# Turbulent Astroplasma Replicator Accessories (*TARA*)

# A package for simulating astrophysical plasmas

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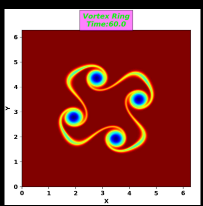
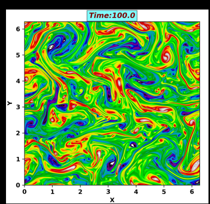

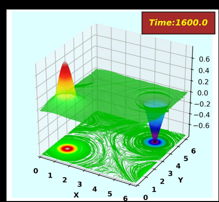

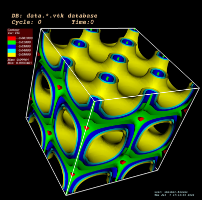
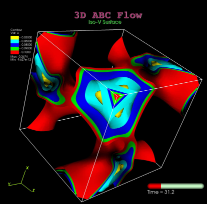


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**About This File**

This user manual about TARA package is prepared and maintained by *Mr. Shishir Biswas* [Institute for Plasma Research, India].

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SECTION 1

# Introduction to TARA

The *TARA* simulation architecture is a multi-dimensional pseudo-spectral solver for weakly compressible and incompressible magneto-hydrodynamic flows. *TARA* is flexible for adding higher order fluid-moment equations with minimal computational overhead. This framework runs efficiently on multiple-GPU architecture. In addition, the performance scales efficiently under MPI on massively parallel shared or distributed-memory computers.

The *TARA* simulation framework have been used for many different applications in astrophysical studies as well as terrestrial laboratory plasma simulations.

## 1.1 Downloading & Compiling *TARA* on your own system

First make a clone of the master branch using the following command:

```
gitclonehttps://github.com/RupakMukherjee/TARA.git
```

Then enter inside the TARA directory:

```
cd TARA
```

Now complile and build the TARA code:

```
make subsystems
make all
```

Upon successful compilation, run the code using following command:

```
make run
```

## 1.2 Prerequisites

### 1.2.1 Prerequisites for running on a single CPU core [Serial]:

1. GNU Compiler (Version 4.0.0 or above) [https://gcc.gnu.org/]
2. FFTW library (Version 3.3.3 or above) [http://www.fftw.org/]
3. git [https://git-scm.com/]

### 1.2.2 Prerequisites for running on multiple CPU cores [OpenMP & MPI]:

1. GNU Compiler (Version 4.0.0 or above) [https://gcc.gnu.org/]
2. OpenMP and MPI architechture
3. FFTW library (Version 3.3.3 or above) [http://www.fftw.org/]
4. git [https://git-scm.com/]

### 1.2.3 Prerequisites for running on a single GPU card [Single-GPU]:

1. CUDA Toolkit (Version 9.1 or above) [https://docs.nvidia.com/cuda/cuda-compiler-driver-nvcc/index.html]
2. PGI Compilers & Tools (Version 18.1 or above) [https://www.pgroup.com/support/new_rel_80.htm#new]
3. cuFFT library [https://developer.nvidia.com/cufft]
4. git [https://git-scm.com/]

### 1.2.4 Prerequisites for running on multiple CPU cores [OpenMP & MPI]:

1. GNU Compiler (Version 4.0.0 or above) [https://gcc.gnu.org/]
2. OpenMP and MPI architechture
3. FFTW library (Version 3.3.3 or above) [http://www.fftw.org/]
4. git [https://git-scm.com/]

### 1.2.5 Prerequisites for running on multiple-GPU cards [multi-GPU]:

1. CUDA Toolkit (Version 9.1 or above) [https://docs.nvidia.com/cuda/cuda-compiler-driver-nvcc/index.html]
2. PGI Compilers & Tools (Version 18.1 or above) [https://www.pgroup.com/support/new_rel_80.htm#new]

3. AccFFT library [http://accfft.org/about/]
4. PnetCDF library (Version 1.11.0) [https://parallel-netcdf.github.io/]
5. OpenMPI (Version 1.10.7 or above) [https://www.open-mpi.org/]
6. git [https://git-scm.com/]

## 1.3 Visualization Toolkits

TARA Visualization toolkit supports graphical representation in:

1. ParaView
2. VisIt
3. Mayavi
4. Python
5. Gnuplot

SECTION

# 2 Governing Equations

The basic equations that are evolved in the code with different specific initial conditions are as follows:

$$\text{Mass density:} \quad \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0 \tag{1}$$

$$\text{Momentum equation:} \quad \frac{\partial (\rho \vec{u})}{\partial t} + \vec{\nabla} \cdot [\rho \vec{u} \otimes \vec{u}] = \frac{\vec{J} \times \vec{B}}{c} - \vec{\nabla} P + \vec{\nabla} \cdot (2\nu\rho \bar{\bar{S}}) + \rho \vec{F} \tag{2}$$

$$\text{Shear viscosity:} \quad S_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i) - \frac{1}{3}\delta_{ij}\theta \tag{3}$$

$$\text{Dilation:} \quad \theta = \vec{\nabla} \cdot \vec{u} \tag{4}$$

$$\text{Equation of state:} \quad P = \gamma \rho K T = C_s^2 \rho \tag{5}$$

$$\text{Non relativistic Ampere's law:} \quad \vec{J} = \frac{c}{4\pi} \vec{\nabla} \times \vec{B} \tag{6}$$

$$\text{Faraday's law:} \quad \frac{\partial B}{\partial t} = -c \vec{\nabla} \times \vec{E} \tag{7}$$

$$\text{Ohm's law:} \quad \vec{E} + \frac{\vec{u} \times \vec{B}}{c} = \frac{1}{\sigma} \vec{J} \tag{8}$$

$$\text{No magnetic monopole:} \quad \vec{\nabla} \cdot \vec{B} = 0 \tag{9}$$

Putting 6 into 2 we get,

$$\frac{\partial (\rho \vec{u})}{\partial t} + \vec{\nabla} \cdot \left[ \rho \vec{u} \otimes \vec{u} + \left( P + \frac{1}{8\pi} |\vec{B}|^2 \right) \mathbf{I} - \frac{1}{4\pi} \vec{B} \otimes \vec{B} \right] = \vec{\nabla} \cdot (2\nu\rho \bar{\bar{S}}) + \rho \vec{F}$$

Putting 8 into 7 and using 6 & 9 we get,

$$\frac{\partial \vec{B}}{\partial t} = -c\vec{\nabla} \times \left[-\frac{\vec{u}\times\vec{B}}{c} + \frac{1}{\sigma}\vec{J}\right] = \vec{\nabla}\times\vec{u}\times\vec{B} - \frac{c}{4\pi\sigma}\vec{\nabla}\times\vec{\nabla}\times\vec{B} = \vec{\nabla}\times\vec{u}\times\vec{B} + \frac{c}{4\pi\sigma}\nabla^2\vec{B}$$

$$\Rightarrow \quad \frac{\partial \vec{B}}{\partial t} + \vec{\nabla}\cdot\left(\vec{u}\otimes\vec{B} - \vec{B}\otimes\vec{u}\right) = \frac{c}{4\pi\sigma}\nabla^2\vec{B}$$

$$\Rightarrow \quad \frac{\partial \vec{B}}{\partial t} + \vec{\nabla}\cdot\left(\vec{u}\otimes\vec{B} - \vec{B}\otimes\vec{u}\right) = \eta\nabla^2\vec{B}, \ \text{where, } \eta = \frac{c}{4\pi\sigma}$$

The internal energy is evaluated by the time evolution of the following equation:

$$\frac{\partial E}{\partial t} + \vec{\nabla}\cdot\left[(E+P_{tot})\,\vec{u} - \frac{1}{4\pi}\vec{u}\cdot\left(\vec{B}\otimes\vec{B}\right) - 2\nu\rho\vec{u}\cdot\bar{\bar{S}} - \frac{\eta}{4\pi}\vec{B}\times\left(\vec{\nabla}\times\vec{B}\right)\right] = 0$$

where $E = \rho\epsilon_{int} + \frac{1}{2}\rho|\vec{u}|^2 + \frac{1}{8\pi}|\vec{B}|^2$ and $P_{tot} = P + \frac{1}{8\pi}|\vec{B}|^2$.

Thus the complete set of MHD equtations are

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\left(\rho\vec{u}\right) = 0 \tag{10}$$

$$\frac{\partial(\rho\vec{u})}{\partial t} + \vec{\nabla}\cdot\left[\rho\vec{u}\otimes\vec{u} + \left(P + \frac{1}{8\pi}|\vec{B}|^2\right)\mathbf{I} - \frac{1}{4\pi}\vec{B}\otimes\vec{B}\right] = \vec{\nabla}\cdot(2\nu\rho\bar{\bar{S}}) + \rho\vec{F} \tag{11}$$

$$\frac{\partial E}{\partial t} + \vec{\nabla}\cdot\left[(E+P_{tot})\,\vec{u} - \frac{1}{4\pi}\vec{u}\cdot\left(\vec{B}\otimes\vec{B}\right) - 2\nu\rho\vec{u}\cdot\bar{\bar{S}} - \frac{\eta}{4\pi}\vec{B}\times\left(\vec{\nabla}\times\vec{B}\right)\right] = 0 \tag{12}$$

$$\frac{\partial \vec{B}}{\partial t} + \vec{\nabla}\cdot\left(\vec{u}\otimes\vec{B} - \vec{B}\otimes\vec{u}\right) = \eta\nabla^2\vec{B} \tag{13}$$

Hence the complete set of equations in component form for a three dimensional fluid becomes,

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u_x) + \frac{\partial}{\partial y}(\rho u_y) + \frac{\partial}{\partial z}(\rho u_z) = 0$$

$$\begin{aligned}
&\frac{\partial(\rho u_x)}{\partial t} + \frac{\partial}{\partial x}\left[\rho u_x u_x + P_{tot} - \frac{1}{4\pi}B_x B_x - 2\nu\rho S_{xx}\right] \\
&\quad + \frac{\partial}{\partial y}\left[\rho u_x u_y - \frac{1}{4\pi}B_x B_y - 2\nu\rho S_{xy}\right] + \frac{\partial}{\partial z}\left[\rho u_x u_z - \frac{1}{4\pi}B_x B_z - 2\nu\rho S_{xz}\right] = \rho F_x \\
&\frac{\partial(\rho u_y)}{\partial t} + \frac{\partial}{\partial x}\left[\rho u_y u_x - \frac{1}{4\pi}B_y B_x - 2\nu\rho S_{yx}\right] \\
&\quad + \frac{\partial}{\partial y}\left[\rho u_y u_y + P_{tot} - \frac{1}{4\pi}B_y B_y - 2\nu\rho S_{yy}\right] + \frac{\partial}{\partial z}\left[\rho u_y u_z - \frac{1}{4\pi}B_y B_z - 2\nu\rho S_{yz}\right] = \rho F_y \\
&\frac{\partial(\rho u_z)}{\partial t} + \frac{\partial}{\partial x}\left[\rho u_z u_x - \frac{1}{4\pi}B_z B_x - 2\nu\rho S_{zx}\right] \\
&\quad + \frac{\partial}{\partial y}\left[\rho u_z u_y - \frac{1}{4\pi}B_z B_y - 2\nu\rho S_{zy}\right] + \frac{\partial}{\partial z}\left[\rho u_z u_z + P_{tot} - \frac{1}{4\pi}B_z B_z - 2\nu\rho S_{zz}\right] = \rho F_z
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial E}{\partial t} + \frac{\partial}{\partial x}[(E+P_{tot})\,u_x - \frac{1}{4\pi}\left(u_x B_x B_x + u_y B_x B_y + u_z B_x B_z\right) - 2\nu\rho\left(u_x S_{xx} + u_y S_{xy} + u_z S_{xz}\right) \\
&\qquad - \frac{\eta}{4\pi}\left\{B_y\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right) + B_z\left(\frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial B_z}\right)\right\}] \\
&\quad + \frac{\partial}{\partial y}[(E+P_{tot})\,u_y - \frac{1}{4\pi}\left(u_x B_y B_x + u_y B_y B_y + u_z B_y B_z\right) - 2\nu\rho\left(u_x S_{yx} + u_y S_{yy} + u_z S_{yz}\right) \\
&\qquad + \frac{\eta}{4\pi}\left\{B_x\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right) - B_z\left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial B_z}\right)\right\}] \\
&\quad + \frac{\partial}{\partial z}[(E+P_{tot})\,u_z - \frac{1}{4\pi}\left(u_x B_z B_x + u_y B_z B_y + u_z B_z B_z\right) - 2\nu\rho\left(u_x S_{zx} + u_y S_{zy} + u_z S_{zz}\right) \\
&\qquad + \frac{\eta}{4\pi}\left\{B_x\left(\frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z}\right) + B_y\left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial B_z}\right)\right\}] = 0
\end{aligned}$$

$$\frac{\partial B_x}{\partial t} - \left[\frac{\partial}{\partial y}\left(u_x B_y - u_y B_x\right) + \frac{\partial}{\partial z}\left(u_x B_z - u_z B_x\right)\right] = \eta\left(\frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} + \frac{\partial^2 B_x}{\partial z^2}\right)$$
$$\frac{\partial B_y}{\partial t} + \left[\frac{\partial}{\partial x}\left(u_x B_y - u_y B_x\right) - \frac{\partial}{\partial z}\left(u_y B_z - u_z B_y\right)\right] = \eta\left(\frac{\partial^2 B_y}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2} + \frac{\partial^2 B_y}{\partial z^2}\right)$$
$$\frac{\partial B_z}{\partial t} + \left[\frac{\partial}{\partial x}\left(u_x B_z - u_z B_x\right) + \frac{\partial}{\partial y}\left(u_y B_z - u_z B_y\right)\right] = \eta\left(\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial y^2} + \frac{\partial^2 B_z}{\partial z^2}\right)$$

$$P_{tot} = P_{th} + \frac{1}{8\pi}(B_x^2 + B_y^2 + B_z^2);\ \ P_{th} = C_s^2\rho$$
$$S_{xx} = \frac{du_x}{dx} - \frac{1}{3}\left(\frac{du_x}{dx} + \frac{du_y}{dy} + \frac{du_z}{dz}\right);\ S_{xy} = \frac{1}{2}\left(\frac{du_y}{dx} + \frac{du_x}{dy}\right);\ S_{xz} = \frac{1}{2}\left(\frac{du_z}{dx} + \frac{du_x}{dz}\right)$$
$$S_{yx} = \frac{1}{2}\left(\frac{du_x}{dy} + \frac{du_y}{dx}\right);\ S_{yy} = \frac{du_y}{dy} - \frac{1}{3}\left(\frac{du_x}{dx} + \frac{du_y}{dy} + \frac{du_z}{dz}\right);\ S_{yz} = \frac{1}{2}\left(\frac{du_z}{dy} + \frac{du_y}{dz}\right)$$
$$S_{zx} = \frac{1}{2}\left(\frac{du_x}{dz} + \frac{du_z}{dx}\right);\ S_{zy} = \frac{1}{2}\left(\frac{du_y}{dz} + \frac{du_z}{dy}\right);\ S_{zz} = \frac{du_z}{dz} - \frac{1}{3}\left(\frac{du_x}{dx} + \frac{du_y}{dy} + \frac{du_z}{dz}\right)$$

The forcing determinates a spatial averaged velocity of the system ($<|\vec{U}(x,y,z,t)|>_{x,y,z}$) in the presence of dissipation ($\nu \neq 0$). We determine a reference scale of pressure ($P_{ref}$) and density ($\rho_{ref}$) to determine the reference sound speed ($C_{s_{ref}}$). This provides a sonic Mach Number at steady state $\left(M_s = \frac{<|\vec{U}(x,y,z,t)|>_{x,y,z}}{C_{s_{ref}}}\right)$. We also evaluate a dynamic sound speed, $C_s(t) = \sqrt{\frac{\gamma<P(x,y,z,t)>_{x,y,z}}{<\rho(x,y,z,t)>_{x,y,z}}}$. The Alfven speed is determined by $V_A(t) = \frac{<|\vec{B}(x,y,z,t)|>_{x,y,z}}{4\pi\sqrt{<\rho(x,y,z,t)>_{x,y,z}}}$. Thus we evaluate the dynamic Alfven Mach number as $M_A(t) = \frac{<|\vec{U}(x,y,z,t)|>_{x,y,z}}{V_A(t)}$.

## 2.1 Normalisation of the MHD equations

Below we provide the normalisation of MHD equations for a special case. We consider the equation of state as $P = C_s^2\rho$. Here we deal with the equations of unforced turbulence i.e. $F = 0$. For simplicity, we also assume, the off-diagonal components of the viscosity term to be zero. For the sake of convenience, we consider the Lorentz force term is absent in the momentum equation. The reduced set of equations are

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\left(\rho\vec{u}\right) = 0 \tag{14}$$
$$\frac{\partial\vec{u}}{\partial t} + \left(\vec{u}\cdot\vec{\nabla}\right)\vec{u} = -C_s^2\frac{\vec{\nabla}\rho}{\rho} + \frac{\nu}{\rho}\nabla^2\vec{u} \tag{15}$$
$$\frac{\partial\vec{B}}{\partial t} + \vec{\nabla}\times\vec{u}\times\vec{B} = \eta\nabla^2\vec{B} \tag{16}$$

Define $\rho = \rho_0\tilde{\rho}$, $t = t_0\tilde{t}$ and $\vec{r} = L\tilde{\vec{r}}$ where, $L$ is the characteristic length-scale and $t_0$ is a characteristic timescale.

Eq. 14 can be written as

$$\frac{\rho_0}{t_0}\frac{\partial\tilde{\rho}}{\partial\tilde{t}} + \frac{\rho_0 u_0}{L}\tilde{\vec{\nabla}}\cdot\left(\tilde{\rho}\tilde{\vec{u}}\right) = 0$$

Defining, $u_0 = \frac{L}{t_0}$, we get,

$$\frac{\partial\tilde{\rho}}{\partial\tilde{t}} + \tilde{\vec{\nabla}}\cdot\left(\tilde{\rho}\tilde{\vec{u}}\right) = 0$$

Eq. 15 can be written as,

$$
\begin{aligned}
& \frac{L}{t_0^2}\frac{\partial\vec{\tilde{u}}}{\partial\tilde{t}}+\frac{L}{t_0^2}\left(\vec{\tilde{u}}\cdot\vec{\tilde{\nabla}}\right)\vec{\tilde{u}}=-\frac{C_s^2}{L}\frac{\vec{\tilde{\nabla}}\tilde{\rho}}{\tilde{\rho}}+\frac{\nu}{\rho_0 L t_0}\frac{\tilde{\nabla}^2\vec{\tilde{u}}}{\tilde{\rho}} \\
\Rightarrow \quad & \frac{\partial\vec{\tilde{u}}}{\partial\tilde{t}}+\left(\vec{\tilde{u}}\cdot\vec{\tilde{\nabla}}\right)\vec{\tilde{u}}=-\frac{C_s^2}{u_0^2}\frac{\vec{\tilde{\nabla}}\tilde{\rho}}{\tilde{\rho}}+\frac{\nu}{\rho_0 L u_0}\frac{\tilde{\nabla}^2\vec{\tilde{u}}}{\tilde{\rho}} \\
\Rightarrow \quad & \frac{\partial\vec{\tilde{u}}}{\partial\tilde{t}}+\left(\vec{\tilde{u}}\cdot\vec{\tilde{\nabla}}\right)\vec{\tilde{u}}=-\frac{1}{M_s^2}\frac{\vec{\tilde{\nabla}}\tilde{\rho}}{\tilde{\rho}}+\frac{1}{Re}\frac{\tilde{\nabla}^2\vec{\tilde{u}}}{\tilde{\rho}}
\end{aligned}
$$

where, $M_s=\frac{u_0}{C_s}$ and $Re=\frac{\rho_0 u_0 L}{\nu}$
Eq. 16 can be written as

$$
\begin{aligned}
& \frac{B_0}{t_0}\frac{\partial\vec{\tilde{B}}}{\partial\tilde{t}}+\frac{B_0}{t_0}\vec{\tilde{\nabla}}\times\vec{\tilde{u}}\times\vec{\tilde{B}}=\frac{\eta B_0}{L^2}\tilde{\nabla}^2\vec{\tilde{B}} \\
\Rightarrow \quad & \frac{\partial\vec{\tilde{B}}}{\partial\tilde{t}}+\vec{\tilde{\nabla}}\times\vec{\tilde{u}}\times\vec{\tilde{B}}=\frac{\eta t_0}{L^2}\tilde{\nabla}^2\vec{\tilde{B}} \\
\Rightarrow \quad & \frac{\partial\vec{\tilde{B}}}{\partial\tilde{t}}+\vec{\tilde{\nabla}}\times\vec{\tilde{u}}\times\vec{\tilde{B}}=\frac{\eta}{L u_0}\tilde{\nabla}^2\vec{\tilde{B}} \\
\Rightarrow \quad & \frac{\partial\vec{\tilde{B}}}{\partial\tilde{t}}+\vec{\tilde{\nabla}}\times\vec{\tilde{u}}\times\vec{\tilde{B}}=\frac{1}{R_m}\tilde{\nabla}^2\vec{\tilde{B}}
\end{aligned}
$$

where, $R_m=\frac{L u_0}{\eta}$.
Removing the "~" sign from the variables, we write the equations as:

$$
\begin{aligned}
& \frac{\partial\rho}{\partial t}+\vec{\nabla}\cdot\left(\rho\vec{u}\right)=0 \\
& \frac{\partial\vec{u}}{\partial t}+\left(\vec{u}\cdot\vec{\nabla}\right)\vec{u}=-\frac{1}{M_s^2}\frac{\vec{\nabla}\rho}{\rho}+\frac{1}{Re}\frac{\nabla^2\vec{u}}{\rho} \\
& \frac{\partial\vec{B}}{\partial t}+\vec{\nabla}\times\vec{u}\times\vec{B}=\frac{1}{R_m}\nabla^2\vec{B}
\end{aligned}
$$

This normalisation is followed throughout the code.

SECTION

# 3 Physics Models

The following provides you with the preamble/formatting for lessons and exams as well as how to organize your code and document.

## 3.1 Incompressible Hydrodynamics model

In two dimensions, the Navier-Stokes equation in vorticity formalism become,

$$\frac{\partial \omega}{\partial t} = [\psi, \omega] + \frac{1}{R_n}\nabla^2\omega \tag{17}$$

$$\omega = -\nabla^2\psi \tag{18}$$

$$\tag{19}$$

**Definition**

where $\psi(x, y, t)$ is the stream function, which relates the two dimensional velocity field by, $u_x = \partial_y\psi$, $u_y = -\partial_x\psi$ and $\omega(x, y, t) = \vec{\omega}\cdot\hat{z} = \partial_x u_y - \partial_y u_x$ is the scalar vorticity field.

The Poisson bracket of Eq. 17 is defined as,

$$[\psi, \omega] = \partial_x\psi\partial_y\omega - \partial_y\psi\partial_x\omega \tag{20}$$

## 3.2 Compressible Hydrodynamics model

The Navier-Stokes equation in velocity formalism with compressibility effects become,

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla})\vec{u} = \frac{1}{R_n}\nabla^2 \vec{u} - \frac{1}{M_s^2}\frac{\vec{\nabla} P}{\rho} \tag{21}$$

**Definition**

where $\rho$ and $\vec{u}$ are the dimensionless density and velocity of the fluid element respectively. $M_s$ is the dimensionless sonic Mach number defined as $\frac{u_0}{C_s}$, where $C_s$ is the sound speed, which is regarded as independent of position and time. We also define dimensionless number, $R_n = \frac{u_0 L_0}{\nu}$ as fluid Reynolds number, where $\nu$ is shear viscosity.

## 3.3 Compressible MagnetoHydrodynamics model

The equations that we time evolve for compressible MagnetoHydroDynamic flows are as follows.

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \left(\rho \vec{u}\right) = 0 \tag{22}$$

$$\frac{\partial (\rho \vec{u})}{\partial t} + \vec{\nabla} \cdot \left[\rho \vec{u} \otimes \vec{u} + \left(P + \frac{B^2}{2}\right)\mathbf{I} - \vec{B} \otimes \vec{B}\right] = \mu \nabla^2 \vec{u} + \rho \vec{f} \tag{23}$$

$$P = C_s^2 \rho \tag{24}$$

$$\frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \cdot \left(\vec{u} \otimes \vec{B} - \vec{B} \otimes \vec{u}\right) = \eta \nabla^2 \vec{B} \tag{25}$$

**Definition**

where, $\rho$, $\vec{u}$, $P$ and $\vec{B}$ represent the density, velocity, kinetic pressure and magnetic fields respectively. $\mu$ and $\eta$ are the kinematic viscosity and magnetic diffusivity respectively. The normalised parameters are defined as, $R_e = \frac{u_0 L}{\mu}$, $R_m = \frac{u_0 L}{\eta}$, where $R_e$ and $R_m$ are kinetic Reynolds number and magnetic Reynolds number, $u_0$ is the maximum fluid speed. Magnetic Prandtl number is defined as $P_M = \frac{\mu}{\eta}$. We define Alfven speed as, $V_A = \frac{u_0}{M_A}$, here $M_A$ is the Alfven Mach number of the plasma. Sound speed of the fluid is defined as $C_s = \frac{u_0}{M_s}$ where $M_s$ is the sonic mach no of the fluid flow. The initial magnetic field present in the plasma is calculated from relation $B_0 = V_A \sqrt{\rho_0}$, where $\rho_0$ is the initial density of the flow. And time is normalized as $t = t_0 \times t'$, $t_0 = \frac{L}{V_A}$.

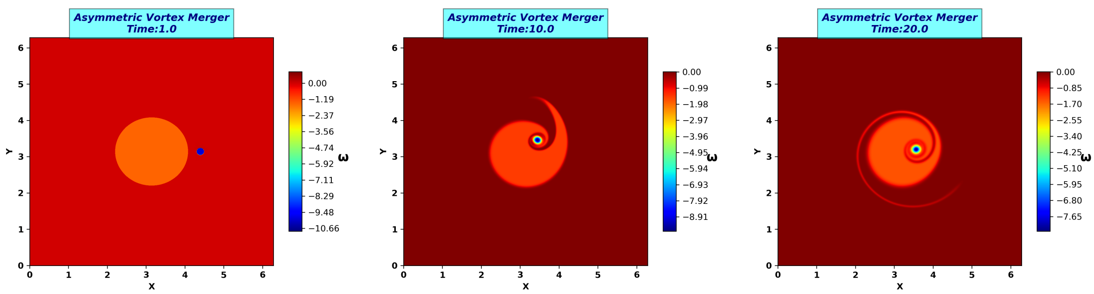
SECTION

4

# Visualization Gallary

## 4.1 2-dimensional cases:

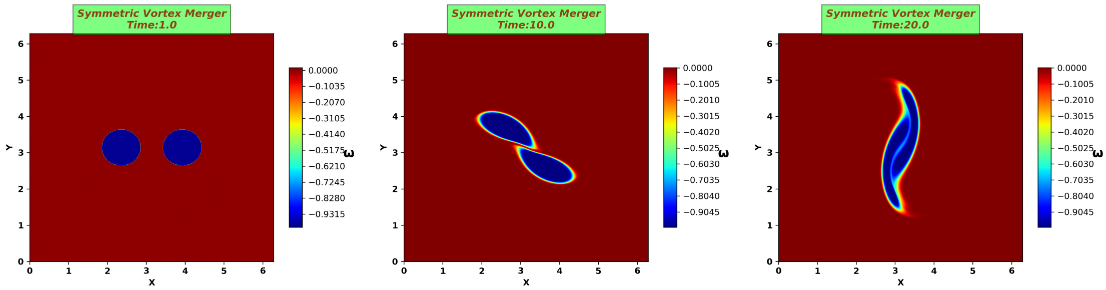
### 4.1.1 Asymmetric Vortex Merger:




(a) Time: 1.0 (b) Time: 10.0 (c) Time: 20.0

Figure 3: The visualization shows the dynamics of an Asymmetric Vortex Merger. [****Visualization Credit:** *Mr. Shishir Biswas*]

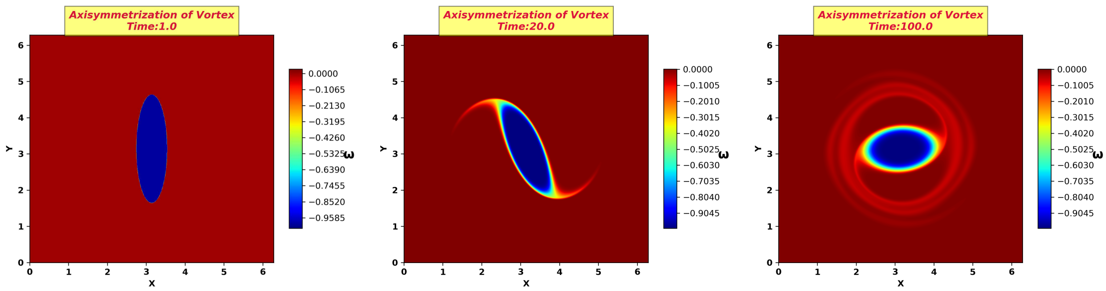
### 4.1.2 Symmetric Vortex Merger:




(a) Time: 1.0 (b) Time: 10.0 (c) Time: 20.0

Figure 4: The visualization shows the dynamics of a Symmetric Vortex Merger. [****Visualization Credit:** *Mr. Shishir Biswas*]

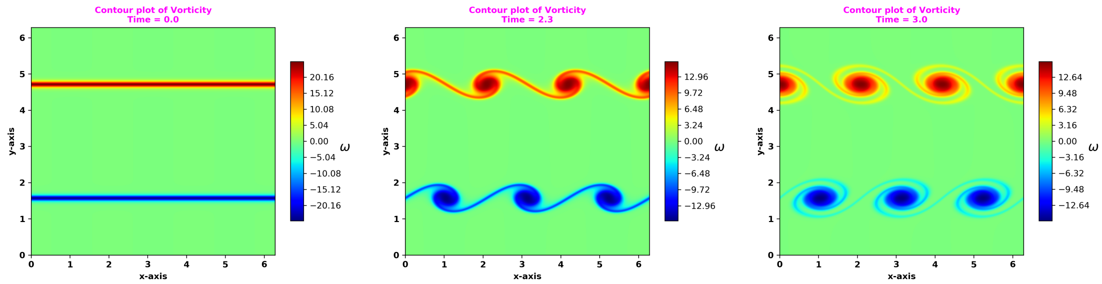
### 4.1.3 Axisymmetrization:




(a) Time: 1.0 (b) Time: 20.0 (c) Time: 100.0

Figure 5: The visualization shows Axisymmetrization of an epileptic Vortex. [****Visualization Credit:** *Mr. Shishir Biswas*]

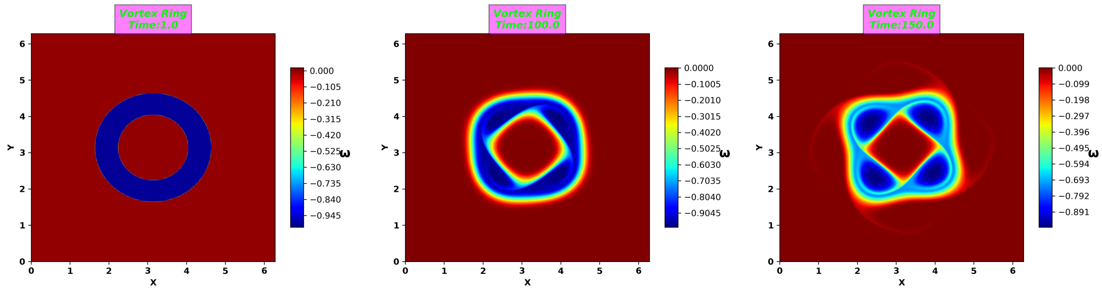
### 4.1.4 Kelvin-Helmholtz (KH) instability:




(a) Time: 0.0 (b) Time: 2.3 (c) Time: 3.0

Figure 6: The visualization shows the dynamics of two Kelvin-Helmholtz (KH) unstable vortex strips. [**Visualization Credit:** *Mr. Shishir Biswas*]

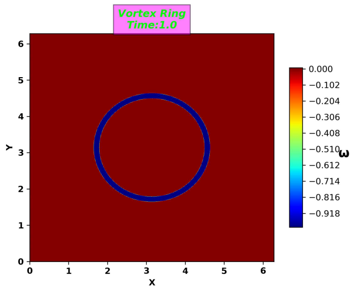
### 4.1.5 Kelvin-Helmholtz (KH) unstable thick ring:



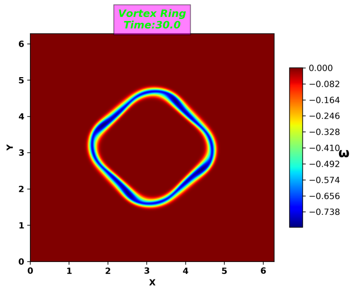

(a) Time: 0.0 (b) Time: 100.0 (c) Time: 150.0

Figure 7: The visualization shows the dynamics of a Kelvin-Helmholtz (KH) unstable thick vortex ring. [**Visualization Credit:** *Mr. Shishir Biswas*]

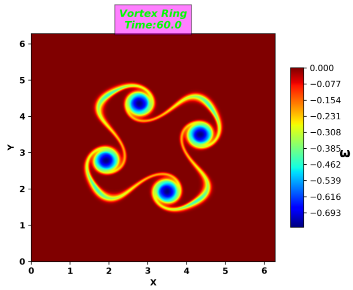
#### 4.1.6 Kelvin-Helmholtz (KH) unstable thin ring:




(a) Time: 1.0




(b) Time: 30.0

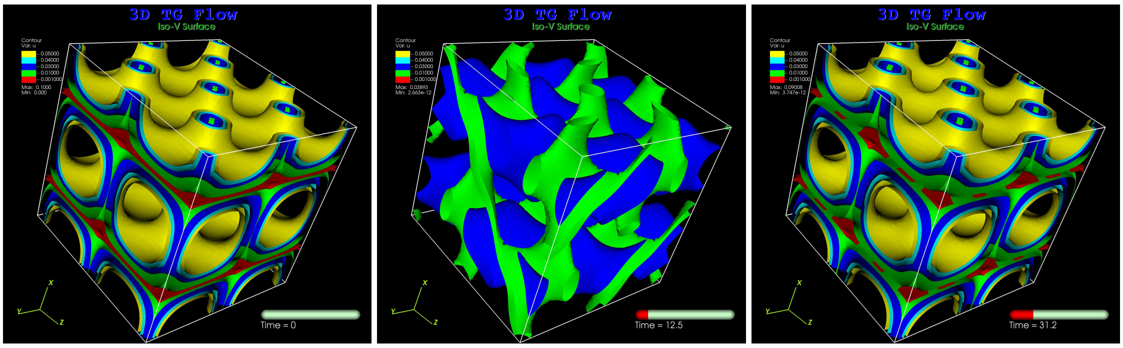



(c) Time: 60.0

Figure 8: The visualization shows the dynamics of a Kelvin-Helmholtz (KH) unstable thin vortex ring. [****Visualization Credit:** *Mr. Shishir Biswas*]

## 4.2 3-dimensional cases:

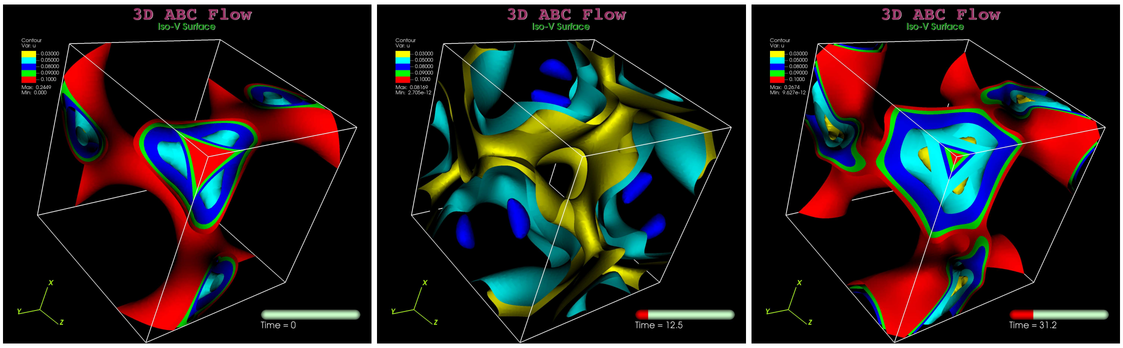
### 4.2.1 Recurring Taylor-Green [TG] Flow:




(a) Time: 0.0 (b) Time: 12.5 (c) Time: 31.2

Figure 9: The visualization shows the dynamics of velocity iso-surface of recurring Taylor-Green [TG] flow. [****Visualization Credit:** *Mr. Shishir Biswas*]

### 4.2.2 Non-Recurring Arnold–Beltrami–Childress [ABC] Flow:




(a) Time: 0.0 (b) Time: 12.5 (c) Time: 31.2

Figure 10: The visualization shows the dynamics of velocity iso-surface of non-recurring Arnold–Beltrami–Childress [ABC] Flow. [****Visualization Credit:** *Mr. Shishir Biswas*]

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