# Computer Vision

## Lecture 3: (Gray) Mathematical Morphology

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# Binary Morphology (reminder)

In short:

- local analysis in a neighbourhood (a.k.a. the structuring element)
- two basic (dual) operations: erosion and dilation
- combined to form more complex operators: opening/closing, gradient, residue, etc.
- scale-space with a series of structuring elements

# Grayscale Morphology

Extending operators from set theory to complete lattices

- translation $f_x(z) = f(z - x)$
- shift $(f + y)(z) = f(z) + y$
- morphological translation $(f_x + y)(z) = f(z - x) + y$
- negative: $f^c(x) = -f(x)$
- reflection: $f(x) = f(-x)$
- $g$ is below $f$ if $D[g] \subset D[f]$ and $g(x) \leq f(x) \quad \forall x \in D[g]$
- minimum: $(f \wedge g)(x) = \min(f(x), g(x))$
- maximum: $(f \vee g)(x) = \max(f(x), g(x))$

## Erosion

$$\varepsilon_g(f)(x) = (f \ominus g)(x) = \max(y : y + g_x \leq f) = \min(f(z) - g_x(z) : z \in D[g_x])$$

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$$\delta_g(f)(x) = (f \oplus \breve{g})(x) = \min(y : y - \breve{g}_x \geq f) = \max(f(z) + \breve{g}_x(z) : z \in D[\breve{g}_x])$$

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Structuring elements can be non-flat (a.k.a. structuring functions) or flat.

Using flat SEs leads to simpler definitions:

- $\varepsilon_B(f)(x) = \min_{z \in B}(f(x+z))$
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Reflection may be omitted if symmetric structuring elements/functions are used.

All other operators extend to greyscale:

- opening
- closing
- alternate sequential filtering (for image smoothing)
- morphological gradient (for edge detection)
- top-hat
- morphological reconstruction, by replacing $\cup$ by $\vee$ and $\cap$ by $\wedge$
- granulometry (with $v$ the image volume, i.e. the sum of pixel values)
- etc.

Some examples follow...

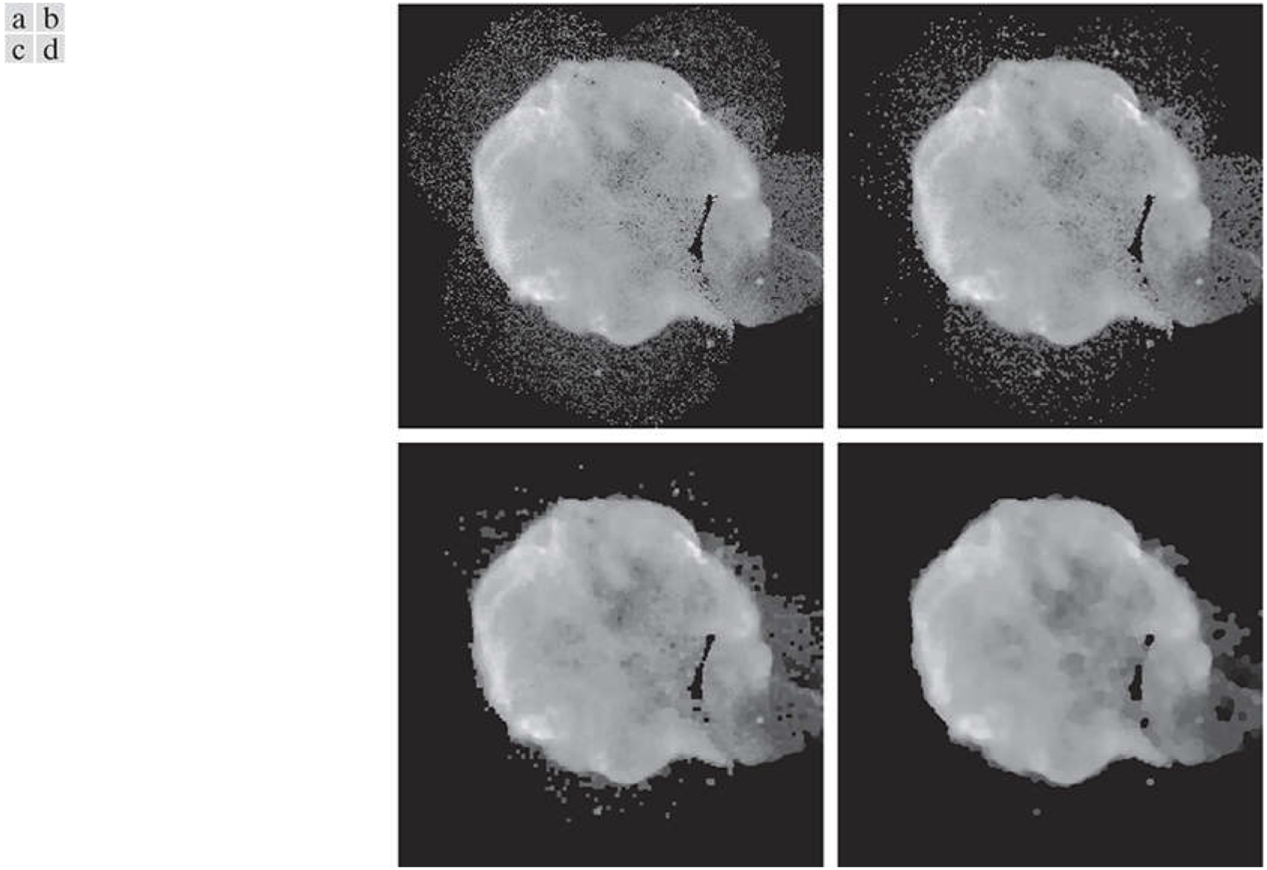



**FIGURE 9.40**

(a) $566 \times 566$ image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively.

*(Original image courtesy of NASA.)*

a b
c d

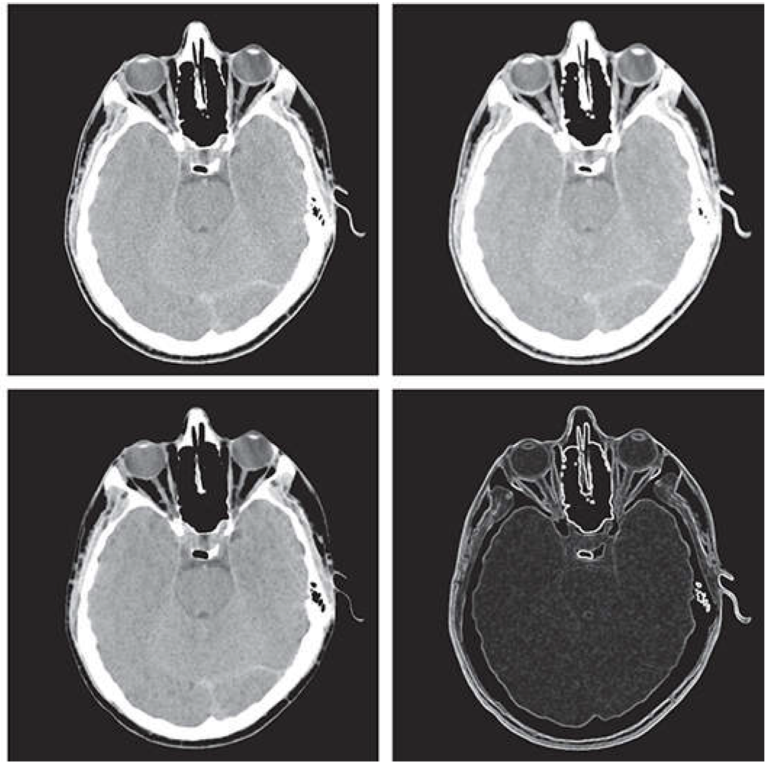


**FIGURE 9.41**

(a) $512 \times 512$ image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c).

*(Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)*

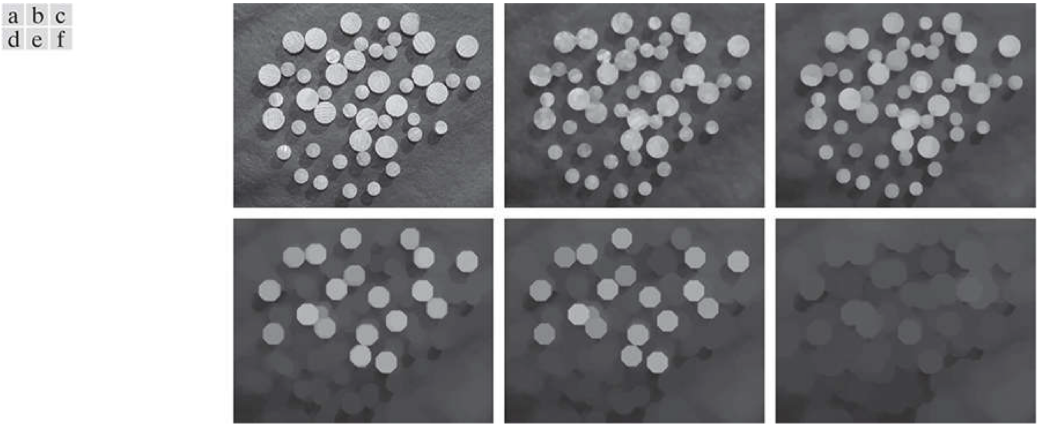


**FIGURE 9.43**

(a) $531 \times 675$ image of wood dowels. (b) Smoothed image. (c)–(f) Openings of (b) with disks of radii equal to 10, 20, 25, and 30 pixels, respectively.

*(Original image courtesy of Dr. Steve Eddins, MathWorks, Inc.)*

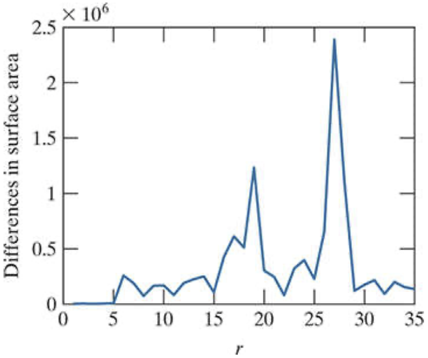
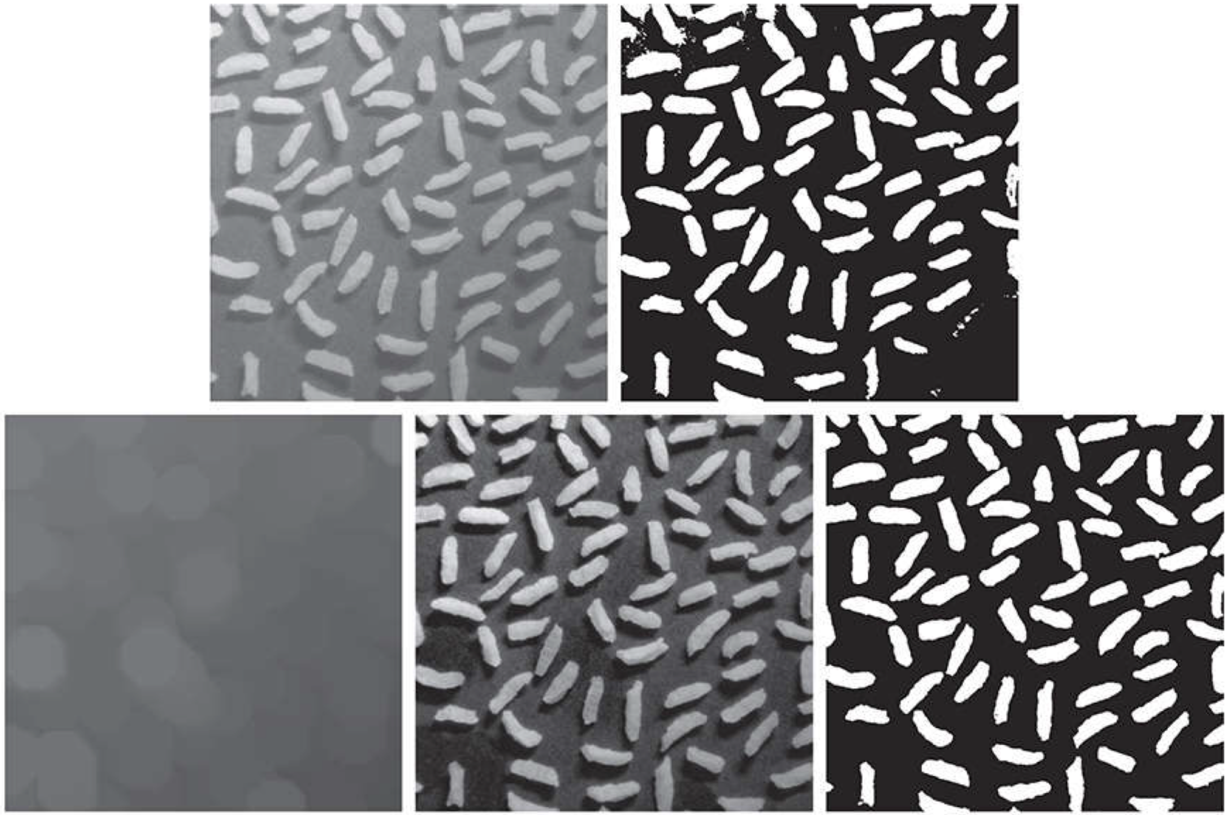



**FIGURE 9.44**

Differences in surface area as a function of SE disk radius, *r*. The two peaks indicate that there are two dominant particle sizes in the image.



a b
c d e

**FIGURE 9.42**

Using the top-hat transformation for *shading correction*. (a) Original image of size $600 \times 600$ pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.

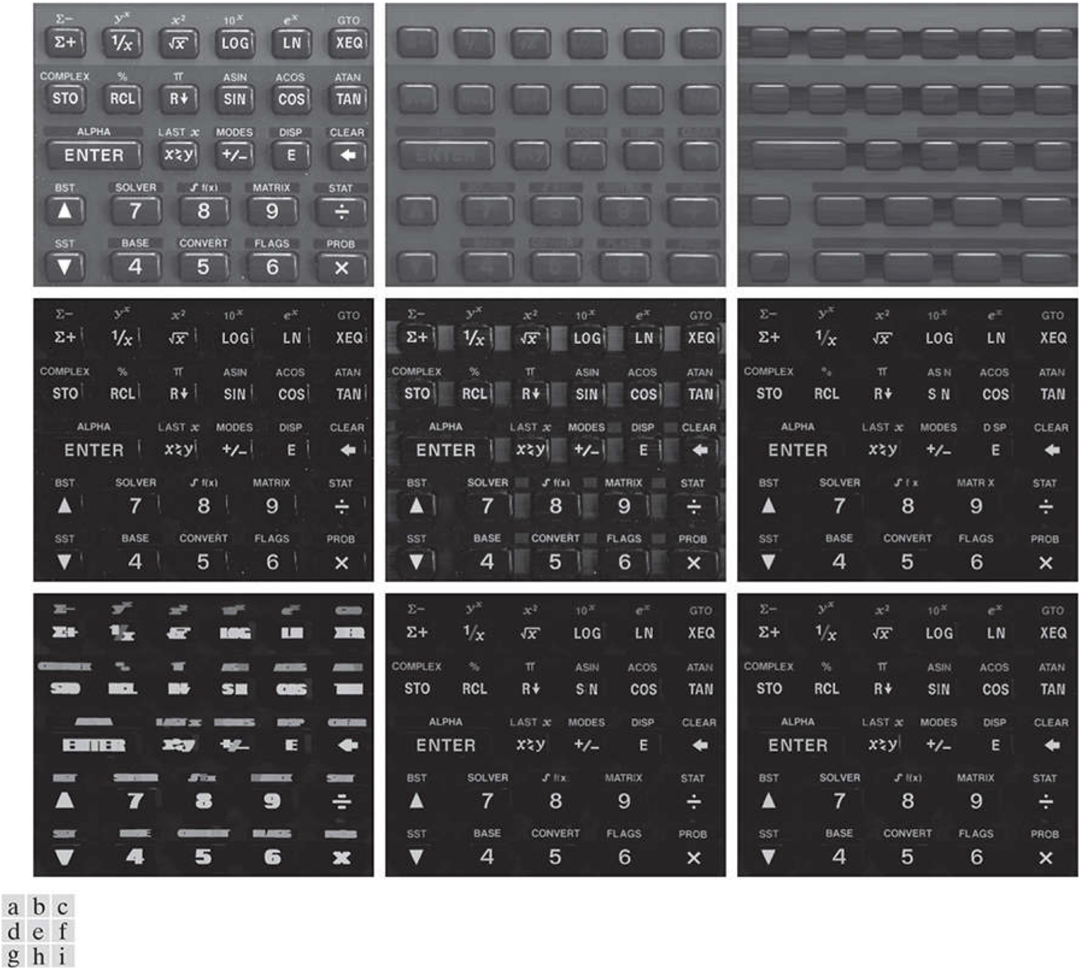


**FIGURE 9.46**

(a) Original image of size $1134 \times 1360$ pixels. (b) Opening by reconstruction of (a), using a structuring element consisting of a horizontal line 71 pixels long in the erosion. (c) Opening of (a) using the same SE. (d) Top-hat by reconstruction. (e) Result of applying just a top-hat transformation. (f) Opening by reconstruction of (d), using a horizontal line 11 pixels long. (g) Dilation of (f) using a horizontal line 21 pixels long. (h) Minimum of (d) and (g). (i) Final reconstruction result.

*(Images courtesy of Dr. Steve Eddins, MathWorks, Inc.)*

# Other morphological-based tools

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For more details, see MM-related references

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- Pesaresi and Benediktsson (2001) A new approach for the morphological segmentation of high-resolution satellite imagery
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A still active topic, with regular special issues,
e.g. IJGI in 2016, JSTARS in 2021.

# Morphological Profiles

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Let us consider the series of openings (by reconstruction).
Not only global measures, but also local features can be derived...
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Both are very popular structural features in remote sensing.

# Assignement

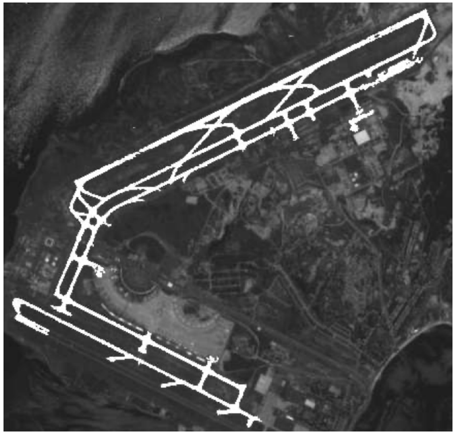
1. Select one paper (old or recent) dealing with MM+RS (e.g. from GoogleScholar)
2. Implement the morphological pipeline
3. Discuss the pros and cons of the method

Deadline: October 3, 2021

# Practice 1

Let us consider the image below.

Design a morphological processing chain to extract the airport.



# Practice 2

Compute and plot the DMP of pixels belonging to different land cover classes.

Discuss the characterization power of the DMP.