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# Conics Assignment

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## Problem Statement:

Consider a branch of hyperbola $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$ with vertex at the point **A**. Let **B** be one of the end point of its latus rectum. If **C** is the focus of the hyperbola nearest to the point **A**, then the area of triangle **ABC** is?

## Construction:

| Symbol | Value | Description |
|---|---|---|
| $\mathbf{C}$ | $\begin{pmatrix}\sqrt{2}\\-\sqrt{2}\end{pmatrix}$ | Center of hyperbola |

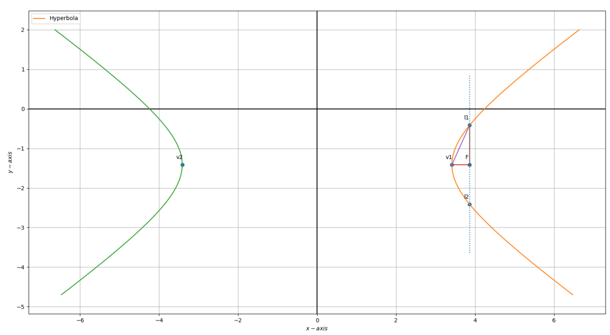

Figure of construction

You can download the python code for generating above hyperbola from the below link

*Githublink : https : //github.com/RupaSaiSreshta/FWC*

## Solution:

The given Hyperbola equation is

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

$$\mathbf{V} = \begin{pmatrix}1 & 0\\0 & -2\end{pmatrix}, \tag{2}$$

$$\mathbf{u} = \begin{pmatrix}-\sqrt{2}\\-2\sqrt{2}\end{pmatrix}, \tag{3}$$

$$f = -6. \tag{4}$$

Eigen values of $\mathbf{V}$ are $\lambda_1$ and $\lambda_2$

Here $\lambda_1$ = -2 , $\lambda_2$ = 1

Now eccentricity of hyperbola is

$$e = \sqrt{1 - \frac{\lambda_1}{\lambda_2}} \tag{5}$$

Center of the hyperbola

$$\mathbf{C} = -\mathbf{V}^{-1}\mathbf{u} \tag{6}$$

By solving we get

$$\mathbf{C} = \begin{pmatrix}\sqrt{2}\\-\sqrt{2}\end{pmatrix} \tag{7}$$

**For vertex:**

Take the major axis equation

$$c = -\sqrt{2} \tag{8}$$

Now, Take the formula for line intersecting to the conic

$$\mathbf{x} = \mathbf{q} + \mu_i \mathbf{m} \tag{9}$$

$$\mathbf{q} = \begin{pmatrix}\sqrt{2}\\-\sqrt{2}\end{pmatrix} \tag{10}$$

$$\mathbf{m} = \begin{pmatrix}1\\0\end{pmatrix} \tag{11}$$

$$\mu_i = \frac{1}{\mathbf{m}^T\mathbf{V}\mathbf{m}}\Big(-\mathbf{m}^T\left(\mathbf{V}\mathbf{q}+\mathbf{u}\right) \pm \sqrt{\left[\mathbf{m}^T\left(\mathbf{V}\mathbf{q}+\mathbf{u}\right)\right]^2 - \left(\mathbf{q}^T\mathbf{V}\mathbf{q}+2\mathbf{u}^T\mathbf{q}+f\right)\left(\mathbf{m}^T\mathbf{V}\mathbf{m}\right)}\Big) \tag{12}$$

By solving we get

$$\mu_i = \pm 2 \tag{13}$$

Now substitute in eq 6 and adding center we get

$$\mathbf{V_1} = \begin{pmatrix} 2+\sqrt{2} \\ -\sqrt{2} \end{pmatrix} \tag{14}$$

$$\mathbf{V_2} = \begin{pmatrix} -2+\sqrt{2} \\ -\sqrt{2} \end{pmatrix} \tag{15}$$

**For Focus:**

$$\mathbf{F} = \pm e\sqrt{\frac{|f_0|}{\lambda_2(1-e^2)}}\mathbf{e_1} \tag{16}$$

By solving and adding center we get

$$\mathbf{F_1} = \begin{pmatrix} \sqrt{6}+\sqrt{2} \\ -\sqrt{2} \end{pmatrix} \tag{17}$$

$$\mathbf{F_2} = \begin{pmatrix} -\sqrt{6}+\sqrt{2} \\ -\sqrt{2} \end{pmatrix} \tag{18}$$

**For Latus rectum:**

Draw the tangent to vertex $\mathbf{V_1}$ and find the direction vector of tangent.

Then draw the line parallel to tangent $\mathbf{V_1}$ through the focus point

Now, Take the formula for line intersecting to the conic

$$\mathbf{x} = \mathbf{q} + \mu_i \mathbf{m} \tag{19}$$

$$\mathbf{q} = \begin{pmatrix} \sqrt{6}+\sqrt{2} \\ -\sqrt{2} \end{pmatrix} \tag{20}$$

$$\mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{21}$$

$$\mu_i = \frac{1}{\mathbf{m}^T\mathbf{V}\mathbf{m}}\Big(-\mathbf{m}^T\left(\mathbf{V}\mathbf{q}+\mathbf{u}\right) \pm \sqrt{\left[\mathbf{m}^T\left(\mathbf{V}\mathbf{q}+\mathbf{u}\right)\right]^2 - \left(\mathbf{q}^T\mathbf{V}\mathbf{q}+2\mathbf{u}^T\mathbf{q}+f\right)\left(\mathbf{m}^T\mathbf{V}\mathbf{m}\right)}\Big) \tag{22}$$

By solving we get

$$\mu_i = \pm 1 \tag{23}$$

Now substitute in eq 15 and adding center

we get

$$\mathbf{L_1} = \begin{pmatrix} \sqrt{6}+\sqrt{2} \\ 1-\sqrt{2} \end{pmatrix} \tag{24}$$

$$\mathbf{L_2} = \begin{pmatrix} \sqrt{6}+\sqrt{2} \\ -1-\sqrt{2} \end{pmatrix} \tag{25}$$

**For Area of triangle:**

For finding area of triangle $\mathbf{V_1L_1F_1}$

$$\mathbf{Ar} = \frac{1}{2}\left\|\left(\left(\mathbf{V_1}-\mathbf{L_1}\right)\times\left(\mathbf{F_1}-\mathbf{L_1}\right)\right)\right\| \tag{26}$$

$$\mathbf{V_1} = \begin{pmatrix} 2+\sqrt{2} \\ -\sqrt{2} \end{pmatrix} \tag{27}$$

$$\mathbf{L_1} = \begin{pmatrix} \sqrt{6}+\sqrt{2} \\ 1-\sqrt{2} \end{pmatrix} \tag{28}$$

$$\mathbf{F_1} = \begin{pmatrix} \sqrt{6}+\sqrt{2} \\ -\sqrt{2} \end{pmatrix} \tag{29}$$

Substitute these values in eq 21

By solving we get

$$\boxed{\mathbf{Ar} = \sqrt{\frac{3}{2}} - 1} \tag{30}$$