

### Chapter 7

# Dielectric Materials and Insulation

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# **Definition of Capacitance**

 $C_o = \frac{Q_o}{V} = \frac{\varepsilon_0 A}{d}$ 

 $C_o$  = capacitance of a parallel plate capacitor in free space

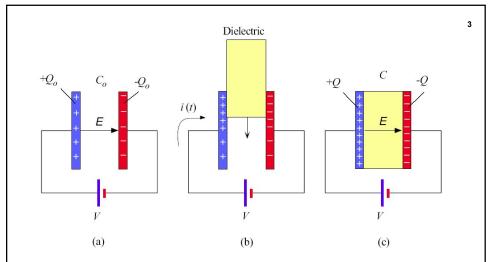
 $Q_o$  = charge on the plates

V = voltage

 $\varepsilon_0$  = absolute permittivity

A = plate area

d = separation between plates



- (a) Parallel plate capacitor with free space between the plates.
- (b) As a slab of insulating material is inserted between the plates, there is an external current flow indicating that more charge is stored on the plates.
- (c) The capacitance has been increased due to the insertion of a medium between the plates.

### **Definition of Relative Permittivity**

 $\varepsilon_r = \frac{Q}{Q_o} = \frac{C}{C_o}$ 

 $\varepsilon_r$  = relative permittivity,

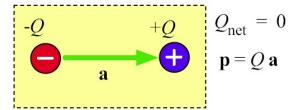
Q = charge on the plates with a dielectric medium,

 $Q_o$  = charge on the plates with free space between the plates,

C= capacitance with a dielectric medium,  $C_o=$  capacitance of a parallel plate capacitor in free space

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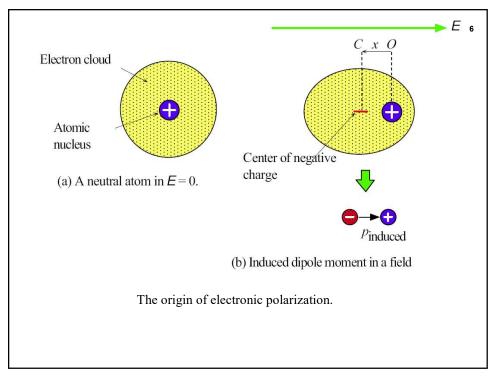
### **Definition of Dipole Moment**



$$\mathbf{p} = Q\mathbf{a}$$

 $\mathbf{p} = \text{electric dipole moment}, \ Q = \text{charge}, \ \mathbf{a} = \text{vector from}$  the negative to the positive charge

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### **Definition of Polarizability**

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$$p_{\text{induced}} = \alpha \mathcal{E}$$

 $p_{\text{induced}}$  = induced dipole moment,  $\alpha$  = polarizability,  $\mathcal{E}$  = electric field

### **Electronic Polarization**

$$p_e = (Ze)x = \left(\frac{Z^2e^2}{\beta}\right)\mathcal{E}$$

 $p_e$  = magnitude of the induced electronic dipole moment, Z = number of electrons orbiting the nucleus of the atom, x = distance between the nucleus and the center of negative charge,  $\beta$  = constant,  $\mathcal{E}$  = electric field

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**Table 7.1** Electronic polarizability  $\alpha_e$  dependence on Z for the inert element atoms

-	Atom					
	He	Ne	Ar	Kr	Xe	Rn*
Z	2	10	18	36	56	
$\alpha_e \times 10^{-40} \; (\text{F m}^2)$	0.18	0.45	1.7	2.7	4.4	5.9
$f_o \times 10^{15}  ({\rm Hz})$	8.90	12.6	8.69	9.76	9.36	10.2

<sup>\*</sup>Rn (radon) gas is radioactive.

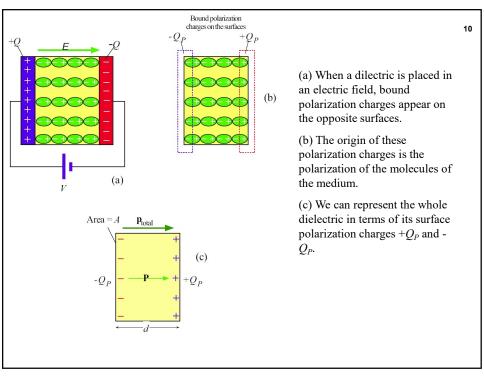
# **Electronic Polarizability**

 $\omega_0 = (\frac{\beta}{Zm_e})^{0.5}$ 

 $\omega_o$  = natural oscillation frequency of the center of mass of electron cloud about nucleus – known as **Electronic Polarization Resonance Frequency** 

$$lpha_e = rac{Ze^2}{m_e \omega_o^2}$$
  $\alpha_e$  = electronic polarizability  $\alpha_e$  = total number of electrons around the nucleus  $\alpha_e$  = mass of the electron in free space  $\alpha_o$  = natural oscillation frequency

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**Definition of Polarization Vector** 

$$\mathbf{P} = \frac{1}{\text{Volume}} \left[ \mathbf{p}_1 + \mathbf{p}_2 + ... + \mathbf{p}_N \right]$$

 $\mathbf{P}$  = Polarization vector,  $\mathbf{p}_1$ ,  $\mathbf{p}_2$ , ...,  $\mathbf{p}_N$  are the dipole moments induced at N molecules in the volume

### **Definition of Polarization Vector**

$$\mathbf{P} = N\mathbf{p}_{\mathrm{av}}$$

 $\mathbf{p}_{av}$  = the average dipole moment per molecule

P = polarization vector, N = number of molecules per unit volume

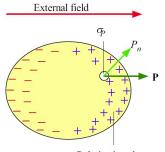
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### Polarization and Bound Surface Charge Density

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$$P = \sigma_p = Q_P / A$$

P = polarization,  $\sigma_p = \text{polarization}$  charge density on the surface



Polarization charge density on the surface of a polarized medium is related to the normal component of the polarization vector.

Polarization charges on the surface of a polarized medium

**Electric Susceptibility** 

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$$P = \chi_e \varepsilon_o \mathcal{E}$$

 $P = \text{polarization}, \ \chi_e = \text{electric susceptibility}, \ \varepsilon_o = \text{permittivity of free space}, \ \mathcal{E} = \text{electric field}$ 

$$P = N. p_{induced} = N. \alpha_e. \mathcal{E}$$

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**Electric Susceptibility and Polarization** 

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$$\chi_e = \frac{1}{\varepsilon_o} N\alpha_e$$

 $\chi_e$  = electric susceptibility,  $\varepsilon_o$  = permittivity of free space, N = number of molecules per unit volume,  $\alpha_e$  = electronic polarizability

**Relative Permittivity and Electronic Susceptibility** 

$$\varepsilon_r = 1 + \chi_e$$

 $\varepsilon_r$  = relative permittivity,  $\chi_e$  = electric susceptibility

Relative Permittivity and Polarizability

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$$\varepsilon_r = 1 + \frac{N\alpha_e}{\varepsilon_o}$$

 $\varepsilon_r$  = relative permittivity

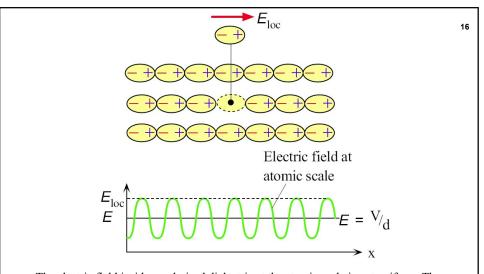
N = number of molecules per unit volume

 $\alpha_e$  = electronic polarizability

 $\varepsilon_o$  = permittivity of free space

Assumption: Only electronic polarization is present

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The electric field inside a polarized dielectric at the atomic scale is not uniform. The local field is the actual field that acts on a molecules. It can be calculated by removing that molecules and evaluating the field at that point from the charges on the plates and the dipoles surrounding the point.

**Local Field in Dielectrics** 

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$$\mathcal{E}_{loc} = \mathcal{E} + \frac{1}{3\varepsilon_o} P$$

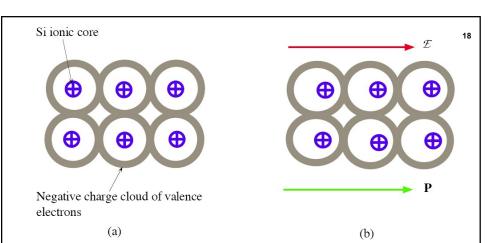
 $\mathcal{E}_{\mathrm{loc}} = \mathrm{local}$  field,  $\mathcal{E} = \mathrm{electric}$  field,  $\mathcal{E}_o = \mathrm{permittivity}$  of free space,  $P = \mathrm{polarization}$ 

### **Clausius-Mossotti Equation**

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha_e}{3\varepsilon_o}$$

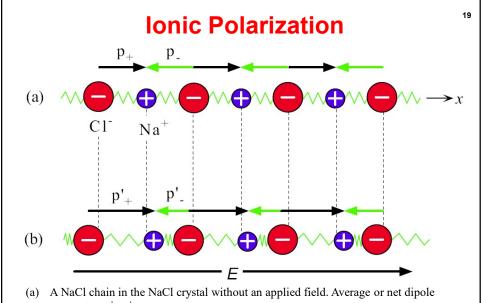
 $\varepsilon_r$  = relative permittivity, N = number of molecules per unit volume,  $\alpha_e$  = electronic polarizability,  $\varepsilon_o$  = permittivity of free space

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- (a) Valence electrons in covalent bonds in the absence of an applied field.
- (b) When an electric field is applied to a covalent solid, the valence electrons in the covalent bonds are shifted very easily with respect to the positive ionic cores. The whole solid becomes polarized due to the collective shift in the negative charge distribution of the valence electrons.

$$\varepsilon_r = 11.9 \ for \ Si$$
  
 $\varepsilon_r = 16 \ for \ Ge$ 



(a) A NaCl chain in the NaCl crystal without an applied field. Average or net dipole moment per ion is zero.

(b) In the presence of an applied field the ions become slightly displaced which leads to a net average dipole moment per ion.

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## **Ionic Polarization**

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### **Clausius-Mossotti Equation**

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N_i \alpha_i}{3\varepsilon_o}$$

 $\varepsilon_r$  = relative permittivity,

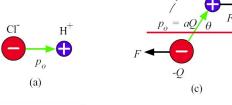
 $N_i$  = number of ion pairs per unit volume,

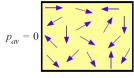
 $\alpha_i$  = ionic polarizability,

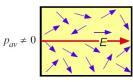
 $\varepsilon_o$  = permittivity of free space

# **Orientational (Dipolar) Polarization**

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(a) A HCl molecule possesses a permanent dipole moment  $p_0$ .

- (b) In the absence of a field, thermal agitation of the molecules results in zero net average dipole moment per molecule.
- (c) A dipole such as HCl placed in a field experiences a torque that tries to rotate it to align  $p_0$  with the field E.
- (d) In the presence of an applied field, the dipoles try to rotate to align with the field against thermal agitation. There is now a net average dipole moment per molecule along the field.

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### **Average Dipole Moment in Orientational Polarization**

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$$p_{\rm av} = \frac{1}{3} \frac{p_o^2 \mathcal{E}}{kT}$$

 $p_{\rm av}$  = average dipole moment,  $p_o$  = permanent dipole moment,  $\mathcal{E}$  = electric field, k = Boltzmann constant, T = temperature

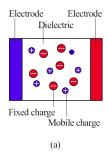
### **Dipolar Orientational Polarizability**

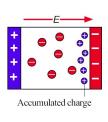
$$\alpha_d = \frac{1}{3} \frac{p_o^2}{kT}$$

 $\alpha_d$  = dipolar orientational polarizability,  $p_o$  = permanent dipole moment

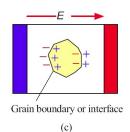
# **Interfacial Polarization**

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(b)



- (a) A crystal with equal number of mobile positive ions and fixed negative ions. In the absence of a field, there is no net separation between all the positive charges and all the negative charges.
- (b) In the presence of an applied field, the mobile positive ions migrate toward the negative charges and positive charges in the dielectric. The dielectric therefore exhibits interfacial polarization.
- (c) Grain boundaries and interfaces between different materials frequently give rise to Interfacial polarization.

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 Table 7.2
 Typical examples of polarization mechanisms

Example	Polarization	Static $\varepsilon_r$	Comment
Ar gas	Electronic	1.0005	Small N in gases: $\varepsilon_r \approx 1$
Ar liquid ( <i>T</i> < 87.3 K)	Electronic	1.53	van der Waals bonding
Si crystal	Electronic polarization due to valence electrons	11.9	Covalent solid; bond polarization
NaCl crystal	Ionic	5.90	Ionic crystalline solid
CsCl crystal	Ionic	7.20	Ionic crystalline solid
Water	Orientational	80	Dipolar liquid
Nitromethane (27 °C)	Orientational	34	Dipolar liquid
PVC (polyvinyl chloride)	Orientational	7	Dipole orientations partly hindered in the solid

**Total Induced Dipole Moment** 

$$p_{\text{av}} = \alpha_e \mathcal{E}_{\text{loc}} + \alpha_i \mathcal{E}_{\text{loc}} + \alpha_d \mathcal{E}_{\text{loc}}$$

 $p_{\rm av}$  = average dipole moment,  $\mathcal{E}_{\rm loc}$  = local electric field,  $\alpha_e$  = electronic polarizability,  $\alpha_i$  = ionic polarizability,  $\alpha_d$  = dipolar (orientational) polarizability

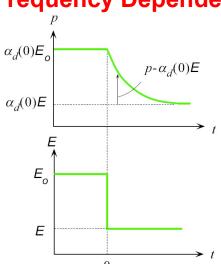
### **Clausius-Mossotti Equation**

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{1}{3\varepsilon_o} (N_e \alpha_e + N_i \alpha_i)$$

 $\varepsilon_r$  = dielectric constant,  $\varepsilon_o$  = permittivity of free space,  $N_e$  = number of atoms or ions per unit volume,  $\alpha_e$  = electronic polarizability,  $N_i$  = number of ion pairs per unit volume,  $\alpha_i$  = ionic polarizability

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# **Frequency Dependence**



The dc field is suddenly changed from  $\mathcal{E}_o$  to  $\mathcal{E}$  at time t=0. The induced dipole moment p has to decrease from  $\alpha_d(0)\mathcal{E}_o$  to a final value of  $a_d(0)\mathcal{E}$ . The decrease is achieved by random collisions of molecules in the gas.

**Dipolar Relaxation Equation** 

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$$\frac{dp}{dt} = -\frac{p - \alpha_d(0)\mathcal{E}}{\tau}$$

p = dipole moment, dp/dt = rate at which the induced dipole moment is changing,  $\alpha_d$  = dipolar orientational polarizability,  $\mathcal{E}$  = electric field,  $\tau$  = relaxation time

### **Orientational Polarizability and Frequency**

$$\alpha_d(\omega) = \frac{\alpha_d(0)}{1 + j\omega\tau}$$

 $\alpha_d(\omega)$  = dipolar orientational polarizability as a function of  $\omega$ ,  $\omega$  = angular frquency of the applied field,  $\tau$  = relaxation time, j is  $\sqrt{(-1)}$ .

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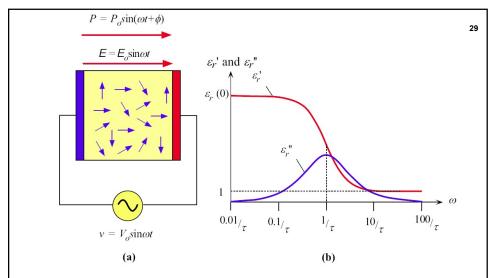
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### **Complex Relative Permittivity**

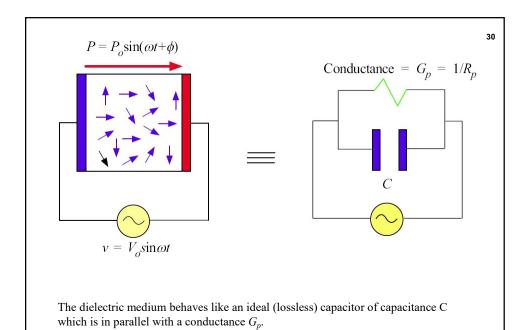
$$\varepsilon_r = \varepsilon_r' - j\varepsilon_r''$$

 $\varepsilon_r$  = dielectric constant

 $\mathcal{E}'_r$  = real part of the complex dielectric constant  $\mathcal{E}''_r$  = imaginary part of the complex dielectric constant j = imaginary constant  $\sqrt{(-1)}$ 



- (a) An ac field is applied to a dipolar medium. The polarization P(P = Np) is out of phase with the ac field.
- (b) The relative permittivity is a complex number with real  $(\varepsilon_r')$  and imaginary  $(\varepsilon_r'')$  parts that exhibit frequency dependence.



### **Admittance of a Parallel Plate Capacitor**

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$$Y = j\omega C + G_P$$

Y = admittance,  $\omega =$  angular frequency of the applied field , C = capacitance,  $G_P =$  conductance

### **Loss Tangent (Loss Factor)**

$$\tan \delta = \frac{\varepsilon_r^{\prime\prime}}{\varepsilon_r^{\prime}}$$

 $\tan \delta = \log t$  tangent or loss factor,  $\mathcal{E}'_r = \text{real part of the complex dielectric constant}$ ,  $\mathcal{E}''_r = \text{imaginary part of the complex dielectric constant}$ 

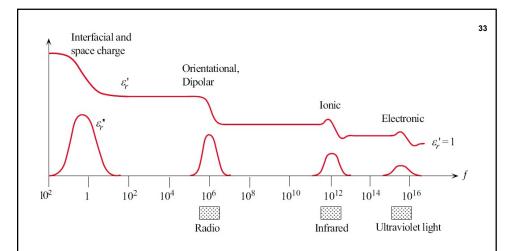
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### Dielectric Loss per Unit Volume

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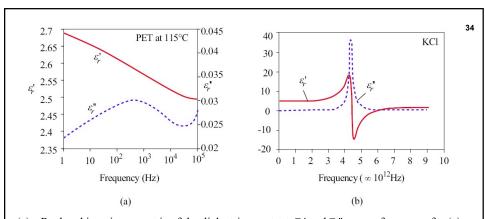
$$W_{\rm vol} = \omega \mathcal{E}^2 \varepsilon_o \varepsilon_r' \tan \delta$$

 $W_{\mathrm{vol}} = \mathrm{dielectric}$  loss per unit volume,  $\omega = \mathrm{angular}$  frquency of the applied field,  $\mathcal{E} = \mathrm{electric}$  field,  $\mathcal{E}_o = \mathrm{permittivity}$  of free space,  $\mathcal{E}_r = \mathrm{real}$  part of the complex dielectric constant,  $\mathrm{tan} \delta = \mathrm{loss}$  tangent or loss factor



The frequency dependence of the real and imaginary parts of the dielectric constant in the presence of interfacial, orientational, ionic, and, electronic polarization mechanisms.

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(a) Real and imaginary part is of the dielectric constant,  $\mathbb{Z}_r$ ' and  $\mathbb{Z}_r$ " versus frequency for (a) a polymer, PET, at 115  $\mathbb{Z}$ C and (b) an ionic crystal, KCl, at room temperature. both exhibit relaxation peaks but for different reasons.

SOURCE: Data for (a) from author's own experiments using a dielectric analyzer (DEA), (b) from C. Smart, G.R. Wilkinson, A. M. Karo, and J.R. Hardy, International Conference on lattice Dynamics, Copenhagen, 1963, as quoted by D. G. Martin, "The Study of the Vibration of Crystal Lattices by Far Infra-Red Spectroscopy," *Advances in Physics*, 14, no. 53-56, 1965, pp. 39-100.

**Table 7.3** Dielectric properties of three insulators

Material	f = 60  Hz				f = 1  MHz	ı.
	$\varepsilon_r'$	tan δ	$\omega \tan \delta$	$\varepsilon_r'$	$ an \delta$	$\omega$ tan $\delta$
Polycarbonate	3.17	$9 \times 10^{-4}$	0.34	2.96	$1 \times 10^{-2}$	$6.2 \times 10^{4}$
Silicone rubber	3.7	$2.25 \times 10^{-2}$	8.48	3.4	$4 \times 10^{-3}$	$2.5 \times 10^{4}$
Epoxy with mineral filler	5	$4.7 \times 10^{-2}$	17.7	3.4	$3 \times 10^{-2}$	$18 \times 10^{4}$

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### **Debye Equations**

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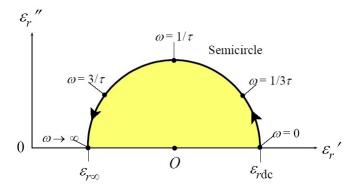
$$\varepsilon_r' = 1 + \frac{\left[\varepsilon_r(0) - 1\right]}{1 + \left(\omega\tau\right)^2} \qquad \varepsilon_r'' = \frac{\left[\varepsilon_r(0) - 1\right]\omega\tau}{1 + \left(\omega\tau\right)^2}$$

 $\varepsilon_r$  = dielectric constant (complex)

 $\mathcal{E}'_r$  = real part of the complex dielectric constant  $\mathcal{E}''_r$  = imaginary part of the complex dielectric constant  $\omega$  = angular frequency of the applied field  $\tau$  = relaxation time

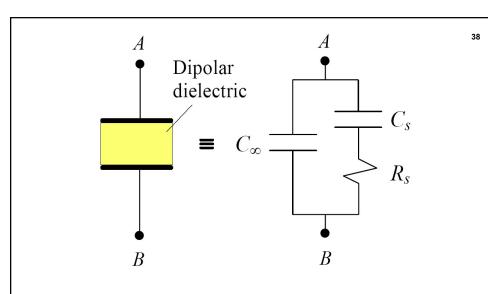
# **Cole-Cole plots**

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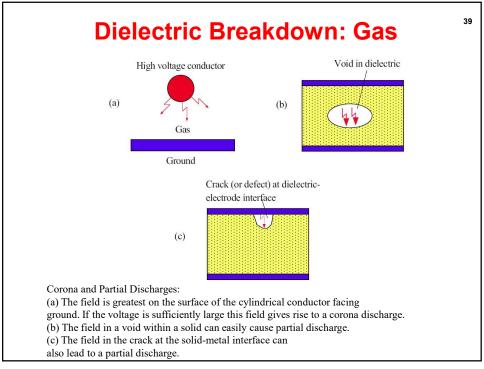


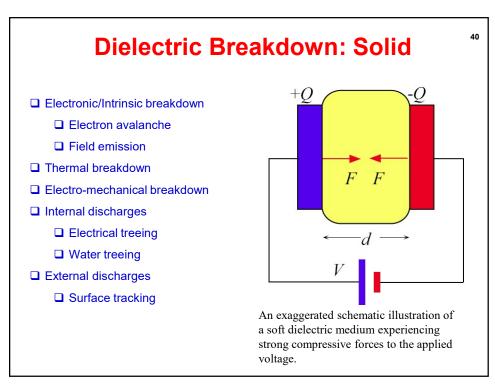
*Cole-Cole* plot is a plot of  $\mathcal{E}'_r$  vs.  $\mathcal{E}'_r$  as a function of frequency,  $\omega$ . As the frequency is changed from low to high frequencies, the plot traces out a circle if Debye equations are obeyed.

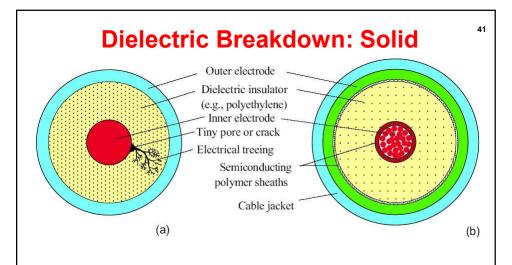
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A capacitor with a dipolar dielectric and its equivalent circuit in terms of an ideal Debye relaxation.



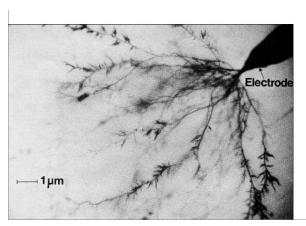




- (a) A schematic illustration of electrical treeing breakdown in a high voltage coaxial cable which was initiated by a partial discharge in the void at the inner conductor dielectric interface.
- (b) A schematic diagram of a typical high voltage coaxial cable with semiconducting polymer layers around the inner conductor and around the outer surface of the dielectric.

# Dielectric Breakdown: Solid

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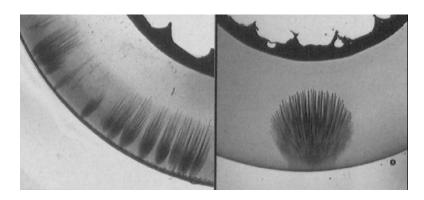


Electrical breakdown by treeing (formation of discharge channels) in a low-density polyethylene insulction when a 50 Hz, 20 kV (rms) voltage is applied for 200 minutes to an electrode embedded in the insulation.

SOURCE: J. W. Billing and D. J. Groves, Proceedings of the Institution of Electrical Engineers, **212**, 1974, p. 1451.

# Dielectric Breakdown: Solid

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Some typical water trees found in field aged cables. (Left: Trees in a cable with tape and graphite insulation. Right: Trees in a cable with strippable insulation.)

SOURCE: P. Werellius, P. Tharning, R. Eriksson,B. Holmgren. J. Gafvert, "Dielectric Spectroscopy for Diagnosis of Water Tree Deterioration in XLPE Cables" *IEEE Transactions on Dielectrics and Electrical Insulation*, Vol. 8, February 2001, p 34, Figure 10 (© IEEE, 2001)

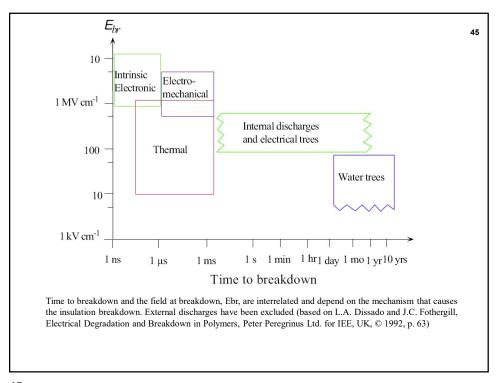
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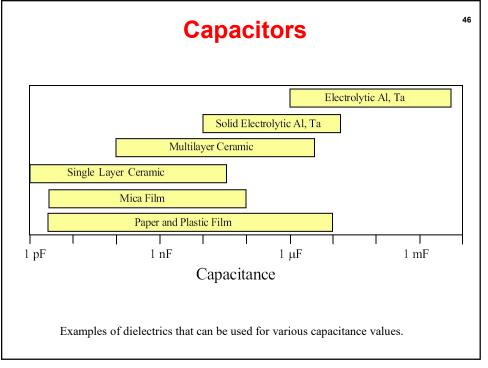
# Dielectric Breakdown: Solid

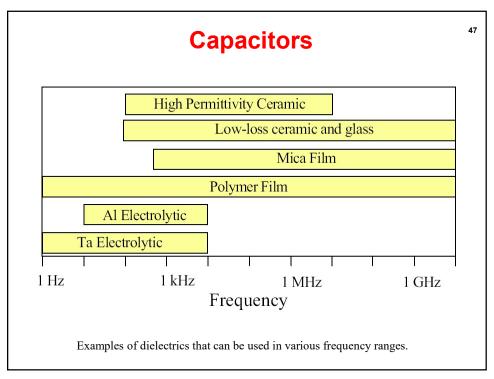
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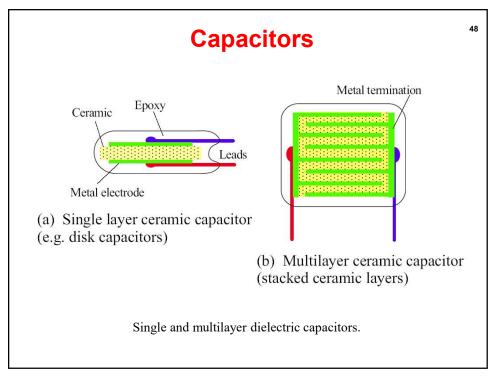


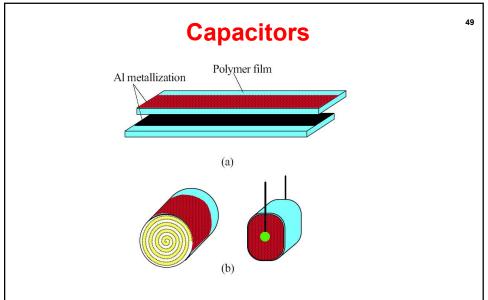
Coaxial cable connector with traces of corona discharge; electrical treeing. SOURCE: M. Mayer and G.H. Schröder , "Coaxial 30 kV Connectors for the RG220/U Cable: 20 Years of Operational Experience" *IEEE Electrical Insulation Magazine* , Vol. 16, March/April 2000, p 11, Figure 6. (© IEEE, 2000)





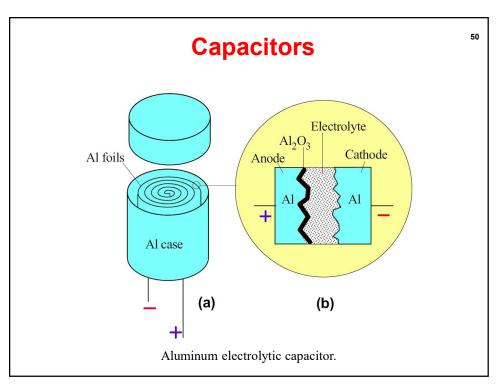


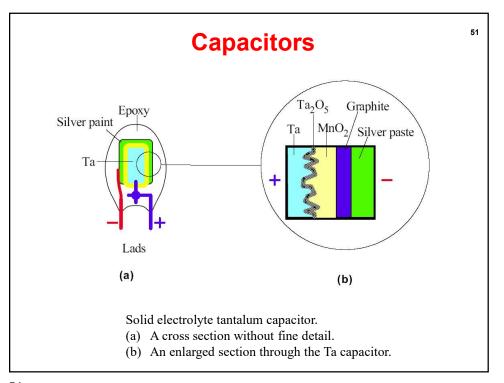




Two polymer tapes in (a), each with a metallized film electrode on the surface (offset from Other), can be rolled together (like a Swiss roll) to obtain a polymer film capacitor as in (b). As the two separate metal films are lined at opposite edges, electroding is done over the whole side surface.

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Capacitor name	Polypropylene	Polyester	Mica	Aluminum, electrolytic	Tantalum, electrolyt ic, solid	High-K ceramic
Dielectric	Polymer film	Polymer film	Mica	Anodized Al <sub>2</sub> O <sub>3</sub> film	Anodized Ta <sub>2</sub> O <sub>5</sub> film	X7R BaTiO <sub>3</sub> base
$\mathcal{E}_{r}'$	2.2 – 2.3	3.2 – 3.3	6.9	8.5	27	2000
$ an\delta$	4 × 10 <sup>-4</sup>	4 × 10 <sup>-3</sup>	2 × 10 <sup>-4</sup>	0.05 - 0.1	0.01	0.01
$\mathcal{E}_{br}$ (kV mm <sup>-1</sup> ) DC	100 - 350	100 - 300	50 - 300	400 - 1000	300 - 600	10
d (typical minimum)	3 - 4 μm	1 μm	2 - 3 μm	0.1 μm	0.1 μm	10 μm
C <sub>vol</sub> (μF cm <sup>-3</sup> )	2	30	15	7,500 <i>a</i>	24,000a	180
$R_p = 1/G_p$ ; $C = 1 \mu F$ ; 1000 Hz	400 kΩ	40 kΩ	800 kΩ	1.5 - 3 kΩ	16 kΩ	16 kΩ
$E_{\rm vol}$ (mJ cm <sup>-3</sup> ) $b$	10	15	8	1000	1200	100
Polarization	Electronic	Electronic and Dipolar	Ionic	Ionic	Ionic	Large ionic displacemen

NOTES: Typical values. h=3 assumed. The table is for comparison purposes only. Breakdown fields are typical DC values, and can vary substantially, by at least an order of magnitude; Ebr depends on the thickness, material quality and the duration of the applied voltage. a Proper volumetric calculations must also consider the volumes of electrodes and the electrolyte necessary for these dielectrics to work; hence the number would have to be decreased. b Evol depends very sensitively on Ebr and the choice of h; hence it can vary substantially. Polyester is PET, or polyethlylene terephthalate. Mica is potassium aluminosilicate, a muscovite crystal. X7R is the name of a particular BaTiO3-based ceramic solid solution.

Capacitance per unit volume

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$$C_{\text{vol}} = \frac{\mathcal{E}_o \mathcal{E}_r}{d^2}$$

 $C_{\text{vol}}$  = capacitance per unit volume,  $\varepsilon_o$  = permittivity of free space,  $\varepsilon_r$  = dielectric constant, d = separation of the capacitor plates

### Maximum energy per unit volume

$$E_{\text{vol}} = \frac{1}{2} C V_m^2 \times \frac{1}{Ad} = \frac{\varepsilon_o \varepsilon_r'}{2\eta^2} \mathcal{E}_{\text{br}}^2$$

 $E_{\mathrm{vol}} = \mathrm{maximum}$  energy stored per unit volume,  $C = \mathrm{capacitance}$ ,  $V_m = \mathrm{maximum}$  voltage,  $A = \mathrm{surface}$  area of the capacitor plates,  $d = \mathrm{separation}$  of the capacitor plates,  $\varepsilon_o = \mathrm{permittivity}$  of free space,  $\varepsilon'_r = \mathrm{real}$  part of the complex dielectric constant,  $\eta = \mathrm{safety}$  factor,  $\varepsilon_{\mathrm{br}} = \mathrm{breakdown}$  electric field

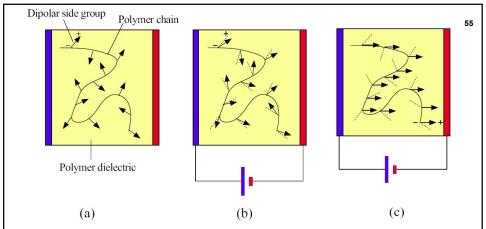
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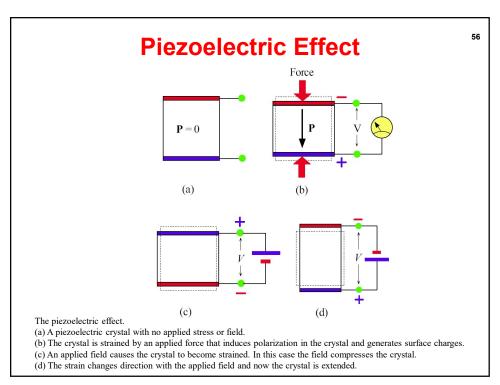
### Dielectric loss per unit volume

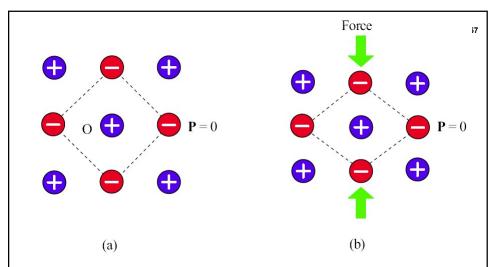
$$W_{\text{vol}} = \frac{\mathcal{E}_{\text{br}}^{2}}{\eta^{2}} \omega \varepsilon_{o} \varepsilon_{r}' \tan \delta$$

 $W_{\rm vol}$  = dielectric loss per unit volume,  $\mathcal{E}_{\rm br}$  = breakdown electric field,  $\eta$  = safety factor,  $\varepsilon_o$  = permittivity of free space,  $\varepsilon_r'$  = real part of the complex dielectric constant,  $\tan \delta$  = loss tangent or loss factor,  $\omega$  = angular frequency of the applied field

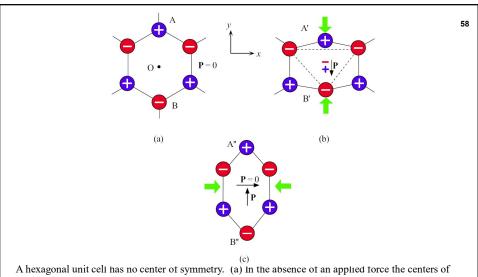


- (a) A polymer dielectric that has dipolar side groups attached to the polymer chains. With no applied field, the dipoles are randomly oriented.
- (b) In the presence of an applied field, some very limited rotation enables dipolar polarization to take place.
- (c) Near the softening temperature of the polymer, the molecular motions are rapid and there is also sufficient volume between chains for the dipoles to align with the field. The dipolar contribution to  $\varepsilon_r$  is substantial, even at high frequencies.





- A NaCl-type cubic unit cell has a center of symmetry.
- (a) In the absence of an applied force, the centers of mass for positive and negative ions coincide.
- (b) This situation does not change when the crystal is strained by an applied force.



A hexagonal unit cell has no center of symmetry. (a) In the absence of an applied force the centers of mass for positive and negative ions coincide. (b) Under an applied force along y the centers of mass for positive and negative ions are shifted which results in a net dipole moment P along y. (c) When the force is along a different direction, along x, there may not be a resulting net dipole moment in that direction though there may be a net P along a different direction (y).

Piezoelectric Effect

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$$P_i = d_{ij} T_j$$

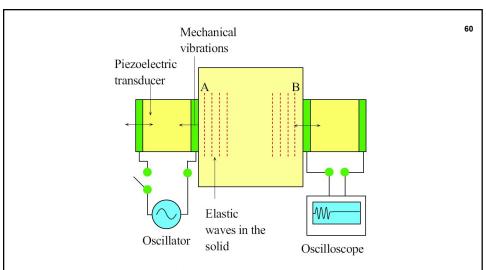
 $P_i$  = induced polarization along the i direction,  $d_{ij}$  = piezoelectric coefficients,  $T_j$  = mechanical stress along the j direction

### **Converse Piezoelectric Effect**

$$S_j = d_{ij} \mathcal{E}_i$$

 $S_j$  = induced stain along the j direction,  $d_{ij}$  = piezoelectric coefficients,  $\mathcal{E}_i$  = electric field along the i direction

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Piezoelectric transducers are widely used to generate ultrasonic waves in solids and also to detect such mechanical waves. The transducer on the left is excited from an ac source and vibrates mechanically. These vibrations are coupled to the solid and generate elastic waves. When the waves reach the other end they mechanically vibrate the transducer on the right which converts the vibrations to an electrical signal.

### **Electro-Mechanical Coupling Factor**

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 $k^2 = \frac{\text{Electricalenergyconverted to mechanical energy}}{\text{Input of electrical energy}}$ 

k = electromechanical coupling factor

### **Electro-Mechanical Coupling Factor**

 $k^2 = \frac{\text{Mechanical energy converted to electrical energy}}{\text{Input of mechanical energy}}$ 

k = electromechanical coupling factor

61



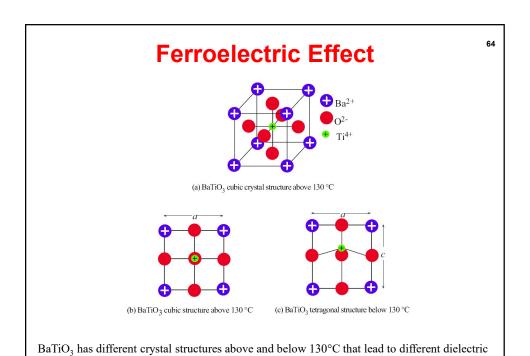


Piezoelectric transducers |SOURCE: Photo by SOK

**Table 7.8** Piezoelectric materials and some typical values for d and k

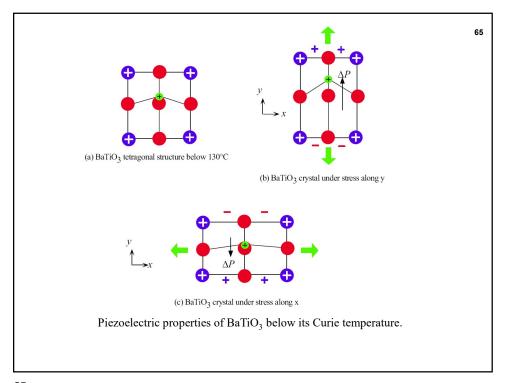
Crystal	$d~({\rm m~V^{-1}})$	$\boldsymbol{k}$	Comment
Quartz (crystal SiO <sub>2</sub> )	$2.3 \times 10^{-12}$	0.1	Crystal oscillators, ultrasonic transducers, delay lines, filters
Rochelle salt (NaKC <sub>4</sub> H <sub>4</sub> O <sub>6</sub> · 4H <sub>2</sub> O)	$350\times10^{-12}$	0.78	
Barium titanate (BaTiO <sub>3</sub> )	$190\times 10^{-12}$	0.49	Accelerometers
PZT, lead zirconate titanate (PbTi <sub>1-x</sub> Zr <sub>x</sub> O <sub>3</sub> )	$480 \times 10^{-12}$	0.72	Wide range of applications including earphones, microphones spark generators (gas lighters, car ignition), displacement transducers, accelerometers
Polyvinylidene fluoride (PVDF)	$18 \times 10^{-12}$	_	Must be poled; heated, put in an electric field and then cooled.  Large area and inexpensive

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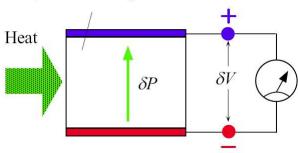
properties.



# **Pyroelectric Effect**

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Temperature change =  $\delta T$ 



The heat absorbed by the crystal increases the temperature by  $\Delta T$  which induces a change  $\Delta P$  in the polarization. This is the pyroelectric effect. The change  $\Delta P$  gives rise to a change  $\Delta V$  in the voltage which can be measured.

**Pyroelectric Coefficient** 

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$$p = \frac{dP}{dT}$$

p = pyroelectric coefficient, P = polarization, T = temperature

# **Pyroelectric Current Density**

$$J_p = \frac{dP}{dt} = p \frac{dT}{dt}$$

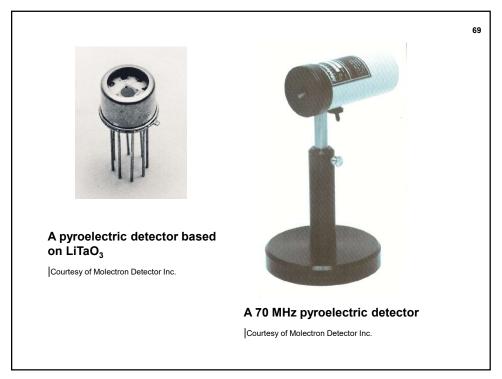
 $J_p$  = pyroelectric current density, dT/dt = rate of change of temperature

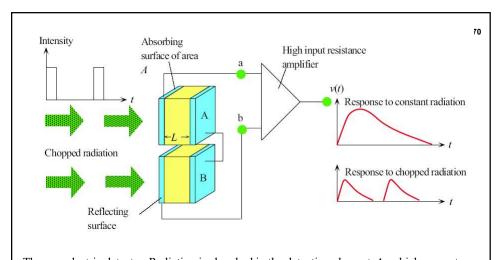
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Table 7.9 Some pyroelectric (and also ferroelectric) crystals and typical properties

Material	$arepsilon_r'$	$ an \delta$	$\begin{array}{c} Pyroelectric \\ Coefficient \\ (\times 10^{-6}~C~m^{-2}~K^{-1}) \end{array}$	Curie Temperature (°C)
BaTiO <sub>3</sub>	4100 ⊥ polar axis; 160 //polar axis	$7 \times 10^{-3}$	20	130
LiTaO <sub>3</sub>	47	$5 \times 10^{-3}$	230	610
PZT modified for pyroelectric	290	$2.7 \times 10^{-3}$	380	230
PVDF, polymer	12	0.01	27	80





The pyroelectric detector. Radiation is absorbed in the detecting element, A, which generates a pyroelectric voltage that is measured by the amplifier. The second element, B, has a reflecting electrode and does not absorb the radiation. It is a dummy element that compensates for the piezoelectric effects. Piezoelectric effects generate equal voltages in both A and B which across a and b (the input of the amplifier) cancel each other.

Pyroelectric Detector Output Voltage (Rough Estimate)

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$$\Delta V = \frac{p \, \mathcal{I} \Delta t}{\rho c \, \varepsilon_r \varepsilon_o} = \left(\frac{p}{\rho c \, \varepsilon_r \varepsilon_o}\right) \mathcal{I} \Delta t$$

 $\Delta V$  = voltage change, p = pyroelectric coefficient,  $\mathcal{I}$  = incident radiation intensity,  $\Delta t$  = time interval (e.g. irradiation time in chopping input radiation),  $\rho$  = density, c = specific heat capacity (heat capacity per unit mass),  $\varepsilon_o$  = permittivity of free space,  $\varepsilon_r$  = relative permittivity

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### **Pyroelectric Responsivity (R)**

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$$R_I = \frac{\text{Pyroelectric current generated}}{\text{Input radiation power}}$$

 $R_I$  = Pyroelectric current responsivity

$$R_{V} = \frac{\text{Pyroelectric output votage generated}}{\text{Input radiation power}}$$

 $R_V$  = Pyroelectric voltage responsivity

# Nanogenerators for Energy Harvesting Piezoelectric Nanogenerators: Triboelectric Nanogenerators: