Beam optical design of achromatic bends for charged particles in particle accelerators

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Abstract

The HEBT (High-Energy Beam Transport) section of the High-Current Injector(HCI) at IUAC transports a heavy ion beam from drift tube linac (DTL) to the superconducting linear accelerator (SC-LINAC). The transport path was designed considering outer space restrictions of the beam-hall and has four 90 degree bends. The beam coming from the DTL has a maximum energy of 1.8 MeV/u and are expected to have some energy spread. So, it is necessary to make the bends achromatic in order to prevent beam loss and emittance growth. This project attempts to simulate the beam in the first achromat, which has a configuration of Q1Q2Q3MQ4MQ3Q2Q1 where M represents a 45-degree bending magnet and 'Q1', 'Q2', 'Q3', 'Q4' represent the four types of quadrupole magnets used in the achromat. The project also attempts to simulate the beam in the actual achromat used in the HEBT, which is not exactly ideal, and to compare it with the data obtained through experiments in the HCI. The simulations have been done using a computer program called 'TRACK'.

1 INTRODUCTION

1.1 High Current Injector

High Current Injector (HCI) is an ongoing project at Inter-University Accelerator Center, New Delhi. Its basic aim is to provide heavy ion beams of 1.8 MeV/u energy with beam currents of hundreds of nano amperes to Superconducting LINAC (SC-LINAC). The HCI consists of a dc accelerating section, radiofrequency quadrupole and drift tube linacs to energies the ion beam. The beam transport section connects these accelerating section. The low, medium, and high energy beam transport section (LEBT, MEBT and HEBT) are responsible for transporting the beam up to SC-LINAC[4].

The present project focus on achromatic bend design of the HEBT section. The HEBT section is very long around 50 meters. It consists of four achromatic bends of 90 degrees. Out of these four, three bends are identical and one is different. It is due to the geometrical constraints of the building and existing beamlines.

1.2 Achromatic bends

Achromatic bends are bending systems in which all the particles reach certain transverse positions(x and y values) irrespective of their initial position(in transverse space) and initial momenta(in transverse space). As a result, the final beam coming out of the achromat is independent of the position and momentum spread in the initial wave(the spreads refer to only x and y directions). Achromatic bends prevent transverse emittance growth and beam loss(which happens when the acceptance is less than emittance). As shown in equation 2, in a certain bending magnet, the radius of curvature of a particle is directly proportional to the square root of it's energy. As a result, the path-length traversed by a higher energy particle in a bending magnet is higher than lower energy particles because the bending angle is the same for both. Due to the difference of curvatures in particles with different energies, the particles lose parallelism once they are out of the bending magnet. This will cause an increase in transverse emittance growth. If the emittance increases beyond the acceptance, there will be beam loss too. So, in order to prevent that, an achromatic bend is made in two parts. The entire lattice has to be mirror-symmetric about the centre so that the effect produced by one side is canceled by the other and beam passes without any beam loss or emittance growth. Ideally, the entire lattice must be mirror-symmetric in terms of the arrangement of the accelerator elements as well as the characteristics of the elements(length, aperture, field strength, etc). So, a 90 degree achromatic bend is made in two parts, each consisting of a 45 degree bending magnet.

1.3 Beam Characteristics

A 'beam' is a collective group of particles that have close positions and close velocities(in both magnitude and direction).In accelerator physics, a beam usually refers to a charged particle beam. It is of two types: 'DC beam' is one in which a continuous stream of particles are being emitted. A beam which is emitted in lumps of particles after certain intervals, is called 'pulsed beam'. This process of emitting the beam in lumps is called 'bunching'.[1]

The beam, as a whole, travels in a particular direction which is conventionally taken to be the z-direction. But the individual particles of the beam, in addition to their velocity component in the z-direction, also have velocity components in the x and y directions. Thus, the beam possesses transverse dynamics and can get focused and defocused as it travels along a particular trajectory. The state of a single particle in a beam, is described by a column vector called *'orbit vector'*[2]

$$\mathbf{X} = \begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{pmatrix}$$

where

- x: It is the x-coordinate of the particle
- y: It is the y-coordinate of the particle
- x': Angle made by the particle with the z-axis on the z-x plane. $x' = v_x/v_z$ as described in Eqn.(1).
- y':Angle made by the particle with the z-axis on the y-z plane. $y' = v_y/v_z$ as described in Eqn.(1).

- l:Path-length difference between the central trajectory of the beam and the trajectory of the particle.

- δ: Fractional momentum deviation of the particle.

In accelerators the transverse components of velocity is much less than the longitudinal component.Therefore by definition[2],

$$\theta_x \approx \tan\theta_x = \frac{v_x}{v_z} \approx \frac{v_x}{v} \tag{1}$$

If the forces acting on the particles in a beam, are linear, inside an accelerator element, then any of the parameters of the orbit vector of the particle at the exit of the element can be represented as a linear combination of all the parameters of the orbit vector at the entrance of the element.So, the relationship between the entrance vector and exit vector can be expressed using a (6X6)-matrix called the*'transfer matrix'*. Every accelerator element has a transfer matrix if the forces inside the elements are linear or atleast can be approximated as linear under certain conditions. The two orbit vectors are called *entrance vector* and *exit vector* respectively.[2]

Entrance vector(V_0): $\begin{pmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \\ l_0 \\ \delta_0 \end{pmatrix}$ and exit vector (V_1): $\begin{pmatrix} x_1 \\ x'_1 \\ y_1 \\ y'_1 \\ l_1 \\ \delta_1 \end{pmatrix}$

$$V_1 = RV_0$$

where R is the transfer matrix[2].

$$R = \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} & R_{26} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} & R_{36} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} & R_{46} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} & R_{56} \\ R_{61} & R_{62} & R_{63} & R_{64} & R_{65} & R_{66} \end{pmatrix}$$

Since, the motions in the three directions are independent, transfer matrix in one dimension can also be written as a (2X2)-matrix.e.g. $\begin{pmatrix} x_1 \\ x'_1 \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$.

The state of a beam can be described by the distribution of it's particles in six-dimensional phase space for three coordinates, consisting of two parameters for each coordinate. Since it is not possible to draw a six-dimensional phase space, two-dimensional phase space distributions are drawn for the three independent degrees of freedom.e.g. (x, x'), (y, y'), $(dW/W, P)$ phase spaces, where the last one is the phase space of fractional energy deviation and phase respectively.Usually, the phase space distributions are elliptical as shown in figure 1. When a beam moves through an element, this distribution changes. But as long as the energy of the beam is constant, the hypervolume occupied by the distribution in six-dimensional phase space is constant, which is a consequence of *Liouville's Theorem* in statistical mechanics. Since the motions in the three directions are independent, it implies that the ellipses in the two-dimensional phase spaces can only stretch and rotate, but the area has to be conserved. Since the elements involved in designing the concerned achromat does not involve any change in the average energy of the beam, the ellipses should be equal in the area as it traverses through different elements in the achromat.

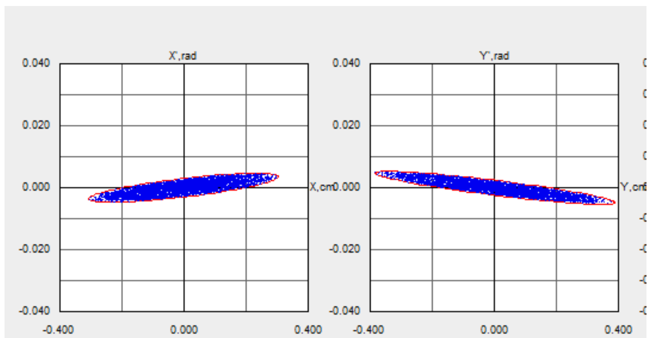

Figure 1: The x, x' phase space(*left*) and y, y' phase space(*right*)

In figure 1, clearly, x' of particles is positive(or equivalently v_x is positive) for positive values of x and negative for negative values of x.This means that the beam is defocusing in x and by a similar logic, the beam is focusing in y.

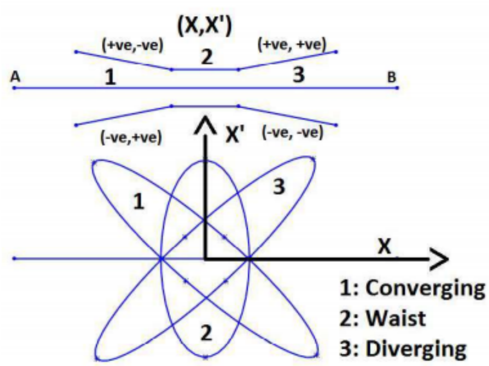

Figure 2: The phase space ellipse in different forms of the beam

A beam is characterized by the values of certain parameters, called *'beam parameters'*. They are: the beam energy(E_b), the beam current(I_B), the charge of an individual particle(q), the mass of the particle(m), Longitudinal emittance(ϵ_z), Transverse emittances(ϵ_x, ϵ_y).Emittance is defined as the area of the ellipse in the phase-space involving a particular degree of freedom.

$$\epsilon_x = \pi x x'$$

where x and x' are the thickness of the beam and the x'-spread (or equivalently the velocity spread) of the beam respectively. Emittance is easy to calculate when the beam forms a waist, since, it is just the product of the lengths of the major axis and the minor axis of the ellipse(x'-axis intercept times the x-intercept in the waist of figure 2).Normalised emittance, given by:

$$\epsilon_{nx} = \beta\epsilon_x = \beta(\pi x x')$$

ϵ_{nx} is constant even if the kinetic energy of the beam changes, as β cancels out the longitudinal velocity factor in the denominator of x'. $\beta = \frac{v}{c}$, where c is the speed of light in vacuum .

1.4 Beamline Elements

"Beamline", also called "Lattice", refers to the arrangement of the various components of an accelerator along the longitudinal path of the beam. These components that comprise the lattice, include Drift spaces, Dipole magnets, Quadrupole magnets, Multipole magnets(Sextupoles, Octopoles,etc), Solenoids, RFQs(Radio Frequency Quadrupoles), DTLs(Drift Tube Linacs), etc. Out of them only the Quadrupole Magnets, Dipole Magnets, and Drift Spaces are relevant for the design of the achromat concerned. So, the following subsections attempt to briefly explain each of them.

1.4.1 Drift Spaces

Drift spaces are accelerator elements that only drift the charged particles and the beam passes undisturbed by any force fields. As a result the trajectory of a beam passing through a drift space, is straight and does not involve any bending. It is characterized by only its length and its transfer matrix only involves the length l. If V_0 is the entrance vector of a drift space and V_1 is the corresponding exit vector, then:

$$x' = \theta_x \approx \tan\theta_x \implies x_1 = x_0 + x'.l$$

$$\& \;\; x'_1 = x'_0$$

Then the 2X2 transfer matrix for x and x' is: $\begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$ and similar for y and y'. Since, drift spaces do not influence the longitudinal dynamics of the beam, the 6X6 transfer matrix is:

$$\begin{pmatrix} 1 & l & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & l & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

1.4.2 Dipole Magnets

Bending Magnets are basically dipole magnets or charged electric plates with opposite polarities, separated by a distance. Dipole magnets exert a transverse force on the charged particles, which makes them bend along the desired trajectory and are thus called *'bending magnets'*[6]. The charged particles experience Lorentz force upon entering the dipole magnet. The Lorentz force acts as the centripetal force to bend the beam(as this force is perpendicular to the direction of motion of the beam and is thus capable of bending it). Since, **B** and **r** have the same direction, **B**.**r**= Br.

$$\frac{mv^2}{r} = qvB$$

$$\implies \mathbf{B.r} = 0.1439\sqrt{\frac{mE}{q}} \tag{2}$$

Here, **B.r**, E, m and q are in units of Tm, MeV, amu and electronic charge units respectively.

Clearly, magnitudes of B, m and q are fixed for a given beam through a given dipole. The radius of curvature is directly proportional to the square root of the energy of the particle.

The transfer matrix of a dipole magnet is given by:

$$\begin{pmatrix} \cos\alpha & \rho\sin\alpha & 0 & 0 & 0 & \rho(1-\cos\alpha) \\ -\frac{\sin\alpha}{\rho} & \cos\alpha & 0 & 0 & 0 & \sin\alpha \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \sin\alpha & \rho(1-\cos\alpha) & 0 & 0 & 1 & -L+\rho\sin\alpha \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Here, α, ρ and L are the bending angle, the bending radius and the effective length of the dipole magnet respectively.

$$L = 6g + \rho\alpha$$

where g is the pole gap in the dipole magnet[5]. The diagram of a dipole bending magnet along with the distribution of magnetic field lines inside a magnetic dipole, has been shown in figure 3.

1.4.3 Quadrupole Magnets

Quadrupoles are basically magnets with four poles, two of each polarity[7]. These can be both magnetic or electric(two positively charged poles and two negatively charged poles).The use of quadrupole magnets is to focus or defocus a beam. Quadrupole magnets are made such that the direction of the beam('z') is normal to the cross-section of the quadrupole. The charged particles in the beam experience Lorentz Force($\mathbf{F} = q(\mathbf{E} + \mathbf{v} * \mathbf{B})$ in case of electro-magnetic

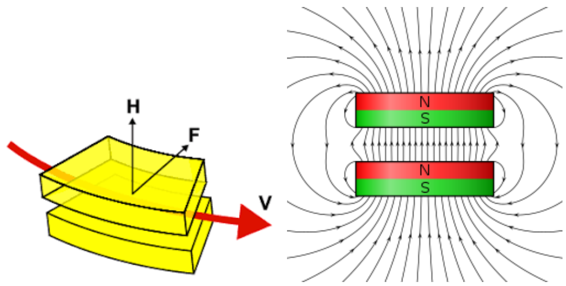

Figure 3: The direction of the magnetic field in a bending magnet and the force experienced by a negatively charged particle is shown (*left*) and the magnetic field lines inside a bending magnet is also shown (*right*).

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forces or $q(\mathbf{v} * \mathbf{B})$ (here * indicates cross-product) in case of purely magnetic force) upon entering the quadrupole and since magnetic forces act in a direction perpendicular to the direction of the motion of charged particle, as a result, the charged particle beams experience force in the x-y plane. An important feature of quadrupoles is that if it focuses the beam in x-direction, it will defocus the beam in y-direction and vice-versa.

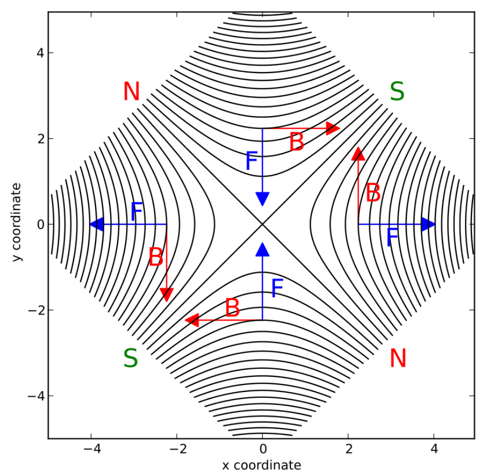

Figure 4: The magnetic field of a quadrupole. Clearly, there is focusing in y-direction and a defocusing in x-direction.

In figure 4, the directions of magnetic field are shown and the forces are indicated for a positively charged particle going into the plane of the paper, perpendicular to it.

The transfer matrix of a quadrupole can be written as:

$$\begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = \begin{pmatrix} \cos kl & \frac{\sin kl}{k} \\ -k\sin kl & \cos kl \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_1' \end{pmatrix} = \begin{pmatrix} \cosh kl & \frac{\sinh kl}{k} \\ k\sinh kl & \cosh kl \end{pmatrix} \begin{pmatrix} y_0 \\ y_0' \end{pmatrix}$$

where $k = \sqrt{\frac{qB}{amv}}$ and B is the magnetic field intensity at a distance of a from the central axis of the quadrupole in the $x - y$ plane, a is the radius of the aperture.

Clearly, it can be seen that the transfer matrix represents a quadrupole that is focusing in x and defocusing in y, which can be distinguished by the presence of 'sin' function in one dimension and 'sinh' in the other.

The net 6X6 transfer matrix is given by:

$$\begin{pmatrix} \cos kl & \frac{\sin kl}{k} & 0 & 0 & 0 & 0 \\ -k\sin kl & \cos kl & 0 & 0 & 0 & 0 \\ 0 & 0 & \cosh kl & \frac{\sinh kl}{k} & 0 & 0 \\ 0 & 0 & k\sinh kl & \cosh kl & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

1.5 TRACK software- Overview

TRACK is a computer program created by the ANL, which can simulate a beam passing through a certain beamline. Using TRACK, a user can define the order of accelerator elements as well as the parameters for a single element to form the beamline through which the beam passes. Users can also define the initial parameters of the beam before it passes through the beamline elements. The TRACK file named 'sclinac.dat' defines all the parameters of the accelerator elements and the sequence of elements in the lattice. The beamline can be altered by making changes to this file. Similarly, the file 'track.dat' defines the initial parameters of the beam and can be altered as and when required. The output is generated after running TRACK.exe file, which shows the phase-space portraits in (x, x'), (y, y') and $(dW/W, P\ )$ phase spaces and the total and rms profiles of the beam in x and y directions, as the beam passes through the given lattice. The simulation can be paused and resumed in order to view the phase space distribution at any of the lattice elements.[5]

2 DESIGN OF THE HCI ACHROMAT

The HEBT consists of four 90 degree achromatic bends, which were designed as per the input beam characteristics and the geometrical restrictions imposed by the outer beam hall[4]. Out of the four achromats, the first three are of the same kind and the fourth one is different. Only one of the first three achromatic bends, has been simulated. At first an ideal achromat(similar to the one actually used) was simulated and then the real achromat was simulated using TRACK[3].

2.1 Simulation values of the achromatic bend

The achromat has a configuration of Q1Q2Q3MQ4MQ3Q2Q1 where 'M' represents a 45 degree bending magnet and 'Q1', 'Q2', 'Q3', 'Q4' represent the four types of quadrupole magnets used in the achromat and has suitable drift spaces between any two of the elements[3]. This configuration(including the drifts) is mirror symmetric about the centre of the beamline(which is Q4 in this arrangement). The ideal achromat was simulated using the beam characteristics given in table 1.

Energy of the beam	1.8 MeV/u
Mass to charge Ratio(A/q)	6
$\beta\epsilon_x$, $\beta\epsilon_y$ and ϵ_z	0.058π cm.mrad, 0.62 deg.
Energy spread	1%
Phase spread	6.2 deg

Table 1: Beam Characteristics

The characteristics of the quadrupoles and the bending magnets simulated are given in table 2 and table 3 respectively.

Quadrupole	**Effective Length(mm)**	**Half Aperture(mm)**	**Field Gradient(T/m)**
Q1	106	26.5	-11.564297
Q2	212	26.5	14.697231
Q3	106	26.5	-13.483392
Q4	212	26.5	13.181310

Table 2: Quadrupole Characteristics

Bending angle	45 degree
Bending radius	850 mm
Pole gap	40 mm
Entrance, Exit angles	24 deg., 24 deg.
Radius of curvature at entrance, exit	850 mm, 850 mm

Table 3: Dipole Characteristics

To confirm these simulations by first order linear beam optics using matrix multiplication, we carry out the matrix multiplication of whole achromat and total R-matrix of full achromat comes out to be as follows,

$$\begin{pmatrix} 1 & 0.05 & 0 & 0 & 0 & 0.000204 \\ -0.03 & 1 & 0 & 0 & 0 & 0.008 \\ 0 & 0 & -1 & -0.561 & 0 & 0 \\ 0 & 0 & -0.029 & -1 & 0 & 0 \\ -0.008 & -0.0002 & 0 & 0 & 1 & -0.127 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

If one compares with standard R-matrix as given in the above section, then R_{16}= 0.000204 and R_{26}= 0.008, thus their values are close to zero and the achromatic bend is independent of initial position and momenta of the particles.

2.2 Realistic beam tuning in Achromatic bend

The arrangement of the accelerator elements were the same as in the ideal achromat discussed before. The only difference was that the magnetic fields produced by the quadrupoles were not same for quadrupoles equally away from the central quadrupole(Q4). The configuration used was: Q1Q2Q3MQ4MQ5Q6Q7, where the 'M' refers to a bending magnet and 'Q' refers to a certain quadrupole.

The real achromat was simulated using the beam characteristics given in table 4.

Energy of the beam	175 keV/u
Mass to charge Ratio(A/q)	2.8
$\beta\epsilon_x$, $\beta\epsilon_y$ and ϵ_z	0.058π cm.mrad, 0.62 deg.
Energy spread	1%
Phase spread	6.2 deg.

Table 4: Beam Characteristics

The characteristics of the quadrupoles are given in table 5

Quadrupole	**Effective Length(mm)**	**Half Aperture(mm)**	**Field Gradient(T/m)**
Q1	106	26.5	-1.7475
Q2	212	26.5	1.0444
Q3	106	26.5	-1.1068
Q4	212	26.5	0.5911
Q5	106	26.5	-1.8552
Q6	212	26.5	0.9959
Q7	106	26.5	-0.8305

Table 5: Quadrupole Characteristics

The bending magnets simulated had the same specifications as before.

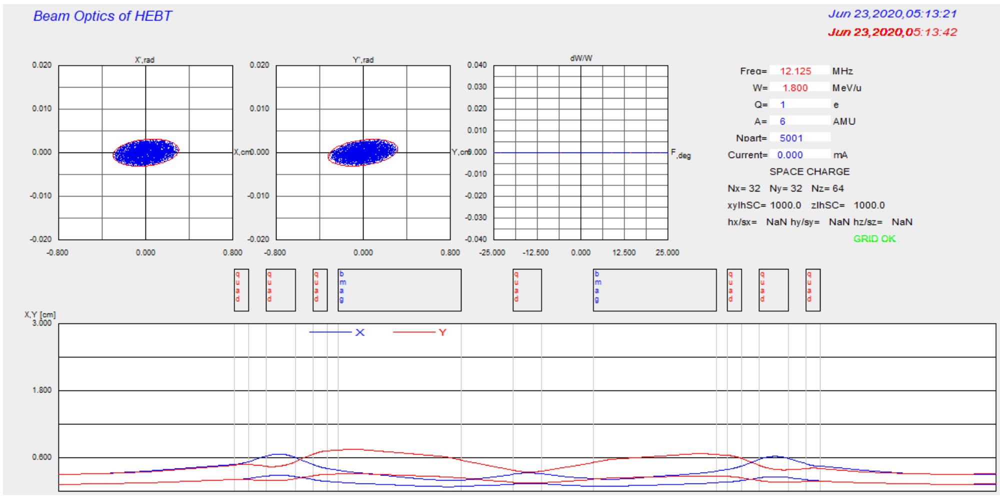

Figure 5: The simulation of beam in the designed achromat, corresponding to Table-1,2,3

3 SIMULATIONS

3.1 Analysis

The ideal achromat and the real achromat(used in HEBT) were simulated and the beam passing through their lattices is depicted in figure 5 and 6 respectively.

Each of the simulations features four profiles, which include the total x(the x-thickness of the beam) and the RMS value of x coordinate of the particles in *blue*. Similarly, it shows the y-total and y-RMS values in *red*. The RMS value is always lower than the total value. The lower graph of a certain color represents the RMS graph.

In figure 5, the lattice simulated, is an ideal 90-degree achromatic bend, which has mirror symmetry about the center of the beamline(which in this case happens to be the fourth quadrupole(Q4) of the lattice) i.e. the nth element from the beginning is equivalent to the nth element from the end. As a result, the four profiles in the output also possess mirror-symmetry about the central quadrupole of the lattice. e.g. The x-total profile starts from a certain value(appears to be close to 0.3) and converges to the same value at the end of the lattice. And similarly for other profiles as well. The profiles are straight in the drift spaces as expected. There appears to be a waist formed x-direction in the centre of Q4.

In figure 6, the achromat is the one which is actually used in HEBT section of HCI. It has the same arrangement of accelerator elements, but the values of the magnetic fields produced in quadrupoles are different for quadrupoles equally away from the centre. The profiles appear very different from the ideal case(because of different beam parameters and different parameters of quadrupoles

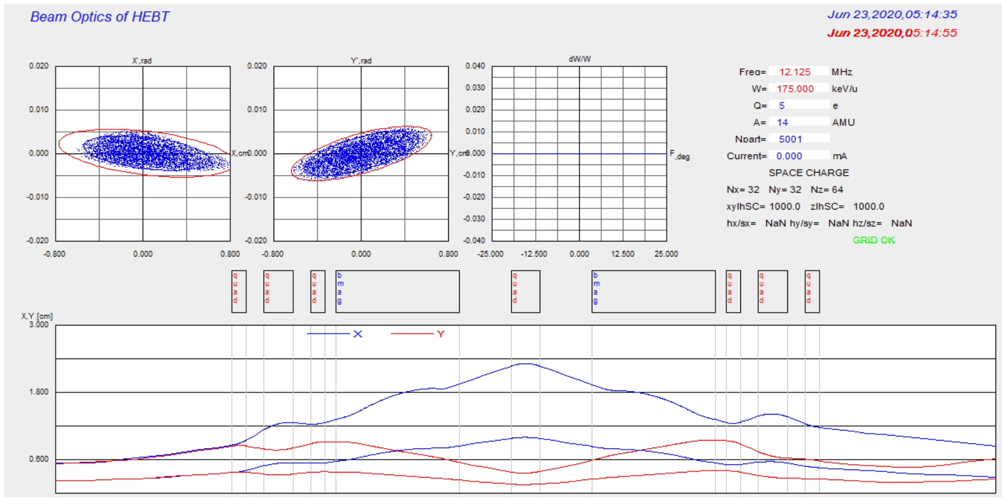

Figure 6: Simulation of the realistic beam with realtime tuning values of the magnet in the achromatic bend, corresponding to Table-4,5

used). The profiles are not exactly the same on both sides of Q4, but appear to be mirror-symmetric for a major portion of the graph. The profiles for x-total only slightly deviates from where it started(about 0.6cm), at the end of the achromat. But the deviation in other profiles is insignificant. There appears to be a waist formed in the y-direction in the last drift space, which is not present in the first. The profiles are straight in the drift spaces as expected. All the deviations are much less than the initial size of the beam(0.6cm). So, in practice, this lattice is sufficient for being used as an achromat.

4 Conclusion

The transverse beam dynamics is being studied using TRACK code for the achromatic bend in the HEBT section of HCI. The beam is first simulated at the designed energy of 1.8 MeV/u and is passed through the whole achromatic bend without any emittance growth and beam losses. Now the realistic values of the tuning parameters of the achromatic bend are taken for 175keV/u N^{5+} ion beam and it is also passed successfully through whole achromat using TRACK code. The validation of realistic values of the tuning parameters with simulation values clearly indicates the successful design implementation of the achromatic bend.

5 Acknowledgement

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