

Circle Assignment

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Problem Statement

From the point A(0,3) on the circle $x^2 + 4x + (y + 3)^2 = 0$. A chord AB is drawn and extended to a point M. Such that AM=2AB. Find the equation of locus of M.

Construction

Symbol	Value	Description
A	$\begin{pmatrix} 0 \\ 3 \end{pmatrix}$	Point on given circle
M	$\begin{pmatrix} h \\ k \end{pmatrix}$	Point outside circle
B	$\begin{pmatrix} \frac{h}{2} \\ \frac{k+3}{2} \end{pmatrix}$	Mid point of A and M
r_1	2	Radius of given circle
C	$\begin{pmatrix} -2 \\ 3 \end{pmatrix}$	Center of given circle

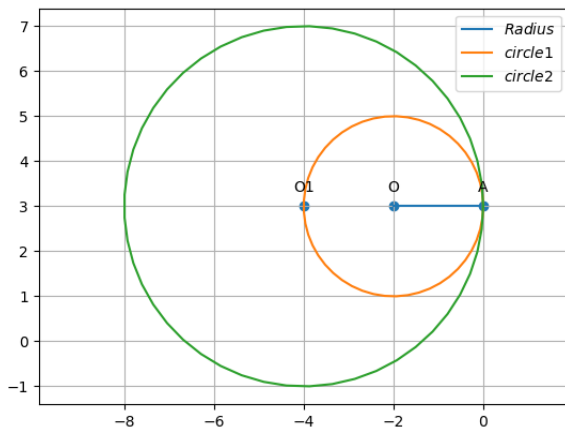


Figure of construction

You can download the python code for generating above circle from the below link

Githublink : <https://github.com/RupaSaiSreshta/FWC>

Solution

Given $A = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

AM=2AB

From this condition B is the midpoint of A and M.

The given circle equation is

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (1)$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$\mathbf{u} = \begin{pmatrix} 2 \\ -3 \end{pmatrix},$$

f = 9.

Center of the circle

$$\mathbf{C} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \quad (2)$$

Radius of the circle

$$r_1 = 2 \quad (3)$$

Let us assume

$$\mathbf{M} = \begin{pmatrix} h \\ k \end{pmatrix}$$

As B is the midpoint of A and M

Then

$$\mathbf{B} = \begin{pmatrix} \frac{h}{2} \\ \frac{k+3}{2} \end{pmatrix}$$

Consider

$$\mathbf{x} = \begin{pmatrix} \frac{h}{2} \\ \frac{k+3}{2} \end{pmatrix}$$

Now substitute \mathbf{x} , \mathbf{V} , \mathbf{u} , f in Equation 1

$$\begin{pmatrix} \frac{h}{2} & \frac{k+3}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{h}{2} \\ \frac{k+3}{2} \end{pmatrix} + 2 \begin{pmatrix} 2 & -3 \end{pmatrix} \begin{pmatrix} \frac{h}{2} \\ \frac{k+3}{2} \end{pmatrix} + 9 = 0 \quad (4)$$

By solving

We get

$$\begin{pmatrix} h & k \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} h \\ k \end{pmatrix} + 2 \begin{pmatrix} 4 & -3 \end{pmatrix} \begin{pmatrix} h \\ k \end{pmatrix} + 9 = 0$$

Now write this equation in terms of x and y

Then

$$\begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x & y \end{pmatrix} + 2 \begin{pmatrix} 4 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 9 = 0$$

The locus equation of M is

$$\boxed{x^2 + y^2 + 8x - 6y + 9 = 0} \quad (5)$$