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Circle Assignment

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Problem Statement:

From the point A(0,3) on the circle $x^2 + 4x + (y+3)^2 = 0$. A chord AB is drawn and extended to a point M.Such that AM=2AB. Find the equation of locus of M.

Construction:

Symbol	Value	Description
$\mathbf{A}$	$\begin{pmatrix} 0 \\ 3 \end{pmatrix}$	Point on given circle
$\mathbf{B}$	$\left(\frac{\mathbf{A}+\mathbf{M}}{2}\right)$	Mid point of A and M
$\mathbf{r_1}$	2	Radius of given circle
$\mathbf{C}$	$\begin{pmatrix} -2 \\ 3 \end{pmatrix}$	Center of given circle

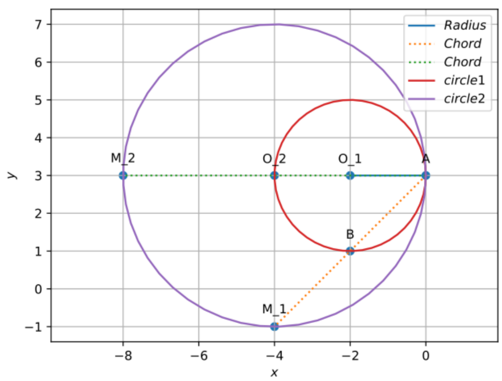

Figure of construction

You can download the python code for generating above circle from the below link

$$\boxed{Githublink : https : //github.com/RupaSaiSreshta/FWC}$$

Solution:

Given $\mathbf{A}$=$\begin{pmatrix} 0 \\ 3 \end{pmatrix}$

$$\mathbf{AM} = 2\mathbf{AB} \tag{1}$$

From this condition B is the midpoint of A and M.

The given circle equation is

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{2}$$

$\mathbf{V}$=$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$,

$\mathbf{u}$=$\begin{pmatrix} 2 \\ -3 \end{pmatrix}$,

f = 9.
Center of the circle

$$\mathbf{C} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \tag{3}$$

Radius of the circle

$$\mathbf{r_1} = 2 \tag{4}$$

Now,

$$\mathbf{B} = \frac{\mathbf{A}+\mathbf{M}}{2} \tag{5}$$

Then,

$$\mathbf{B}^\top \mathbf{V}\mathbf{B} + 2\mathbf{u}^\top \mathbf{B} + f1 = 0 \tag{6}$$

Now substitute Equation 5 in Equation 6

Then,

$$\frac{1}{4}\left(\mathbf{A}+\mathbf{M}\right)\left(\mathbf{A}+\mathbf{M}\right)^\top + \mathbf{u}^\top\left(\mathbf{A}+\mathbf{M}\right) + f \tag{7}$$

By solving

We get

$$\boxed{\mathbf{M}^\top \mathbf{M} + 2\mathbf{u}^\top \mathbf{M} + f = 0} \tag{8}$$

Proof:

Now, Let us take randomly two points of $\mathbf{M}$ with respectively to the locus equation

Let two points be $\mathbf{M_1}$ and $\mathbf{M_2}$

$$\mathbf{M_1} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \tag{9}$$

$$\mathbf{u} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \tag{10}$$

$$f = 9 \tag{11}$$

Now substitute $\mathbf{M_1}$,$\mathbf{u}$, f in Equation 7 Then, This equation will become zero.

$\therefore$ This equation is satisfied.

$$\mathbf{M_2} = \begin{pmatrix} -8 \\ 3 \end{pmatrix} \tag{12}$$

Now substitute $\mathbf{M_2}$,$\mathbf{u}$, f in Equation 7

Then, This equation will become zero.

$\therefore$ This equation is also satisfied.

From this we can say that equation of locus is correct.

Now for B point substitute $\mathbf{A}$ and $\mathbf{M_1}$ in equation 5.

Then $\mathbf{B}$ points will be

$$\mathbf{B} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \tag{13}$$

Now substitute $\mathbf{B}$ in circle equation

Then equation is satisfied.

From this we can say that $\mathbf{B}$ is point on the circle.