

Line Assignment

Name: Rupa Sai Sreshta Vallabhaneni

0.1 Problem

The area of triangle is 5. Two of its vertices are A(2,1) and B(3,-2). The third vertex lies on $y=x+3$. Find C.

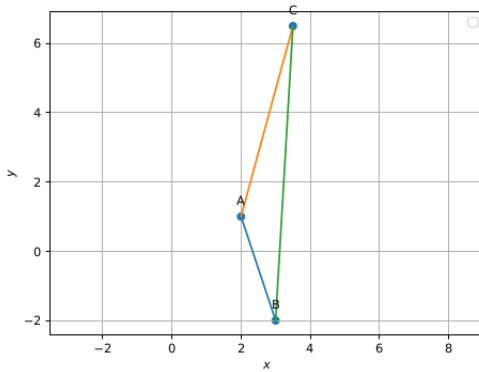


Figure of Construction

The following python code is used for finding third vertex of triangle

Githublink : <https://github.com/RupaSaiSreshta/FWC>

$$3x + y = 17 \quad (1)$$

$$x - y = -3 \quad (2)$$

$$\Rightarrow \begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 17 \\ -3 \end{pmatrix} \quad (3)$$

The above matrix is in the form of $A\mathbf{x}=\mathbf{B}$

The augmented matrix for the above matrix equation is

$$\left(\begin{array}{cc|c} 3 & 1 & 17 \\ 1 & -1 & -3 \end{array} \right) \quad (4)$$

$$\xleftrightarrow{R_2 \leftarrow 3R_2 - R_1} \left(\begin{array}{cc|c} 3 & 1 & 17 \\ 0 & -4 & -20 \end{array} \right) \quad (5)$$

$$\xleftrightarrow{R_1 \leftarrow 4R_1 + R_2} \left(\begin{array}{cc|c} 12 & 0 & 42 \\ 0 & -4 & -26 \end{array} \right) \quad (6)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 * 1/12} \left(\begin{array}{cc|c} 1 & 0 & 3.5 \\ 0 & -4 & -20 \end{array} \right) \quad (7)$$

$$\xleftrightarrow{R_2 \leftarrow R_2 * 1/4} \left(\begin{array}{cc|c} 1 & 0 & 3.5 \\ 0 & 1 & 6.5 \end{array} \right) \Rightarrow \mathbf{x} = \begin{pmatrix} 3.5 \\ 6.5 \end{pmatrix} \quad (8)$$

Now the vertex of C is

$$C(3.5, 6.5)$$

0.2 Solution

Construction:

Input parameters for this construction

Symbol	Value
A	(2,1)
B	(3,-2)
Area	5
C lies on	$y=x+3$

Solution:

Let us assume $C(x,y)$

Area of triangle is

$$Ar = \frac{1}{2} |\mathbf{ABXAC}|$$

By solving we get

$$3x+y=17 \dots (1)$$

$$\text{The vertex C lies on } y=x+3 \dots (2)$$

By solving (1) and (2) equations using matrix reduction method

0.3 Proof

Proof: Here The area of triangle is given as 5. Now we have to calculate the area of triangle using three vertices A, B and C. We need to prove that the area of triangle is 5. Area of triangle is

$$Ar = \frac{1}{2} |\mathbf{ABXAC}|$$

By substituting the values of A, B and C

$$Ar = \frac{1}{2} [2(-8.5) + 3(5.5) + 3.5(3)]$$

$$Ar = \frac{1}{2} [-17 + 16.5 + 10.5]$$

$$Ar = \frac{1}{2} [10.5 - 0.5]$$

$$\boxed{Ar = 5}$$

Hence verified.