Line Assignment

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Problem

The area of triangle is 5. Two of its vertices are A(2,1) and B(3,-2). The third vertex lies on y=x+3. Find C.

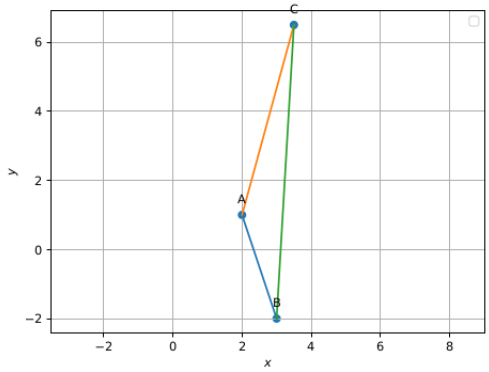

Figure of Construction

The following python code is used for finding third vertex of triangle

Githublink : *https://github.com/RupaSaiSreshta/FWC*

Solution

Construction:

Input parameters for this construction

Symbol	Value	Description
A	$\begin{pmatrix}2\\1\end{pmatrix}$	Given vertex of triangle
B	$\begin{pmatrix}3\\-2\end{pmatrix}$	Given vertex of triangle
Ar	**5**	Area of triangle

Solution:

Let us assume $\mathbf{C}\begin{pmatrix}x\\y\end{pmatrix}$

Area of triangle is

$$Ar = \frac{1}{2}\left\|((\mathbf{A}-\mathbf{B})\times(\mathbf{A}-\mathbf{C}))\right\| \tag{1}$$

By solving we will get the below matrix

$$\implies \begin{pmatrix}3 & 1\\1 & -1\end{pmatrix}\mathbf{x} = \begin{pmatrix}17\\-3\end{pmatrix} \tag{2}$$

The above matrix is in the form of **Ax=B**

The augmented matrix for the above matrix equation is

$$\left(\begin{array}{cc|c}3 & 1 & 17\\1 & -1 & -3\end{array}\right) \tag{3}$$

$$\xleftrightarrow{R_2\leftarrow 3R_2-R_1}\left(\begin{array}{cc|c}3 & 1 & 17\\0 & -4 & -20\end{array}\right) \tag{4}$$

$$\xleftrightarrow{R_1\leftarrow 4R_1+R_2}\left(\begin{array}{cc|c}12 & 0 & 42\\0 & -4 & -26\end{array}\right) \tag{5}$$

$$\xleftrightarrow{R_1\leftarrow R_1*1/12}\left(\begin{array}{cc|c}1 & 0 & 3.5\\0 & -4 & -20\end{array}\right) \tag{6}$$

$$\xleftrightarrow{R_2\leftarrow R_2*1/4}\left(\begin{array}{cc|c}1 & 0 & 3.5\\0 & 1 & 6.5\end{array}\right) \implies \mathbf{x} = \begin{pmatrix}3.5\\6.5\end{pmatrix} \tag{7}$$

Now the vertex of C is

$$\mathbf{C}\begin{pmatrix}3.5\\6.5\end{pmatrix}$$

Proof

Proof: Here The area of triangle is given as 5.Now we have to calculate the area of triangle using three vertices A,B and C. We need to prove that the area of triangle is 5. Area of triangle is

$$Ar = \frac{1}{2}\left\|((\mathbf{A}-\mathbf{B})\times(\mathbf{A}-\mathbf{C}))\right\| \tag{8}$$

By substituting the values of A,B and C

$$Ar = \frac{1}{2}[2(-8.5)+3(5.5)+3.5(3)] \tag{9}$$

$$Ar = \frac{1}{2}[-17+16.5+10.5] \tag{10}$$

$$Ar = \frac{1}{2}[10.5-0.5] \tag{11}$$

$$\boxed{Ar = 5}$$

Hence verified.