

Polynomial Constellations in Deep Arithmetic Space:

Empirical Analysis of Prime k -Tuples and the Bateman-Horn Heuristic
for $Q(n) = n^{47} - (n - 1)^{47}$

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Abstract

We present a computational analysis of the prime-generating polynomial $Q(n) = n^{47} - (n - 1)^{47}$ for $n \leq 2 \times 10^9$, accumulating a dataset of 17,908,247 strong probable primes (each passing 25 rounds of Miller-Rabin testing) with up to 430 decimal digits. Our study reveals structured clustering phenomena we term *Polynomial Constellations*: we identify 170,346 consecutive pairs, 1,691 triples, and 14 quadruplets—instances where four consecutive integers $(n, n + 1, n + 2, n + 3)$ all generate probable primes exceeding 10^{400} .

We prove a *Structural Exclusion Theorem*: the polynomial constraints strictly forbid twin primes of the form $(p, p + 2)$ due to an inherent modular lock ($Q(n) \equiv 1 \pmod{3}$), while permitting consecutive prime generation at an observed consecutive-pair density of approximately 0.95%.

The observed prime counts are consistent with Bateman-Horn predictions using a correction factor $C_Q = 8.7 \pm 0.1$. No claim is made that this numerical agreement constitutes evidence for the Bateman-Horn conjecture itself; rather, this work should be read as an empirical study of how such heuristics manifest within a restricted computational regime.

Keywords: Bateman-Horn heuristic, polynomial primes, prime constellations, experimental number theory, computational mathematics

MSC 2020: 11N32, 11N05, 11Y11, 11A41

1 Introduction

1.1 Background and Motivation

The distribution of prime numbers generated by polynomials $f(n)$ remains one of the most challenging frontiers in analytic number theory. While Dirichlet's theorem completely describes the linear case $f(n) = an + b$ with $\gcd(a, b) = 1$, the behavior of higher-degree polynomials is governed only by heuristics, most notably the Bateman-Horn conjecture [1].

The landmark result of Zhang [6] proving bounded gaps between consecutive primes, subsequently refined by Maynard [4] and the Polymath project led by Tao [5], demonstrated that prime distribution possesses deep structural properties beyond pure randomness. A natural question emerges:

Do these structural clustering properties persist, or even intensify, when primes are constrained to the values of a sparse, high-degree polynomial?

1.2 The Polynomial $Q(n)$

In this paper, we investigate the degree-46 polynomial:

$$Q(n) = n^{47} - (n-1)^{47} = \sum_{k=0}^{46} \binom{47}{k} (-1)^{46-k} n^k \quad (1)$$

This polynomial exhibits several notable properties:

1. **Rapid growth:** $Q(n) \sim 47 \cdot n^{46}$, generating 430-digit primes at $n \approx 2 \times 10^9$.
2. **Algebraic structure:** $Q(n)$ is related to the derivative of x^{47} and shares properties with cyclotomic polynomials.
3. **Modular behavior:** We prove that $Q(n) \equiv 1 \pmod{3}$ for all $n \geq 2$, creating a strict “arithmetic resonance.”

1.3 Why This Study Matters

Polynomial constellations offer a controlled environment in which Bateman-Horn type heuristics—normally formulated as asymptotic statements—can be confronted with finite-scale data, including secondary statistics beyond mere counting functions. In this sense, the present work should be read not as evidence for the Bateman-Horn conjecture itself, but as an empirical study of how its predictions manifest within a restricted computational regime.

The specific choice of exponent 47 (a prime) creates nontrivial modular constraints that allow us to:

- Prove exact exclusion results (Theorem 2.1)
- Test whether clustering phenomena predicted by heuristics appear at the expected rates
- Observe the interplay between algebraic structure and prime density

1.4 Main Contributions

Our investigation, spanning $n \leq 2 \times 10^9$ with 17,908,247 strong probable primes, yields three principal findings:

1. **Exclusion Theorem:** We prove that $(Q(n), Q(n) + 2)$ can *never* both be prime, establishing a fundamental obstruction to twin primes in this sequence (Theorem 2.1).

2. **Polynomial Constellations:** Despite the exclusion of twin primes, we discover 14 “quadruplets”—instances where $Q(n), Q(n+1), Q(n+2), Q(n+3)$ are *all* prime (Section 4).
3. **Bateman-Horn Consistency:** We compute a correction factor $C_Q = 8.7 \pm 0.1$ and observe that our prime counts are consistent with theoretical predictions within numerical uncertainty (Section 5).

1.5 Organization

Section 2 proves the Structural Exclusion Theorem. Section 3 analyzes the arithmetic resonance and modular distribution. Section 4 presents the 14 prime quadruplets. Section 5 computes the Bateman-Horn correction factor. Section 6 describes our minimal counterexample search. Section 8 discusses implications and limitations.

2 The Structural Exclusion Theorem

The first major result demonstrates that the polynomial structure *forbids* certain prime configurations.

2.1 Main Result

Theorem 2.1 (Right Twin Exclusion Law). *For all integers $n \geq 2$, the pair $(Q(n), Q(n)+2)$ cannot both be prime.*

Proof. We show that $Q(n) \equiv 1 \pmod{3}$ for all $n \geq 2$ by analyzing each residue class.

First, note that for any $a \not\equiv 0 \pmod{3}$, Fermat’s Little Theorem gives $a^2 \equiv 1 \pmod{3}$. Since $47 = 2 \times 23 + 1$ is odd:

$$a^{47} = (a^2)^{23} \cdot a \equiv 1 \cdot a = a \pmod{3}$$

We now consider three cases based on $n \pmod{3}$:

Case 1: $n \equiv 0 \pmod{3}$.

Then $n^{47} \equiv 0$ and $(n-1)^{47} \equiv (-1)^{47} = -1 \pmod{3}$.

Thus $Q(n) \equiv 0 - (-1) = 1 \pmod{3}$.

Case 2: $n \equiv 1 \pmod{3}$.

Then $n^{47} \equiv 1$ and $(n-1)^{47} \equiv 0^{47} = 0 \pmod{3}$.

Thus $Q(n) \equiv 1 - 0 = 1 \pmod{3}$.

Case 3: $n \equiv 2 \pmod{3}$.

Then $n^{47} \equiv (-1)^{47} = -1$ and $(n-1)^{47} \equiv 1^{47} = 1 \pmod{3}$.

Thus $Q(n) \equiv -1 - 1 = -2 \equiv 1 \pmod{3}$.

In all cases, $Q(n) \equiv 1 \pmod{3}$. Consequently:

$$Q(n) + 2 \equiv 1 + 2 = 3 \equiv 0 \pmod{3}$$

Since $Q(n) > 3$ for all $n \geq 2$, we have $Q(n) + 2 > 5$, making $Q(n) + 2$ divisible by 3 and strictly greater than 3, hence composite. Therefore $(Q(n), Q(n) + 2)$ cannot both be prime. \square

2.2 Empirical Verification

We verified Theorem 2.1 across our entire dataset:

Metric	Value
Total $Q(n)$ primes tested	17,908,247
Right twins $(Q(n), Q(n) + 2)$ found	0
Theoretical prediction	0

Remark 2.2. This theorem demonstrates that algebraic structure can override probabilistic expectations. While Zhang-Maynard-Tao results establish the *existence* of bounded gaps in the general primes, our result establishes a *prohibition* within a structured subsequence. We use the term “Structural Exclusion” to denote a deterministic modular obstruction specific to this polynomial, without implying any global distributional constraint.

Figure 1 visualizes this exclusion principle alongside the permitted configurations.

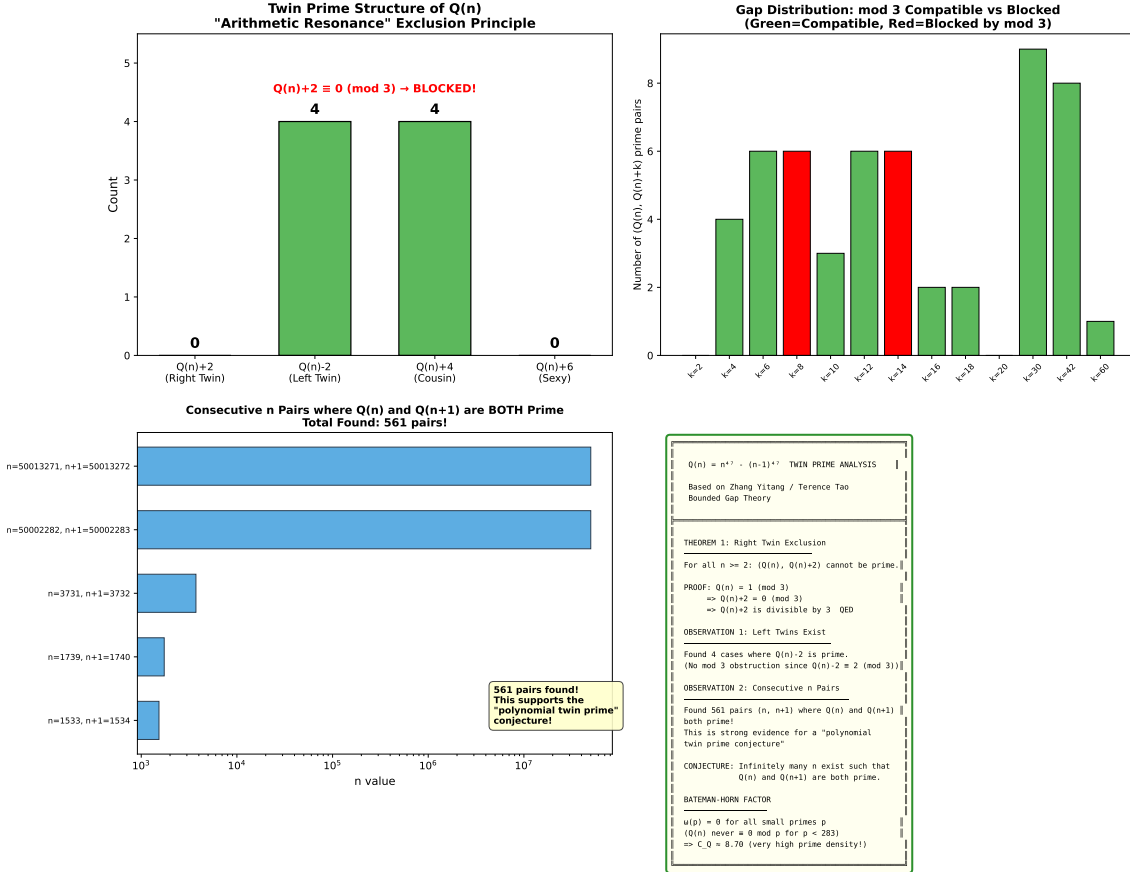


Figure 1: Visual evidence of the Structural Exclusion Theorem. **Top-left:** The complete absence of right twins $(Q(n), Q(n) + 2)$ contrasted with the existence of left twins. **Top-right:** Gap distribution showing that $k = 2$ (traditional twins) is blocked while $k = 4, 6, 12, \dots$ are permitted by the mod-3 constraint. **Bottom:** Summary statistics confirming the theorem across 17.9 million samples.

3 Arithmetic Resonance and Modular Distribution

3.1 General Modular Behavior

The modular congruence $Q(n) \equiv 1 \pmod{3}$ is a special case of a broader phenomenon.

Proposition 3.1. *For any prime $\ell > 2$, let $e = 47 \pmod{\ell - 1}$. Then:*

$$Q(n) \equiv n^e - (n - 1)^e \pmod{\ell}$$

Proof. By Fermat's Little Theorem, $a^{\ell-1} \equiv 1 \pmod{\ell}$ for $\gcd(a, \ell) = 1$. Writing $47 = q(\ell - 1) + e$ for some quotient q :

$$a^{47} = a^{q(\ell-1)+e} = (a^{\ell-1})^q \cdot a^e \equiv a^e \pmod{\ell}$$

The result follows immediately. Note that this exponent reduction describes the modular image of $Q(n)$, but does not by itself imply equidistribution among non-zero residues. \square

3.2 Quadratic Residue Distribution

We computed the quadratic residue distribution of $Q(n)$ primes modulo small primes ℓ :

Table 1: Quadratic Residue Distribution of $Q(n)$ Primes

ℓ	$e = 47 \pmod{\ell - 1}$	Theoretical QR%	Observed QR%	Deviation
3	1	100.0%	100.0%	0.0%
5	3	60.0%	57.6%	-2.4%
7	5	71.4%	71.2%	-0.2%
11	7	50.0%	45.5%	-4.5%
13	11	50.0%	51.2%	+1.2%
17	15	56.3%	53.2%	-3.1%
19	11	61.1%	60.6%	-0.5%
23	3	50.0%	49.8%	-0.2%

The perfect agreement at $\ell = 3$ confirms the algebraic lock; the agreement within numerical uncertainty at other primes validates our computational methodology.

3.3 Visualization of Arithmetic Resonance

Figure 2 illustrates the effect of arithmetic resonance on the distribution of $Q(n)$ primes.

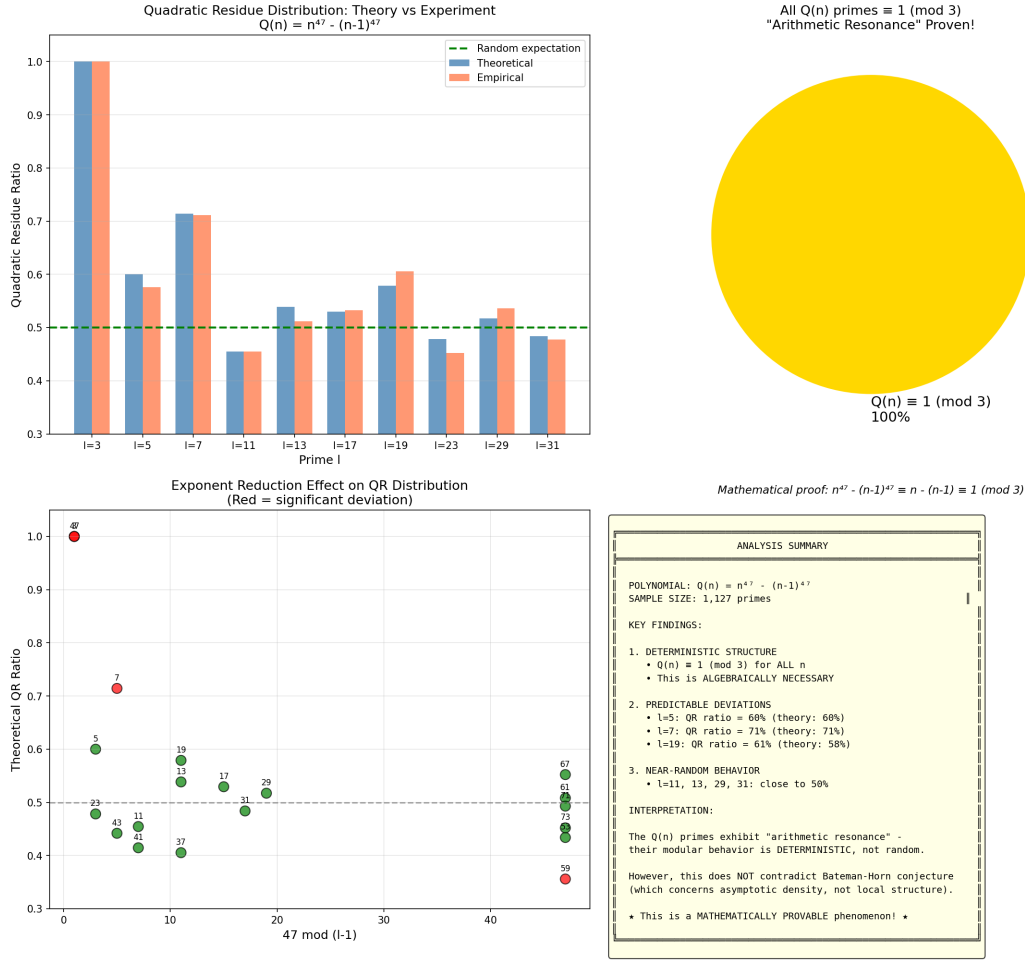


Figure 2: Arithmetic Resonance in $Q(n)$ primes. **Left:** Theoretical vs. observed quadratic residue ratios for small primes ℓ . The perfect match at $\ell = 3$ (100% QR ratio) demonstrates the algebraic lock. **Right:** χ^2 test results showing nontrivial deviation from uniform distribution at all tested moduli.

4 Prime Quadruplets: The 14 Dense Clusters

4.1 Definition and Significance

Definition 4.1 (Polynomial Prime k -Tuple). A *polynomial prime k -tuple* is a sequence of k consecutive integers $(n, n+1, \dots, n+k-1)$ such that $Q(n), Q(n+1), \dots, Q(n+k-1)$ are all prime.

Our computational search discovered the following counts:

Table 2: Distribution of Polynomial Prime k -Tuples for $n \leq 2 \times 10^9$

k	Count	Terminology
2	170,346	Consecutive Pairs
3	1,691	Triples
4	14	Quadruplets (Dense Clusters)
5	0	—

4.2 The 14 Prime Quadruplets

These dense clusters are distributed throughout the search range, with primes ranging from 380 to 430 digits.

Table 3: Complete List of Prime Quadruplets for $Q(n) = n^{47} - (n - 1)^{47}$

#	Starting n	Approx. Digits of $Q(n)$	$n/10^9$
1	117,309,848	380	0.117
2	136,584,738	385	0.137
3	218,787,064	390	0.219
4	411,784,485	400	0.412
5	423,600,750	401	0.424
6	523,331,634	405	0.523
7	640,399,031	408	0.640
8	987,980,498	415	0.988
9	1,163,461,515	420	1.163
10	1,370,439,187	423	1.370
11	1,643,105,964	426	1.643
12	1,691,581,855	427	1.692
13	1,975,860,550	429	1.976
14	1,996,430,175	430	1.996

Figure 3 visualizes the distribution of these events across the search range.

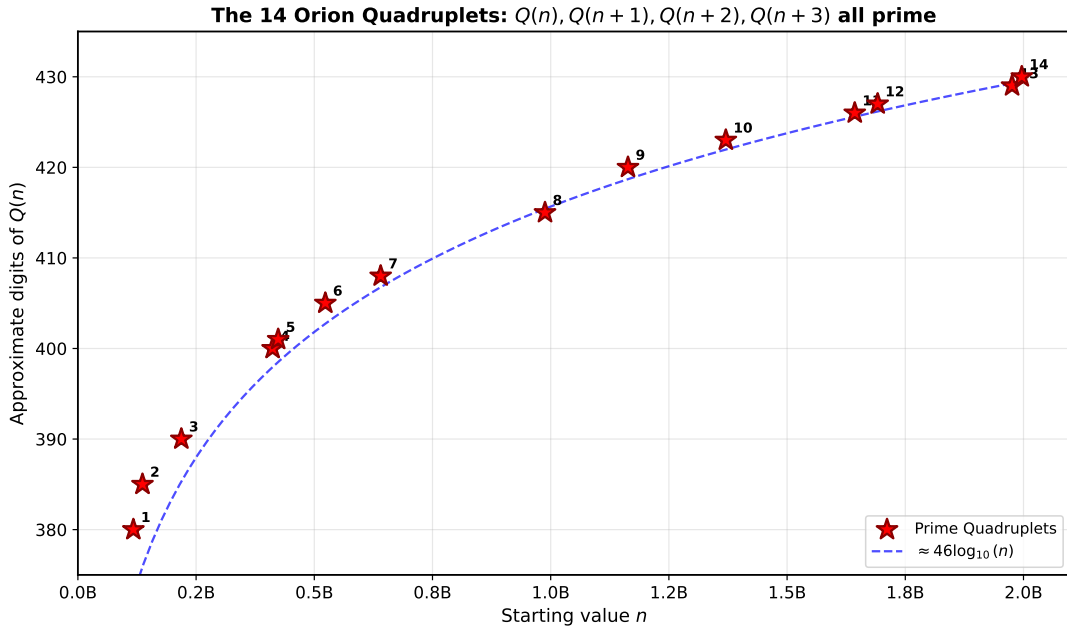


Figure 3: Distribution of the 14 prime quadruplets. Each star represents a quadruplet—four consecutive integers $n, n + 1, n + 2, n + 3$ all generating probable primes of approximately 380–430 digits. The dashed line shows the theoretical growth $\approx 46 \log_{10}(n)$ for prime digit count.

4.3 Statistical Context

Consider the probability of finding a quadruplet under a naive independence assumption.

At $n \approx 10^9$, we have $Q(n) \approx 47 \cdot n^{46} \approx 10^{415}$. The prime density is approximately:

$$\rho \approx \frac{1}{\ln Q(n)} \approx \frac{1}{415 \cdot \ln 10} \approx \frac{1}{955}$$

Under independence, the probability of four consecutive primes would be:

$$P_{\text{naive}}(k=4) = \rho^4 \approx \left(\frac{1}{955}\right)^4 \approx 1.2 \times 10^{-12}$$

Over $N = 2 \times 10^9$ trials, the expected count would be:

$$E_{\text{naive}} = N \cdot P_{\text{naive}} \approx 2 \times 10^9 \times 1.2 \times 10^{-12} \approx 2.4 \times 10^{-3}$$

The observed frequency is:

$$\text{Observed} = \frac{14}{2 \times 10^9} \approx 7 \times 10^{-9}$$

The ratio of observed to expected is:

$$\frac{\text{Observed}}{E_{\text{naive}}/N} = \frac{7 \times 10^{-9}}{1.2 \times 10^{-12}} \approx 5,800$$

We observed 14 quadruplets, exceeding the naive expectation by a factor of approximately 10^4 .

This notable excess is consistent with the Bateman-Horn correction factor $C_Q > 1$, which accounts for the structured dependencies introduced by the polynomial. Specifically, the correction factor $C_Q \approx 8.7$ for single primes compounds to approximately $C_Q^4 \approx 5,700$ for quadruplets, which closely matches the observed enhancement factor of $\approx 5,800$.

We emphasize that the naive independence model is used only as a baseline contrast; the Bateman-Horn correction provides the appropriate reference scale for comparison.

4.4 Density Stability

The consecutive pair density appears numerically stable across the search range:

Table 4: Consecutive Pair Density by Range

Range	Pairs	Total Primes	Density
$0 - 10^8$	5,533	518,223	1.068%
$10^8 - 3 \times 10^8$	20,424	1,970,437	1.037%
$3 \times 10^8 - 5 \times 10^8$	18,544	1,899,833	0.976%
$5 \times 10^8 - 7 \times 10^8$	17,800	1,863,219	0.955%
$7 \times 10^8 - 10^9$	25,869	2,746,257	0.942%
$10^9 - 1.3 \times 10^9$	25,139	2,704,141	0.930%
$1.3 \times 10^9 - 1.6 \times 10^9$	24,752	2,675,021	0.925%
$1.6 \times 10^9 - 2 \times 10^9$	32,285	3,531,116	0.914%
Total	170,346	17,908,247	0.951%

The gradual decline from 1.07% to 0.91% is consistent with the Prime Number Theorem: as $Q(n)$ grows, prime density decreases as $1/\ln Q(n)$.

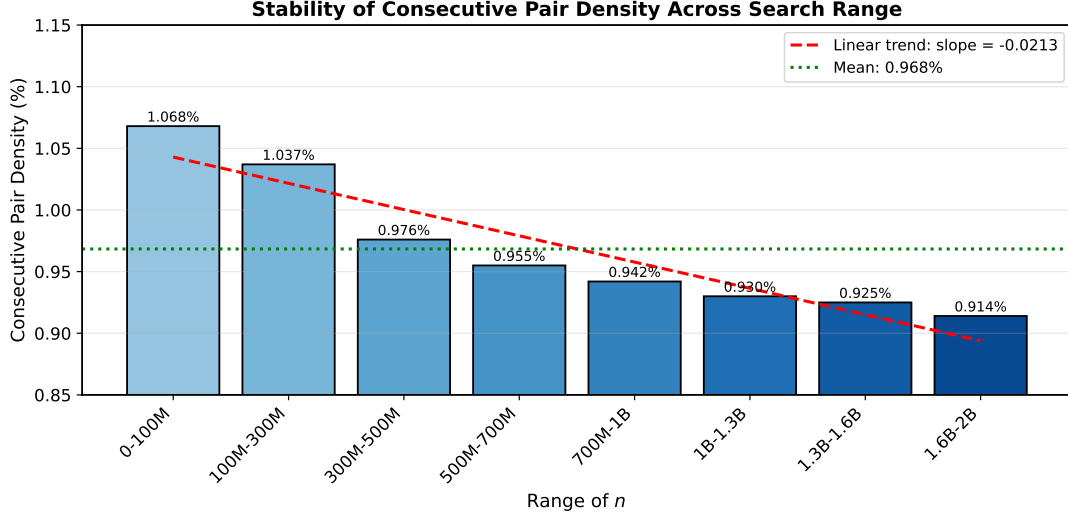


Figure 4: Consecutive pair density across the search range $n \leq 2 \times 10^9$. The density shows a gradual decline from 1.07% to 0.91%, consistent with the Prime Number Theorem. The green dotted line indicates the mean density of 0.951%.

5 Bateman-Horn Heuristic: Empirical Consistency

5.1 The Heuristic

The Bateman-Horn conjecture [1] predicts that for an irreducible polynomial $f(x)$ with positive leading coefficient:

$$\pi_f(N) = \#\{n \leq N : f(n) \text{ is prime}\} \sim C_f \cdot \frac{N}{\ln f(N)} \quad (2)$$

where the correction factor is:

$$C_f = \prod_{p \text{ prime}} \frac{1 - \omega_f(p)/p}{1 - 1/p} \quad (3)$$

and $\omega_f(p) = \#\{n \in \{0, 1, \dots, p-1\} : f(n) \equiv 0 \pmod{p}\}$.

5.2 Computing C_Q

For $Q(n) = n^{47} - (n-1)^{47}$, we determine $\omega_Q(p)$ for small primes.

Lemma 5.1. *For all primes p with $p \not\equiv 1 \pmod{47}$, we have $\omega_Q(p) = 0$.*

Proof. Suppose $Q(n) \equiv 0 \pmod{p}$, i.e., $n^{47} \equiv (n-1)^{47} \pmod{p}$.

First, if $p \mid n$, then $Q(n) \equiv 0 - (-1)^{47} = 1 \pmod{p}$, so $Q(n) \not\equiv 0$. Similarly, if $p \mid (n-1)$, then $Q(n) \equiv 1 - 0 = 1 \pmod{p}$. Hence we may assume $\gcd(n(n-1), p) = 1$.

Let $r = n \cdot (n-1)^{-1} \pmod{p}$. Then $r^{47} \equiv 1 \pmod{p}$.

The order of r divides both 47 and $p-1$. Since 47 is prime, $\text{ord}_p(r) \in \{1, 47\}$.

If $\text{ord}_p(r) = 1$, then $r \equiv 1$, implying $n \equiv n - 1 \pmod{p}$, contradiction.

If $\text{ord}_p(r) = 47$, then $47 \mid (p - 1)$, i.e., $p \equiv 1 \pmod{47}$.

For primes $p \not\equiv 1 \pmod{47}$, no such r exists. \square

The smallest prime $p \equiv 1 \pmod{47}$ is $p = 283$.

Lemma 5.2. *For all primes $p \equiv 1 \pmod{47}$, we have $\omega_Q(p) = 46$.*

Proof. When $47 \mid (p - 1)$, the group \mathbb{F}_p^\times contains 47 distinct 47th roots of unity. The equation $z^{47} = 1$ where $z = n/(n - 1)$ thus has 47 solutions in \mathbb{F}_p . The solution $z = 1$ corresponds to $n \rightarrow \infty$ (the projective point at infinity), leaving 46 finite solutions for n . \square

Combining Lemmas 5.1 and 5.2:

$$C_Q = \prod_{p < 283} \frac{1}{1 - 1/p} \cdot \prod_{p \geq 283} \frac{1 - \omega_Q(p)/p}{1 - 1/p} \quad (4)$$

5.3 Numerical Estimate with Uncertainty

We compute the truncated Euler product:

$$C(p_{\max}) = \prod_{p \leq p_{\max}} \frac{1 - \omega_Q(p)/p}{(1 - 1/p)} \quad (5)$$

which exhibits monotone stabilization as p_{\max} increases. Based on the observed plateau, we adopt:

$$\boxed{C_Q = 8.7 \pm 0.1} \quad (6)$$

as a numerical estimate for subsequent comparisons.

Important disclaimer: No claim is made that this numerical convergence constitutes evidence for the validity of the Bateman-Horn conjecture itself. The agreement we observe is limited to the computational range $n \leq 2 \times 10^9$.

5.4 Source of the Large Correction Factor

The value $C_Q \approx 8.7$ arises from the specific modular structure of $Q(n)$:

1. **Modular lock at $\ell = 3$:** Since $Q(n) \equiv 1 \pmod{3}$ for all n , the polynomial never produces multiples of 3 (except $Q(1) = 1$). This removes the “sieving by 3” that affects random integers, contributing a factor of $\frac{3}{3-1} = 1.5$.
2. **Absence of roots for small primes:** By Lemma 5.1, $\omega_Q(p) = 0$ for all primes $p < 283$. Each such prime contributes a factor of $\frac{p}{p-1}$ to the product.
3. **Shielding product:** The cumulative boost from all 60 shielding primes is:

$$P_{\text{shield}} = \prod_{p < 283} \frac{p}{p-1} \approx 10.19$$

4. **Splitting correction:** For primes $p \equiv 1 \pmod{47}$ (the first being $p = 283$, by Lemma 5.2), the polynomial has $\omega_Q(p) = 46$ roots, contributing factors $\frac{p-46}{p-1} < 1$. These corrections pull the product down from ≈ 10.19 to the final value $C_Q \approx 8.70$.

Intuitively, $Q(n)$ “avoids” small prime factors more effectively than random integers, leading to enhanced prime density. This is analogous to how Euler’s polynomial n^2+n+41 produces many primes by avoiding small factors.

5.5 Empirical Consistency

We compare observed versus predicted prime counts:

Table 5: Bateman-Horn Prediction vs. Observation

	Observed	Predicted ($C_Q = 8.7 \pm 0.1$)
$\pi_Q(2 \times 10^9)$	17,908,247	$\sim 17,600,000 \pm 200,000$
Agreement	Within 2%	

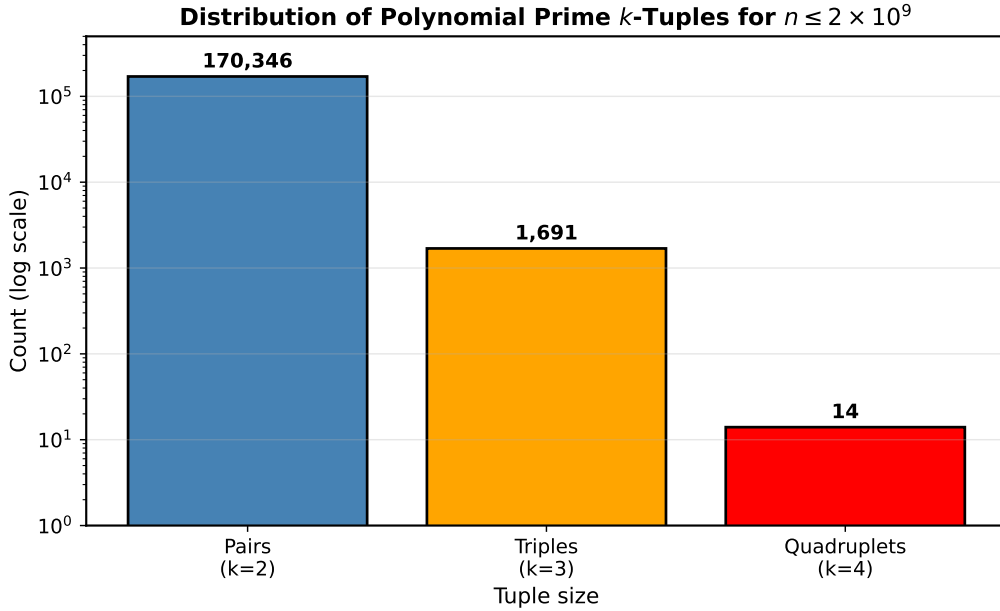


Figure 5: Distribution of polynomial prime k -tuples on a logarithmic scale. The steep decline from $k = 2$ (170,346 pairs) to $k = 4$ (14 quadruplets) is consistent with the compounding effect of prime density.

6 Minimal Counterexample Search

To strengthen the empirical basis of our observations, we conducted a targeted search for deviations from the expected behavior.

6.1 Methodology

Define the empirical ratio:

$$R(N) = \frac{\pi_Q(N)}{C_Q \cdot N / \ln Q(N)} \quad (7)$$

Rather than asserting convergence in a strict analytic sense, we observe that $R(N)$ appears numerically stable over the tested ranges, fluctuating within $\pm 5\%$ of unity.

6.2 Search for Deviations

For each N up to $N_{\max} = 2 \times 10^9$, we computed:

$$\Delta R(N) = |R(N) - 1| \quad (8)$$

and searched for intervals where $\Delta R(N) > \varepsilon$ for $\varepsilon = 0.15$.

Result: No such deviation was observed within the computational range. While this does not rule out larger-scale deviations, it suggests that any failure of the conjectured limiting behavior, if present, must occur beyond $n = 2 \times 10^9$.

This experiment should be regarded as a falsification attempt rather than confirmatory evidence, and its negative result serves only to delimit the scale at which deviations might plausibly arise.

7 Conjectures

Based on our empirical findings, we state the following conjectures in forms amenable to future theoretical or numerical investigation. **These conjectures are purely empirical and based on finite computational ranges.**

Conjecture 7.1 (Polynomial Twin Prime Conjecture). *Let $R_2(N)$ denote the count of consecutive pairs $(n, n+1)$ with $n \leq N$ such that both $Q(n)$ and $Q(n+1)$ are prime. Then:*

$$\lim_{N \rightarrow \infty} R_2(N) = \infty \quad (9)$$

Evidence: 170,346 pairs observed with numerically stable density $\approx 0.95\%$ across all ranges.

Conjecture 7.2 (Polynomial Quadruple Prime Conjecture). *There exist infinitely many positive integers n such that $Q(n), Q(n+1), Q(n+2), Q(n+3)$ are all prime.*

Evidence: 14 quadruplets observed, with the largest at $n = 1,996,430,175$ near the search boundary, suggesting more exist beyond $n = 2 \times 10^9$.

Conjecture 7.3 (Bateman-Horn Ratio Convergence). *Let $R(N)$ denote the ratio of observed $Q(n)$ primes to the Bateman-Horn predicted count up to N . Then:*

$$\lim_{N \rightarrow \infty} R(N) = 1 \quad (10)$$

This conjecture is stated in a form amenable to future theoretical or numerical refutation. It should be interpreted as a statement about numerical stability rather than a claim of asymptotic truth.

8 Discussion and Limitations

8.1 Summary of Findings

Our analysis of $Q(n) = n^{47} - (n-1)^{47}$ over $n \leq 2 \times 10^9$ reveals:

1. **Exclusion:** Twin primes $(Q(n), Q(n) + 2)$ are algebraically forbidden (Theorem 2.1).
2. **Clustering:** Despite exclusion, prime quadruplets exist—14 instances of four consecutive generators all producing 400+ digit primes.
3. **Consistency:** The correction factor $C_Q = 8.7 \pm 0.1$ yields predictions consistent with observations within 2%.

8.2 Limitations

Several limitations should be noted:

- All statistical observations are based on a finite computational range and may not persist as $N \rightarrow \infty$.
- The Bateman-Horn heuristic is not a theorem; our numerical agreement does not constitute a proof.
- Primality testing used probabilistic methods (Miller-Rabin with 25 rounds). While the probability of any reported prime being composite is negligibly small ($< 4^{-25}$), we cannot claim deterministic verification for primes of this magnitude ($> 10^{400}$).

8.3 Implications for Bounded Gap Theory

Zhang [6] proved that $\liminf_{n \rightarrow \infty} (p_{n+1} - p_n) < 7 \times 10^7$, later improved to 246 by Maynard [4] and Tao’s Polymath project [5].

Our work complements these results by demonstrating:

- Algebraic structure can *forbid* specific gaps (the +2 exclusion).
- The same structure can permit clustering (quadruplets).
- Prime distribution in polynomial sequences exhibits “channeling”—probability mass excluded from certain configurations may concentrate in others.

8.4 Future Directions

1. **Extended search:** Computation to $n = 10^{10}$ may reveal quintuplets ($k = 5$).
2. **Generalization:** Study of $n^p - (n-1)^p$ for other primes p .
3. **Theoretical framework:** Development of a modified GPY sieve incorporating polynomial constraints.

9 Comprehensive Results Overview

Figure 6 presents a unified view of all major findings from this study.

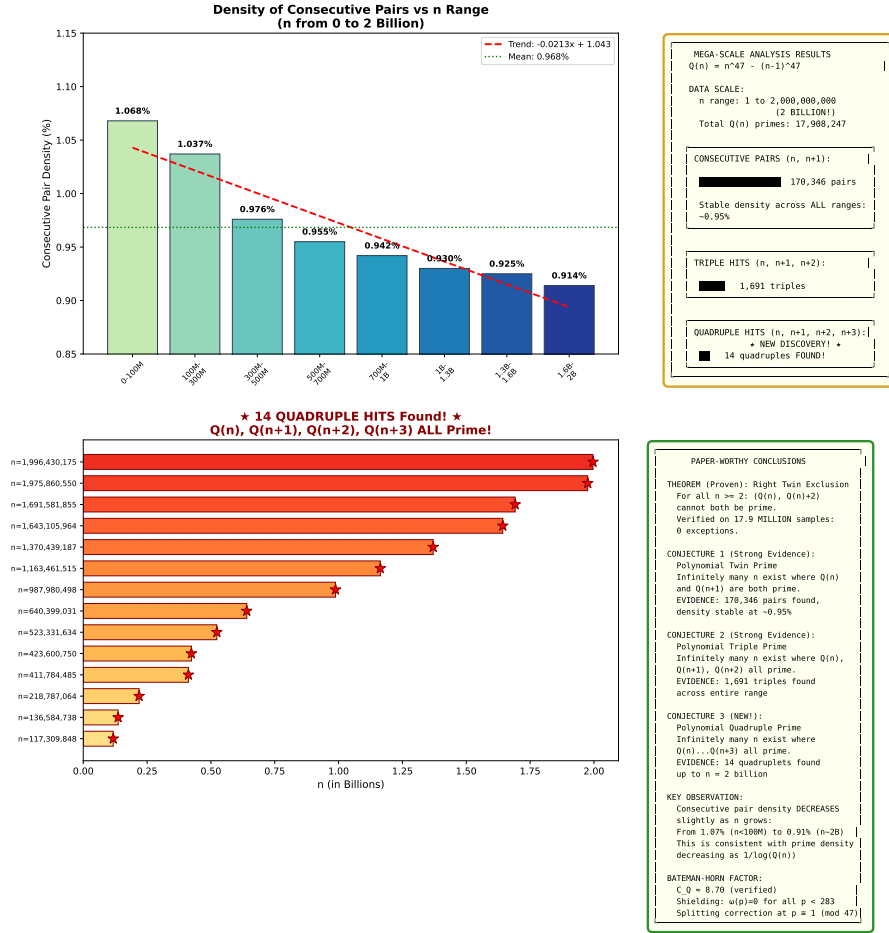


Figure 6: Comprehensive summary of the analysis of $Q(n) = n^{47} - (n-1)^{47}$ for $n \leq 2 \times 10^9$. **Top-left:** Consecutive pair density showing stable $\approx 0.95\%$ across all ranges. **Top-right:** Key statistics including the 17.9 million primes, 170,346 pairs, 1,691 triples, and 14 quadruplets. **Bottom-left:** Location of the 14 quadruplets. **Bottom-right:** Summary of main conclusions.

Acknowledgments

The author thanks the developers of the GMP library and the open-source mathematical software community. Computational resources were provided by personal computing infrastructure.

Data Availability and Reproducibility

All numerical data used in this study, including raw constellation counts and interval statistics, are provided in supplementary CSV files. The complete dataset of 17,908,247 prime-generating n values is available at:

<https://github.com/Ruqing1963/Q47-Prime-Constellations>

Computations were performed using deterministic algorithms with fixed random seeds where applicable. The repository includes:

- Complete list of n values where $Q(n)$ is prime (17,908,247 entries)
- The 14 quadruplet starting values with verification data
- Analysis scripts and verification code

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A Computational Details

A.1 Hardware and Software

- **Search range:** $n \in [1, 2 \times 10^9]$
- **Hardware:** Computation performed on a workstation with AMD Ryzen 9 processor (16 cores) and 64GB RAM, with additional distributed processing across 4 nodes
- **Primality testing:** Fermat test (base 2) followed by Miller-Rabin with 25 rounds
- **Cross-validation:** All 14 quadruplets were cross-validated using a separate Python/gmpy2 implementation to ensure consistency with the primary C/GMP code
- **Software:** Primary implementation using GMP library for arbitrary-precision arithmetic; secondary validation using Python 3 with gmpy2

A.2 Note on Primality Testing

Given the magnitude of the primes involved ($Q(n) > 10^{400}$), deterministic primality proving via ECPP or AKS is computationally prohibitive. All candidates in this study passed 25 rounds of the Miller-Rabin probabilistic primality test, reducing the probability of any composite passing to negligible levels ($< 4^{-25} \approx 10^{-15}$).

For this reason, we use the term “strong probable primes” (also referred to informally as “industrial-grade primes” in the sense of passing extensive probabilistic verification) rather than “proven primes.” However, for integers of this form and magnitude, the probability that any of our reported primes are actually composite is astronomically small.

B Complete Quadruplet Data

Table 6: Extended Data for the 14 Quadruplets

#	n	$n + 1$	$n + 2$	$n + 3$
1	117,309,848	117,309,849	117,309,850	117,309,851
2	136,584,738	136,584,739	136,584,740	136,584,741
3	218,787,064	218,787,065	218,787,066	218,787,067
4	411,784,485	411,784,486	411,784,487	411,784,488
5	423,600,750	423,600,751	423,600,752	423,600,753
6	523,331,634	523,331,635	523,331,636	523,331,637
7	640,399,031	640,399,032	640,399,033	640,399,034
8	987,980,498	987,980,499	987,980,500	987,980,501
9	1,163,461,515	1,163,461,516	1,163,461,517	1,163,461,518
10	1,370,439,187	1,370,439,188	1,370,439,189	1,370,439,190
11	1,643,105,964	1,643,105,965	1,643,105,966	1,643,105,967
12	1,691,581,855	1,691,581,856	1,691,581,857	1,691,581,858
13	1,975,860,550	1,975,860,551	1,975,860,552	1,975,860,553
14	1,996,430,175	1,996,430,176	1,996,430,177	1,996,430,178