

## Circle Assignment

Name: Rupa Sai Sreshta Vallabhaneni

### Problem Statement

From the point  $A(0,3)$  on the circle  $x^2 + 4x + (y - 3)^2 = 0$ . A chord AB is drawn and extended to a point M. Such that  $AM=2AB$ . Find the equation of locus of M.

### Construction

Symbol	Value	Description
A	$\begin{pmatrix} 0 \\ 3 \end{pmatrix}$	Point on given circle
M	$\begin{pmatrix} h \\ k \end{pmatrix}$	Point outside circle
B	$\begin{pmatrix} \frac{h}{2} \\ \frac{k+3}{2} \end{pmatrix}$	Mid point of A and M
$r_1$	2	Radius of given circle
C	$\begin{pmatrix} -2 \\ 3 \end{pmatrix}$	Center of given circle

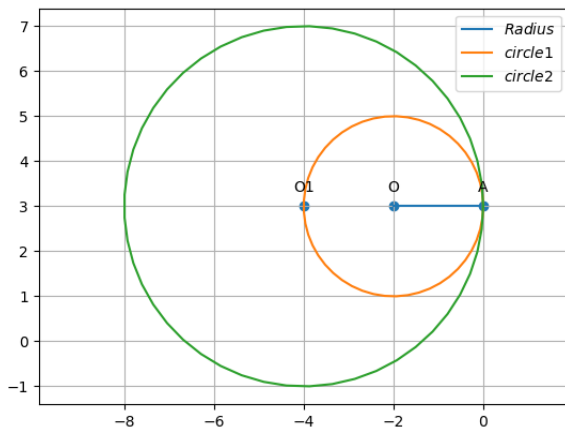


Figure of construction

You can download the python code for generating above circle from the below link

Githublink : <https://github.com/RupaSaiSreshta/FWC>

### Solution

Given  $A = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

$AM=2AB$

From this condition B is the midpoint of A and M.

The given circle equation is

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (1)$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$\mathbf{u} = \begin{pmatrix} 2 \\ -3 \end{pmatrix},$$

$f = 9$ .

Center of the circle

$$\mathbf{C} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \quad (2)$$

Radius of the circle

$$r_1 = 2 \quad (3)$$

Let us assume

$$\mathbf{M} = \begin{pmatrix} h \\ k \end{pmatrix}$$

As B is the midpoint of A and M

Then

$$\mathbf{B} = \begin{pmatrix} \frac{h}{2} \\ \frac{k+3}{2} \end{pmatrix}$$

Consider

$$\mathbf{x} = \begin{pmatrix} \frac{h}{2} \\ \frac{k+3}{2} \end{pmatrix}$$

Now substitute  $\mathbf{x}$ ,  $\mathbf{V}$ ,  $\mathbf{u}$ ,  $f$  in Equation 1

$$\begin{pmatrix} \frac{h}{2} & \frac{k+3}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{h}{2} \\ \frac{k+3}{2} \end{pmatrix} + 2 \begin{pmatrix} 2 & -3 \end{pmatrix} \begin{pmatrix} \frac{h}{2} \\ \frac{k+3}{2} \end{pmatrix} + 9 = 0 \quad (4)$$

By solving

We get

$$\begin{pmatrix} h & k \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} h \\ k \end{pmatrix} + 2 \begin{pmatrix} 4 & -3 \end{pmatrix} \begin{pmatrix} h \\ k \end{pmatrix} + 9 = 0$$

Now write this equation in terms of x and y

Then

$$\begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x & y \end{pmatrix} + 2 \begin{pmatrix} 4 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 9 = 0$$

The locus equation of M is

$$\boxed{x^2 + y^2 + 8x - 6y + 9 = 0} \quad (5)$$