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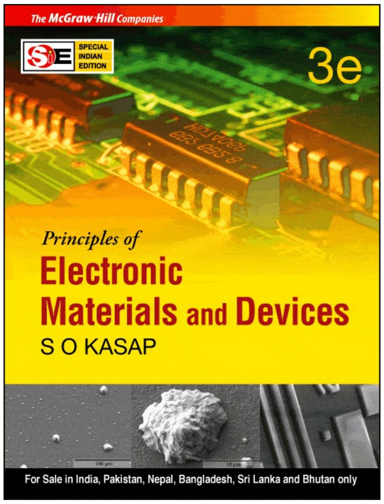

Chapter 8

Magnetic Properties and Superconductivity

1

Atomic Magnetic Moments

Orbital magnetic moment of the electron

$$\mu_{\text{orb}} = -\frac{e}{2m_e}L$$

Orbital angular momentum

Spin magnetic moment of the electron

$$\mu_{\text{spin}} = -\frac{e}{m_e}S$$

Intrinsic angular momentum

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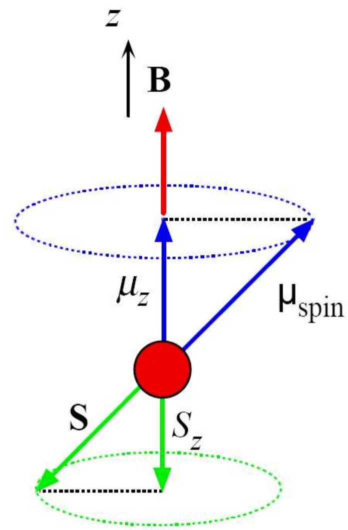

The spin magnetic moment processes about an external magnetic field along *z* and has an average value of μ_z along *z*.

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Atomic Magnetic Moments

Magnetic moment along the field

$$\mu_z = -\frac{e}{m_e} S_z = -\frac{e}{m_e}(m_s \hbar) = \frac{e\hbar}{2m_e} = \beta$$

The quantity $\beta = \frac{e\hbar}{2m_e}$ is called the **Bohr magneton** and has the value 9.27×10^{-24} A m^2 or J T^{-1}.

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Atomic Magnetic Moments

Magnetization and surface currents

Total magnetic moment = (Total current) × (Cross-sectional area) = $I_m \ell A$

Equating the two total magnetic moments, we find

$$M = I_m$$

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Magnetic Permeability

Definition of magnetic permeability

Magnetic field

$$\mu = \frac{B}{H}$$

Magnetizing field

Definition of relative permeability

$$\mu_r = \frac{B}{B_o} = \frac{B}{\mu_o H}$$

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Magnetic Susceptibility

Definition of magnetic susceptibility

$$\mathbf{M} = \chi_m \mathbf{H}$$

Relative permeability and susceptibility

$$\mu_r = 1 + \chi_m$$

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Table 8.1 Magnetic quantities and their units

Magnetic Quantity	Symbol	Definition	Units	Comment
Magnetic field; magnetic induction	$\mathbf{B}$	$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$	T = tesla = webers m^{-2}	Produced by moving charges or currents, acts on moving charges or currents.
Magnetic flux	Φ	$\Delta\Phi = B_{normal}\,\Delta A$	Wb = weber	$\Delta\Phi$ is flux through ΔA and B_{normal} is normal to ΔA. Total flux through any closed surface is zero.
Magnetic dipole moment	$\boldsymbol{\mu}_m$	$\mu_m = IA$	A m^2	Experiences a torque in $\mathbf{B}$ and a net force in a nonuniform $\mathbf{B}$.
Bohr magneton	β	$\beta = e\hbar/2m_e$	A m^2 or J T^{-1}	Magnetic moment due to the spin of the electron. $\beta = 9.27 \times 10^{-24}$ A m^2
Magnetization vector	$\mathbf{M}$	Magnetic moment per unit volume	A m^{-1}	Net magnetic moment in a material per unit volume.
Magnetizing field; magnetic field intensity	$\mathbf{H}$	$\mathbf{H} = \mathbf{B}/\mu_o - \mathbf{M}$	A m^{-1}	$\mathbf{H}$ is due to external conduction currents only and is the cause of $\mathbf{B}$ in a material.
Magnetic susceptibility	χ_m	$\mathbf{M} = \chi_m\mathbf{H}$	None	Relates the magnetization of a material to the magnetizing field $\mathbf{H}$.
Absolute permeability	μ_o	$c = [\varepsilon_o\mu_o]^{-1/2}$	H m^{-1} = Wb m^{-1} A^{-1}	A fundamental constant in magnetism. In free space, $\mu_o = B/H$.
Relative permeability	μ_r	$\mu_r = B/\mu_o H$	None	
Magnetic permeability	μ	$\mu = \mu_o\mu_r$	H m^{-1}	Not to be confused with magnetic moment.
Inductance	L	$L = \Phi_{total}/I$	H (henries)	Total flux threaded per unit current.
Magnetostatic energy density	E_{vol}	$dE_{vol} = H\,dB$	J m^{-3}	dE_{vol} is the energy required per unit volume in changing B by dB.

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Table 8.2 Classification of magnetic materials

Type	χ_m (typical values)	χ_m versus T	Comments and Examples
Diamagnetic	Negative and small (-10^{-6})	T independent	Atoms of the material have closed shells. Organic materials, *e.g.*, many polymers; covalent solids, *e.g.*, Si, Ge, diamond; some ionic solids, *e.g.*, alkalihalides; some metals, *e.g.*, Cu, Ag, Au.
	Negative and large (-1)	Below a critical temperature	Superconductors
Paramagnetic	Positive and small (10^{-5}–10^{-4})	Independent of T	Due to the alignment of spins of conduction electrons. Alkali and transition metals.
	Positive and small (10^{-5})	Curie or Curie–Weiss law, $\chi_m = C/(T - T_C)$	Materials in which the constituent atoms have a permanent magnetic moment, *e.g.*, gaseous and liquid oxygen; ferromagnets (Fe), antiferromagnets (Cr), and ferrimagnets (Fe_3O_4) at high temperatures.
Ferromagnetic	Positive and very large	Ferromagnetic below and paramagnetic above the Curie temperature	May possess a large permanent magnetization even in the absence of an applied field. Some transition and rare earth metals, Fe, Co, Ni, Gd, Dy.
Antiferromagnetic	Positive and small	Antiferromagnetic below and paramagnetic above the Néel temperature	Mainly salts and oxides of transition metals, *e.g.*, MnO, NiO, MnF_2, and some transition metals, α–Cr, Mn.
Ferrimagnetic	Positive and very large	Ferrimagnetic below and paramagnetic above the Curie temperature	May possess a large permanent magnetization even in the absence of an applied field. Ferrites.

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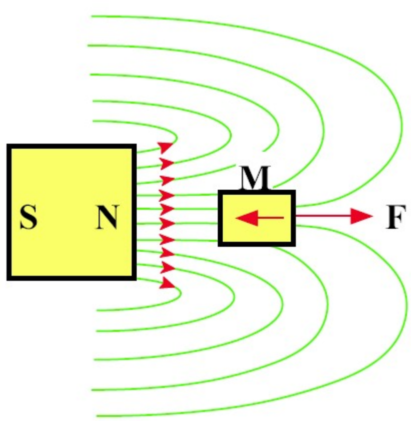
Diamagnetic Materials



A diamagnetic material placed in a non-uniform magnetic field experiences a force towards smaller fields. This repels the diamagnetic material away from a permanent magnet.

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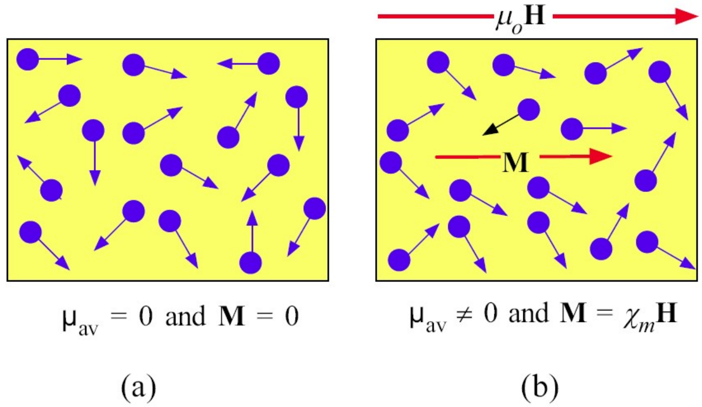
Paramagnetic Materials



(a) In a paramagnetic material each individual atom possesses a permanent magnetic moment due to thermal agitation there is no average moment per atom and **M** = 0.
(b) In the presence of an applied field, individual magnetic moments take alignments along the applied field and **M** is finite and along **B**.

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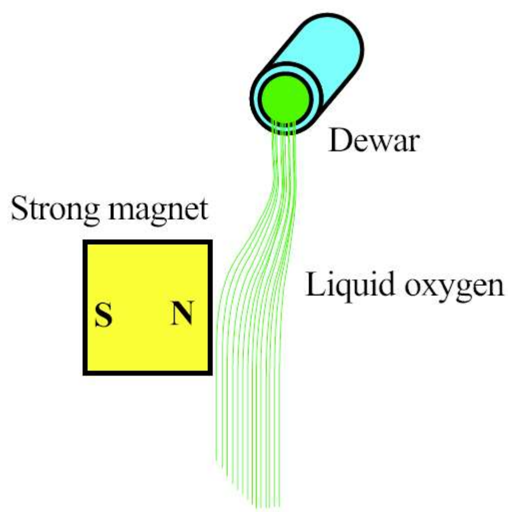

A paramagnetic material placed in a non-uniform magnetic field experiences a force towards greater fields. This attracts the paramagnetic material (e.g. liquid oxygen) towards a permanent magnet.

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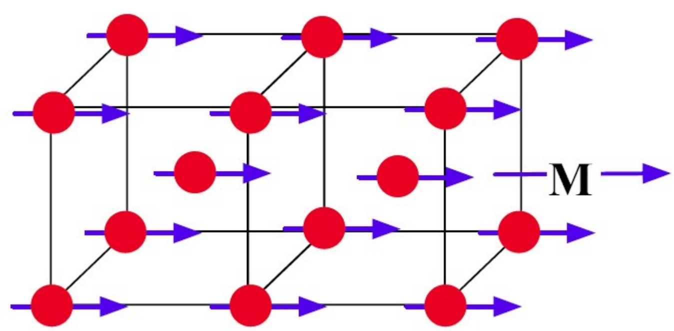
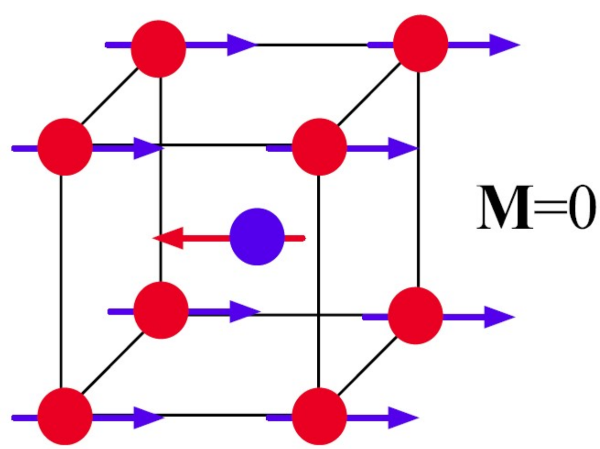
Ferromagnetic Materials



In a magnetized region of a ferromagnetic material such as iron all the magnetic moments are spontaneously aligned in the same direction. There is a strong magnetization vector **M** even in the absence of an applied field.

Antiferromagnetic Materials



In this antiferromagnetic BCC crystal (Cr) the magnetic moment of the center atom is cancelled by the magnetic moments of the corner atoms (an eighth of the corner atom belongs to the unit cell).

Ferrimagnetic Materials

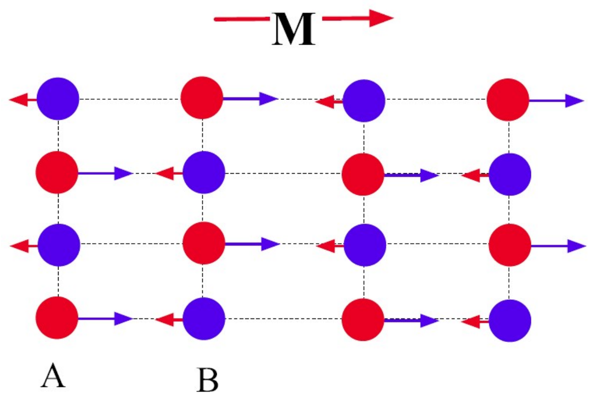


Illustration of magnetic ordering in a ferrimagnetic crystal. All A-atoms have their spins Aligned in one direction an all B-atoms have their spins aligned in the opposite direction. As The magnetic moment of an A-atom is greater than that of a B-atom, there is net magnetization, **M**, in the crystal.

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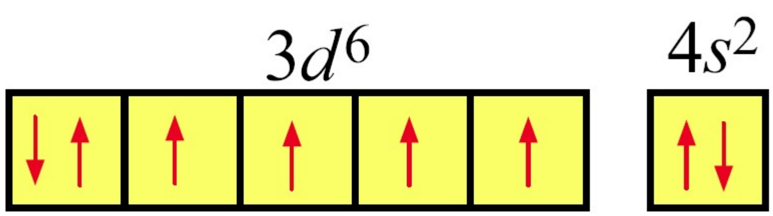
The isolated Fe atom has a spin magnetic moment of 4β



The isolated Fe atom has 4 unpaired spins and a spin magnetic moment of 4β

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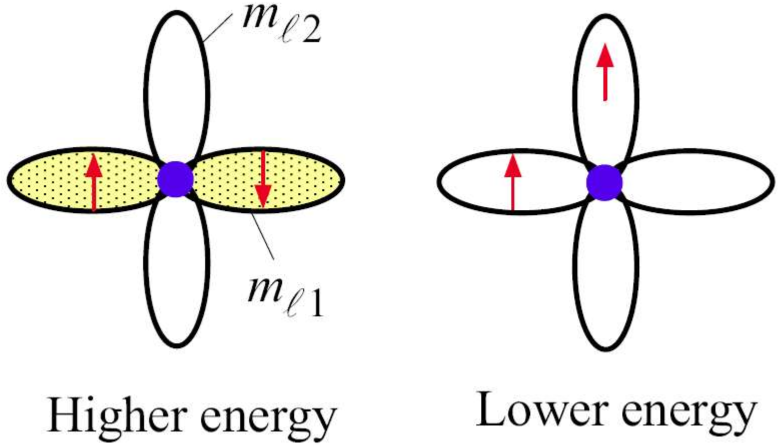

Hund's rule for an atom with many electrons is based on the *exchange interaction*.

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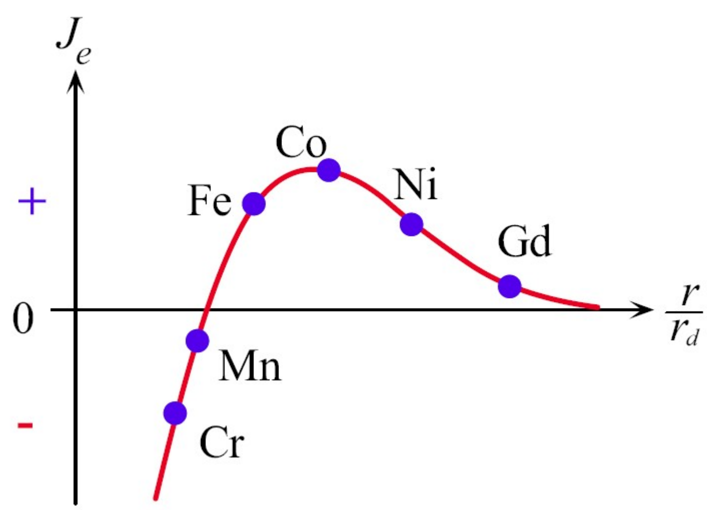

The exchange integral as a function of r/r_d, where r is the interatomic distance and r_d the radius of the *d*-orbit (or the average *d*-subshell radius. Cr to Ni are transition metals. For Gd, the *x*-axis is r/r_f where r_f is the radius of the *f*-orbit.

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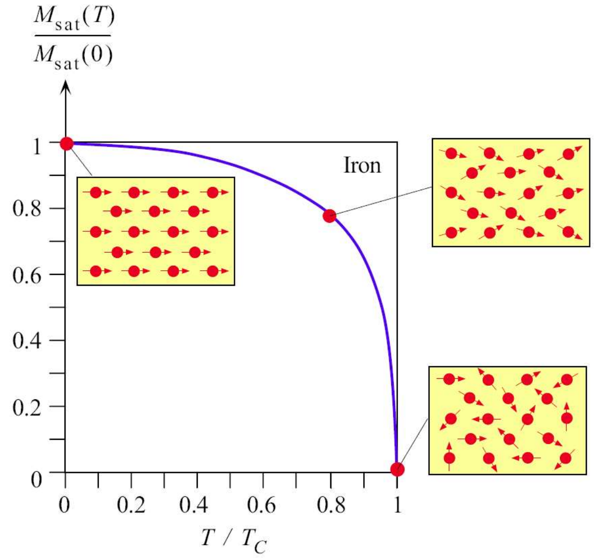

Normalized saturated magnetization vs. reduced temperature T/T_C where T_C is the Curie temperature (1043 K).

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Table 8.3 Properties of the ferromagnets Fe, Co, Ni, and Gd

	Fe	Co	Ni	Gd
Crystal structure	BCC	HCP	FCC	HCP
Bohr magnetons per atom	2.22	1.72	0.60	7.1
$M_{sat}(0)$ (MA m^{-1})	1.75	1.45	0.50	2.0
$B_{sat} = \mu_o M_{sat}(T)$	2.2	1.82	0.64	2.5
T_C	770 °C	1127 °C	358 °C	16 °C
	1043 K	1400 K	631 K	289 K

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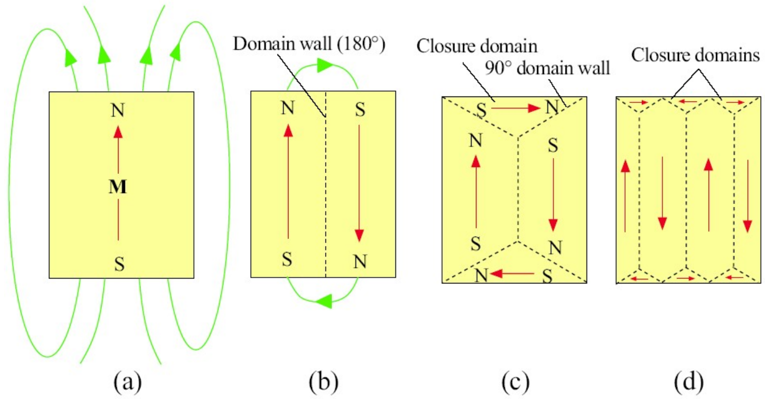

(a) Magnetized bar of ferromagnet in which there is only one domain and hence an external magnetic field.
(b) Formation of tow domains with opposite magnetizations reduces the external field. There are, however, field lines at the ends.
(c) Domains of closure fitting at the ends eliminates the external fields at the ends.
(d) A speciment with several domains and closure domains. There is not external Magnetic field and the specimen appear sunmagnetized.

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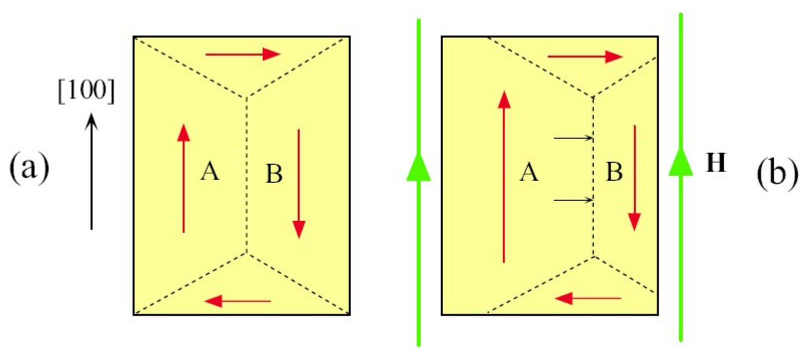

(a) An unmagnetized crystal of iron in the absence of an applied magnetic field. Domains A and B are the same size and have opposite magnetizations.
(b) When an external field is applied the domain wall migrates into domain B which enlarges A and B. The result is that the specimen now acquires net magnetization.

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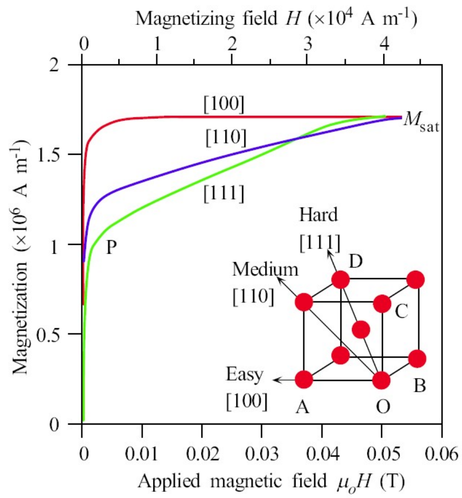

Magnetocrystalline anisotropy in a single iron crystal. *M* vs. *H* depends on the crystal on the crystal direction and is easiest along [100] and hardest along [111]

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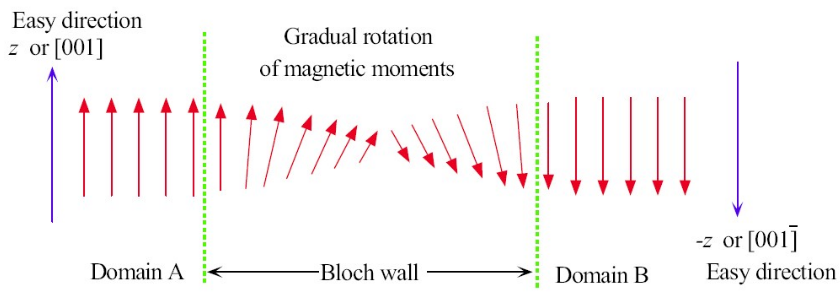

In a Bloch wall the neighboring spin magnetic moments rotate gradually and it takes several hundred atomic spacings to rotate the magnetic moment by 180.

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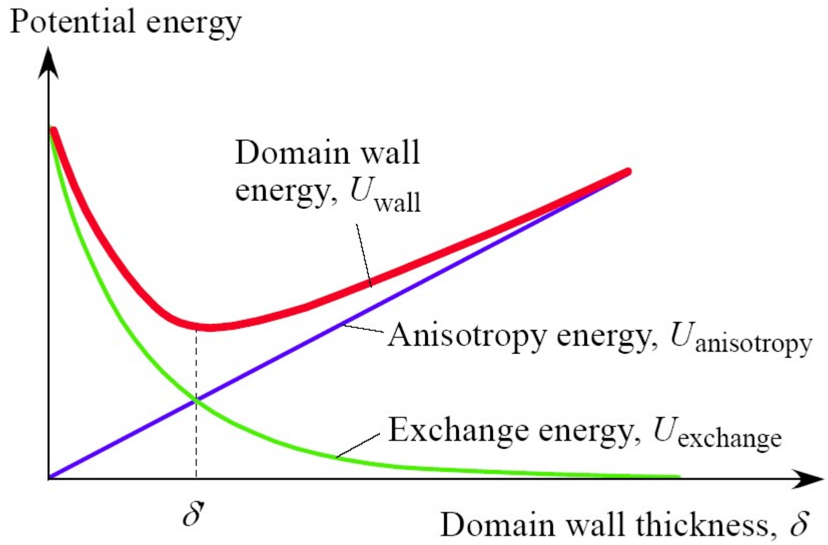

Potential energy of a domain wall depends on the exchange and anisotropy energies

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Bloch Wall

Potential energy of a Bloch wall depends on its thickness δ

$$U_{\text{wall}} \approx \frac{\pi^2 E_{\text{ex}}}{2a\delta} + K\delta$$

Optimal Bloch wall thickness minimizes the total PE of the wall, U_{wall}

$$\delta' = \left(\frac{\pi^2 E_{\text{ex}}}{2a\delta} \right)^{1/2}$$

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Magnetostriction

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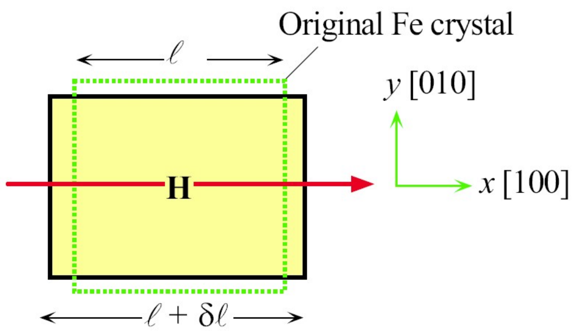

Magnetostriction means that the iron crystal in a magnetic field along *x*, an easy direction, elongates along x but contracts in the transverse directions.

Magnetostrictive constant : In the range of $10^{-5} – 10^{-6}$
Could be very large > 10^{-4}, for Co-ferrite
Either positive (Fe) or negative (Ni) or zero – if properly alloyed (Ni-15%Fe)

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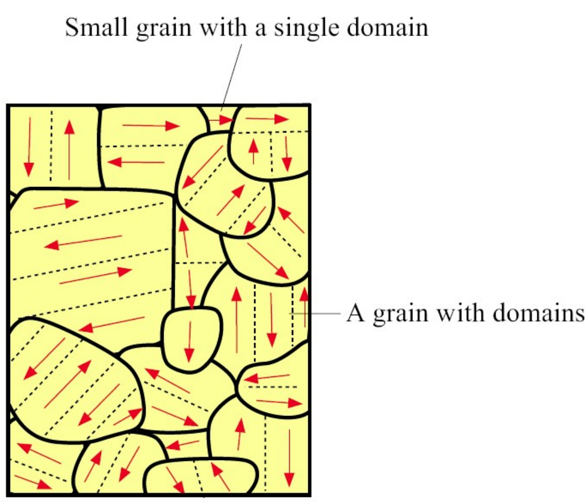

Schematic illustration of magnetic domains in the grains of an unmagnetized polycrystalline iron sample. Very small grains have single domains.

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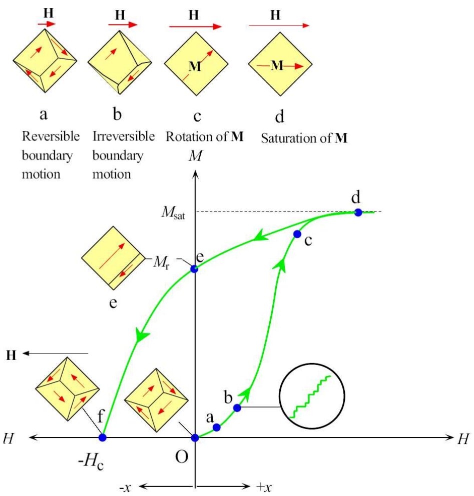

M vs. H behavior of a previously unmagnetized polycrystalline iron specimen. An example grain in the unmagnetized specimen is that at O. (a) Under very small fields the domain boundary motion is reversible. (b) The boundary motions are irreversible and occur in sudden jerks. (c) Nearly all the grains are single domains with saturation magnetizations in the easy directions. (d) Magnetizations in individual grains have to be rotated to align with the field, H. (e) When the field is removed the specimen returns along d to e. (f) To demagnetize the specimen we have to apply a magnetizing field of H_c in the reverse direction.

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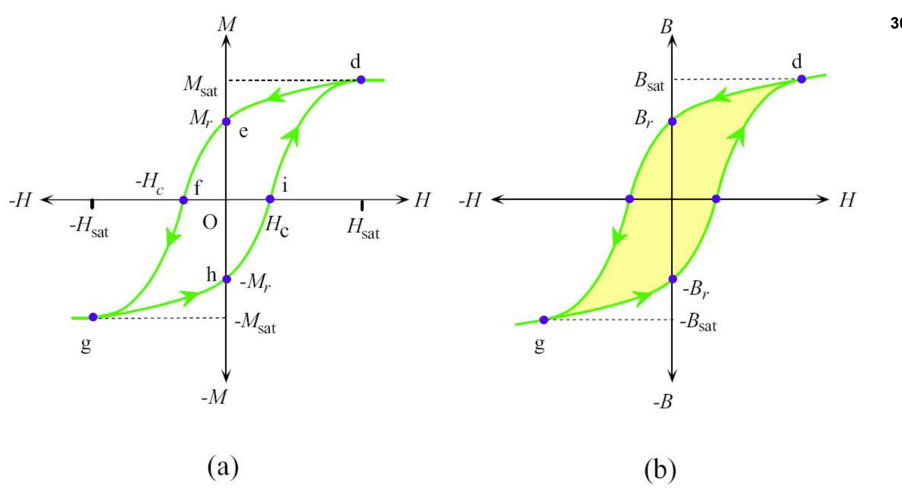

(a) A typical M vs. H hysterisis curve
(b) The corresponding B vs. H hysterisis curve. The shaded area inside the hysterisis loop Is the energy loss per unit volume per cycle.

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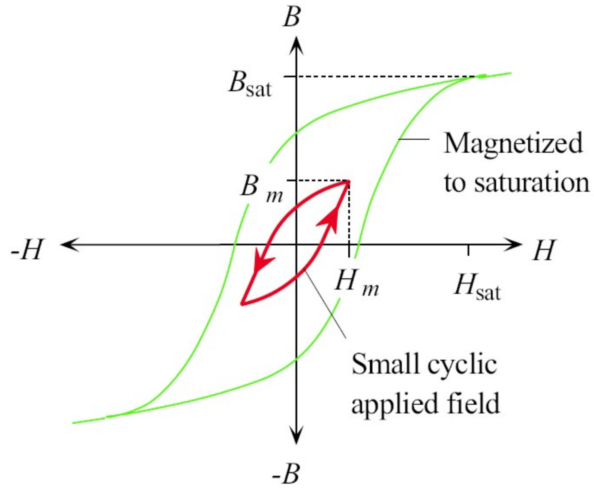

The B vs. H hysterisis loop depends on the magnitude of the applied field in addition to the Material and sample and size.

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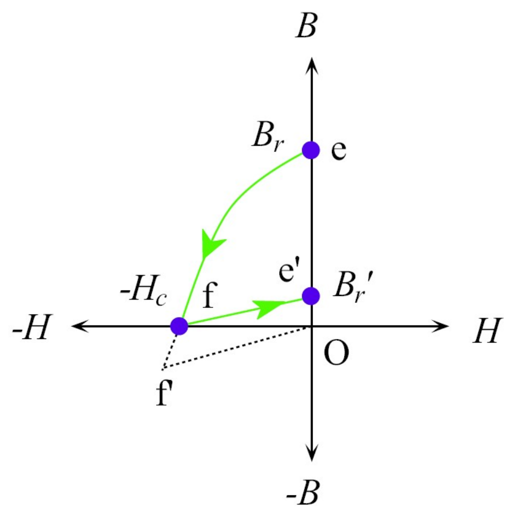

Removal of the demagnetizing field at f does not necessarily result in zero magnetization as the sample recovers along f-e'

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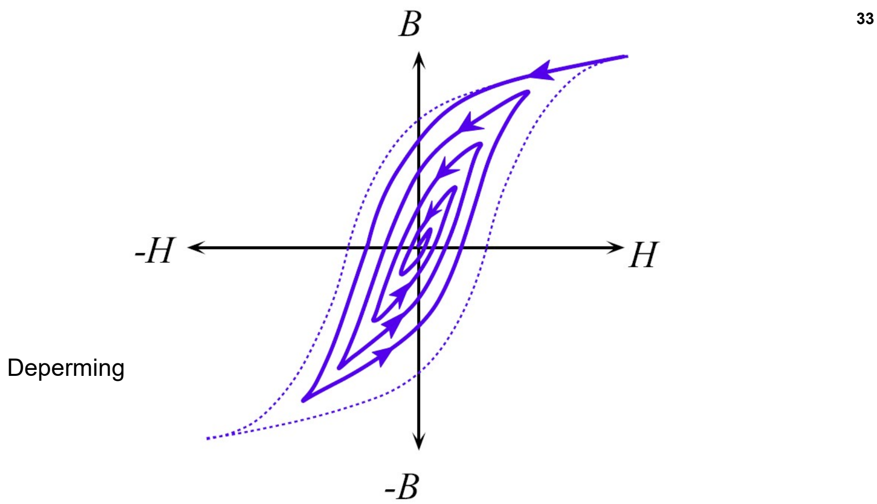

A magnetized specimen can be demagnetized by cycling the field intensity with a decreasing magnitude, i.e. tracing out smaller and smaller *B*-*H* loops until the origin is reached, *H*=0.

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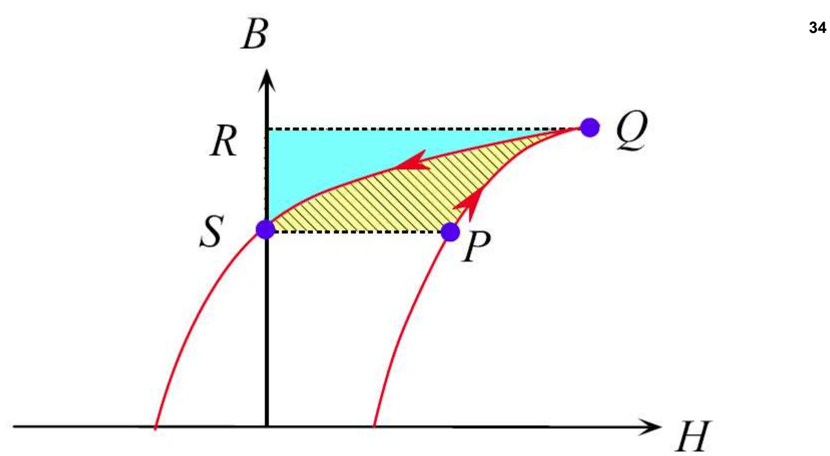

The area between the *B*-*H* curve and the *B*-axis is the energy absorbed per unit volume in magnetization or released during demagnetization.

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Hysteresis

Hysteresis power loss per unit volume

$n = 1.6$

$$P_h = K f B_m^n$$

Constant ≈ 150.7

Frequency

Maximum magnetic field

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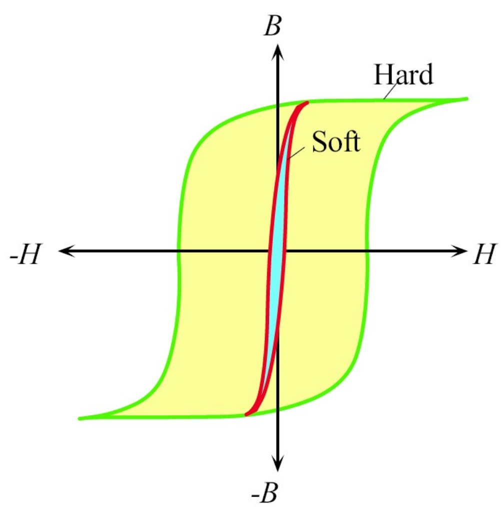

Soft and hard magnetic materials

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Table 8.5 Selected soft magnetic materials and some typical values and applications

Magnetic Material	$\mu_o H_c$ (T)	B_{sat} (T)	B_r (T)	μ_{ri}	$\mu_{r,max}$	W_h	Typical Applications
Ideal soft	0	Large	0	Large	Large	0	Transformer cores, inductors, electric machines, electromagnet cores, relays, magnetic recording heads.
Iron (commercial) grade, 0.2% impurities)	$<10^{-4}$	2.2	<0.1	150	10^4	250	Large eddy current losses. Generally not preferred in electric machinery except in some specific applications (e.g., some electromagnets and relays).
Silicon iron (Fe: 2–4% Si)	$<10^{-4}$	2.0	0.5–1	10^3	10^4–4×10^5	30–100	Higher resistivity and hence lower eddy current losses. Wide range of electric machinery (e.g., transformers).
Supermalloy (79% Ni–15.5% Fe–5% Mo–0.5% Mn)	2×10^{-7}	0.7–0.8	<0.1	10^5	10^6	<0.5	High permeability, low-loss electric devices, e.g., specialty transformers, magnetic amplifiers.
78 Permalloy (78.5% Ni–21.5% Fe)	5×10^{-6}	0.86	<0.1	8×10^3	10^5	<0.1	Low-loss electric devices, audio transformers, HF transformers, recording heads, filters.
Glassy metals, Fe–Si–B	2×10^{-6}	1.6	$<10^{-6}$	—	10^5	20	Low-loss transformer cores.
Ferrites, Mn–Zn ferrite	10^{-5}	0.4	<0.01	2×10^3	5×10^3	<0.01	HF low-loss applications. Low conductivity ensures negligible eddy current losses. HF transformers, inductors (e.g., pot cores, E and U cores), recording heads.

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Table 8.6 Hard magnetic materials and typical values

Magnetic Material	$\mu_o H_c$ (T)	B_r (T)	$(BH)_{max}$ (kJ m^{-3})	Examples and Uses
Ideal hard	Large	Large	Large	Permanent magnets in various applications.
Alnico (Fe–Al–Ni–Co–Cu)	0.19	0.9	50	Wide range of permanent magnet applications.
Alnico (Columnar)	0.075	1.35	60	
Strontium ferrite (anisotropic)	0.3–0.4	0.36–0.43	24–34	Starter motors, dc motors, loudspeakers, telephone receivers, various toys.
Rare earth cobalt, *e.g.*, Sm_2Co_{17} (sintered)	0.62–1.1	1.1	150–240	Servo motors, stepper motors, couplings, clutches, quality audio headphones.
NdFeB magnets	0.9–1.0	1.0–1.2	200–275	Wide range of applications, small motors (*e.g.*, in hand tools), walkman equipment, CD motors, MRI body scanners, computer applications.
Hard particles, γ-Fe_2O_3	0.03	0.2		Audio and video tapes, floppy disks.

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This small neodymium-iron-boron permanent magnet (diameter about the same as one-cent coin) is capable of lifting up to 10 pounds. Nd-Fe-B magnets typically have large $(BH)_{max}$ values (200–275 kJ m^{-3}).

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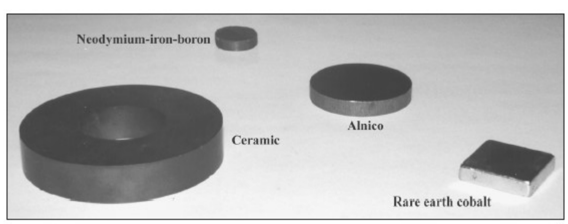

Permanent magnets; Ceramic, outside diameter 7 cm; Nd-Fe-B, diameter 2 cm; Alnico, diameter 4 cm; rare earth cobalt.

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Hard Magnetic Materials

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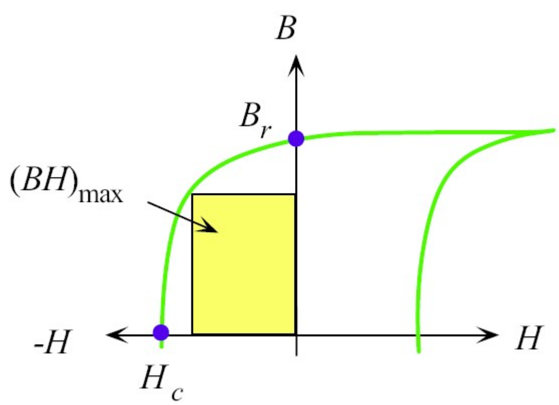

Hard magnetic materials and $(BH)_{max}$.

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A Single Domain Fine Particle

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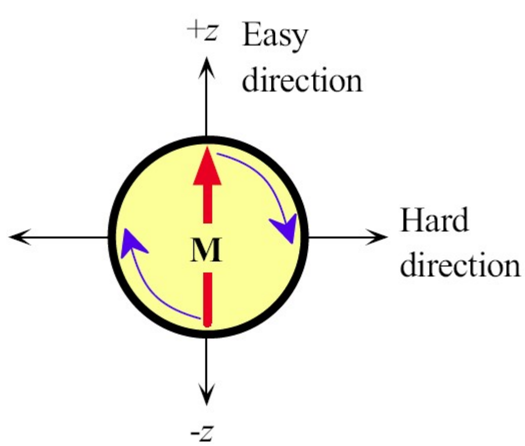

A single domain fine particle.

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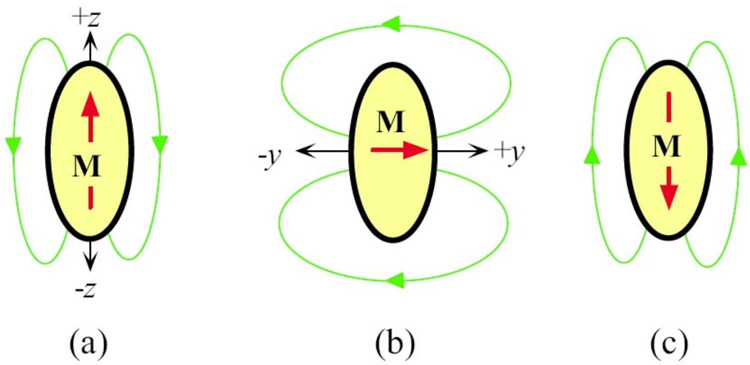
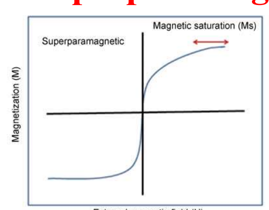
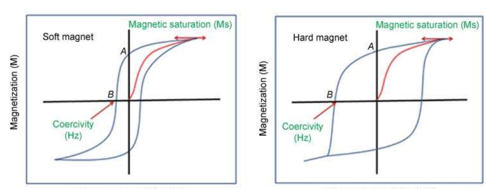
Shape Anisotropy



A single domain elongated particle. Due to shape anisotropy, magnetization prefers to be along The long axis as in (a) Work has to be done to change **M** from (a) to (b).

Superparamagnetism





Superparamagnetism is a form of magnetism which appears in small ferromagnetic or ferrimagnetic nanoparticles. In sufficiently small nanoparticles, magnetization can randomly flip direction under the influence of temperature. The typical time between two flips is called the Néel relaxation time. In the absence of an external magnetic field, when the time used to measure the magnetization of the nanoparticles is much longer than the Néel relaxation time, their magnetization appears to be in average zero; they are said to be in the superparamagnetic state. In this state, an external magnetic field is able to magnetize the nanoparticles, similarly to a paramagnet. However, their magnetic susceptibility is much larger than that of paramagnets.

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Superconductors

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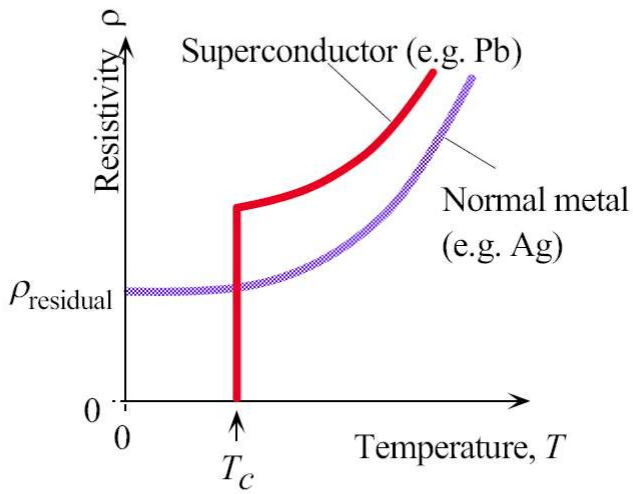

A superconductor such as lead evinces a transition to zero resistvity at a critical temperature T_c (7.2 K for Pb) whereas a normal conductor such as silver does not, and exhibits residual resistivity at the lowest temperatures.

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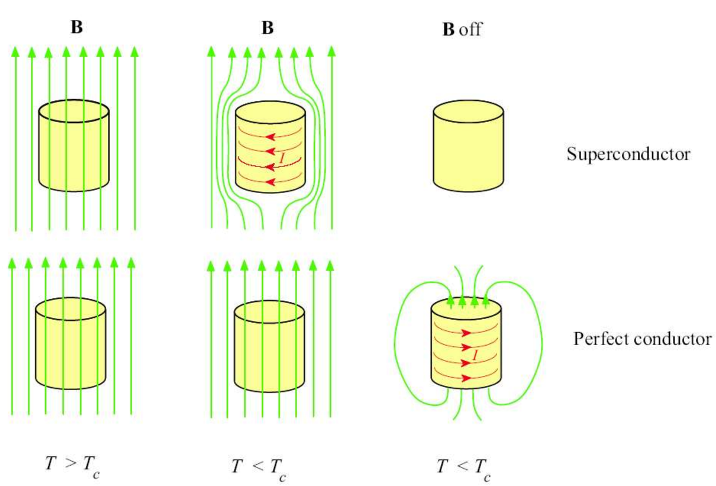

The Meissner effect. A superconductor cooled below its critical temperature expels all magnetic field lines from the bulk by setting up a surface current. A perfect conductor ($\sigma = \infty$) shows no Meissner effect.

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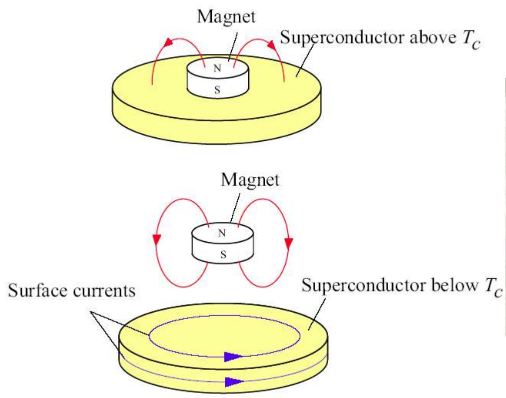

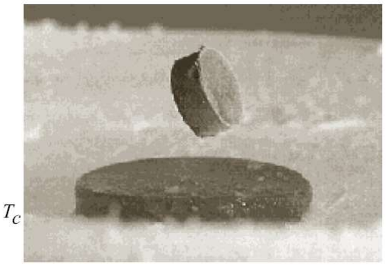

Left: A magnet over a superconductor becomes levitated. The superconductor is a perfect Diamagnet which means that there can be no magnetic field inside the superconductor. Right: Photograph of a magnet levitating above a superconductor immersed in liquid nitrogen (77 K). This is the Meissner effect. (SOURCE: Photo courtesy of Professor Paul C.W. Chu.)

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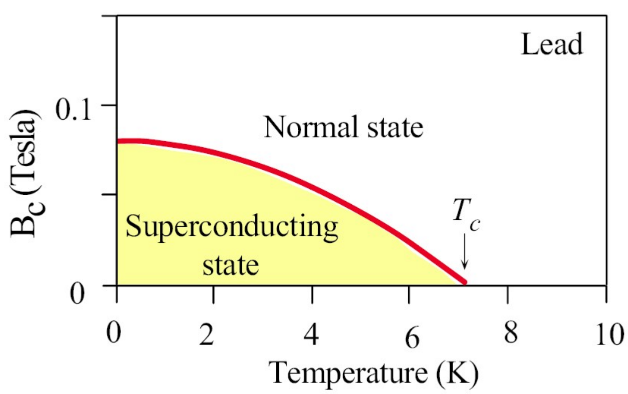
Type I Superconductor



The critical field vs. temperature in Type I superconductors.

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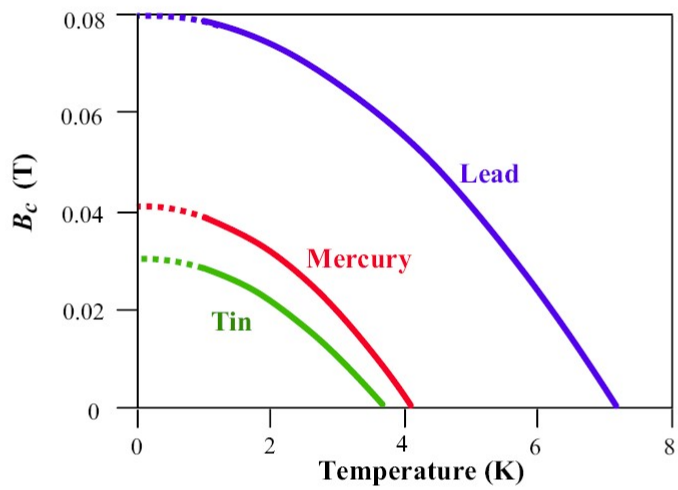
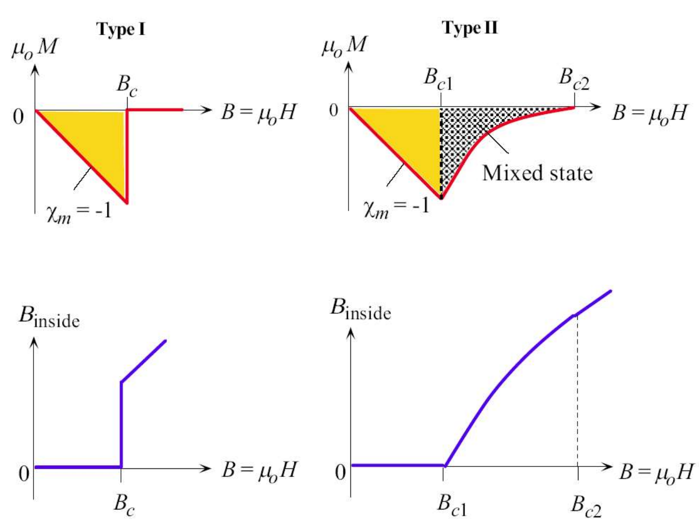
Type I Superconductor



The critical field vs. temperature in three examples of Type I superconductors.



Characteristics of Type I and Type II superconductors. $B = \mu_o H$ is the applied field and M is the overall magnetization of the sample. Field inside the sample, $B_{\text{inside}} = \mu_o H + \mu_o M$, which is zero only for $B < B_c$ (Type I) and $B < B_{c1}$ (Type II).

Type II Superconductor

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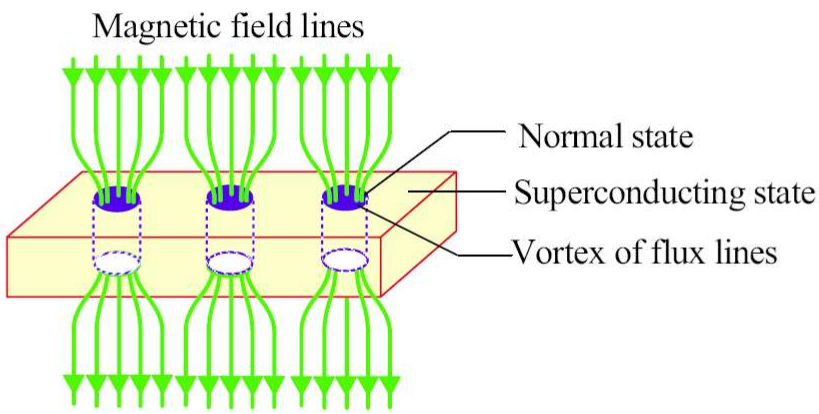

The mixed or vortex state in a Type II superconductor.

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Type II Superconductor

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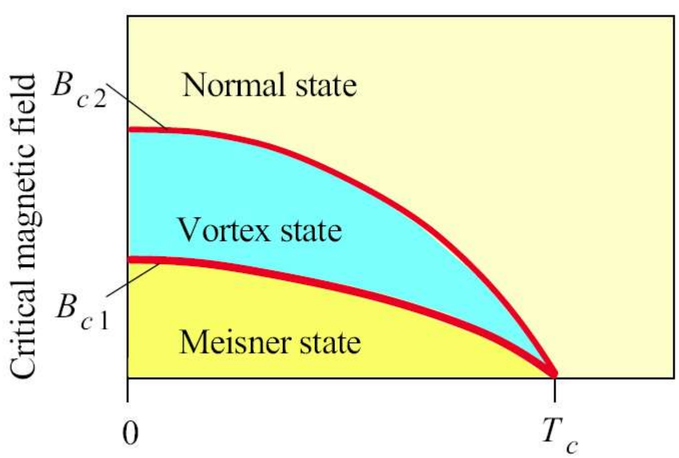

Temperature dependence of B_{c1} and B_{c2}.

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Type II Superconductor

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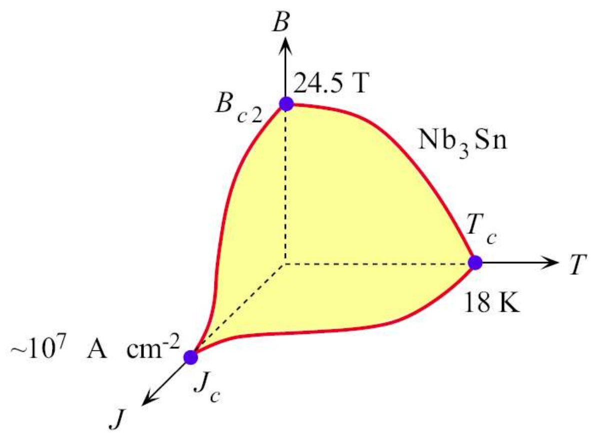

The critical surface for a niobium-tin alloy which is a Type II superconductor.

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Table 8.7 Examples of Type I and Type II superconductors

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Type I	Sn	Hg	Ta	V	Pb	Nb
T_c (K)	3.72	4.15	4.47	5.40	7.19	9.2
B_c (T)	0.030	0.041	0.083	0.14	0.08	0.198

Type II	Nb_3Sn	Nb_3Ge	$Ba_{2-x}Br_xCuO_4$	Y–Ba–Cu–O ($YBa_2Cu_3O_7$)	Bi–Sr–Ca–Cu–O ($Bi_2Sr_2Ca_2Cu_3O_{10}$)	Hg–Ba–Ca–Cu–O
T_c (K)	18.05	23.2	30–35	93–95	122	130–135
B_{c2} (Tesla) at 0 K	24.5	38	~150	~300		
J_c (A cm^{-2}) at 0 K	~10^7			10^4–10^7		

In 1986 J. George Bednorz (right) and K. Alex Müller, at IBM Research Laboratories in Zurich, discovered that a copper oxide based ceramic-type compound (La-Ba-Cu-O) which normally has high resistivity becomes superconducting when ooled below 35 K This Nobel prize winning discovery opened a new era of hightemperature-superconductivity research; now there are various ceramic compounds that are superconducting above the liquid nitrogen (an inexpensive cryogen) temperature (77 K).
|SOURCE: IBM Zürich Research Laboratories.

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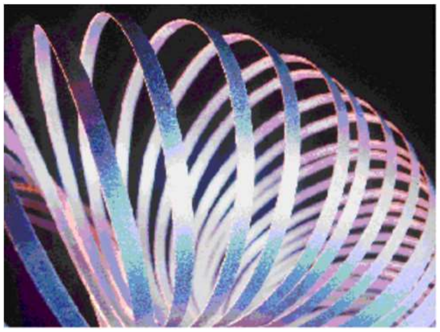

These high temperature superconductor (HTS) flat tapes are based on $(Bi_{2-x}Pb_x)Sr_2Ca_2Cu_3O_{10-d}$(Bi-2223). The tape has an outer surrounding protective metallic sheath. Right: HTS tapes having ac power loss below 10 mW/m have a major advantage over equivalent-sized metal conductors, in being able to transmit considerably higher power loads. Coils made from HTS tape can be used to create more compact and efficient motors, generators, magnets, transformers and energy storage devices.

| SOURCE: Courtesy of Australian Superconductors.

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A Cooper Pair

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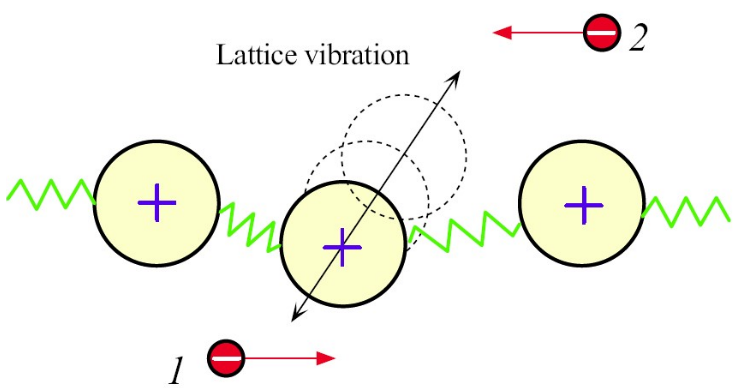

A pictorial and intuitive view of an indirect attraction between two oppositely traveling electrons via a lattice distortion and vibration.

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