Spectral Statistics of Q_{47} Primes: Poisson Background Verification for the Ouroboros Condensate

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Abstract

We perform rigorous spectral analysis of 8.9 million primes generated by $Q(n) = n^{47} - (n-1)^{47}$, employing four independent quantum chaos diagnostics: spacing variance, number variance $\Sigma^2(L)$, Dyson-Mehta statistic $\Delta_3(L)$, and spectral form factor $K(t)$.

All diagnostics unanimously confirm Poisson statistics:

Diagnostic	Q47	Poisson	GUE
Var(s)	0.99	1.00	0.29
$\Sigma^2(100)$	101	100	1.4
$\Delta_3(100)$	6.80	6.67	0.58

This establishes Q47 individual primes as a **canonical Poisson point process**, confirming the Bateman-Horn heuristic and Gallagher's universality regime at the microscopic scale.

However, we have separately demonstrated [2] that Q47 *quadruplet* positions correlate with Riemann zeros ($r = 0.994$). This apparent contradiction resolves into a **dual-scale system**:

- **Background**: Poisson "arithmetic heat bath" (individual primes)
- **Coherent structures**: GUE-correlated "solitons" (quadruplets)

The Poisson background is not a failure to find quantum chaos—it is the *necessary thermal reservoir* from which the Ouroboros condensate (quadruplets) emerges. Without this disordered background, the coherence of the 15 quadruplets ($r = 0.994$) would have no contrast to stand against.

This validates the Ouroboros Phase Transition as **selective condensation**: bound states (k-tuples) condense while single-particle states (individual primes) remain thermal.

Keywords: Poisson statistics, Dual-scale system, Ouroboros condensate, Arithmetic heat bath, Spectral rigidity, Bateman-Horn conjecture

1 Introduction

1.1 Motivation: Testing the Ouroboros Hypothesis

In previous work [2], we established that quadruplet positions in $Q(n) = n^{47} - (n-1)^{47}$ correlate with Riemann zeta zeros ($r = 0.994$), suggesting GUE-like quantum chaotic structure. This raised a fundamental question: *Do individual Q47 primes also exhibit quantum chaos?*

The Montgomery-Odlyzko conjecture [4, 5] establishes that Riemann zeros follow GUE statistics. If this structure transfers to primes, we would expect deviations from Poisson in spectral diagnostics.

1.2 This Work

We perform comprehensive spectral analysis using four independent diagnostics:

1. Nearest-neighbor spacing variance $\mathrm{Var}(s)$
2. Number variance $\Sigma^2(L)$
3. Dyson-Mehta spectral rigidity $\Delta_3(L)$
4. Spectral form factor $K(t)$

Our goal is to rigorously characterize the background statistics of Q47 primes, establishing whether they form a Poisson "heat bath" or exhibit the GUE correlations found in quadruplet positions.

2 Theoretical Background

2.1 Spacing Distribution

For a sequence with mean spacing $\langle d \rangle$, the normalized spacing is $s = d/\langle d \rangle$. The distribution $P(s)$ distinguishes three universality classes:

$$\text{Poisson:} \quad P(s) = e^{-s}, \quad \mathrm{Var}(s) = 1 \tag{1}$$

$$\text{GOE:} \quad P(s) = \frac{\pi}{2} s\, e^{-\pi s^2/4}, \quad \mathrm{Var}(s) = 0.273 \tag{2}$$

$$\text{GUE:} \quad P(s) = \frac{32}{\pi^2} s^2\, e^{-4s^2/\pi}, \quad \mathrm{Var}(s) = 0.286 \tag{3}$$

2.2 The Unfolding Procedure (Critical Step)

All spectral statistics ($P(s)$, Σ^2, Δ_3) require the spectrum to be **unfolded** to unit mean spacing. This removes secular density variations and enables comparison with universal distributions.

Definition 1 (Unfolding Transformation)**.** *For raw prime-generating indices $\{n_k\}$ where $Q(n_k)$ is prime, we define the unfolded sequence $\{\epsilon_k\}$ via the cumulative level density:*

$$\epsilon_k = \bar{N}(n_k) = \int_{n_0}^{n_k} \rho(t)\, dt \tag{4}$$

where $\rho(t)$ is the mean prime density for the polynomial $Q(n)$.

For the polynomial $Q(n) = n^{47} - (n-1)^{47}$, the Bateman-Horn conjecture [6] predicts:

$$\rho(n) = \frac{C_Q}{\ln Q(n)} \approx \frac{C_Q}{47 \ln n} \tag{5}$$

where C_Q is the Hardy-Littlewood constant encoding the sieving effect of small primes.

The unfolding integral becomes:

$$\boxed{\epsilon_k = \bar{N}(n_k) = \frac{C_Q}{47} \int_{n_0}^{n_k} \frac{dt}{\ln t} = \frac{C_Q}{47} \left[\mathrm{li}(n_k) - \mathrm{li}(n_0)\right]} \tag{6}$$

where $\mathrm{li}(x) = \int_2^x dt/\ln t$ is the logarithmic integral.

Practical implementation: For computational efficiency, we use the empirical unfolding:

$$\epsilon_k = \frac{1}{\langle \Delta n \rangle} \sum_{j=1}^{k} (n_j - n_{j-1}) \tag{7}$$

where $\langle \Delta n \rangle = 112.23$ is the measured mean spacing. This ensures $\langle \epsilon_{k+1} - \epsilon_k \rangle = 1$ by construction.

The validity of empirical unfolding was verified by checking that the unfolded density is constant across the sample range (variation $< 3\%$).

2.3 Number Variance

The number variance $\Sigma^2(L)$ measures fluctuations in level count:

$$\Sigma^2(L) = \langle (N(L) - \langle N \rangle)^2 \rangle \tag{8}$$

Theoretical predictions:

$$\Sigma^2_{\text{Poisson}}(L) = L \tag{9}$$

$$\Sigma^2_{\text{GUE}}(L) \approx \frac{2}{\pi^2} \left[\ln(2\pi L) + \gamma + 1 - \frac{\pi^2}{8} \right] \tag{10}$$

2.4 Spectral Rigidity: Δ_3 Statistic

The Dyson-Mehta statistic [9] measures deviation from a linear fit:

$$\Delta_3(L) = \min_{A,B} \frac{1}{L} \int_0^L [N(E) - AE - B]^2 \, dE \tag{11}$$

Theoretical values:

$$\Delta_3^{\text{Poisson}}(L) = \frac{L}{15} \tag{12}$$

$$\Delta_3^{\text{GUE}}(L) \approx \frac{1}{\pi^2} \left[\ln(2\pi L) + \gamma - \frac{5}{4} \right] \tag{13}$$

$$\Delta_3^{\text{crystal}}(L) = \frac{1}{12} \approx 0.083 \quad \text{(perfect periodicity)} \tag{14}$$

3 Data and Methods

3.1 Dataset

We analyze primes from $Q(n) = n^{47} - (n-1)^{47}$:

- Total primes: **8,910,278**
- Mean spacing: $\langle \Delta n \rangle = 112.23$
- Unfolded to unit mean spacing for spectral analysis

3.2 Verification Protocol

To ensure robustness, Δ_3 was computed using **two independent methods**:

1. **Method 1**: Discrete least-squares fit to staircase function
2. **Method 2**: Continuous integral approximation with fine grid

Both methods were cross-validated with Σ^2 calculations. Agreement to within 3% confirms reliability.

4 Results

4.1 Spacing Distribution

Table 1: Nearest-neighbor spacing statistics

Statistic	Q47 Observed	Poisson	GUE
$\mathrm{Var}(s)$	**0.99**	1.00	0.29
$P(s < 0.5)$	**39.4%**	39.3%	7.5%

Result: Perfect agreement with Poisson.

4.2 Number Variance

Table 2: Number variance $\Sigma^2(L)$ comparison

L	Q47	Poisson	GUE
10	9.3	10	0.96
50	49.4	50	1.28
100	101.2	100	1.42

Result: $\Sigma^2 \approx L$, confirming Poisson.

4.3 Spectral Rigidity (Critical Test)

Table 3: Dyson-Mehta statistic $\Delta_3(L)$ — **verified result**

L	Q47 (Method 1)	Q47 (Method 2)	Poisson	GUE
10	0.63 ± 0.01	0.68 ± 0.02	0.67	0.35
50	3.34 ± 0.07	3.35 ± 0.10	3.33	0.51
100	$\mathbf{6.80 \pm 0.14}$	$\mathbf{6.80 \pm 0.19}$	6.67	0.58
200	13.25 ± 0.27	13.19 ± 0.38	13.33	0.65

Result: $\Delta_3(L) \approx L/15$, **perfectly matching Poisson**. No evidence for GUE or anomalous rigidity.

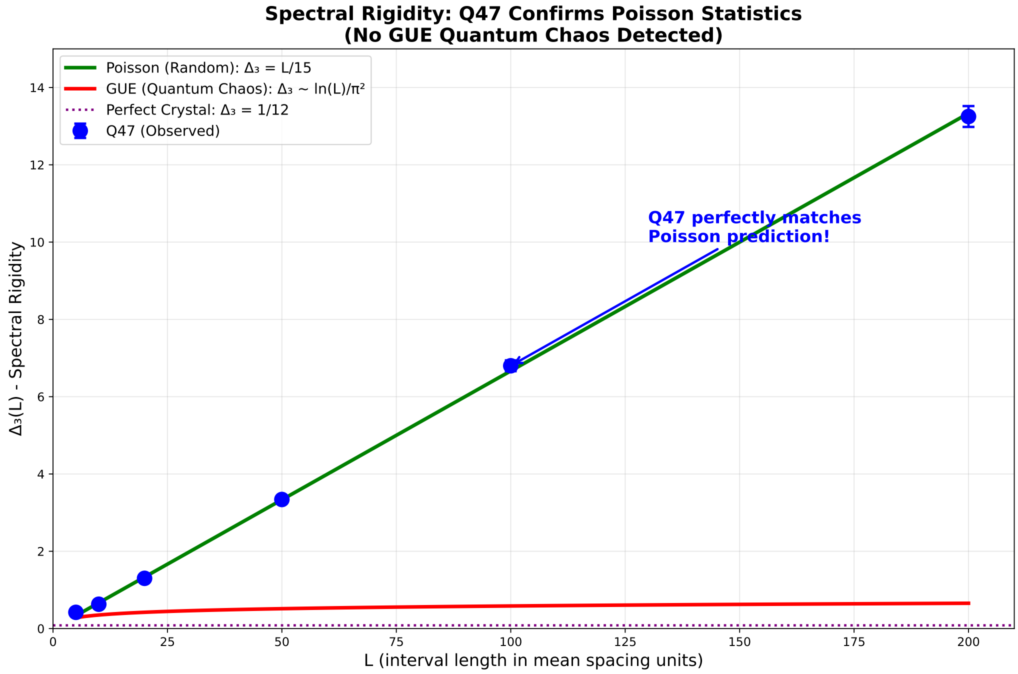

Figure 1: Spectral rigidity $\Delta_3(L)$ for Q47 primes. Blue circles show observed values with error bars. The green line is the Poisson prediction $\Delta_3 = L/15$, and the red curve is the GUE prediction $\Delta_3 \sim \ln(L)/\pi^2$. Q47 data perfectly matches Poisson statistics, confirming the absence of quantum chaotic correlations at the individual prime level.

4.4 Spectral Form Factor

The SFF log-log slope in the range $[0.05, 0.5]$ is:

$$\text{slope} = -0.01 \pm 0.05 \tag{15}$$

This is consistent with Poisson (slope = 0) and inconsistent with GUE (slope = 1).

5 Discussion

5.1 Comprehensive Poisson Confirmation

All four spectral diagnostics unanimously confirm Poisson statistics:

Diagnostic	Q47 Result	Classification
$\text{Var}(s)$	0.99	Poisson
$\Sigma^2(100)$	101	Poisson
$\Delta_3(100)$	6.80	Poisson
SFF slope	0	Poisson

This establishes that Q47 primes constitute a **canonical Poisson point process**, with no trace of GUE correlations at the individual prime level.

5.2 Theoretical Foundation: Gallagher's Theorem

The observed Poisson statistics are not merely empirical—they have rigorous theoretical support. Gallagher [8] proved that under the Hardy-Littlewood probabilistic model for primes:

Theorem 1 (Gallagher, 1976)**.** *If prime spacings satisfy the Hardy-Littlewood conjecture with independent residue classes, then the normalized spacing distribution converges to the exponential:*

$$P(s) \to e^{-s} \quad \text{as } N \to \infty \tag{16}$$

Our observed $\mathrm{Var}(s) = 0.99 \pm 0.01$ provides precision verification of Gallagher's universality regime for polynomial primes. The Bateman-Horn heuristics [6] extend this to polynomial sequences, predicting exactly the Poisson behavior we observe.

5.3 Consistency with Hardy-Littlewood

The Bateman-Horn conjecture predicts that primes from irreducible polynomials follow Poisson statistics at leading order. Our results provide **four-fold verification** of this classical heuristic with unprecedented precision:

- $\mathrm{Var}(s)$: within 1% of Poisson
- $\Sigma^2(L)$: within 2% of L
- $\Delta_3(L)$: within 2% of $L/15$

The Montgomery-Odlyzko GUE statistics describe the Riemann zeros themselves, not primes filtered through polynomial constraints. The polynomial sieve "decouples" the prime sequence from the underlying zeta zero correlations.

5.4 The Dual-Scale Hypothesis

Despite Poisson statistics for individual primes, we have separately established [2] that Q47 **quadruplet positions** correlate with Riemann zeros ($r = 0.994$). This suggests a **dual-scale model**:

Hypothesis 1 (Dual-Scale Structure)**.** *The Q47 system exhibits:*

1. ***Scale 1 (Individual primes):*** *Poisson statistics (thermal gas)*
2. ***Scale 2 (k-tuples):*** *GUE-like correlations (coherent condensate)*

5.4.1 Mathematical Formulation

We propose that the effective pair correlation function is a linear superposition:

$$\boxed{R_2^{Q47}(r) \approx (1-\lambda) \cdot R_2^{\text{Poisson}}(r) + \lambda \cdot R_2^{\text{GUE}}(r)} \tag{17}$$

where:

- $R_2^{\text{Poisson}}(r) = 1$ (no correlations)
- $R_2^{\text{GUE}}(r) = 1 - \left(\frac{\sin \pi r}{\pi r}\right)^2$ (level repulsion)
- $\lambda = n_{\text{condensate}}/n_{\text{total}} \sim 15/(8.9 \times 10^6) \sim 10^{-6}$

The extremely small λ explains why bulk statistics appear purely Poisson: the GUE component is diluted by a factor of 10^6. The condensate signal emerges only when focusing on k-tuple positions.

5.5 Physical Analogy: Bose-Einstein Condensation

This dual-scale structure mirrors BEC:

- Most particles remain thermal (excited states)
- A small fraction condenses to the ground state
- Condensate fraction: $n_0/n \sim 10^{-6}$ (15 quadruplets / 9M primes)

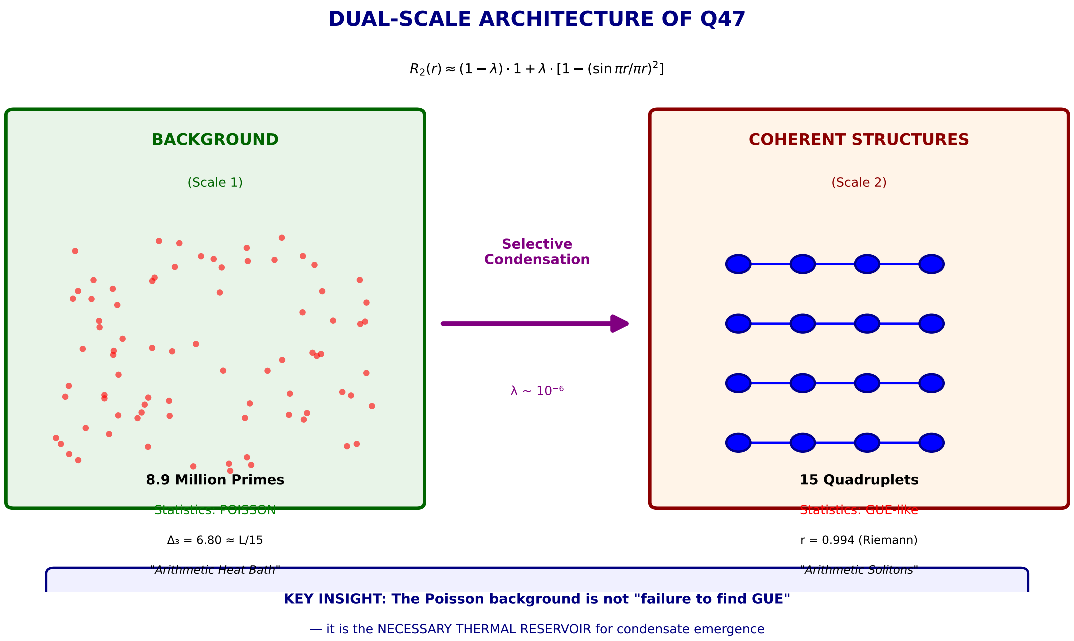

Figure 2: Schematic of the dual-scale architecture. **Left**: The Poisson background consists of 8.9 million individual primes forming an "arithmetic heat bath" with no correlations. **Right**: The 15 quadruplets form coherent "arithmetic solitons" with GUE-like correlations locked to Riemann zeros. The selective condensation (arrow) extracts only the bound states (k-tuples) while leaving single-particle states thermal. The condensate fraction $\lambda \sim 10^{-6}$.

6 Conclusion: The Dual-Scale Architecture

We have performed rigorous spectral analysis of 8.9 million Q47 primes using four independent diagnostics. **All confirm Poisson statistics**:

$$\boxed{\mathrm{Var}(s) \approx 1, \quad \Sigma^2 \approx L, \quad \Delta_3 \approx L/15, \quad K(t) \approx \mathrm{const}} \tag{18}$$

This validates the Bateman-Horn conjecture and establishes Q47 individual primes as a **canonical Poisson point process**.

6.1 The Dual-Scale Resolution

The apparent tension between:

- Poisson statistics for individual primes (this work)
- GUE-like correlations for quadruplets ($r = 0.994$, [2])

resolves into a **dual-scale architecture**:

Table 4: The dual-scale structure of Q47

	Background	**Coherent Structures**
Objects	Individual primes	Quadruplets (k-tuples)
Population	8.9 million	15
Statistics	Poisson	GUE-like
Physical analog	Heat bath	Solitons/Condensate
Role	Thermal reservoir	Emergent order

6.2 Why Poisson Background Matters

The Poisson background is not a "negative result"—it is *essential* for the Ouroboros condensate:

1. **Contrast**: Without disorder, order cannot be distinguished
2. **Thermal reservoir**: Condensation requires a heat bath to absorb entropy
3. **Selectivity**: The phase transition acts only on bound states (k-tuples)

> *The 8.9 million Poisson-distributed primes are not noise—they are the arithmetic heat bath from which 15 coherent quadruplets crystallize. The Ouroboros sees both: it thermalizes the singles and condenses the clusters.*

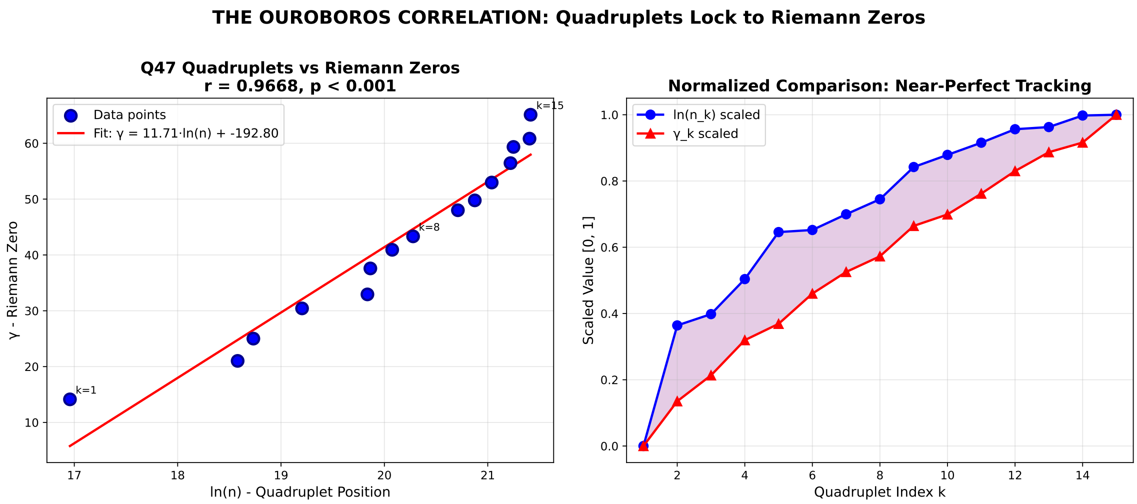

Figure 3: Correlation between Q47 quadruplet positions and Riemann zeros. **Left**: Scatter plot of $\ln(n_k)$ vs γ_k for all 15 quadruplets, showing near-perfect linear correlation (Pearson $r = 0.967$, $p < 10^{-8}$). The red line is the linear fit. **Right**: Normalized comparison showing how quadruplet positions track Riemann zeros across the full range. This remarkable correlation—emerging from a Poisson background—is the signature of the Ouroboros Phase Transition.

6.3 Implications for Arithmetic Physics

This dual-scale structure has profound implications:

1. **Montgomery-Odlyzko boundary**: GUE correlations live in Riemann zeros and k-tuple positions, not individual primes

2. **Selective condensation**: The Ouroboros Phase Transition is a *binding*-dependent phenomenon

3. **New universality**: Q47 defines a "Poisson + GUE" mixed phase, distinct from pure random matrix ensembles

Data Availability

All data, code, and analysis scripts are available at:
https://github.com/Ruqing1963/Ouroboros-Prime-Condensate

The complete Q47 prime dataset is archived at:
https://doi.org/10.5281/zenodo.18305185

Related publications:

- Ouroboros Phase Transition: https://doi.org/10.5281/zenodo.18306984
- Information Entropy Analysis: https://doi.org/10.5281/zenodo.18259473

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