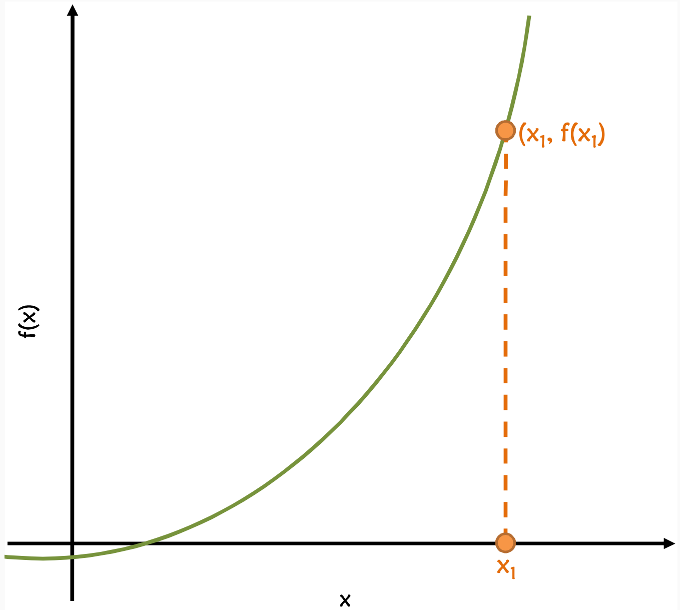
Newton Raphson-Method



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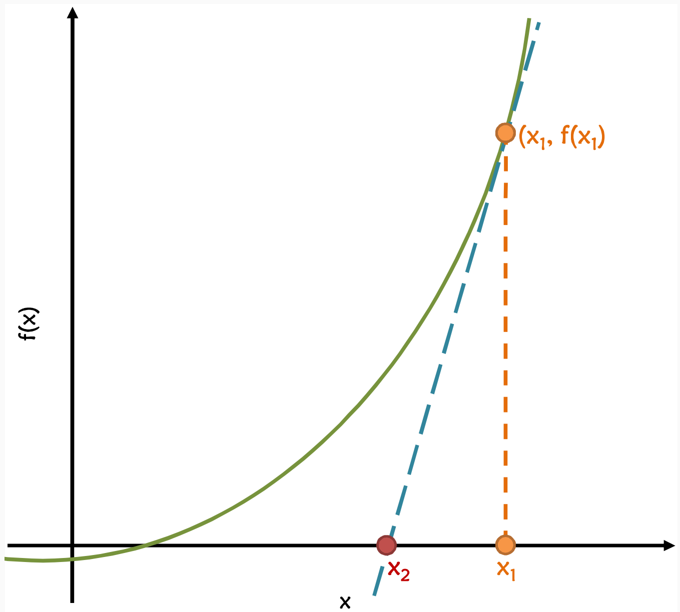

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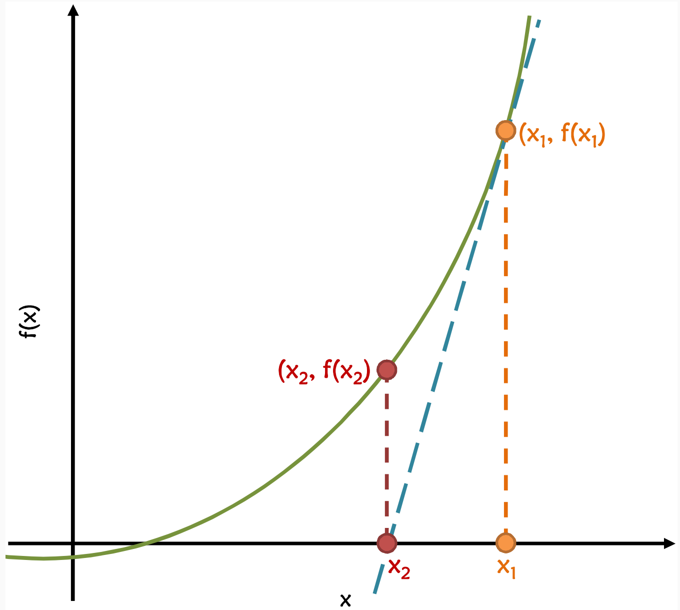

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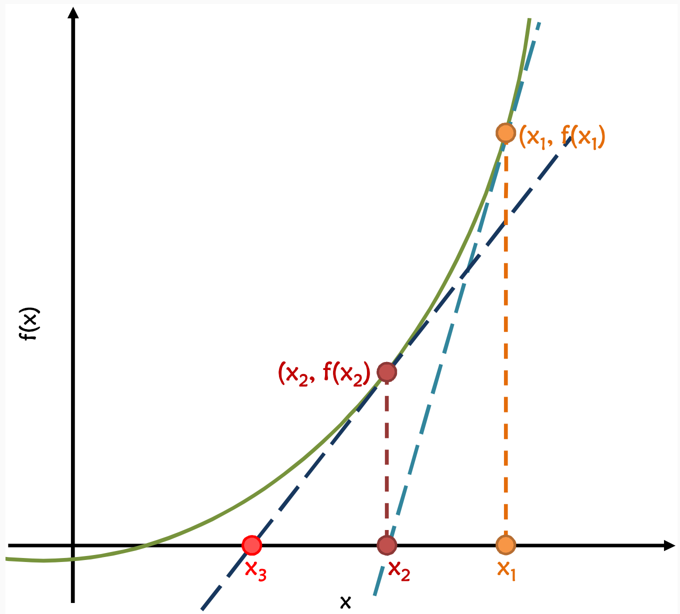

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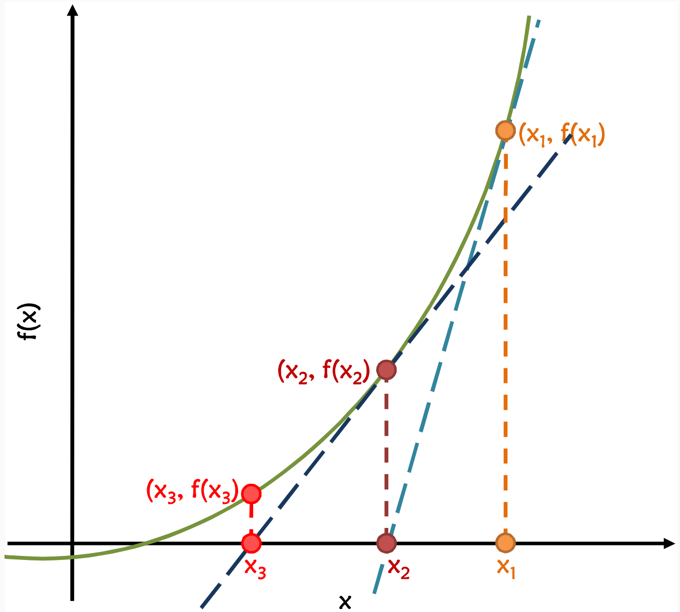

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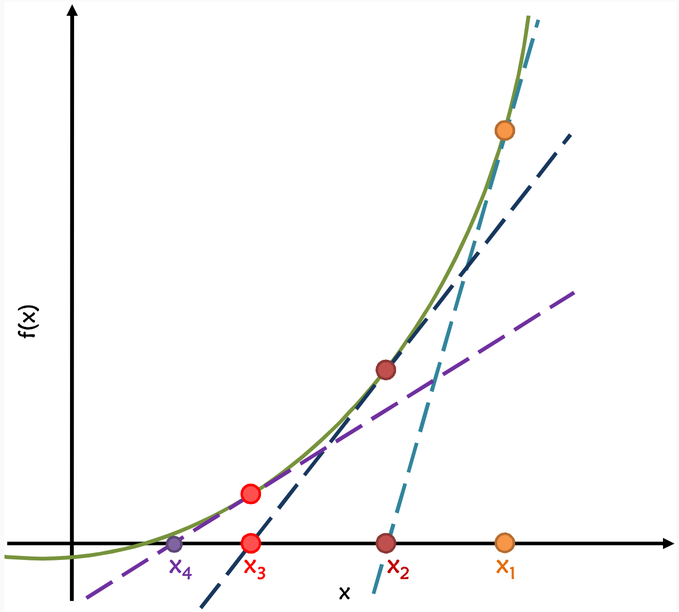

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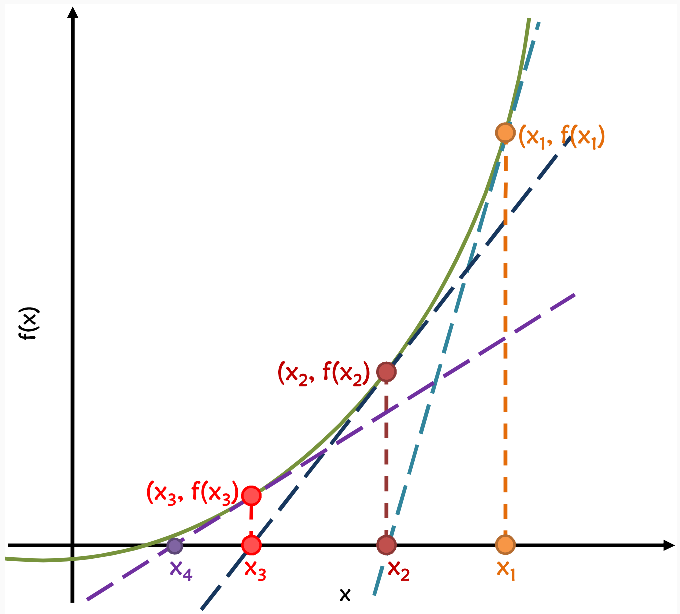

Newton-Raphson Method

Let us suppose that x_0 is the initial estimate for the root α of $f(x) = 0$. Draw a tangent at the point $(x_0, f(x_0))$ of the curve $y = f(x)$,

$$y - f(x_0) = f'(x_0)(x - x_0) \tag{1}$$

The point where the tangent (1) cuts the axis ($y = 0$), say $x = x_1$, is the next estimate of the root i.e.,

$$-f(x_0) = f'(x_0)(x_1 - x_0)$$

$$\implies \qquad x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

This process may be repeated to get the next estimate x_2 using x_1 and so on. Let us suppose that we have computed the n^{th} estimate x_n, the next estimate may be computed from

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, \cdots \tag{2}$$

Newton-Raphson Method

Let x_0 be an estimate for the root α and h be the error in it such that $x_0 + h = \alpha$. The Taylor's series expansion gives,

$$f(\alpha) = f(x_0 + h) = f(x_0) + hf'(x_0) + \frac{h^2}{2!} f''(x_0) + \cdots \tag{3}$$

Neglecting second and higher powers of h and remembering that $f(\alpha) = 0$, we get approximate value of h, as

$$h \simeq -\frac{f(x_0)}{f'(x_0)}, \tag{4}$$

and the next estimate,

$$x_1 = x_0 + h = x_0 - \frac{f(x_0)}{f'(x_0)}. \tag{5}$$

In the general may be written as,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \; n = 0, 1, \cdots \tag{6}$$