Midterm Guide

- 20% of points are from definitions, thus study defintions!!! (Topic 0: Review part)

Sample Midterm 2019 Solution

> - Note: This was done 2 weeks earlier & is missing the LLSP topic (we will be tested on this). 2019 version only contains review (topic 0) + QR methods (topic 1)

Q1: Definitions (20% of the grade)

a)

Define span of vectors in a V.S.

Solution:

Given $(V, \mathbb{K}, +, \cdot)$ a V.S and a subset $S \subseteq V$ of vectors.

$$span(S) = \{a_1 s_1 + \cdots + a_n s_n : s_i \in S, a_i \in \mathbb{K}, n \in \mathbb{N}\}$$

which is the set of all finite linear combinations.

b)

Let $(V, \mathbb{K}, +, \cdot, < \cdot, \cdot >)$ be an Inner Product Space. State the properties satisfied by the Inner Prodcut map

Solution:

1. $< u, v >=< v, u >$ (symmetry)
2. $< au, v >= a < u, v >, < u + v, w >=< u, w > + < v, w >$ (Linearity in the first and second component)

3. $< u, u > \geq 0 \iff u = \vec{0}$ (positivity)

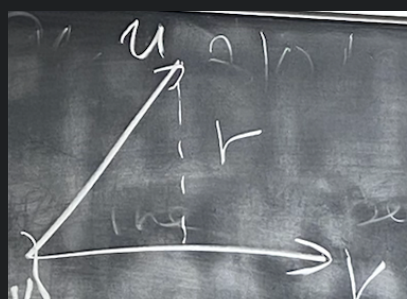
Note: The descriptions in the bracket are good to have, but not required in the exam.

c)

Consider $\mathbb{R}^n$ equipped with the usual vector addition, scalar product and inner product $< u, v >= u^T v$, and let $u, v \in \mathbb{R}^n$, and $W \subset R^n$ be a subspace. Define orthogonal projection of u onto v. Define orthogonal projection of u onto W

Solution:

use a formula or explain in words by expressing the appropriate condition.



In formula: $proj_v(u) = \frac{(v^T u)}{v^T v} v$

In words: $proj_v(u)$ is the vector u_v in $span\{v\}$ s.t the residual $r = u - u_v \perp span\{v\}$

UNFINISHED, CONTINUE NEXT TIME