ECE 233: Wireless Commun. System Design, Modeling, and Implementation	**Spring 2023**
Project — due Friday, June 16, 2023	
Instructor: Prof. Danijela Cabric	

Instructions

- A project can be done in a group of two or individually.

- There are three group projects to choose from, and there are three individual projects to choose from. There is also an option to propose your own project.

- E-mail the names of group members (or the individual) and the project selected to Prof. Danijela Cabric <danijela@ee.ucla.edu> and Reader Yen Chin Wang <ycwang85@ucla.edu>. This is due by **Sunday, May 28, 2023**.

- An archive file, i.e., a zip file, should be submitted by one of the group members on gradescope. Name of the zip file should be $lastname\_firstname$.zip with the name of the student submitting the file. The zip file should contain two things: a PDF report and MATLAB code.

- The report should be formatted as follows

  - **Cover page**: Includes the project title, group members, and their student IDs
  - **Abstract**: A short description of the project (length: approx. 200 words)
  - **System model**: Description of the model used and the assumptions considered
  - **Main part**: Description of the main techniques, algorithms, or architectures used in the project
  - **Results and Discussion**: Figures showing the simulation results, and a short discussion for each one
  - **Conclusion**: A summary of the main findings of this project

Project list for group submissions

Project 1: Hybrid Analog-Digital Beamforming Design

MIMO systems at sub-6GHz frequencies rely on fully digital arrays for beamforming. Digital arrays, however, are unfeasible and power inefficient at millimeter-wave (mmWave) frequencies, where a large number of antenna elements are used. For this reason, hybrid analog-digital arrays are considered to be an acceptable alternative at mmWave frequencies. Unlike digital arrays, hybrid arrays have an analog front-end with many

antennas connected to a smaller number of radio-frequency (RF) chains. In this project, you will study the design of beamformers for hybrid arrays at mmWave frequencies assuming a point-to-point scenario (communication between one base station and one user).

- **System model**: Consider the system model described in [R-1, Sec. II], for $K = 1$ user. Both the base station and user are assumed to be equipped with hybrid arrays, as illustrated in [R-1, Fig. 1].

- **Channel model**: Use the narrowband channel model described in [R-1, Sec. VII]. Assume the same channel parameters as in [R-1, Sec. VII].

- **Design of hybrid precoders**: Design the base station beamformers, including analog RF precoders and digital precoders, assuming that the numbers of RF chains N^{RF} and data streams N_s are equal, i.e., $N^{\mathrm{RF}} = N_s$. The design of digital precoders is described in [R-1, Sec. IV-A], while the design of analog RF precoders is described in [R-1, Sec. IV-B] and summarized in [R-1, Sec. IV-B, Alg. 1].

- **Design of hybrid combiners**: Design the user beamformers, including analog RF combiners and digital combiners, assuming $N^{\mathrm{RF}} = N_s$. The design of digital and analog RF combiners is described in [R-1, Sec. IV-C].

- **Design extension**: Extend the design of hybrid beamformers for the case when $N_s < N^{\mathrm{RF}} < 2N_s$. The extension is described in [R-1, Sec. IV-D], while the overall design of hybrid beamformers is summarized in [R-1, Sec. IV-B, Alg. 2].

- **Results**: Study achievable spectral efficiency with hybrid beamformers and compare it with that of fully digital arrays. The spectral efficiency is calculated based on the expression [R-1, (8)]. The signal-to-noise ratio (SNR) is defined as in [R-1, Sec. VII].

  1. Plot the spectral efficiency vs. SNR in the range -10 dB to 6 dB, assuming a 64×16 MIMO system and $N^{\mathrm{RF}} = N_s = 6$. Refer to the black and blue curves in [R-1, Fig. 2] for comparison with your results.

  2. Plot the spectral efficiency vs. SNR in the range 0 dB to 30 dB, assuming a 10×10 MIMO system, $N^{\mathrm{RF}} = N_s = 2$, and phase shifters with 1-bit and infinite resolutions. Refer to the black and blue curves in [R-1, Fig. 3] for comparison with your results.

  3. Plot the spectral efficiency vs. SNR in the range -10 dB to 6 dB, assuming a 64×16 MIMO system, $N_s = 4$, $N^{\mathrm{RF}} = \{N_s, N_s + 1, N_s + 3\}$, and phase shifters with 1-bit and infinite resolutions. Refer to the black and blue curves in [R-1, Fig. 4] for comparison with your results.

0.0.1 Relevant References

[R-1] F. Sohrabi and W. Yu, "Hybrid Digital and Analog Beamforming Design for Large-Scale Antenna Arrays," in IEEE Journal of Selected Topics in Signal Processing, vol. 10, no. 3, pp. 501-513, April 2016 (IEEE Xplore link)

Project 2: Flexible Frequency-Dependent beamforming with Analog TTD arrays

Millimeter-wave and terahertz systems offer large bandwidths which can facilitate simultaneous data communication with multiple spatially separated users occupying non-overlapping sub-bands. Such sub-band-specific data communication calls for the deployment of directional beams with a sub-band-specific spatial response, where all subcarriers within a sub-band form a directional beam to serve a particular user, as shown in Fig. 1.
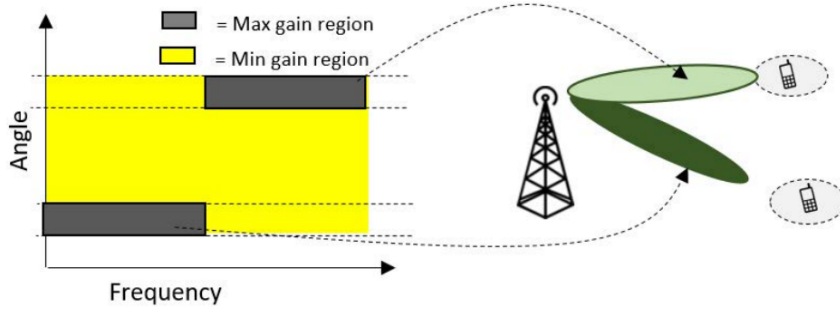

Figure 1. Example of sub-band specific beams to support data communication with two users.

- **System model:** Consider an OFDM communication system comprising a Base Station that seeks to simultaneously send data packets to K users (UE) located at angles $\theta^{(k)} \, \forall \, k = 1, ..., K$ with respect to the BS. The BS operates over the bandwidth BW with a total of M_{tot} subcarriers and carrier frequency f_c, whereas each UE operates over a non-overlapping BW/K wide sub-band with a total of M_{tot}/K subcarriers. The BS is equipped with a critically spaced $N_T \times 1$ True-Time-Delay (TTD) array with a single RF chain. The TTD combiner $\mathbf{w}(f) \in \mathbb{C}^{N_T \times 1}$ is hence given as shown in (1), where the vectors $\boldsymbol{\tau}, \boldsymbol{\Phi} \in \mathbb{R}^{N_T \times 1}$ denote the per-antenna time-delays and phase shifts respectively, $\alpha(f)$ denotes frequency-dependent unit-magnitude complex coefficients, and $f_m = f_c - \frac{BW}{2} + BW \frac{f-1}{M_{tot}-1}$ is the frequency of the fth subcarrier $\forall f = 1, ..., M_{tot}$.

$$\mathbf{w}(f) = \frac{1}{\sqrt{N_T}} \alpha(f).exp(-j(2\pi f_m \boldsymbol{\tau} - \boldsymbol{\Phi})) \quad \forall f = 1, ..., M_{tot} \tag{1}$$

- **Target frequency-angle mapping:** $\boldsymbol{\Theta}(f) \, \forall \, f = 1, ..., M_{tot}$ denotes the desired pointing direction of the fth subcarrier.

- **Algorithm to design delays and phases:** This project studies two algorithms to design the delay and phase vectors $\boldsymbol{\tau}, \boldsymbol{\Phi} \in \mathbb{R}^{N_T \times 1}$, and complex coefficients $\alpha(f)$ to achieve beams with the required sub-band-angle mapping.

  1. JPTA architecture-based alternating minimization algorithm [R-2, Sec V-Algorithm 1], which takes as input the target frequency-dependent beamformer $\mathbf{b}_f \, \forall \, f = 1, ..., M_{tot}$, given in (2), where $f_m = f_c - \frac{BW}{2} + BW \frac{f-1}{M_{tot}-1}$.

$$\mathbf{b}_f = \frac{1}{\sqrt{N_T}} \exp(-j\pi \frac{f_m}{f_c} \sin \Theta(f)) \tag{2}$$

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>*Note:* In each iteration, adapt the per antenna delays τ_n based on [R-2, eqn. 9] (simplify to obtain a closed-form solution), phases ϕ_n based on [R-2, eqn. 8b], and complex coefficients $\alpha(f)$ based on [R-2, eqn. 11]. Ensure that $|\alpha(f)| = 1 \; \forall f$.

2. mmFlexible [R-3, Appendix A, Theorem 2] provides a closed-form expression for designing per-antenna delays τ_n [R-3, eqn. 36] and phases ϕ_n [R-3, eqn. 35] for the given sub-band-angle mapping. For this algorithm, consider $\alpha(f) = 1 \; \forall f$. Note that you need to add a phase correction term of $2\pi f_c \tau_n$ to the per-antenna phases ϕ_n in [R-3, eqn. 35].

- **Goodness of fit:** The goodness of fit of combiner $\mathbf{w}(f) \in \mathbb{C}^{N_T \times 1}$ with respect to the target beam $\mathbf{b}_f$ is defined as $\frac{1}{M_{tot}} \sum_f |\mathbf{w}^H(f)\mathbf{b}_f|$.

- **Results:** Implement the alternating minimization algorithm and mmFlexible closed-form solution to optimize the TTD combiner of the BS array to achieve the target frequency-angle map for a system with $N_T = 32$, $BW = 2\text{GHz}$, $M_{tot} = 4096$ and $f_c = 60\text{GHz}$, and answer the questions that follow.

    1. Plot the beamforming-gain of beams obtained by performing 20 iterations of alternating minimization for the target frequency-angle maps $\Theta(f)$ specified in (a)-(c), on a frequency-angle grid as shown in [R-2, Fig. 4]. For parts (a)-(c), plot separately the beamforming-gain heat maps for beams generated using mmFlexible [R-3, Appendix A, Theorem 2]. Additionally, show the beamforming heatmaps of target beams on a separate frequency-angle grid. Be sure to label the heat maps correctly. In each case (a)-(c), compute and mention the goodness of fit of the beams generated for each algorithm implemented.

        (a) $\Theta(f) = \begin{cases} 45° \; ; & f_c - BW/2 \le f < f_c \\ -15° \; ; & f_c \le f \le f_c + BW/2 \end{cases}$

        (b) $\Theta(f) = \begin{cases} -30° \; ; & f_c - BW/2 \le f < f_c - BW/6 \\ 0° \; ; & f_c - BW/6 \le f < f_c + BW/6 \\ 30° \; ; & f_c + BW/6 \le f \le f_c + BW/2 \end{cases}$

        (c) $\Theta(f) = \begin{cases} -30° \; ; & f_c - BW/2 \le f < f_c - BW/6 \\ 23° \; ; & f_c - BW/6 \le f < f_c + BW/6 \\ 30° \; ; & f_c + BW/6 \le f \le f_c + BW/2 \end{cases}$

    2. For K=2 users, plot the cumulative distribution function (CDF) of the goodness of fit of beams generated by both algorithms, recorded over 625 realizations, with $\theta^{(1)}$ and $\theta^{(2)}$ each sweeping across 25 values in the range of $(-60°, 60°)$ for two values of BW: (a) $BW = 1\%$ of f_c (solid curve) & (b) $BW = 10\%$ of f_c (dashed curve). Hence, the CDF plot should have two curves for each method, making it a total of 4 curves.

    3. Repeat the steps in part 2 for $K = 4$ users, where the goodness of fit of beams for both algorithms is recorded across 10^4 realizations, with each $\theta^{(k)}|_{k=1,\dots,4}$ sweeping across 10 values in the range of $(-60°, 60°)$.

    4. List three main observations about the proposed algorithms that you make based on the results in parts 1 to 3. Provide adequate justification backed by evidence to substantiate your inferences.

0.0.2 Relevant References

[R-2] V. V. Ratnam, J. Mo, A. Alammouri, B. L. Ng, J. Zhang and A. F. Molisch, "Joint Phase-Time Arrays: A Paradigm for Frequency-Dependent Analog Beamforming in 6G," in IEEE Access, vol. 10, pp. 73364-73377, 2022, doi: 10.1109/ACCESS.2022.3190418. link)

[R-3] I. K. Jain, R. R. Vennam, R. Subbaraman, and D. Bharadia, "mmflexible: Flexible directional frequency multiplexing for multi-user mmwave networks," arXiv preprint arXiv:2301.10950, 2023. link

Project 3: Compressive Estimation of Millimeter-Wave Channels

Channel estimation is one of the most important tasks in millimeter-wave (mmWave) MIMO systems. Due to severe propagation loss, mmWave channels are sparse, i.e., they consists of a low number of propagation paths. This sparsity can be exploited for efficient channel estimation with a small number of training pilots by leveraging compressive digital signal processing (DSP) techniques. In this project, you will study mmWave channel estimation using a compressive sensing method that requires a smaller number of training pilots than conventional estimation techniques.

Compressive sensing is a mathematical concept focused on underdetermined systems of linear equations. Namely, it says that it is possible to solve an underdetermined system of linear equation if the unknown vector is sparse, i.e., it has a small number of non-zero elements. Let $\mathbf{y} \in \mathbb{C}^M$ be a vector of measurements, $\mathbf{A} \in \mathbb{C}^{M \times N}$, $M \ll N$, be a known "fat" sensing matrix, and $\mathbf{x} \in \mathbb{C}^N$ be a large unknown sparse vector with K, $K \ll N$, non-zero values, related as follows:

$$\mathbf{y} = \mathbf{A}\mathbf{x}. \tag{3}$$

Although underdetermined, the system in (3) can be solved using different reconstruction algorithms. In this project, you will implement and study one of them in the context of mmWave channel estimation.

- **System and mmWave channel models**: Consider the orthogonal frequency division multiplexing (OFDM) based system model described in [R-4, Sec. II]. Both the base station and user are assumed to be equipped with hybrid arrays, as illustrated in [R-4, Fig. 1]. Wideband channel is assumed to have L propagation paths with different delays. Its frequency-domain representation is provided in [R-4, Sec. II].

- **Vectorization of taken measurements**: Given the received signal model in [R-4, (7)], vectorize the measurements across different training frames to obtain a linear model suitable for application of compressive sensing reconstruction algorithms. The vectorization process is described in [R-4, Sec. III-A].

- **OMP-based reconstruction algorithm**: Study and implement the Simultaneous Weighted Orthogonal Matching Pursuit (SW-OMP) algorithm, described in [R-4, Sec. III-B] and [R-4, Sec. III-C]. The algorithm is summarized in [R-4, Sec. III-C, Alg. 1]. The SW-OMP is a greedy iterative algorithm, which estimates one propagation path in each iteration. After estimation, the gains of all paths estimated so far are refined and their "contribution" is subtracted from the original vector of measurements. The algorithm iterates until an energy-based stopping criterion is satisfied.

- **Results**: Study normalized mean square error (NMSE) of channel estimation as a function of signal-to-noise ratio (SNR) and number of training frames. Additionally, study spectral efficiency as a function of the SNR. The definition of the NMSE is provided in [R-4, (34)], while the definition of spectral efficiency is provided in [R-4, (36)]. Assume the same channel parameters as in [R-4, Sec. IV].

  1. In the same figure, plot the NMSE vs. SNR in the range -15 dB to 10 dB, assuming on-grid channel angles and $M = 80$ and $M = 120$ training frames. Refer to the blue curves in [R-4, Fig. 2(a)] and [R-4, Fig. 2(b)] for comparison with your results.

  2. In the same figure, plot the NMSE vs. number of training frames M in the range 20 to 100, for the following SNR values SNR $= \{-10 \text{ dB}, -5 \text{ dB}, 0 \text{ dB}\}$, assuming on-grid channel angles. Refer to the blue curves in [R-4, Fig. 5(a)], [R-4, Fig. 5(b)], and [R-4, Fig. 5(c)] for comparison with your results.

  3. Plot the spectral efficiency vs. SNR in the range -15 dB to 10 dB, assuming on-grid channel angles and $M = 60$. Refer to the blue curve in [R-4, Fig. 6(a)] for comparison with your results. Note that your results may be slightly different from [R-4, Fig. 6(a)], where the authors of the paper considered a more realistic channel model.

0.0.3 Relevant References

[R-4] J. Rodríguez-Fernández, N. González-Prelcic, K. Venugopal and R. W. Heath, "Frequency-Domain Compressive Channel Estimation for Frequency-Selective Hybrid Millimeter Wave MIMO Systems," in IEEE Transactions on Wireless Communications, vol. 17, no. 5, pp. 2946-2960, May 2018 (IEEE Xplore link)

Project list for individual submissions

Project 4: Beam Training with Analog True-Time-Delay Arrays

Beam training in a wideband millimeter-wave (mmWave) system with conventional arrays based on phase shifters requires exhaustive beam sweeping to find the dominant propagation directions in the channel. With such type of arrays, all frequency components of the signal are steered to or combined from the same direction because the same phase shifts are applied to all signal frequencies. Recent advances in the design of true-time-delay (TTD) arrays allowed their adoption in wideband mmWave systems. Compared to the arrays based on phase shifters, TTD arrays have the ability to synthesize frequency-dependent beams by delaying the signal in all antenna array branches. A user with a TTD array can combine different frequencies from different angular directions at the same time and thus conclude the dominant propagation direction in a fast way using simple frequency-domain digital signal processing (DSP). In this project, you will study TTD-based beam training on the user side. Specifically, you will design a frequency-dependent beam training codebook for analog TTD arrays, implement a dictionary-based angle estimation algorithm, and study the impact of hardware impairments on beam training performance.

- **System model**: Consider the orthogonal frequency division multiplexing (OFDM) based system model described in [R-5, Sec. II]. The base station is assume to be equipped with a digital array. It uses a fixed precoder $\mathbf{v}$, which is designed according to a known angle of departure and it is the same for all subcarriers. The user is assumed to have a TTD array with delay elements implemented in baseband domain and frequency-dependent beams described in [R-5, Sec. II-A].

- **Channel model**: Implement a wideband channel model with $L = 3$ multipath components, where the dominant path is 10 dB stronger than each of the other two. The channel model is described in [R-5, Sec. II].

- **TTD codebook design**: Design frequency-dependent codebooks for analog TTD arrays, as explained in [R-5, Sec. III]. To set up a codebook, you need to design delay and phase taps in all antenna branches.

- **Angle estimation algorithm**: Implement the dictionary-based estimation algorithm described in [R-5, Sec. IV] and summarized in [R-5, Sec. IV, Alg. 1].

- **Impact of hardware impairments**: Study the impact of hardware impairments, including the gain and phase errors, on the beam training performance with analog TTD arrays. The models of delay and phase errors are described in [R-5, Sec. II-A], while their impacts on TTD combiners are included in [R-5, (5), second equation].

- **Results**: Study beam training with analog TTD arrays. Specifically, visualize the designed training codebook and evaluate the root mean square error (RMSE) of angle of arrival estimation. Assume the system parameters as in [R-5, Sec. V].

   1. Plot the beam patterns of designed codebook for analog TTD arrays.

2. Plot the RMSE of angle estimation vs, Signal-to-Noise Ratio (SNR) in the range of -20dB to 20dB for diversity factor $R = 1, 2, 4$, in the absence of hardware impairments (i.e. without delay, gain and phase errors).

3. Plot the RMSE of angle estimation vs. standard deviation of gain error σ_A in the range 0 dB to 4.5 dB. Refer to the red dashed line with stars in [R-5, Fig. 4] for comparison with your results.

4. **Bonus:** Plot the RMSE of angle estimation vs. standard deviation of phase error σ_P in the range $0°$ to $50°$. Refer to the red dashed line with diamonds in [R-5, Fig. 4] for comparison with your results.

0.0.4   Relevant References

[R-5] V. Boljanovic, H. Yan, E. Ghaderi, D. Heo, S. Gupta and D. Cabric, "Design of Millimeter-Wave Single-Shot Beam Training for True-Time-Delay Array," 2020 IEEE 21st International Workshop on Signal Processing Advances in Wireless Communications (SPAWC), pp. 1-5, 2020 (IEEE Xplore link)

Project 5: Beam Training with Compressive Sensing and Machine Learning

Similar to Project 4, in Project 5 you will also study how novel algorithms can speed up beam training for millimeter-wave users. However, instead of studying methods utilizing alternative array architectures, Project 5 will study a combination of compressive sensing and machine learning to reduce the overhead for conventional analog phased-arrays, even under array impairments.

     Beam training in a millimeter-wave (mmWave) system with conventional arrays based on phase shifters typically requires exhaustive beam sweeping to find the dominant propagation directions in the channel. Since mmWave channels are often sparse, containing few significant angles-of-arrival (AoAs), compressive estimation methods can be used to reduce the number of measurement beams required to estimate the AoAs. By measuring a few compressive, analog beams instead of all directional beams, we can estimate the best directional beam for data communications more quickly.

- **System model**: Consider a base station transmitting omnidirectionally to a multi-antenna user. We consider a simplified model of the base station using only one antenna and a user with a 32-element, half-wavelength spaced, uniform linear, analog phased array. Due to hardware constraints, the user can only measure the received signal strength (RSS) from known transmitted pilot symbols. In this project, we will abstract out the fixed base station precoding and pilot symbols by considering a simple "1" symbol being transmitted from a single antenna base station.

- **Channel model**: Implement a simple geometric SIMO (Single-Input Multiple-Output) channel with one or two paths ($L = 1$ or $L = 2$). The secondary path should have -15 dB gain relative to the primary path, though you may want to make this an adjustable parameter for the bonus result. The path AoAs should randomly distributed between -60 and 60 degrees from boresight and should be separated by at least 10 degrees. The per-antenna Signal-to-Noise Ratio (SNR) should range from 0 dB to 30 dB in 5 dB steps (i.e. SNRs from the set {0 dB, 5 dB, 10 dB, 15 dB, 20 dB, 25 dB, 30 dB}).

- **Codebook Design**: Design the directional and compressive codebooks based on the description in [R-6, Sec. 3.1]. The directional codebook should contain 64 beams over the specified AoA range.

The compressive codebook should use 4-bit phase quantization and contain 12 codes, though you will use 4, 8, or 12 of those for 4, 8, or 12 measurements.

- **Array impairments**: Study the impact of hardware impairments, including the gain and phase errors, on the sensing codebooks and the beam training performance with mmRAPID. In this project, we will model one instance of impairments, where the gain error (linear factor) $g_{err}(i)$ and phase error $p_{err}(i)$ (in degrees) are described per element index i:

$$g_{err}(i) = 0.25 \sin{(0.4\pi i)}$$

$$p_{err}(i) = 15 \sin{(0.3\pi i)}$$

- **Angle estimation algorithm**: Implement the mmRAPID algorithm for compressive beam training, as described in [R-6, Sec. 4.2].

- **Dataset generation**: Generate 1400 channel instances and randomly split the dataset into a training set of 1000 channels and test set of 400 channels. Use the 1000 training channels with 30 dB SNR to create a 1000 point training dataset. Use the 400 test channels with each SNR, creating a 400 point test set for each of the SNR levels. Note that for each data point, you will need: the index of directional beam with the strongest RSS (label), the compressive beam measurements (input data vector of 12 real scalars), and the true AoA. You must retrain mmRAPID for each number of measurements and different/no impairments. In other words, you will need to train mmRAPID 6 times for the following combinations of situations: 4 measurements with impairments, 4 measurements without impairments, 8 measurements with impairments, 8 measurements without impairments, 12 measurements with impairments, and 12 measurements without impairments. All performance curves should use your test data set.

- **Results**: Study beam training with mmRAPID. Specifically, visualize the designed codebooks and evaluate the AoA estimation error. We will use root mean square error (RMSE) as our AoA estimation metric.

  1. Plot the beam patterns of one code from the directional codebook and one code from the compressive codebook, each with and without array impairments. You should have two total figures with the results with and without impairments overlaid; one figure of the directional beam's pattern with and without impairments and a second figure of the compressive beam's pattern with and without impairments. Discuss how the impairments affect each beam.

  2. Plot the RMSE of AoA estimation vs SNR when using 8 measurements with a -15 dB secondary path relative gain, with and without impairments. Discuss how the impairments affect the estimation performance.

  3. Plot the RMSE of AoA estimation vs number of measurements with a -15 dB secondary path relative gain and 20 dB SNR, with and without impairments. Comment on how the number of measurements impacts estimation error.

  4. **Bonus:** Plot the RMSE of AoA estimation vs the secondary path relative gain with 8 measurements, 20 dB SNR, and no impairments. Adjust the secondary path gain between -15 dB to -3 dB in 3 dB steps (i.e. relative gains from the set {-15 dB, -12 dB, -9 dB, -6 dB, -3 dB}). Do not retrain mmRAPID, just use the same 6 trained instances of mmRAPID as in the previous parts.

0.0.5 Relevant References

[R-6] H. Yan, B. W. Domae, and D. Cabric, "mmRAPID: Machine Learning assisted Noncoherent Compressive Millimeter-Wave Beam Alignment," 4th ACM Workshop on Millimeter-Wave Networks and Sensing Systems (mmNets'20), Article 3, pp. 1–6, 2020 (ACM Digital Library link)

Project 6: Scaling up MIMO: Massive MIMO Communication

Massive multiple-input multiple-output (MIMO) wireless communications refers to the idea of equipping cellular base stations (BSs) with a very large number of antennas. Roughly speaking, we think of systems that use antenna arrays with an order of magnitude more elements than in systems being built today, say 100 antennas or more. Massive MIMO has been shown to potentially allow for orders of magnitude improvement in spectral and energy efficiency using relatively simple (linear) processing. After Lecture 10 and HW2 were finished, we have already built some good understanding of MIMO communication. In this project, we will extend it to MU-MIMO and multi-cell. You will study how the number of antennas affects the channel condition, SIR, and capacity.

- **System model**: We consider a MU-MIMO scenario where the Base station is equipped with M antennas and it is serving $K = 10/15$ users. The detailed system configuration is described in the numerical result section of [R-7], page 16.

- **Channel model**: Rayleigh fading is assumed, i.e., the channel matrix is chosen to have independent components that are distributed as $\mathcal{CN}(0, 1)$. The channel model is described at the beginning of each numerical result section in [R-7].

- **Linear precoders**: We will consider 3 different linear precoders, which are Matched Filter (MF), Zero forcing (ZF), and Regularized ZF. The math formulation of these 3 precoders are described in the section "Precoding in the forward link: The ultimate limit of noncooperative multicell MIMO with large arrays" of [R-7]

- **Results**: We will study the effect of increasing the number of antenna in MIMO and reproduce some of the results in [R-7]

  1. Using the simulation parameters in [R-7], calculate the average condition number of the channel matrix and the sum rate. Plot the them as a function of the number of antenna M. Show the results of Figure 5 (a) and (b) in [R-7]. You are only required to recreate the solid IID Rayleigh curve.

  2. Same as 1, use the simulation parameters to calculate the CDFs of ordered eigenvalues of the channel matrix. Note that only the CDFs curves of the largest and smallest eigenvalues are required. 3 MIMO settings, which are $(6 \times 6), (6 \times 64), (6 \times 128)$, are considered. Show the results of Figure 6 in [R-7]. You are only required to recreate the solid IID $(6 \times 6), (6 \times 64), (6 \times 128)$ curves.

  3. Simulate the SIR performance of "Precoding in the forward link: The ultimate limit of noncooperative multicell MIMO with large arrays" as described in [R-7]. Plot the Cumulative distribu-

tions on the SIR for the MF precoder, the ZF precoder, and a regularized ZF precoder. Namely, you are required to recreate the curves of Figure 10.

0.0.6 Relevant References

[R-7] F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, F. Tufvesson, "Scaling Up MIMO: Opportunities and Challenges with Very Large Arrays," in IEEE Signal Processing Magazine, vol. 30, no. 1, pp. 40-60, Jan. 2013, link)