Indian Institute of Technology Hyderabad

ELECTRICAL ENGINEERING
National Resource Centre
IIT HYDERABAD

Optimization Assignment

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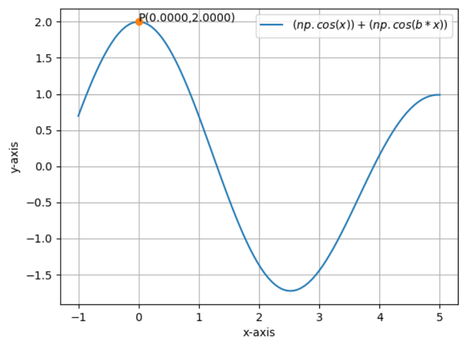
Problem Statement:

The number of values of x, where the function f(x)=cosx+cos$\sqrt{2}$x **attains its maximum is**



Solution

Given function is,

$$f(x) = cosx + cos\sqrt{2}x \tag{1}$$

$$\tag{2}$$

Objective function:

$$f(x) = \max_{x} cosx + cos\sqrt{2}x \tag{3}$$

$$\tag{4}$$

constraints:

$$x \in \mathbb{R} \tag{5}$$

Calculation using normal differentiation

Differentiating (1) yields,

$$\nabla f(x) = -\sqrt{2}sin\sqrt{2}x - sinx \tag{6}$$

Calculation of Maxima using gradient ascent algorithm

Maxima of the above equation (1), can be calculated from the following expression,

$$x_{n+1} = x_n + \alpha \nabla f(x_n) \tag{7}$$

Calculation of Maxima using gradient ascent algorithm

$$f(x) = cosx + cos\sqrt{2}x \tag{8}$$

$$f'(x) = -\sqrt{2}sin\sqrt{2}x - sinx \tag{9}$$

we have to attain the maximum value of x. This can be seen in Figure.Using gradient ascent method we can find its maxima.

$$\implies x_{n+1} = x_n + \alpha(-\sqrt{2}sin\sqrt{2}x - sinx) \tag{10}$$

Taking $x_0 = 0.5, \alpha = 0.001$ and precision $= 0.00000001$, values obtained using python are:

$$\boxed{\text{Maxima} = 1.9999} \tag{11}$$

$$\boxed{\text{Maxima Point} = 0.0000, 2.0000} \tag{12}$$

Theoritical proof

Here, f(x) can never be bigger than 2 as it is the sum of two functions who are always less than or equal to 1.

Then, f(0)=2, hence f(x) has a maximum value of 2.

Next note for f(x)=2 we need

$$x = 2\pi n \tag{13}$$

$$\sqrt{2}x = 2\pi m, \tag{14}$$

for some n,m $\in \mathbb{Z}$ (n,m are integers).

This only has 1 solution at x=0.

To see this say x≠0.

Then n,m≠0.

Now we substitute the 12^{th} equation into the 13^{th} to get

$$\sqrt{2}(2\pi n) = 2\pi m \tag{15}$$

so,

$$\sqrt{2}n = m \tag{16}$$

Since n,m≠0 this would imply $\sqrt{2}$ is rational which is clearly a contradiction.

Hence, after attaining maximum value at x=0 ,i.e. f(0)=2 f(x) will not attain any maximum value.

Conclusion

1. At first, the given function has been differentiated and it is solved by setting f'(x) equal to zero. By using x values, f(x) values are calculated.

2. Later, the given function f(x) is solved by gradient ascent algorithm to find maxima and the point at which f(x) is maximum.

3. Then, the given function f(x) is solved by gradient descent algorithm to find minima and the point at which f(x) is is minimum.

From this we can say that there is 1 maximum point for the function f(x) Download the code

$\boxed{Githublink : https : //github.com/RupaSaiSreshta/FWC}$