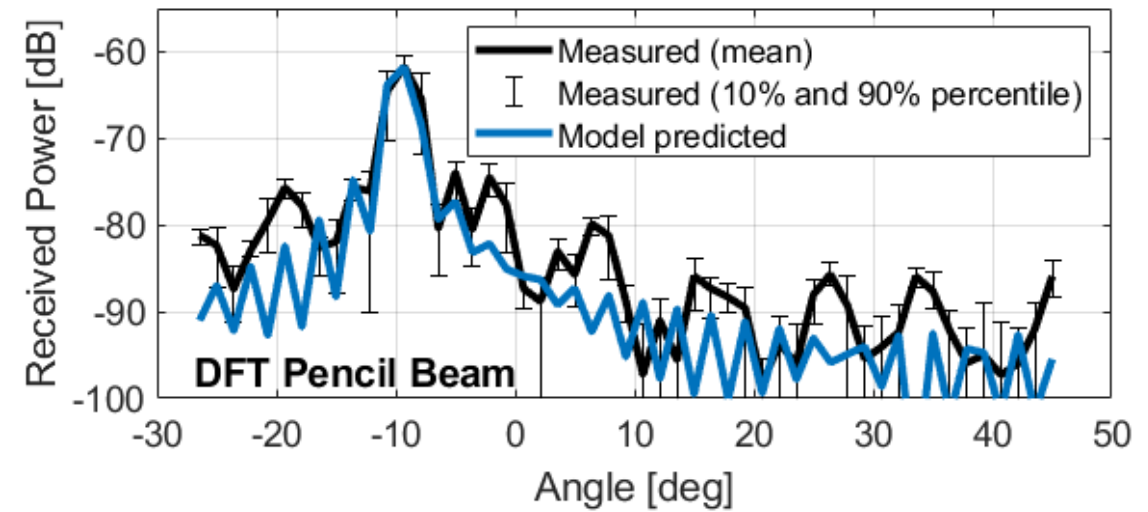
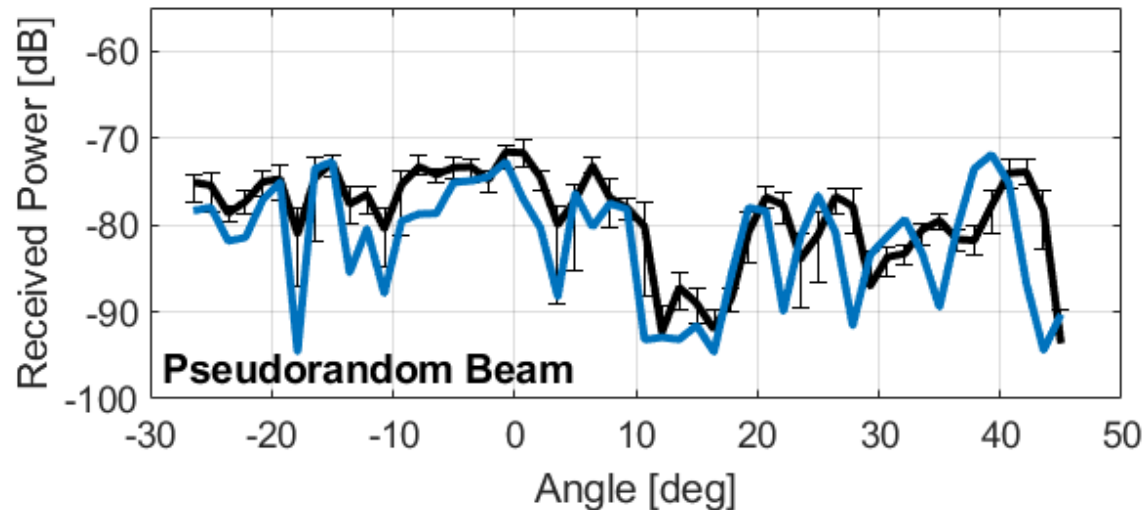
All required plots: w/ and w/o impairments results overlaid

1. Directional beam pattern
2. PN beam pattern
3. RMSE vs SNR (8 meas.)
4. RMSE vs # of meas. (20 dB SNR)
5. (Bonus) RMSE vs 2nd path gain
(8 meas., 20 dB SNR, w/o impairments)

Measurement of PN Beam Mismatch

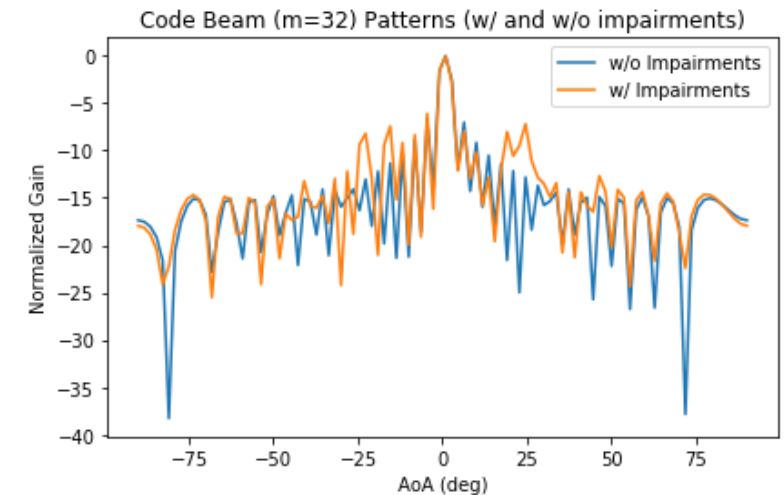
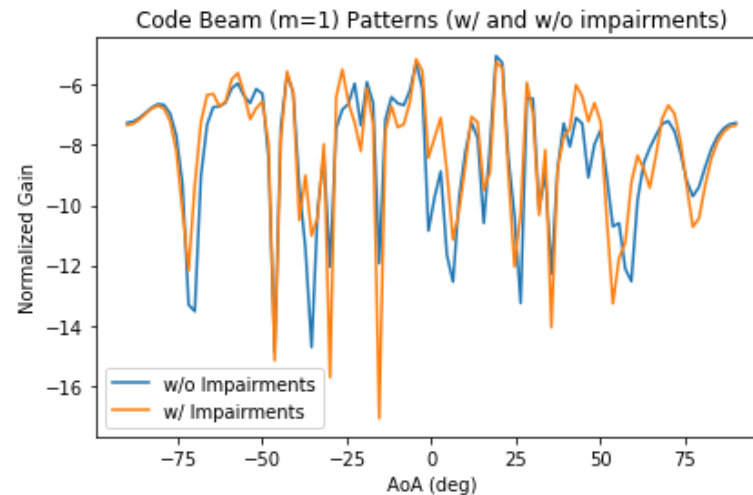
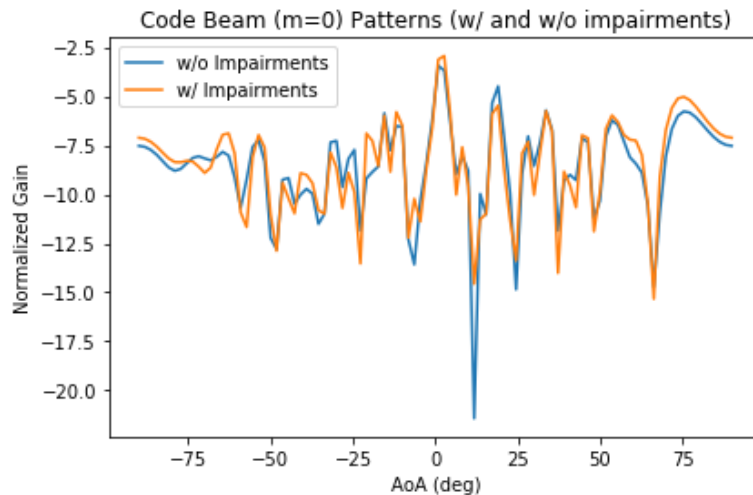


- **Demonstrated array impairment → beam design mismatch**
 - DFT beams aren't as sensitive
- **PN beam spatial fingerprint:**
 - Set of responses for each AoA for all measurement beams
 - Hardware impairment dependent (need to learn for each device)



Beam Patterns w/ Impairments

- 2 PN beam examples
- 1 DFT beam example
- Note: $B = I_{N_r}$ (identity matrix) when there are no impairments

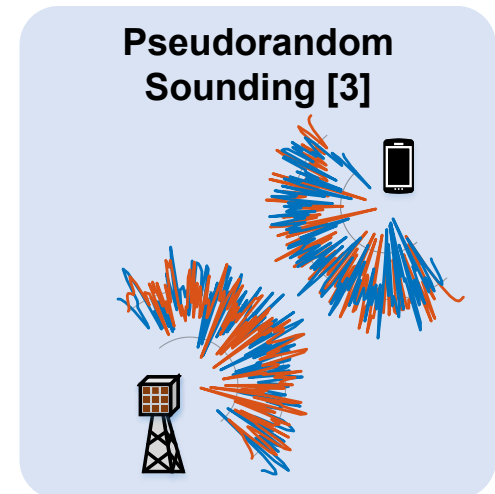
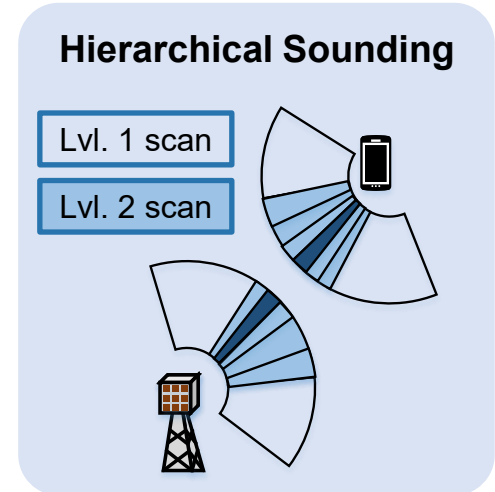
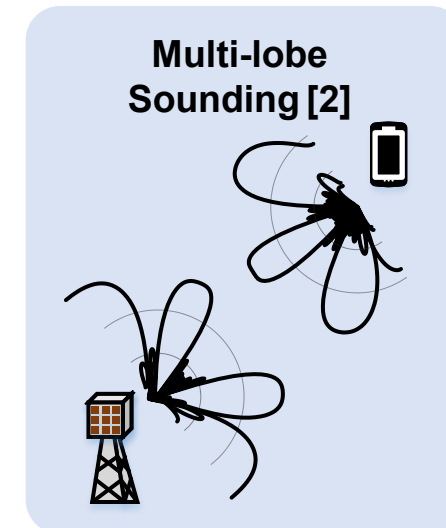


Extra Slides

Beam Alignment (BA) in mmW Systems



- **Key procedure: beam alignment**
 - Goal: find best beam steering directions
 - Challenge in mmW evolution
 - Larger Tx/Rx arrays will be used
 - Increased training overhead & complexity
- **Approaches are constrained by array**
 - Advanced array architecture – fast alignment but high power and complexity
 - Fully-digital array, true-time-delay array, etc.
 - **Phased array are preferred for mobile terminals**
 - **Beam alignment is a bottleneck**

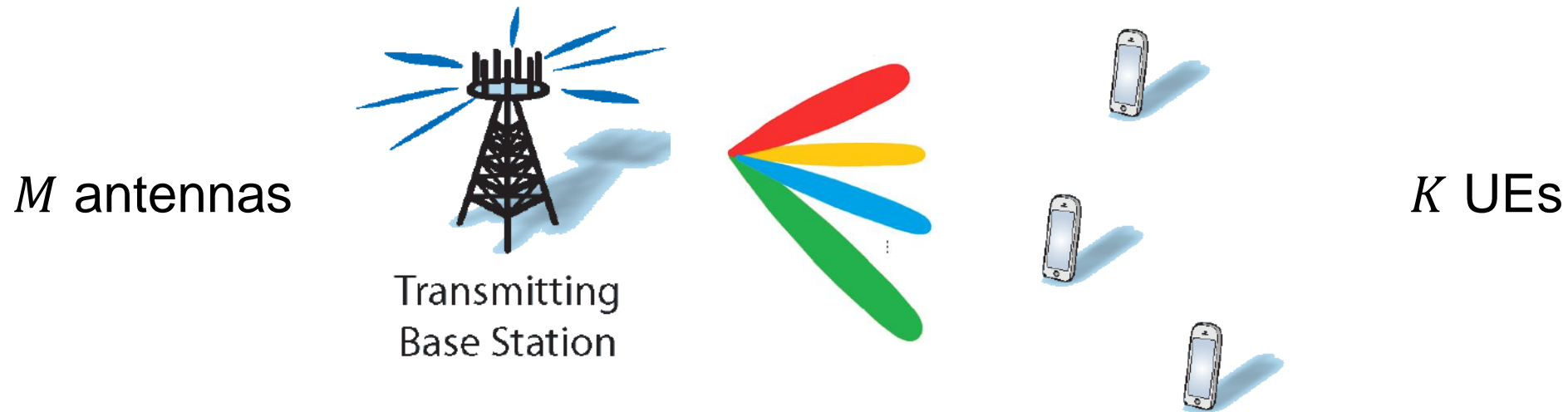


Project 6:

Scaling up MIMO: Massive MIMO Communication

May. 25, 2023

Downlink MU-MIMO System Model



- Received signal model:

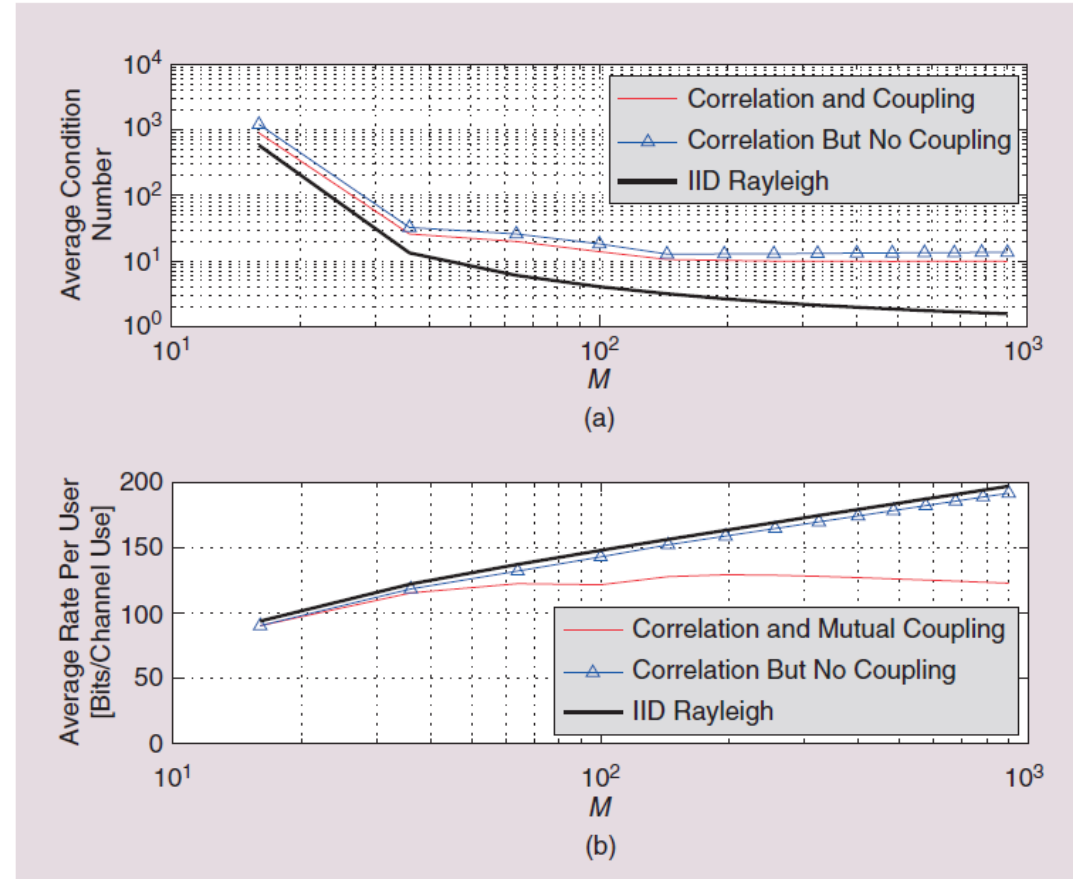
$$\mathbf{x}_f = \mathbf{G}^T \mathbf{W} \mathbf{s} + \mathbf{n}$$

$$\mathbf{x}_f, \mathbf{n}, \mathbf{s} \in \mathbb{C}^{K \times 1}, \mathbf{H}^T \in \mathbb{C}^{K \times M}, \mathbf{W} \in \mathbb{C}^{M \times K}, M > K$$

Benefit of Massive MIMO

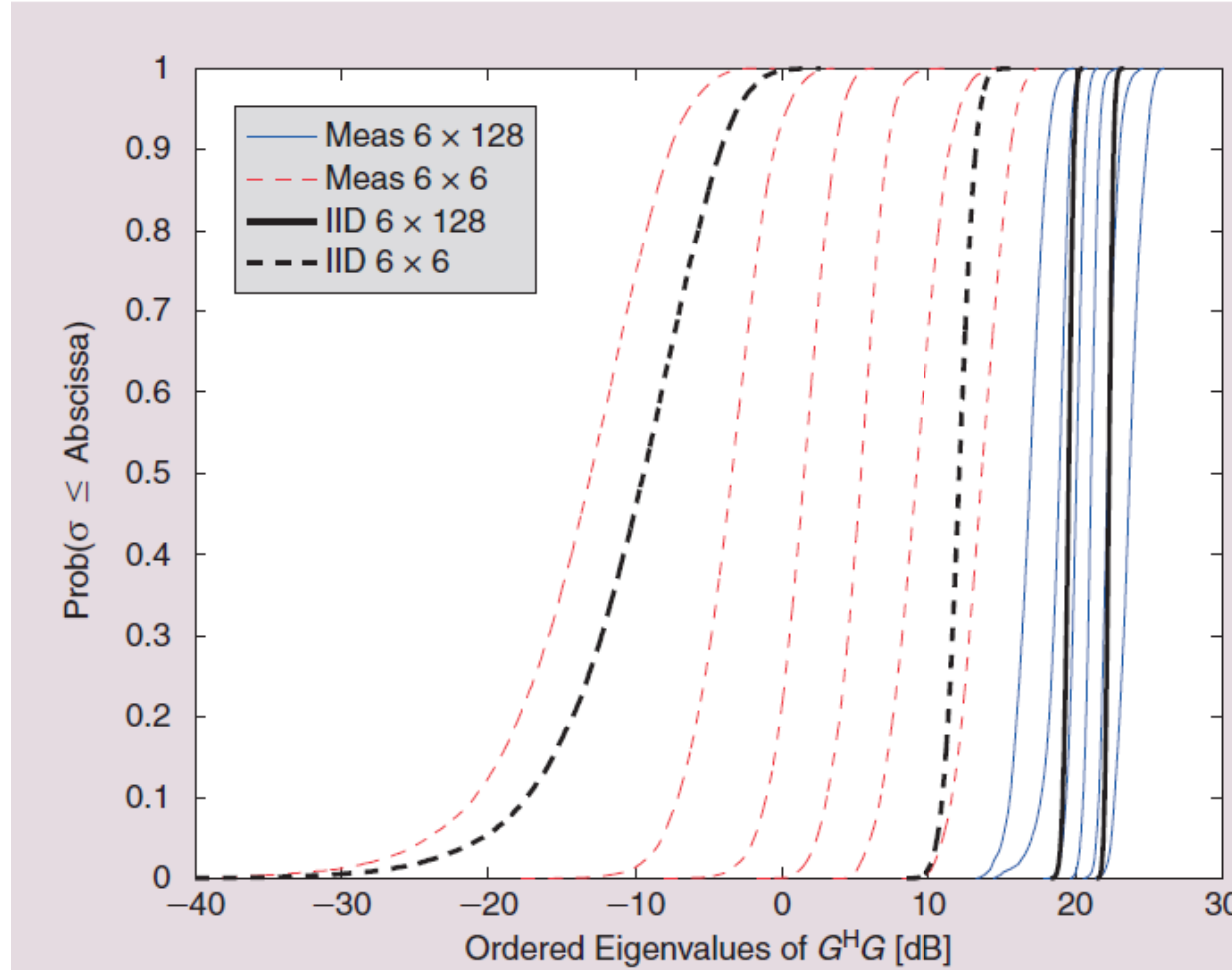
- Channel condition and capacity [1] as number of antenna grows up

$$C_{mc} = \log_2 \det \left(I_n + \frac{\rho}{n_t} \hat{\mathbf{G}}_{mc} \hat{\mathbf{G}}_{mc}^H \right)$$



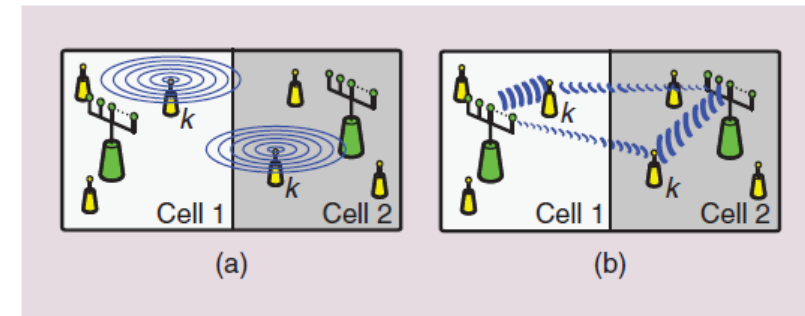
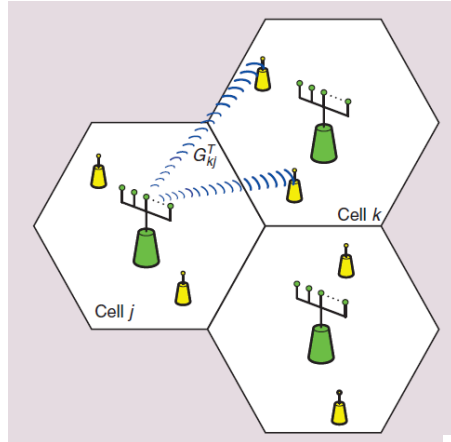
Benefit of Massive MIMO

- Eigenvalue distribution [1] as number of antenna grows up



Issues of Massive MIMO

- When the number of antenna gets large, the channel estimation becomes challenging
- Number of orthogonal pilots are limited and will be reused by all cells
- Multi-cell DL MU-MIMO with pilot contamination [1]



[FIG8] Illustration of the pilot contamination concept. (a) During

$$\hat{G}_{nn}^T = \sqrt{\rho_p} G_{nn}^T + \sqrt{\rho_p} \sum_{i \neq n} G_{in}^T + V_n^T$$

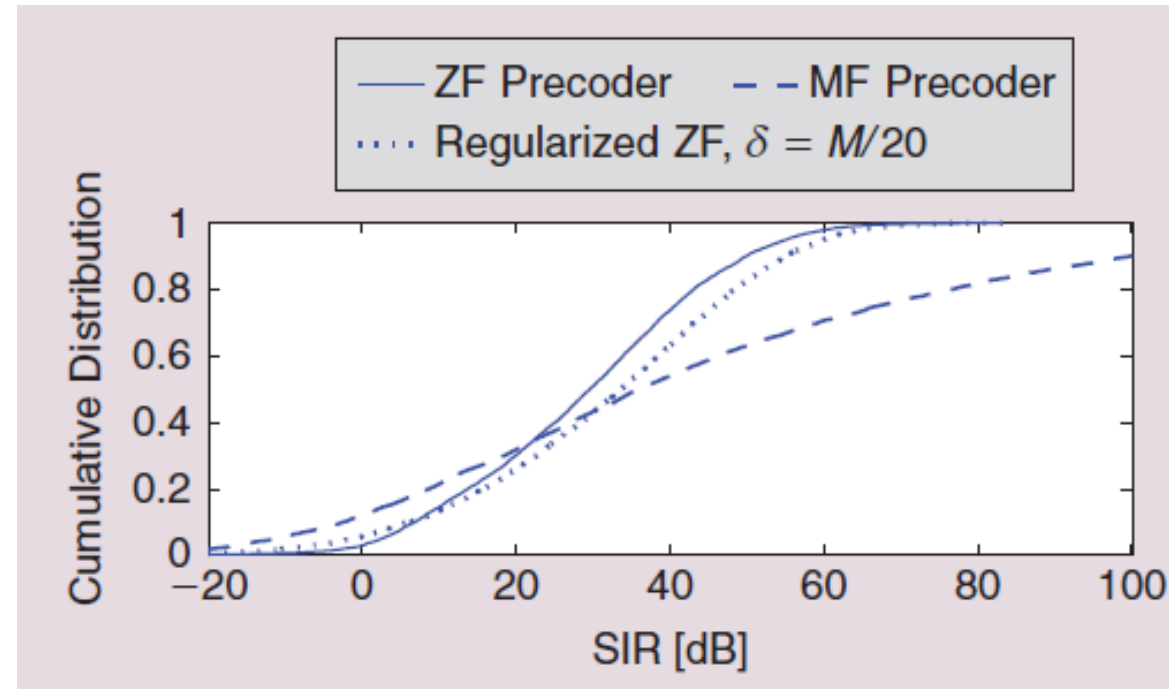
$$x_{fj} = \sqrt{\rho_f} \sum_n G_{jn}^T \hat{G}_{nn}^* q_{fn} + w_{fj}$$

$$= \sqrt{\rho_f} \sum_n G_{jn}^T \left[\sqrt{\rho_p} \sum_i G_{in}^T + V_n^T \right]^H q_{fn} + w_{fj}$$

Issues of Massive MIMO

- Signal to Interference ratio (SIR) performance of different precoders [1]

$$\text{SIR} = \frac{\beta_{jj\ell}^2 / \left(\sum_i \beta_{ij\ell} + \frac{1}{\rho_p} \right)^2}{\sum_{n \neq j} \beta_{jn\ell}^2 / \left(\sum_i \beta_{in\ell} + \frac{1}{\rho_p} \right)^2}$$



$$\text{SIR} = \frac{\beta_{jj\ell}^2}{\sum_{n \neq j} \beta_{jn\ell}^2}$$