UCLA Samueli School of Engineering

Hybrid Digital and Analog Beamforming Design for Large-Scale Antenna Arrays

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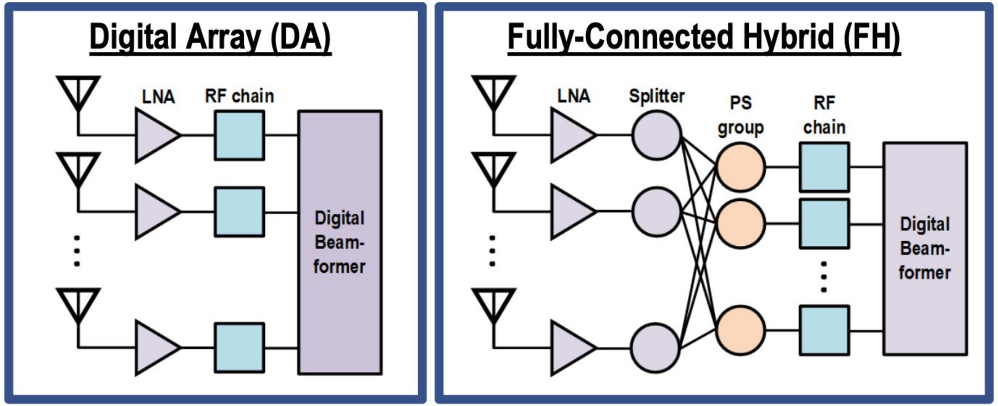
Introduction

Digital beamformer followed by RF beamformer

RF beamformer implemented with analog phase shifters

Two scenarios are considered in this project:

- $N_t^{RF} = N_r^{RF} = N^{RF} = N_S$
- $N_S < N^{RF} < 2N_S$



System model

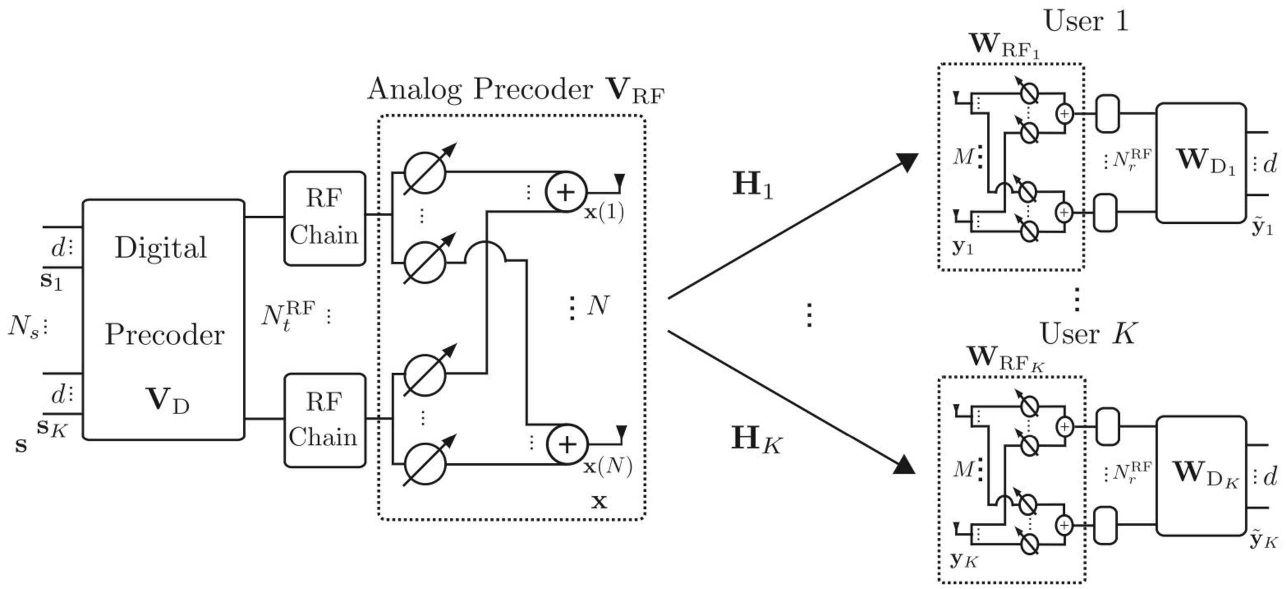


Fig. 1. Block diagram of a multi-user MIMO system with hybrid beamforming architecture at the BS and the user terminals.

Main part - mathematical objective

$$\begin{aligned} \underset{\mathbf{V}_{\mathrm{RF}}, \mathbf{V}_{\mathrm{D}}, \mathbf{W}_{\mathrm{RF}}, \mathbf{W}_{\mathrm{D}}}{\text{maximize}} \quad & \sum_{k=1}^{K} \beta_k R_k \\ \text{subject to} \quad & \mathrm{Tr}(\mathbf{V}_{\mathrm{RF}} \mathbf{V}_{\mathrm{D}} \mathbf{V}_{\mathrm{D}}^H \mathbf{V}_{\mathrm{RF}}^H) \leq P \\ & |\mathbf{V}_{\mathrm{RF}}(i,j)|^2 = 1, \forall i, j \\ & |\mathbf{W}_{\mathrm{RF}_k}(i,j)|^2 = 1, \forall i, j, k, \end{aligned}$$

Spectral efficiency for user K:

$$R_k = \log_2 \left| \mathbf{I}_M + \mathbf{W}_{\mathrm{t}_k} \mathbf{C}_k^{-1} \mathbf{W}_{\mathrm{t}_k}^H \mathbf{H}_k \mathbf{V}_{\mathrm{t}_k} \mathbf{V}_{\mathrm{t}_k}^H \mathbf{H}_k^H \right|$$

Assuming $N_t^{RF} = N_r^{RF} = N^{RF} = N_S$ and K=1 :

$$R = \log_2 \left| \mathbf{I}_M + \frac{1}{\sigma^2} \mathbf{W}_{\mathrm{t}} (\mathbf{W}_{\mathrm{t}}^H \mathbf{W}_{\mathrm{t}})^{-1} \mathbf{W}_{\mathrm{t}}^H \mathbf{H} \mathbf{V}_{\mathrm{t}} \mathbf{V}_{\mathrm{t}}^H \mathbf{H}^H \right|$$

Main part - RF precoder design

$$\max_{\mathbf{V}_{\mathrm{RF}},\mathbf{V}_{\mathrm{D}}} \quad \log_2\left|\mathbf{I}_M + \frac{1}{\sigma^2}\mathbf{H}\mathbf{V}_{\mathrm{RF}}\mathbf{V}_{\mathrm{D}}\mathbf{V}_{\mathrm{D}}^H\mathbf{V}_{\mathrm{RF}}^H\mathbf{H}^H\right|$$
$$\text{s.t.} \quad \mathrm{Tr}(\mathbf{V}_{\mathrm{RF}}\mathbf{V}_{\mathrm{D}}\mathbf{V}_{\mathrm{D}}^H\mathbf{V}_{\mathrm{RF}}^H) \le P,$$
$$|\mathbf{V}_{\mathrm{RF}}(i,j)|^2 = 1, \quad \forall i,j.$$

Assuming : $V_D V_D^H \approx \gamma^2 I$

$$\max_{\mathbf{V}_{\mathrm{RF}}} \quad \log_2\left|\mathbf{I} + \frac{\gamma^2}{\sigma^2}\mathbf{V}_{\mathrm{RF}}^H\mathbf{F}_1\mathbf{V}_{\mathrm{RF}}\right|$$
$$\text{s.t.} \quad |\mathbf{V}_{\mathrm{RF}}(i,j)|^2 = 1, \forall i,j,$$

where $\mathbf{F}_1 = \mathbf{H}^H\mathbf{H}$.

Algorithm 1. Design of $\mathbf{V}_{\mathrm{RF}}$ by solving (12)

Given: $\mathbf{F}_1$, γ^2, σ^2

1: Initialize $\mathbf{V}_{\mathrm{RF}} = \mathbf{1}_{N\times N^{\mathrm{RF}}}$.
2: **for** $j = 1 \to N^{\mathrm{RF}}$ **do**
3: Calculate $\mathbf{C}_j = \mathbf{I} + \frac{\gamma^2}{\sigma^2}(\bar{\mathbf{V}}_{\mathrm{RF}}^j)^H\mathbf{F}_1\bar{\mathbf{V}}_{\mathrm{RF}}^j$.
4: Calculate $\mathbf{G}_j = \frac{\gamma^2}{\sigma^2}\mathbf{F}_1 - \frac{\gamma^4}{\sigma^4}\mathbf{F}_1\bar{\mathbf{V}}_{\mathrm{RF}}^j\mathbf{C}_j^{-1}(\bar{\mathbf{V}}_{\mathrm{RF}}^j)^H\mathbf{F}_1$.
5: **for** $i = 1 \to N$
6: Find $\eta_{ij} = \sum_{\ell\ne i}\mathbf{G}_j(i,\ell)\mathbf{V}_{\mathrm{RF}}(\ell,j)$.
7: $\mathbf{V}_{\mathrm{RF}}(i,j) = \begin{cases} 1, & \text{if } \eta_{ij} = 0, \\ \frac{\eta_{ij}}{|\eta_{ij}|}, & \text{otherwise.}\end{cases}$
8: **end for**
9: **end for**
10: Check convergence. If yes, stop; if not go to Step 2.

Main part - digital precoder design

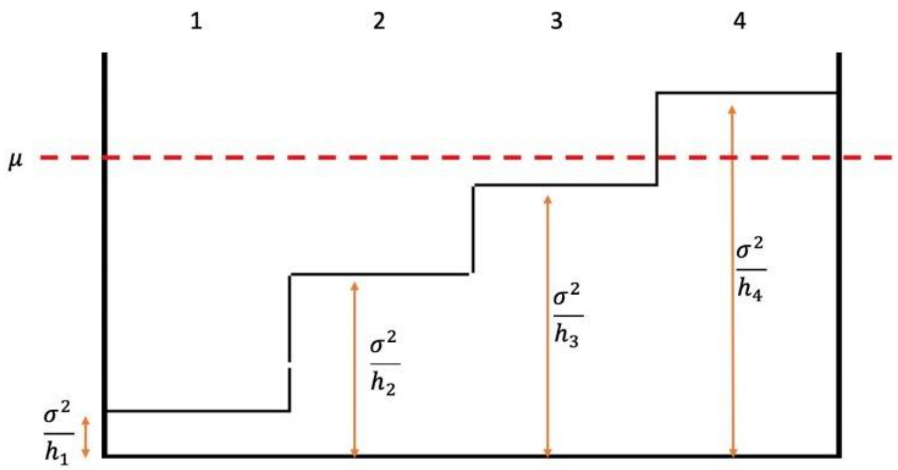
$$\max_{\mathbf{V}_{\mathrm{D}}} \quad \log_2 \left| \mathbf{I}_M + \frac{1}{\sigma^2} \mathbf{H}_{\mathrm{eff}} \mathbf{V}_{\mathrm{D}} \mathbf{V}_{\mathrm{D}}^H \mathbf{H}_{\mathrm{eff}}^H \right|$$
$$\text{s.t.} \quad \mathrm{Tr}(\mathbf{Q}\mathbf{V}_{\mathrm{D}}\mathbf{V}_{\mathrm{D}}^H) \leq P,$$

Where $Q = V_{RF}^H V_{RF}$ and $H_{eff} = HV_{RF}$

Using a water filling solution:

$$\begin{cases} \sum_{i=1}^{m} P_i = P \\ P_i = \max\left(\left(\mu - \frac{\sigma^2}{h_i}\right), 0\right) \end{cases}$$

$$\mu = \frac{P + \sum_{i=1}^{m} \frac{\sigma^2}{h_i}}{m}$$



The matrix of digital precoder can be calculated by: $V_D = Q^{-1/2} U_e \Gamma_e$

Main part - RF combiner design

$$\underset{\mathbf{V}_{\mathrm{RF}},\mathbf{V}_{\mathrm{D}},\mathbf{W}_{\mathrm{RF}},\mathbf{W}_{\mathrm{D}}}{\text{maximize}} \quad \sum_{k=1}^{K} \beta_k R_k \tag{5a}$$

$$\text{subject to} \quad \mathrm{Tr}(\mathbf{V}_{\mathrm{RF}}\mathbf{V}_{\mathrm{D}}\mathbf{V}_{\mathrm{D}}^{H}\mathbf{V}_{\mathrm{RF}}^{H}) \leq P \tag{5b}$$

$$|\mathbf{V}_{\mathrm{RF}}(i,j)|^2 = 1, \forall i, j \tag{5c}$$

$$|\mathbf{W}_{\mathrm{RF}_k}(i,j)|^2 = 1, \forall i, j, k, \tag{5d}$$

Precoders designed

$$\max_{\mathbf{W}_{\mathrm{RF}}} \quad \log_2 \left| \mathbf{I} + \frac{1}{\sigma^2} (\mathbf{W}_{\mathrm{RF}}^{H}\mathbf{W}_{\mathrm{RF}})^{-1} \mathbf{W}_{\mathrm{RF}}^{H} \mathbf{F}_2 \mathbf{W}_{\mathrm{RF}} \right|$$

$$\text{s.t.} \quad |\mathbf{W}_{\mathrm{RF}}(i,j)|^2 = 1, \forall i, j,$$

$$(W_{RF}^H W_{RF})^{-1} = \frac{1}{M}$$

$$F_2 = H V_t V_t^H H^H$$

$$V_t = V_{RF} V_D$$

Algorithm 1. Design of $\mathbf{V}_{\mathrm{RF}}$ by solving (12)

Given: $\mathbf{F}_1$, γ^2, σ^2

1: Initialize $\mathbf{V}_{\mathrm{RF}} = \mathbf{1}_{N \times N^{\mathrm{RF}}}$.
2: **for** $j = 1 \rightarrow N^{\mathrm{RF}}$ **do**
3: Calculate $\mathbf{C}_j = \mathbf{I} + \frac{\gamma^2}{\sigma^2}(\bar{\mathbf{V}}_{\mathrm{RF}}^{j})^H \mathbf{F}_1 \bar{\mathbf{V}}_{\mathrm{RF}}^{j}$.
4: Calculate $\mathbf{G}_j = \frac{\gamma^2}{\sigma^2}\mathbf{F}_1 - \frac{\gamma^4}{\sigma^4}\mathbf{F}_1 \bar{\mathbf{V}}_{\mathrm{RF}}^{j} \mathbf{C}_j^{-1} (\bar{\mathbf{V}}_{\mathrm{RF}}^{j})^H \mathbf{F}_1$.
5: **for** $i = 1 \rightarrow N$
6: Find $\eta_{ij} = \sum_{\ell \neq i} \mathbf{G}_j(i,\ell)\mathbf{V}_{\mathrm{RF}}(\ell, j)$.
7: $\mathbf{V}_{\mathrm{RF}}(i,j) = \begin{cases} 1, & \text{if } \eta_{ij} = 0, \\ \frac{\eta_{ij}}{|\eta_{ij}|}, & \text{otherwise.} \end{cases}$
8: **end for**
9: **end for**
10: Check convergence. If yes, stop; if not go to Step 2.

Main part - digital combiner design

MMSE solution

$$\mathbf{W}_{\mathrm{D}} = \mathbf{J}^{-1}\mathbf{W}_{\mathrm{RF}}^{H}\mathbf{H}\mathbf{V}_{\mathrm{t}},$$

where $\mathbf{J} = \mathbf{W}_{\mathrm{RF}}^{H}\mathbf{H}\mathbf{V}_{\mathrm{t}}\mathbf{V}_{\mathrm{t}}^{H}\mathbf{H}^{H}\mathbf{W}_{\mathrm{RF}} + \sigma^{2}\mathbf{W}_{\mathrm{RF}}^{H}\mathbf{W}_{\mathrm{RF}}$

Precoders √
Combiners √

$$R = \log_2 \left| \mathbf{I}_M + \frac{1}{\sigma^2}\mathbf{W}_{\mathrm{t}}(\mathbf{W}_{\mathrm{t}}^{H}\mathbf{W}_{\mathrm{t}})^{-1}\mathbf{W}_{\mathrm{t}}^{H}\mathbf{H}\mathbf{V}_{\mathrm{t}}\mathbf{V}_{\mathrm{t}}^{H}\mathbf{H}^{H} \right|$$

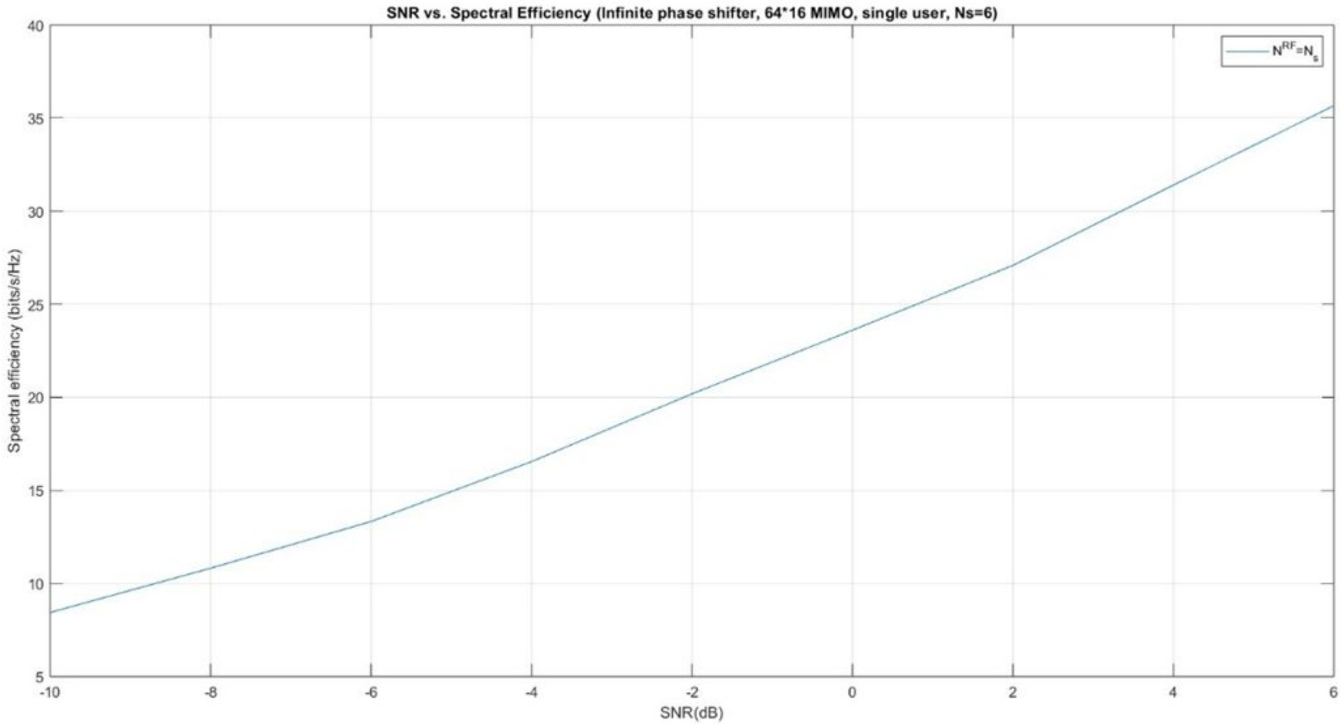
where $\mathbf{V}_{\mathrm{t}} = \mathbf{V}_{\mathrm{RF}}\mathbf{V}_{\mathrm{D}}$ and $\mathbf{W}_{\mathrm{t}} = \mathbf{W}_{\mathrm{RF}}\mathbf{W}_{\mathrm{D}}$.

Performance metrics: Spectral efficiency

Design process repeated for

$$N_S < N^{RF} = \{N_S,\ N_S + 1, N_S + 3\} < 2N_S.$$

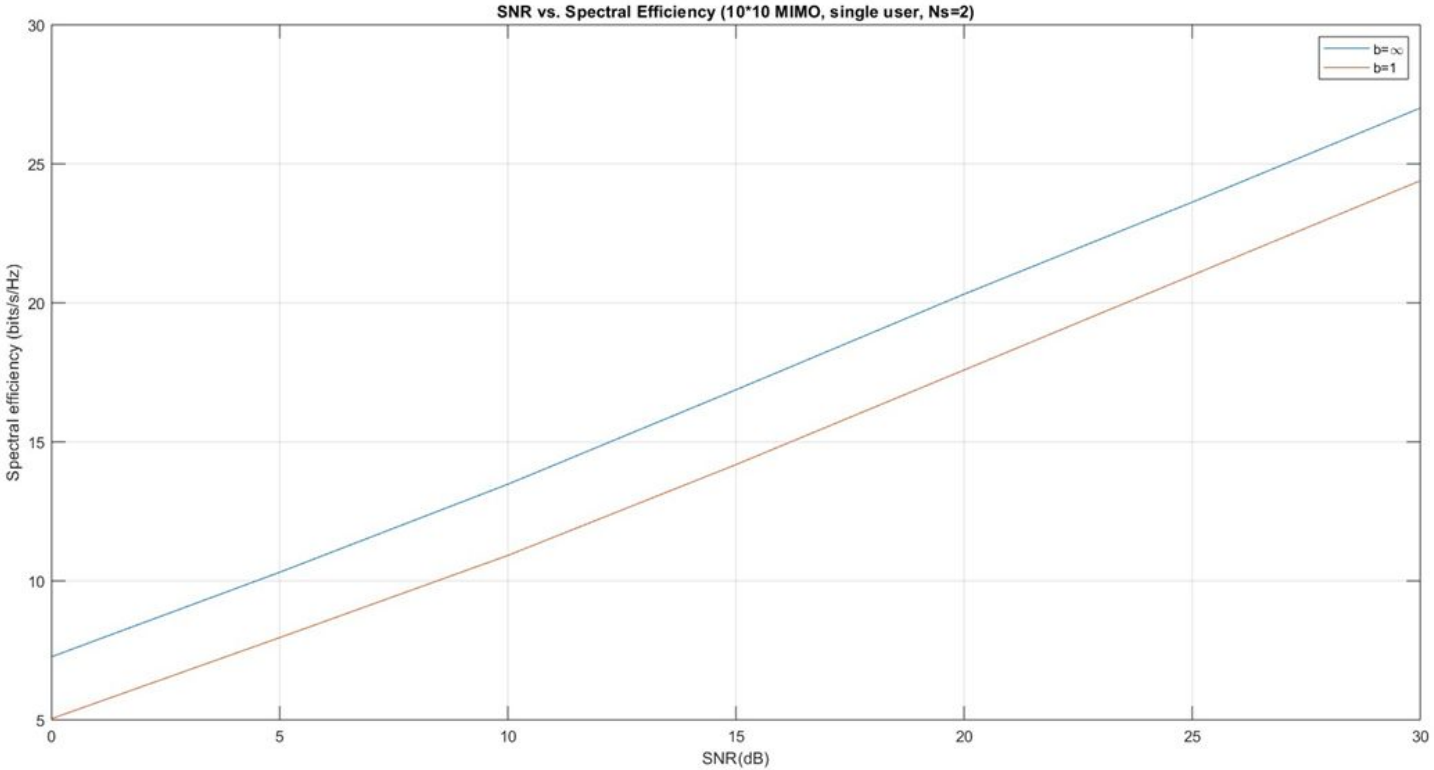
Result 1 - 64*16 MIMO, N_S=N_{RF}=6



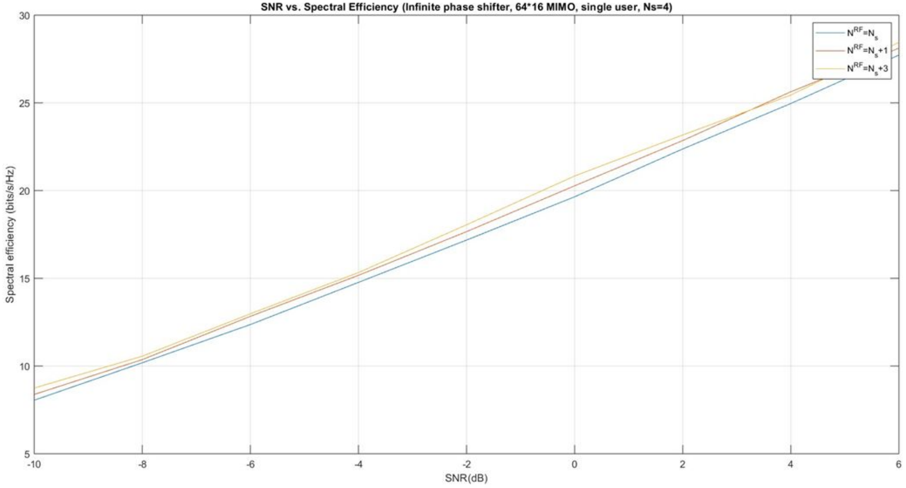
Result 2 - 10*10 MIMO, N_S=N_{RF}=2

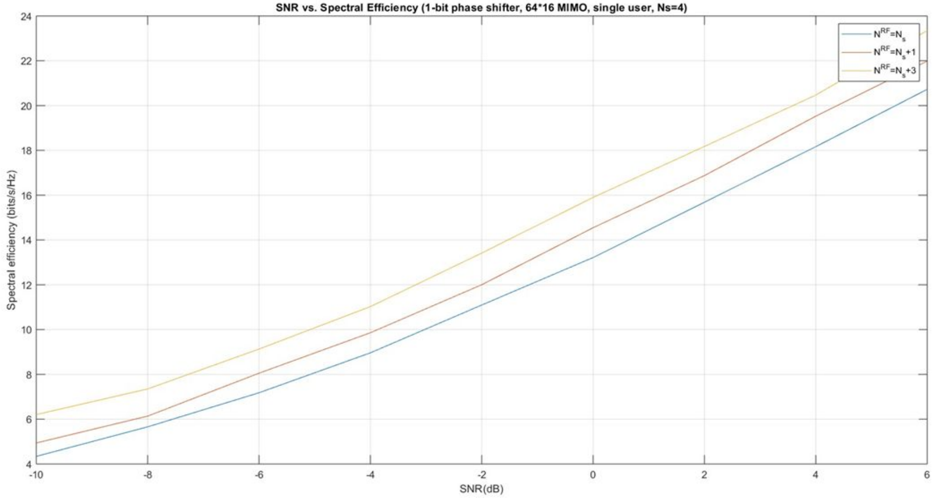
Infinite resolution phase shifter has better performance.

Possible reason: no quantization



Result 3 - 64*16 MIMO, $N_S=4$, $N_{RF}=\{N_S, N_S+1, N_S+3\}$

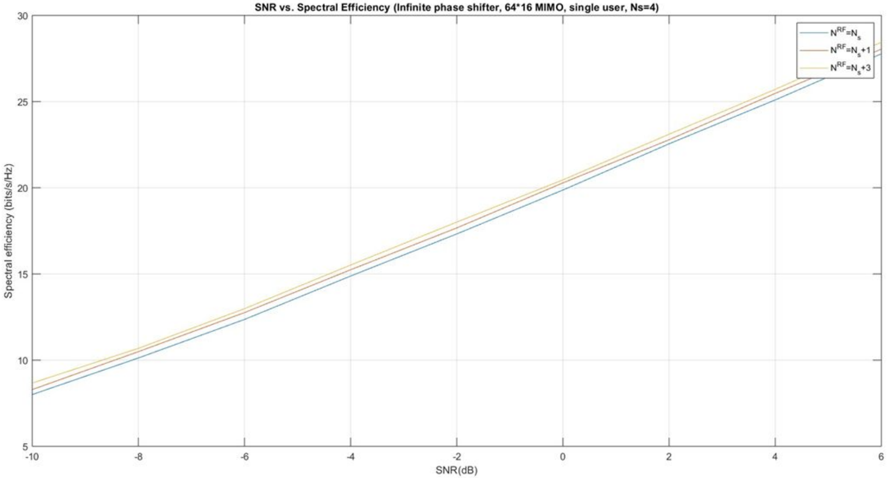


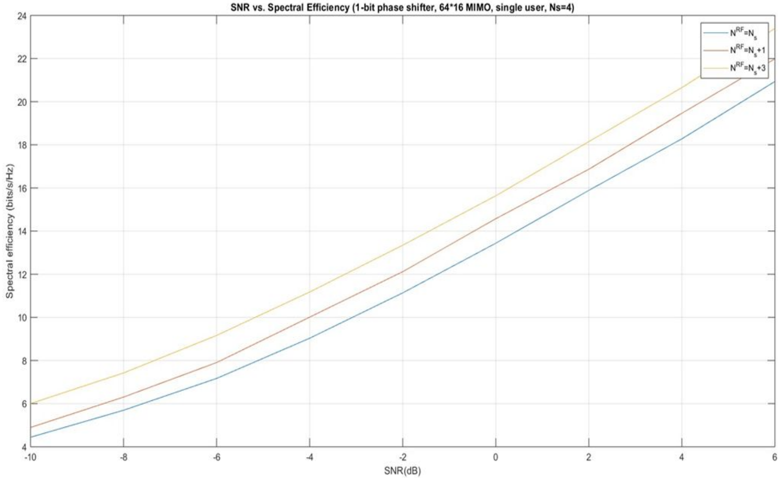


100 Monte Carlo trials

- Spectral efficiency increases as the number of RF chains increase
- 1-bit phase shifter with more RF chains has a performance close to infinite phase shifters

Result 3 - 64*16 MIMO, N_S=4, N_{RF}={N_S, N_S+1, N_S+3}





1000 Monte Carlo trials

- The increased number of RF chains can be used to trade off the accuracy of phase shifters in hybrid beamforming design.

Conclusion

Beamformer design

- Precoders
- Combiners

Number of data streams vs. number of RF chains

- $N_S = N_{RF}$
- $N_S < N_{RF} = \{N_S, N_S+1, N_S+3\} < 2N_S$

Spectral efficiency

- Increasing the number of RF chains can trade off the inaccuracy of phase shifters in hybrid beamformer architectures to achieve a closer performance to that of fully digital beamformer architectures.

Thank you

GitHub: https://github.com/Ruoye36/ECE233_Project_1

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