JACC Vol. 12, No. 5
November 1988:1344–53

Review of Hydrodynamic Principles for the Cardiologist: Applications to the Study of Blood Flow and Jets by Imaging Techniques

AJIT P. YOGANATHAN, PHD, EDWARD G. CAPE, BS, HSING-WEN SUNG, PHD, FRANK P. WILLIAMS, PHD, ABDUL JIMOH, MS

Atlanta, Georgia

An understanding of the basic concepts of the physics of blood flow is of vital importance to the cardiologist as he or she attempts to utilize new blood flow imaging modalities, such as Doppler ultrasound and nuclear magnetic resonance imaging. Concepts such as the Bernoulli equation and its limitations, the continuity equation and volume flow calculations and the theory of free and confined jets have applications in cardiac blood flow-related problems. For example, mitral regurgitant flow may be treated with the free jet theory. Aortic stenosis results in confined jet flow.

It is important that the cardiologist understand the basic principles behind these hydrodynamic concepts so that he or she can use them in appropriate applications. The limitations of the simplification of complex hydrodynamic relations that are used clinically need to be clearly understood so that these simplified principles are not used improperly or used to draw oversimplified conclusions.

(J Am Coll Cardiol 1988;12:1344–53)

Blood flow within the human circulatory system must obey the principles of conservation of mass, momentum and energy (1–3). The principle of conservation of mass as applied to any given region of flow states that whatever mass flows in must flow out. The principle of conservation of mass is used to derive the continuity equation. Similarly, the principle of conservation of energy states that in the absence of applied forces, the total energy of a system is constant (i.e., total energy flowing into the system is equal to the total energy flowing out). The principle of conservation of energy is used to derive the Bernoulli equation. The principle of conservation of momentum translates into Newton's second law, which states that the rate of change of momentum of a body (or fluid element) is equal to the forces acting on that body. From the principle of conservation of momentum are derived the fundamental working equations for solving hydrodynamic problems. These equations are called the Navier-Stokes equations (1).

A majority of the hydrodynamics problems that have been studied by engineers are steady flow problems. In the heart we deal with unsteady or pulsatile flow conditions. However, some of the concepts used in steady fluid flow problems are useful in understanding the physics of blood flow and therefore will be discussed in this article. In addition, the following concepts will be discussed: 1) the Bernoulli equation, 2) the continuity equation and volume flow, and 3) the basic principles of jets.

Steady Flow Concepts

Flow through a cylindrical (that is, circular) tube or vessel may be characterized as laminar, transitional or turbulent. The character of the flow is a result of the various forces acting on the fluid. Fluid particles in steady flow through a cylindrical tube experience two primary forces that characterize the flow field, namely, inertial and viscous forces.

The concept of dynamic similarity. This concept is utilized to characterize the flow in a generalized manner. If we have two flow geometries that are of different sizes but have the same shape (that is, two blood vessels that are circular but of different diameters), when do we expect the flow fields to be identical? The flow fields will be identical when the forces acting on volume elements of fluid at points of identical position have the same ratio. Therefore, for steady flow in two circular tubes, we would expect identical flow fields when the ratio of inertial to viscous forces in both tubes is the same. The ratio of these two forces is a dimensionless number that is known universally as the *Reynolds number* (Re). It is defined as

From the Cardiovascular Fluid Mechanics Laboratory, School of Chemical Engineering, Georgia Institute of Technology, Atlanta, Georgia.

Manuscript received October 26, 1987; revised manuscript received February 18, 1988, accepted April 11, 1988.

Address for reprints: Ajit P. Yoganathan, PhD, School of Chemical Engineering, Georgia Institute of Technology, Atlanta, Georgia 30332-0100.

0735-1097/88/$3.50

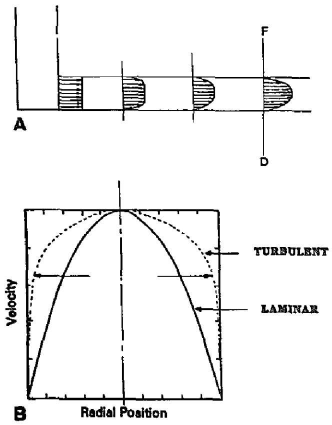


Figure 1. Laminar and turbulent flow. **A**, Developing velocity profiles in a cylindrical tube. At line F-D the velocity profile has a parabolic shape. **B**, The relative flatness of a turbulent velocity profile as compared with a laminar velocity profile.

$$Re = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{\rho \bar{V} d}{\mu},$$

where ρ = fluid density; $\bar{V}$ = cross-sectionally averaged fluid velocity (that is, volumetric flow rate/tube cross-sectional area); d = tube diameter; and μ = fluid viscosity. A large Reynolds number indicates that inertial effects are dominant, whereas a small number indicates that viscous effects are dominant. Therefore, the Reynolds number is used to characterize a flow field as laminar, transitional or turbulent.

Laminar flow. Typically, if the Reynolds number is <1,200, the flow is characterized as laminar. Consider fluid entering a long tube from a relatively large reservoir (Fig. 1a). At the entrance of the tube, the velocity profile is flat. Now, as we move away (that is, distal) from the inlet, the velocity profile begins to have curvature near the wall and has a magnitude of zero at the wall (that is, the *no slip* condition). The profile in the center remains flat. As the fluid passes line F-D, the profile has assumed a parabolic shape. The section of tube before line F-D is called the *entrance length* or *flow development region*. The section of tube past line F-D is called the *fully developed* flow region.

In the laminar flow region (Reynolds number <1,200), the fluid moves in what may be thought of as concentric cylindrical shells. Radial and tangential velocities are negligible so fluid particles remain in the same radial position as they move through the tube. The fluid enters the tube with a volume flow rate Q and its development is governed by two basic constraints: 1) because the tube is of constant cross section (i.e., constant diameter), the *average* velocity at any position along the tube is constant by continuity; and 2) the fluid particles adjacent to the wall are motionless with respect to the wall. As a result of these constraints, the profile shown in Figure 1 develops. The motionless "shell" of fluid adjacent to the wall retards the motion of the next shell through shearing forces. These forces act in the direction opposite to the direction of flow and on the surface area of the shell. Because the motion of the second shell is retarded, that of the next shell is retarded, and so on. However, because of the continuity constraint, the fluid near the center is accelerated to maintain the same average velocity as that at the entrance. This simultaneous retardation and acceleration continues until the profile becomes parabolic at line F-D.

The variable velocity region that gradually consumes the flat profile is called the boundary layer. It is a direct consequence of *viscous forces*. If viscous forces did not exist, the profile would remain flat and proceed along from the entrance with its original inertia (that is, inertial force) and none of the fluid elements would be retarded. We should then expect to be able to characterize the entrance length by the Reynolds number (Re). Indeed, the entrance length X for laminar flow in a straight tube is given by (3): X = 0.03d(Re). As expected, for low Reynolds numbers we have higher viscous forces and so the boundary layer consumes the flat profile more rapidly (i.e., shorter entrance length). *The parabolic profile in the fully developed region for laminar flow can be expressed as follows (Fig. 1b)*:

$$\frac{V}{V_o} = \left(1 - \frac{r^2}{R^2}\right),$$

where V_o = centerline velocity; V = velocity at a radial location, r; and R = radius of tube or vessel.

For fully developed steady laminar flow in a circular tube, the volumetric flow rate through the tube is related to the pressure drop (Δp) measured over a length L by the expression (1):

$$\frac{\Delta p}{L} = 128 \frac{\mu}{\pi d^4} Q,$$

where Q = volumetric flow rate; μ = fluid viscosity; and d = tube diameter. This expression is known as the *Haagen-Poiseuille* equation.

One final note on laminar flow concerns the shear stress on the fluid particles. The shear stress is an important variable for blood flow-related problems, because excessive shear stress can cause sublethal or lethal damage, or both, to blood cells and endothelial cells. For laminar flow, the shear stress (τ) is given by Newton's law of viscosity:

$$\tau = -\mu \frac{dV}{dr}.$$

Fluids that obey this law are called Newtonian. The fact that steeper velocity gradients correspond to higher shear stress will become especially important in turbulent flow considerations. Note that the shear stress (τ) on the tube wall can be calculated by the following equation:

$$\tau \text{ (at the wall)} = \frac{\Delta p}{L}\frac{d}{4}.$$

The above expression for the wall shear stress is also valid for steady turbulent flow in a circular tube.

Transitional flow. Between Reynolds numbers of 1,200 and 2,300, transitional flow occurs. An ink stream injected into the flow would begin to show oscillations. Radial and possibly tangential velocities would begin to show definite magnitudes in the transitional regime and the flow cannot be considered purely laminar or turbulent.

Turbulent flow. Above Reynolds numbers of 2,300, the flow is characterized as turbulent. Turbulence is essentially characterized by randomly *fluctuating* velocity and pressure at a position. Instead of all transport being on the molecular or particle level, "packets" of fluid called *eddies* randomly move about the tube. A stream of ink injected into turbulent flow in a tube would be rapidly dissipated from the centerline to the wall. However, even with this random motion, there is a well defined *average* velocity profile in the tube.

Instantaneous velocity (V) at a point is given by the time-averaged velocity ($\overline{V}$) plus the fluctuating velocity at that instant (V′):

$$V = \overline{V} + V'.$$

The root mean square magnitude of the fluctuating velocity (V_{rms}) is directly proportional to the level of turbulence at a given spatial location in the flow field (1):

$$V_{rms} = \sqrt{\overline{V'^2}}.$$

Turbulence intensity (I) is defined as:

$$I = \frac{V_{rms}}{\overline{V}} \times 100\%.$$

Two spatial locations having the same magnitude of mean velocity ($\overline{V}$) may have very different levels of turbulence (that is, different magnitudes of V_{rms}). The turbulent fluctuations result in additional physical forces known as turbulent or Reynolds shear stresses. Elevated levels of turbulent shear stresses (>50 N/m^2) can cause sublethal or lethal damage, or both, to the formed elements of blood (that is, red cells and platelets).

The velocity profile for fully developed turbulent flow in a circular tube can be expressed by the following *empirical* expression (Fig. 1b):

$$\frac{\overline{V}}{\overline{V}_0} = (1 - \frac{r}{R})^{1/7},$$

where the velocities are time-averaged values (1). The flatter central profile results in steeper profiles (that is, velocity gradients) near the wall and therefore higher shear stresses in the boundary layer.

Because of the lateral interactions, the turbulent boundary layer develops more quickly than the laminar boundary layer. Therefore, the entrance length is shorter. It may be represented by the expression (3): $X = 0.693d\ Re^{1/4}$.

The Reynolds number ranges given previously are guidelines. With obstructions or other disturbances such as vibrations, the turbulent phenomena begin to occur at lower Reynolds numbers. For very smooth tubes or exceptionally low levels of disturbance, the turbulent phenomena do not begin until higher Reynolds numbers.

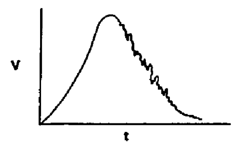


Figure 2. The effect of acceleration and deceleration on flow stability and development of turbulence: velocity (v) versus time (t).

Unsteady Flow Concepts

Steady flow analyses can be useful in certain cases, especially at the peak flow phase in the cardiac cycle, where pseudo-steady state flow conditions may be assumed. However, such analyses neglect the effects of pulsatility on the measured quantities such as velocity and pressure and these effects can change the flow field significantly.

Role of acceleration and deceleration in turbulent flow. The presence of acceleration and deceleration in a flow field is particularly important with regard to the development of turbulence. It has been shown, for example, that acceleration exerts a stabilizing effect on a flow field. In contrast, deceleration is destabilizing. Consider the velocity trace in Figure 2. The velocity at a position is plotted as a function of time during a pulsatile flow cycle. As the velocity ascends to its peak value, the curve is smooth; i.e., there are negligible fluctuations. As the velocity descends, turbulent fluctuations are observed. Because turbulence was observed during deceleration, that means that the fluid was above a critical (turbulent) Reynolds number as it neared its peak in acceleration, yet it remained relatively smooth. Over consecutive cardiac cycles, acceleration can cause *relaminarization* of turbulence left over from the previous cycle.

Role of pulse frequency in turbulence. The frequency of the pulses in unsteady flow is also important to the development of turbulence. A time is required for turbulence to develop. It is possible in some cases of pulsatile flow that there may not be enough time for turbulence to occur even with the deceleration period. The flow reaccelerates and is

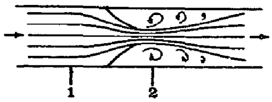


Figure 3. Flow velocity increase through a stenotic valve. The increase in kinetic energy between points 1 and 2 results in a decrease in potential energy (i.e., pressure drop) between the two locations and can be expressed by the Bernoulli equation.

stabilized before the turbulence invited by the deceleration develops. Therefore, the critical Reynolds number for laminar to turbulent transition in pulsatile flow depends on the rate of change of the velocity field. Quantitative studies of the laminar to turbulent transition in pulsatile flow express the critical Reynolds number as a function of the frequency variable called the Womersley number (N_w). The Womersley number is dimensionless and is defined as:

$$N_w = \frac{d}{2}\sqrt{\frac{\rho\omega}{\mu}},$$

where ω = circular frequency of heartbeat; d = tube or vessel diameter; ρ = density of fluid; and μ = viscosity of fluid. The Womersley number may be regarded as an "unsteady Reynolds number" in that it describes the relative importance of inertial to viscous forces.

For example, the Womersley number in the ascending aorta of humans (d = 2.54 cm) at a rest heart rate (60 beats/min) is about 17.5. In an in vivo dog study by Nerem and Seed (4), it was shown that as the Womersley number increased, the peak systolic Reynolds number required to cause the flow field to become turbulent also increased. Under rest cardiac output conditions in a normal healthy human, turbulence is *not* observed. From the study of Nerem and Seed, it may be extrapolated that a peak systolic Reynolds number on the order of 8,000 would be required to cause turbulence in the ascending aorta of humans.

Role of different locations in turbulent flow. Furthermore, turbulence can also be found at different locations in a pulsatile flow field. In a separation region, for example, the flow may become turbulent because of deceleration whereas the main flow remains laminar. Relaminarization can then occur during forward flow in the region. Turbulence may occur at different times during a cardiac cycle and at different places at one instant in time, with heart rate affecting both of these variables. This is a clear illustration of the complexities of the human system and the difficulties involved in applying our idealized principles of steady flow.

Bernoulli Equation

On the basis of the principle of conservation of energy, the Bernoulli equation may be derived. For pulsatile flow in the cardiovascular system (Fig. 3), the pressure difference between two locations 1 and 2 along a streamline may be expressed by the Bernoulli equation (2,5):

$$P_1 - P_2 = 1/2 \cdot \rho (V_2^2 - V_1^2) + \rho \int_1^2 \frac{dV}{dt} \cdot ds + R(v), \quad [1]$$

where P_1 = pressure at proximal location 1; P_2 = pressure at distal location 2; V_1 = velocity at proximal location 1; V_2 = velocity at distal location 2; ρ = density of fluid;

$$\rho \int_1^2 \frac{dV}{dt} \cdot ds$$

= flow acceleration between locations 1 and 2; R(v) = viscous friction loss between locations 1 and 2; and $1/2 \cdot \rho (V_2^2 - V_1^2)$ = convective acceleration.

Estimation of pressures and pressure gradient. *When applying the Bernoulli equation to clinical situations, the following simplifications are commonly utilized:* 1) At peak systole or peak diastole the acceleration term on the right-hand side is zero. 2) For most flow conditions encountered in the heart and great vessels, the viscous term can be *neglected*. Therefore,

$$(P_1 - P_2)_{peak} = 1/2 \cdot \rho (V_2^2 - V_1^2)_{peak}. \quad [2]$$

If P_1 and P_2 are expressed in millimeters of mercury, V_1 and V_2 in meters per second and a blood density of 1.07 g/cm^3 is assumed, equation 2 becomes:

$$(P_1 - P_2)_{peak} = 4 (V_2^2 - V_1^2)_{peak}. \quad [3]$$

If the peak distal velocity is much larger than the peak proximal velocity (that is: $V_{2, peak} >> V_{1, peak}$) then

$$(P_1 - P_2)_{peak} = \Delta P_{peak} = 4 V_{2, peak}^2, \quad [4]$$

where ΔP_{peak} is the peak systolic or diastolic pressure gradient. Similarly, when equation 1 is integrated over the duration of systole or diastole, the second and third terms on the right-hand side (of equation 1) become negligible (2,5).

Therefore, the mean systolic or diastolic pressure gradient may be expressed as

$$(\bar{P}_1 - \bar{P}_2) = \Delta\bar{P} = 4 (\overline{V_2^2} - \overline{V_1^2}). \quad [5]$$

Once again, if $\overline{V_2^2} >> \overline{V_1^2}$, then

$$\Delta\bar{P} = 4\, \overline{V_2^2}, \quad [6]$$

where $\Delta\overline{P}$ is the mean systolic or diastolic pressure gradient. Please note that $\overline{V_2^2} \neq (\overline{V}_1)^2$ and $\overline{V_2^2} \neq (\overline{V}_2)^2$. $\overline{V^2}$ is called the square of the root mean square velocity (V_{rms}). Equations 4 and 6 are commonly referred to as the *simplified Bernoulli equation*.

What are viscous losses and why do they occur? Viscous loss is the dissipation of flow energy due to the viscosity of the flowing fluid. If a fluid had no viscosity (an idealized situation) there would be *no* viscous losses. Viscous losses occur 1) because of friction between the moving fluid and solid boundaries, such as vessel walls; and 2) in regions of flow separation and in wakes behind obstacles.

For flow through a circular tube or nozzle type of obstruction (that is, valvular stenosis), viscous effects occur in a boundary layer adjacent to the tube or nozzle wall and decrease as you approach the center of the flow field. As flow through the tube or nozzle decreases or as the viscosity of the fluid increases, the size of the viscous boundary layer region *increases*.

As discussed previously, the larger the Reynolds number, the smaller the viscous effects (i.e., the smaller the viscous boundary layer region). For most flow conditions encountered in the heart and great vessels (except the coronary arteries) the viscous effects are much smaller than the inertial forces and therefore can be neglected.

When does the simplified Bernoulli equation "not work"? (i.e., $\Delta P_{peak} = 4V^2_{2,\ peak}$). 1) When the proximal velocity is of the same order of magnitude as the distal velocity (1–3). Examples are: a) aortic regurgitation in combination with aortic stenosis; and b) prosthetic heart valves. Note, that a 1 to 2 m/s proximal velocity leads to a 4 to 16 mm Hg decrease in pressure gradient. In such cases use equation 3, that is, the Bernoulli equation.

2) *Improper location of the catheter to measure distal pressure.* The maximal pressure drop across an orifice or nozzle-like obstruction occurs at a location immediately downstream of the obstruction called the vena contracta. If the distal pressure is not measured *within* the vena contracta, the measured gradient will be lower than the true maximal gradient. You may then be led to "believe" that the Doppler technique (that is, the simplified Bernoulli equation) is "overestimating" the gradient. Note that this underestimation is possible proximal to the vena contracta, before maximal velocity is reached, or distal to the vena contracta as a result of pressure recovery effects (see confined jet section).

3) *Stenoses in series such as*: a) long coarctations; b) tunnel-like muscular ventricular septal defects.

In all of the preceding situations (i.e., items 1, 2 and 3), the measured maximal pressure gradient (by catheter) will be lower than the true maximal gradient occurring in the flow field, if the catheter is not located in the vena contracta (6). In such clinical situations it may not be practical for the clinician to place the catheter in the vena contracta.

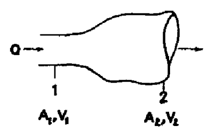


Figure 4. Continuity of fluid flow through an expansion. As the cross-sectional area (A) increases, the spatially averaged velocity ($\overline{V}$) decreases, maintaining a constant volume flow (Q).

In the case of resistances in series, the pressure drops are not additive. This is due to the phenomena of pressure recovery and relaminarization of the flow field downstream of the vena contracta (6). In most cases, the highest gradient will occur immediately downstream of the most restrictive (i.e., most severe) stenosis.

4) *True inapplicability of equations 3 and 4 occurs when viscous forces become significant (1–3).* Under these circumstances, Doppler ultrasound (that is, the simplified Bernoulli equation) will underestimate the true gradient: for example, in long (>10 mm), narrow (<0.10 cm^2) tunnel-like obstructions at very low flow rates (peak Reynolds number <500) (see confined jet section). Stenoses in coronary vessels would fall into this category. The Haagen-Poiseuille equation could be used in such a case (i.e., the case of coronary artery stenosis) to obtain a first order estimate of the pressure drop due to viscous effects, which occurs in addition to that due to convective acceleration.

Continuity Equation and Volume Flow

Continuity equation to estimate stenosis area. As stated previously, the continuity equation is derived from the principle of mass conservation. For steady flow through the geometry shown in Figure 4, the continuity equation states that (1):

$$\rho\,\overline{V}_1 A_1 = \rho\,\overline{V}_2\,A_2,$$

where $\overline{V}_1$ and $\overline{V}_2$ are the cross-sectionally averaged velocities at locations 1 and 2. If the flow is pulsatile, then at any instant in time t:

$$\rho\,\overline{V}_1(t)\ A_1 = \rho\,\overline{V}_2(t)\ A_2,$$

where $\overline{V}_1(t)$ and $\overline{V}_2(t)$ are the cross-sectionally averaged velocities at locations 1 and 2, at time instant t. Therefore,

$$\frac{A_2}{A_1} = \frac{\overline{V}_1\,(t)}{\overline{V}_2\,(t)}.$$

This equation can be used in principle to estimate the area of a stenotic lesion, if the velocity field and flow geometry proximal to the lesion and the velocity field distal to the lesion are known. It should be noted that the distal velocity profile can not be assumed to be flat.

Volume flow rate and flow velocity. Now let us consider the basic fundamentals of volume flow. For flow through a circular pipe under steady flow conditions:

$$\text{Volume flow rate} = Q = \overline{V}A, \quad [7]$$

where, $\overline{V}$ = spatially (i.e., cross-sectionally) averaged velocity in the pipe, and A = cross-sectional area of pipe. If the velocity of the fluid within the pipe can vary with time, then the total volume flow during time T (for example one cardiac cycle) is equal to:

$$Q(T) = \int_T Q(t)\,dt = A \int_T \overline{V}(t)\,dt. \quad [8]$$

where Q(t) and $\overline{V}(t)$ are the volume flow rate and the spatially averaged velocity in the pipe at any instant in time t respectively. For a circular tube, we can write:

$$Q(T) = \int_T \int_0^R 2\pi r \cdot V(r,t)\,dr\,dt. \quad [9]$$

Note that Q(T) is a cumulative number. It is the total volume flow over the time period T. Note that the velocity is dependent on two variables, V(r,t), position and time. By integrating over position and time it is possible to obtain the total volume, Q(T).

To make use of equation 9, one must know or assume the velocity profile. If the velocity profile is radially symmetric, as shown in Figure 1b, then the velocity V(r,t) may be expressed as:

$$V(r,t) = V_o(t)\,[1 - (\tfrac{r}{R})^n], \quad [10]$$

where n is an integer and $V_0(t)$ is the maximal velocity at time t. For a parabolic velocity profile, n = 2. The larger the value of n, the flatter the velocity profile. Also for a parabolic profile (i.e., n = 2), $\overline{V} = 0.5\,V_o$; whereas for n = 4.5 (i.e., turbulent or flat velocity profile, or both,

$$\overline{V} = 0.7\,V_o.$$

Equation 10 clearly indicates that the spatial extent of flow by itself does not necessarily reflect the quantitative amount of forward or regurgitant volume. It is essential to know the spatial distribution of the flow field within the "flow area" of interest as a function of time in the cardiac cycle.

If the flow field is three-dimensional in nature, as is the case in many regurgitant and stenotic lesions, obtaining the spatial extent of flow in a single plane may be inadequate for accurate volume flow calculations. Three orthogonal planes would be required under those circumstances.

The analysis of flow velocity profiles in various physiologic locations is the object of much current research.

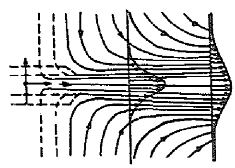


Figure 5. Schematic diagram of a laminar free jet (see text).

Accurate knowledge of velocity profiles allows potential clinical application of equation 9. For example, proximal to the aortic valve in the outflow tract the velocity profile is relatively flat. This leaves the velocity term in equation 9 as a function of time only. We could therefore integrate the maximal Doppler velocity curve in accordance with equation 9 and obtain the cardiac output.

Basic Principles of Jets

Free Jets

A *free jet* is defined as a jet issuing into a relatively stagnant environment where the cross-sectional area of the jet is less than one-fifth of the cross-sectional area of the region or chamber into which it is flowing, and it develops free from influence of external or chamber boundaries (i.e., no wall effects) (7,8). Free jets have been studied extensively in traditional fluid mechanics. Their analysis is also useful from the standpoint of cardiovascular hemodynamics because, for example, a regurgitant jet issuing into the left or right atrium from an incompetent mitral or tricuspid valve may be considered in many cases as a free jet. Now that Doppler flow mapping has allowed us to display jets within the heart, analysis of the fundamental fluid mechanics is important in relating jet volume to regurgitant volume in assessing the severity of the regurgitant lesion.

Features of free jet: momentum and dynamic similarity. A key feature of a free jet important in its analysis is that axial momentum is conserved. Consider the schematic of a free jet in Figure 5. As the jet moves distally from its origin it diffuses radially. The average velocity across an axial position becomes smaller but, simultaneously, mass is entrained from the surrounding reservoir because of viscous effects or turbulent mixing, or both. The product of these two variables—mass and velocity—is the momentum. The property of constant axial momentum for free jets is a consequence of the constant pressure throughout the large receiving reservoir (9).

A second feature of free jets is the phenomenon of dynamic similarity. Consider the plots of velocity profiles at various axial positions in Figure 6a that show axial velocity

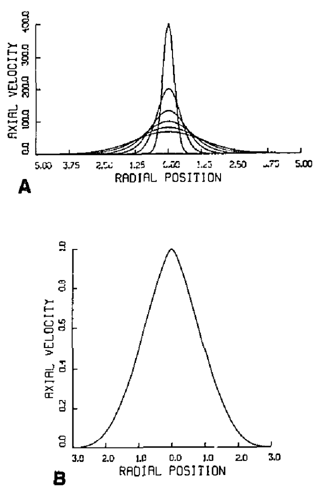


Figure 6. Phenomenon of dynamic similarity of free jets. **A**, Developing jet velocity profile. Free jet velocity profiles spread and decrease in magnitude at the centerline distal to the orifice. **B**, Nondimensional velocity profile. Normalized velocity profile using dynamic similarity. See text.

versus radial position. Now, let us take away from each profile its dimension as follows: First, divide each value of axial velocity by the centerline velocity at that axial position. Second, divide each radial position by the radial position at which the velocity is one-half of the centerline velocity (this radial position is commonly called the half-width). If this is done for each axial profile, all resulting profiles are as shown in Figure 6b. That is, all the normalized profiles collapse onto a single curve. This property allows us to develop useful dimensionless relations for flow or velocity, such as equations 11 and 12 to follow.

These two features of free jets are typically combined with reduced forms of the Navier-Stokes equations to yield a solution for the flow field in a jet. For turbulent jets, an empiric model is also required for the turbulent shear stress. Theoretical solutions for both laminar and turbulent free jets can be found in the engineering studies (8–11).

Jet Reynolds number (Jet Re) is defined as (7):

$$\text{Jet Re} = \frac{\rho V_0 d_0}{\mu},$$

where V_o = orifice jet velocity; d_o = orifice diameter; ρ = fluid density; and μ = fluid viscosity.

Laminar free jet. A laminar free jet is defined as one with a Reynolds number <2,000 but >300. A jet with a Reynolds number <300 exists as a dissipative creeping flow (12). A

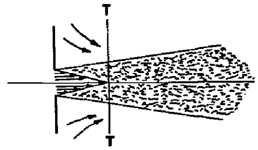


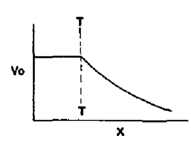
Figure 7. Schematic diagram of a turbulent free jet. To the left of line T–T is the conical potential core region. To the right of line T–T is the fully developed region.

turbulent free jet is one with a Reynolds number >3,000. Some references give 2,000 as the lower limit for turbulent jets (7). The range between 2,000 and 3,000, however, is usually thought of as a transition region. Figure 5 shows a laminar free jet. As the jet travels distal to the orifice, it diffuses radially and entrains surrounding fluid. The centerline velocity decreases and the velocity profiles spread out.

Turbulent free jet (Fig. 7). The physical structure of the turbulent jet is different from that of a laminar jet. The turbulent jet may be divided into two regions: the flow development region and the fully developed region. The two regions are divided by the line T-T in Figure 7. Consider a flat profile with velocity V_o at the orifice surface. As this core moves away from the orifice, it is consumed by the turbulent shear layer. The core eventually disappears at the beginning of the fully developed region. This conical volume of fluid with velocity V_o is called the potential core. In the fully developed region, the centerline velocity begins to decay and the profiles spread as in Figure 6a. Before line T-T, the velocity profiles remain partially flat near the center with a value of V_o. A plot of the centerline velocity would then look similar to Figure 8.

The initial core region is typically about 5 diameters long. Its development is primarily dependent on the intensity of the turbulent shear layer that consumes the core. The core region of the jet is short compared with the potential overall length of a turbulent free jet. It is the fully developed region that is of primary import. The following expressions apply to the fully developed region. *The volume rate of flow at a distance X from the orifice is given by (8):*

Figure 8. Plot of centerline velocity of a turbulent jet. It remains constant through the potential core and then decays through the fully developed region. See text.



$$Q = 0.16 \frac{X}{r_0} Q_0, \qquad [11]$$

where Q_0 and r_0 are the volumetric flow rate and jet radius at the orifice, respectively. *The decay of centerline velocity, $V_{m'}$, is given by (8):*

$$V_m = \frac{12 r_0 V_0}{X}. \qquad [12]$$

The radial expansion of the jet half-width b, is given by b = 0.086X.

Limitations. By examining the references given here, one will observe that theoretical and experimental results on free jets have agreed quite well. The results provided here are some that are very practical. When applying these equations, however, one must keep in mind three primary limitations: 1) the jet must, indeed, be free as defined; 2) the pressure in the large reservoir must be constant or nearly constant, so that axial momentum is conserved; and 3) because the solutions were obtained for circular orifices, the orifice of application needs to be approximately circular.

Clinical implications. By surveying these results and the references provided, it will be noted that the typical approach to free jet analysis in the past has been to consider the development of the flow field for a given orifice size and flow rate. From the standpoint of cardiovascular hemodynamics, we are generally faced with the inverse problem: given a jet, how larger is the orifice and how much flow is passing through it? The analysis of this problem is not a trivial reversal of the known results, because many combinations of orifice size and volumetric flow rate can produce similar jet flow fields. Therefore, a thorough understanding of the present state of jet theory and its limitations is very important to attempt to solve the cardiovascular fluid mechanics problem.

Furthermore it is apparent that the size of the orifice is a critical variable in the described equation. Free jet theory is most applicable to valvular incompetency, such as mitral regurgitation. In these clinical cases, it is highly unlikely that we know the size of or can even visualize the lesion. Therefore, although these types of equations do not allow immediate clinical application, they do provide a basis on which to proceed in developing *quantitative clinical* methods.

Confined Jets

Bounded or confined jets have been extensively studied not only in traditional fluid mechanics but also in the relatively new field of cardiovascular fluid mechanics. Bounded jet information has obvious application to stenotic aortic and pulmonary valve lesions as well as atherosclerotic lesions.

A schematic of a confined jet emerging from a constriction is given in Figure 9. The results of analyses of confined jets have shown that the flow upstream of the jet producing constriction is important in the resulting behavior of the downstream flow field. Therefore, confined jets will be considered from both sides and through the constriction.

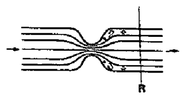


Figure 9. Schematic diagram of a confined jet. Reattachment occurs at point R. See text.

Confined jet and severity of constriction. A confined jet and the severity of the constriction through which it passes is typically described in terms of pressure considerations. The discussion here will primarily consider pressure implications of confined jet flow. As the fluid approaches the constriction (Fig. 9), it begins to accelerate because of the decrease in flow area. This increase in kinetic energy is accompanied by a corresponding loss of potential energy (pressure). As the constriction begins to expand, so does the jet (although it may possess a vena contracta to be discussed later). However, unless the expansion of the constriction is very gradual, the jet will expand more slowly with respect to the axial position. As a consequence, the annular region of recirculation in Figure 9 is observed. The region of recirculation is primarily due to the interaction between jet inertia, viscous effects and axial pressure gradients. The issuing jet experiences a strong adverse pressure gradient. The fluid just leaving the orifice has a relatively high axial momentum and ejects against the gradient. However, as a result of viscous effects (and turbulent mixing if it is present), the jet begins to entrain surrounding fluid, causing it to expand radially. As it expands, it develops progressively smaller and more distributed velocity profiles (similar in concept to those for a free jet in Fig. 6a). The fluid near the center has enough inertia to continue downstream as it loses velocity. The fluid on the outer fringes of the jet, however, has low velocity and cannot overcome the adverse pressure gradient. These fluid particles are pushed back into the recirculation region by the adverse gradient or are pulled into the lower pressure vortices.

At point R in Figure 9, the jet reattaches to the tube wall and tube flow reconstitutes. The reattachment length increases with an orifice Reynolds number up to about 200 and is constant at about 6 to 12 diameters above a Reynolds number of 2,000. In the intermediate range, it is dependent on inlet flow profile and disturbances (8,13).

The vena contracta. As mentioned previously, the jet may pass through a minimal cross-sectional area called the vena contracta before expanding. Let us now consider this phenomenon. If fluid is led smoothly into an orifice, as in

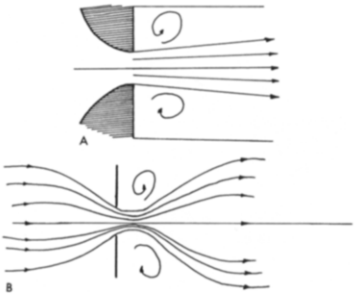


Figure 10. The vena contracta. **A**, Flow led smoothly into an orifice begins to expand immediately distal to the orifice. **B**, Flow entering abruptly into an orifice has its vena contracta distal to the orifice. See text.

Figure 10a, it is likely that the minimal diameter of the jet will be that of the orifice or very near that. On the other hand, if the jet passes through a sharp edged orifice, as in Figure 10b, the abrupt contraction will cause the jet to form with a vena contracta. Past the orifice surface, the jet continues to constrict for a certain length. It then expands radially. The vena contracta is the point at which the jet has its minimal area.

If A_c is the cross-sectional area of the jet at the vena contracta and A_o is that at the orifice, the contraction coefficient is defined as $C = A_c/A_o$ (14). Depending on the nature of the orifice, C can range from 0.60 to 1.00 (15). The contraction of a jet decreases with increasing viscosity. As the ratio of orifice to tube diameter decreases, the contraction of the jet increases. The location of the vena contracta is dependent on the geometry of the orifice and usually independent of flow rate (16).

Clearly, the vena contracta can place the position of maximal pressure gradient slightly downstream from the obstruction. The physical location of the maximal gradient is important when an invasive means of pressure measurement such as catheterization is used. Another phenomenon important in this respect is the recovery of pressure.

Recovery of pressure. To illustrate the concept of pressure recovery, let us consider two types of fluid meters—the orifice and Venturi. These two meters are shown in Figure 11. The orifice meter consists of a flat plate with a central circular hole. The jet emerges from the orifice and begins to diffuse radially. Eventually, it reattaches as described before. As the fluid emerges from the hole, there is a minimum in static pressure at the vena contracta. However, downstream from the vena contracta, pressure begins to recover. That is, the static pressure begins to increase to its original value. It does not, however, reach that value. There is a finite, overall, irrecoverable loss due to viscous effects. For

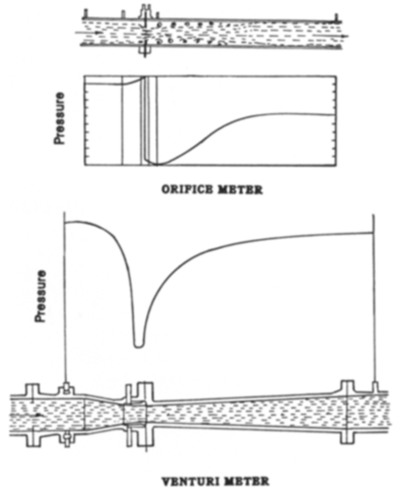


Figure 11. Pressure recovery phenomenon in an orifice meter and a Venturi meter. See text.

the orifice meter, these losses occur primarily in the region of recirculation surrounding the jet.

Consider now the Venturi meter. It, too, experiences a finite overall pressure loss. Its overall loss, however, is much less than that of the orifice meter because the fluid—after passing through the point of maximal constriction—is gradually led back to the original tube diameter. This process is in contrast to the events occurring with the orifice, which opens abruptly back to the original diameter and causes the regions of recirculation.

Now, instead of thinking of an overall pressure loss for a fluid meter, let us think of overall pressure recovery for a stenosis. It is clear, depending on the magnitude and rate of pressure recovery, that a catheter displaced downstream from a vena contracta could significantly underestimate the magnitude of the maximal pressure gradient that is associated with stenosis severity. We might, therefore, expect distally tapering stenoses to show more dramatic pressure recovery in analogy with the Venturi meter.

Viscous term: relation to pressure gradient. The minimal pressure in the jet—or the maximal pressure gradient with respect to the proximal chamber—is primarily due to convective acceleration (that given by the simplified Bernoulli equation). However, the geometry of the orifice and fluid properties can sometimes cause the neglected viscous term to become important. As found by Teirstein et al. (17), the viscous term becomes important as diameter decreases and stenosis or tunnel length increases. The viscous term also, of course, becomes important with increasing viscosity. These results can be simply characterized as a reaffirmation of the Reynolds number definition (i.e., the ratio of inertial to viscous forces). Increasing tunnel length provides a surface on which a boundary layer can develop. This boundary layer is a result of viscous effects and the phenomenon contributes to the pressure loss.

In summary, note that the free jet theory has application primarily to mitral regurgitation in which there are negligible wall effects. Confined jet flow exists in situations such as aortic stenosis. Although the flows discussed here are ideal cases, they are important for understanding the basic concepts of jet flow, which can then be used to analyze more complex physiologic flows.

Conclusions. In this article we have discussed some basic and fundamental concepts of hydrodynamics of value to the cardiologist. Unfortunately, none of the topics could be discussed in detail because that endeavor would require an entire textbook. The reader is strongly urged to read the reference material listed at the end of the article for the necessary details. References 2 and 3 are essential.

The secretarial assistance of Harolyn Ingram, Jamie Lockwood and Cecille Miller in preparing this manuscript is greatly appreciated.

References

1. Bird RB, Stewart WE, Lightfoot EN. Transport Phenomena. New York: John Wiley & Sons, 1960.
2. Fung YC. Biodynamics: Circulation. New York: Springer-Verlag, 1984.
3. Caro CE, Pedley TJ, Schroter RC, Seed WA. The Mechanics of the Circulation. New York: Oxford University Press, 1978.
4. Nerem R, Seed WA. An *in vivo* study of aortic flow disturbances. Cardiovasc Res 1972;6:1–4.
5. Yoganathan AP, Corcoran WH, Harrison EC. Pressure drops across prosthetic aortic heart valves under steady and pulsatile flow: *in vitro* measurements. J Biomech 1979;12:153–64.
6. Yoganathan AP, Valdes-Cruz LM, Schmidt-Dohna J, et al. Continuous wave Doppler velocities and gradients across fixed tunnel obstructions: *in vitro* and *in vivo* studies. Circulation 1987;76:657–66.
7. Perry RH, Green DW, eds. Chemical Engineers' Handbook. 6th ed. New York: McGraw-Hill, 1984:5, 22.
8. Blevins RD. Applied Fluid Dynamics Handbook. New York: Van Nostrand Reinhold, 1984.
9. Schlichting H. Boundary Layer Theory. 7th ed. New York: McGraw-Hill, 1979.
10. Pai S-I. Fluid Dynamics of Jets. New York: D. Van Nostrand, 1954.
11. Abramovich GN. The Theory of Turbulent Jets. Cambridge, MA: MIT Press, 1963.
12. McNaughton KJ, Sinclair CG. Submerged jets in short cylindrical flow vessels. J Fluid Mech 1966;25:367–75.
13. Roschke EJ, Back LH. The influence of upstream conditions on flow reattachment lengths downstream of an abrupt circular channel expansion. J Biomech 1976;9:481–3.
14. Kaufman W. Fluid Mechanics. New York: McGraw-Hill, 1963:47–8.
15. Shames IH. Mechanics of Fluids. 2nd ed. New York: McGraw-Hill, 1982: 192.
16. Bean HS, ed. Fluid Meters: Their Theory and Application. 5th ed. New York: American Society of Mechanical Engineers, 1959.
17. Teirstein PS, Yock PG, Popp RL. The accuracy of Doppler ultrasound measurement of pressure gradients across irregular, dual, and tunnel-like obstructions to blood flow. Circulation 1985;72:577–84.