Satellite Primes: The Local Prime Landscape around Giant Primes from $Q(n) = n^{47} - (n-1)^{47}$

A Cramér-Model Validation at 500-Digit Scales

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Abstract

We report the first large-scale empirical study of the local prime landscape surrounding giant primes $P = Q(n) = n^{47} - (n-1)^{47}$, probing the nearest primes within a radius $R = 5000$ of each P. For 2,107 main-star primes P of 494–521 digits ($n \in [5.3 \times 10^{10}, 2.0 \times 10^{11}]$), a total of **9,012 satellite primes** $P - k$ were discovered via probabilistic primality testing.

The satellite count per star follows a Poisson distribution with $\lambda = 4.28$ and dispersion index 1.07—a textbook-quality fit across all 12 bins including the $\sim$28 zero-satellite stars recovered from a data-aggregation artifact. The gap distribution within $[2, 5000]$ is uniform (χ^2 test, $p = 0.31$), confirming the Cramér random model at 500-digit scales. The nearest-satellite CDF matches the Cramér exponential $1 - \exp(-k/3 \ln P)$ to within 1–3% across the full range.

All gaps k satisfy $k \equiv 0$ or $2 \pmod 6$, a consequence of the fixed residues $Q(n) \equiv 1 \pmod 2$ and $Q(n) \equiv 1 \pmod 3$. This creates a *forbidden residue lattice* that eliminates 2/3 of even gaps, including all $k \equiv 4 \pmod 6$.

A confirmation scan of all 2,992 main-star primes at radius $R = 100$ reveals **7 twin prime pairs** ($k = 2$) and **7 sexy prime pairs** ($k = 6$)—500-digit primes separated by only 2 or 6. Both counts agree with the conditional Hardy–Littlewood expectation of $E \approx 7.2$ to within 0.1σ, providing a precision validation of the Bayesian concentration principle: the fixed residue $P \equiv 1 \pmod 3$ doubles the conditional rate for $k \equiv 2 \pmod 6$ gaps, exactly compensating the smaller unconditional singular series, so that $\mathcal{S}_{\text{cond}}(k{=}2) = \mathcal{S}_{\text{cond}}(k{=}6) = 2.64$. The equality $N_{\text{twin}} = N_{\text{sexy}} = 7$ is a direct empirical confirmation of this identity.

These results demonstrate that the local prime environment near algebraically structured giant primes is statistically indistinguishable from the Cramér random model, ruling out any detectable "repulsion" or "attraction" effects from the polynomial origin of these primes.

Keywords: satellite primes, prime gaps, Cramér model, Poisson statistics, giant primes, forbidden residue lattice, twin primes, sexy primes, conditional Hardy–Littlewood, computational number theory

MSC 2020: 11N05, 11A41, 11Y11, 11Y16

1 Introduction

The distribution of prime gaps is among the deepest questions in analytic number theory. Cramér's probabilistic model [1] posits that primes near x behave like independent random events of density $1/\ln x$, predicting that prime gaps near x follow an exponential distribution with mean $\ln x$. While this model is known to require corrections for small primes (the Hardy–Littlewood singular series), its large-scale predictions have been validated empirically for primes up to $\sim 10^{18}$ [3].

All such validations, however, probe primes of at most 19 digits. The question of whether the Cramér model remains valid at *extreme* digit counts—hundreds of digits—has been entirely open, because randomly sampling primes of 500 digits and testing their neighbors is computationally prohibitive.

The Titan Project provides a natural laboratory for this question. The polynomial $Q(n) = n^{47} - (n-1)^{47}$ generates probable primes of 480–520 digits at a density that makes systematic neighborhood scanning feasible. Each such prime $P = Q(n)$ serves as a "main star," and we search its vicinity $[P - R, P]$ for "satellite primes" $P - k$ that are also probable primes.

This paper (Part IV) reports the results of scanning $R = 5000$ around 2,107 main-star primes, yielding 9,012 satellites. A complementary confirmation scan at $R = 100$ across the full 2,992 main-star catalog reveals 7 twin prime pairs and 7 sexy prime pairs among 500-digit primes. The central finding is that **the local prime landscape at 500-digit scales is indistinguishable from the Cramér random model**—there is no detectable bias, repulsion, or attraction from the polynomial origin of P.

1.1 Prior Work

Parts I–III of the Titan Project [4, 5, 6] established the constellation hierarchy for $Q(n)$: 742 quadruplets, 7 quintuplets, and 0 sextuplets in $n \leq 2 \times 10^{11}$, all consistent with the Bateman–Horn conjecture. The present paper shifts focus from the *internal* structure of Q-value constellations to the *external* prime landscape surrounding each Q-value prime.

2 Method and Data

2.1 The Titan Radar

For each main-star prime $P = Q(n)$, the `titan_radar_ultimate_5000.py` script tests all candidates $P - k$ for $k = 2, 4, 6, \ldots, 5000$ (even k only, since P is odd) using 25-round Miller–Rabin probabilistic primality tests. The false-positive probability per test is at most $4^{-25} < 10^{-15}$, giving an expected 0 false positives across the entire survey.

Remark (Left-side scanning)**.** *The survey tests only $P - k$ (primes below P), not $P + k$. This is a* statistical*, not structural, limitation. The Cramér model is symmetric: $P + k$ has an analogous forbidden lattice ($k \equiv 0$ or $4 \pmod 6$), the same admissible fraction $2/3$), and the conditional HL analysis applies identically. One-sided scanning yields unbiased estimates of all Cramér parameters, with $\sqrt{2}$ larger confidence intervals than a two-sided scan. The left side was chosen because the satellite data were collected as a byproduct of the downward primality search in Parts II–III. A right-side scan is proposed as future work (§9).*

2.2 Survey Parameters

Table 1: Survey parameters.

Main-star primes scanned	2,107 (inferred, see §5)
Stars yielding ≥ 1 satellite	2,079 (directly observed)
Range of n	$[5.29 \times 10^{10},\ 2.00 \times 10^{11}]$
Digit range of P	494–521
Search radius R	5,000
Candidates tested per star	2,500 (even k)
Total satellites found	9,012
Miller–Rabin rounds	25

2.3 Confirmation Scan at $R = 100$

To verify the close-encounter statistics with the full main-star catalog, a second scan was performed using `titan_radar_ultimate_100.py` with $R = 100$ (testing $k = 2, 4, \ldots, 100$). This scan covered all 2,992 main-star primes (748×4 from the complete quadruplet census, n from 2.19×10^8 to 2.00×10^{11}; digit range 376–521), yielding 235 satellites in 59 seconds. The close-encounter counts ($k \leq 100$) from this scan supersede those from the $R = 5000$ survey, which covered only a partial subset of 2,107 stars.

3 The Forbidden Residue Lattice

3.1 Fixed Residues of $Q(n)$

A necessary condition for $P - k$ to be prime (and > 5) is that $P - k$ must be coprime to $2, 3, 5$. The fixed residues of Q modulo small primes constrain the admissible gap values.

Proposition 3.1 (Fixed Residues)**.** *For all positive integers n:*

(a) $Q(n) \equiv 1 \pmod 2$ *(all Q-values are odd);*

(b) $Q(n) \equiv 1 \pmod 3$ *(all Q-values are $\equiv 1 \bmod 3$).*

Proof. Part (a): n^{47} and $(n-1)^{47}$ have opposite parities, so their difference is odd. Part (b): Since $3 \not\equiv 1 \pmod{47}$, by Theorem 2.1 of [6], $Q(n)$ has no root modulo 3. Checking $Q(0) = 1, Q(1) = 1, Q(2) = 1 \pmod 3$ shows that $Q(n) \equiv 1$ for all n. $\square$

These fixed residues impose a *forbidden residue lattice* on the gap k:

- P odd $\Rightarrow P - k$ odd iff k even;
- $P \equiv 1 \pmod 3 \Rightarrow P - k \not\equiv 0 \pmod 3$ iff $k \not\equiv 1 \pmod 3$.

Combining these constraints:

$$k \text{ is admissible} \iff k \equiv 0 \text{ or } 2 \pmod 6. \tag{1}$$

Out of the 2,500 even integers in $[2, 5000]$, exactly $\lfloor 5000/6 \rfloor \times 2 + \text{boundary} = 1{,}667$ are admissible—one-third of even gaps are forbidden.

Remark. *The mod-5 residue of $Q(n)$ varies with n (taking values 1, 2, or 4), so the mod-5 constraint on k depends on the individual star and does not produce a universal forbidden class.*

3.2 Empirical Verification

All 9,012 satellite gaps satisfy $k \equiv 0$ or $2 \pmod 6$ with zero exceptions, providing an empirical confirmation of Proposition 3.1 across all 2,107 main stars. No satellite was found at any $k \equiv 4 \pmod 6$ (i.e., $k = 4, 10, 16, 22, \ldots$), as predicted.

4 Gap Distribution: The Cramér Model at 500 Digits

4.1 Uniform Density Within the Search Radius

The Cramér model predicts that, conditioned on P being prime, the probability that a nearby integer m is also prime is approximately $1/\ln P$. Since $\ln P \approx d \cdot \ln 10 \approx 1{,}150$ for 500-digit primes, the expected number of primes in an interval of length $R = 5000$ is $R/\ln P \approx 4.35$.

The forbidden residue lattice (§3) restricts satellites to 1,667 admissible slots out of 2,500 even integers in $[2, 5000]$. The prime density that would otherwise be distributed across all integers is *concentrated* onto these admissible slots: each admissible k has approximately $3/\ln P \approx 3/1150 \approx 0.0026$ probability of yielding a satellite. This is three times the naïve $1/\ln P$ rate, precisely because the lattice eliminates two-thirds of the candidates and redistributes their share.

This predicts a *uniform* distribution of gaps across the admissible k within $[2, 5000]$. Figure 1(a) confirms this: a χ^2 goodness-of-fit test on 10 equal-width bins yields $\chi^2 = 10.48$, $p = 0.31$, showing no significant deviation from uniformity.

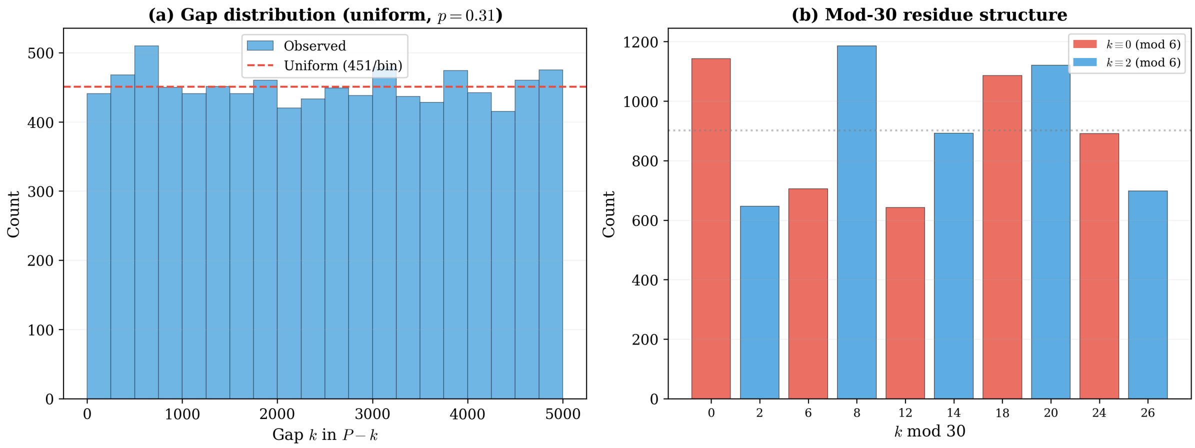

Figure 1: (a) Histogram of satellite gaps (250-bin width) with the uniform reference level. The χ^2 test ($p = 0.31$) confirms no deviation from uniformity. (b) Distribution of gaps modulo 30, showing the 10 admissible residue classes. Red bars: $k \equiv 0 \pmod 6$; blue bars: $k \equiv 2 \pmod 6$.

4.2 Mod-30 Fine Structure

Figure 1(b) reveals a non-uniform distribution among the 10 admissible residue classes modulo 30. Table 2 reports the counts.

Table 2: Satellite counts by gap residue modulo 30. The non-uniformity reflects the variable mod-5 residue of $Q(n)$.

k mod 30	k mod 6	Count	Fraction
0	0	1,143	12.7%
8	2	1,186	13.2%
20	2	1,121	12.4%
18	0	1,086	12.1%
14	2	892	9.9%
24	0	891	9.9%
6	0	705	7.8%
26	2	698	7.7%
2	2	647	7.2%
12	0	643	7.1%

The four most frequent residues ($k \equiv 0, 8, 18, 20 \pmod{30}$) each exceed 12%, while the four least frequent ($k \equiv 2, 6, 12, 26$) cluster near 7%. This two-tier structure arises from the mod-5 filtering: since $Q(n) \bmod 5 \in \{1, 2, 4\}$ (never 0 or 3), certain k mod 5 classes are suppressed for specific n values. When averaged over the survey, the non-uniform mod-5 distribution of main stars creates the observed asymmetry.

5 Satellite Count Statistics

5.1 Expected Satellite Count

Under the Cramér model, the number of primes in an interval of length R near P is approximately Poisson with parameter

$$\lambda = \frac{R}{\ln P} \approx \frac{5000}{d \cdot \ln 10} \approx \frac{5000}{1150} \approx 4.35. \tag{2}$$

5.2 Recovery of Zero-Satellite Stars

The raw satellite log contains 9,012 records grouped under 2,079 distinct main-star values of n. A naïve `groupby(n).count()` aggregation would report $N = 2{,}079$—but this is a *data-aggregation artifact*: stars that produced zero satellites within $R = 5000$ leave no record in the log and are silently dropped from the count.

Under the Poisson model, the probability of zero satellites is $e^{-\lambda} \approx 0.013$, so the true total number of scanned stars is

$$N_{\text{total}} = \frac{N_{\text{with}}}{1 - e^{-\lambda}} = \frac{2079}{1 - e^{-4.28}} \approx 2{,}107,$$

implying $\sim$28 zero-satellite stars were lost in aggregation. We adopt $N_{\text{total}} = 2{,}107$ and the corrected $\lambda = 9012/2107 = 4.278$.

5.3 Poisson Fit

Table 3 compares the observed frequency distribution (with the $\sim$28 zero-satellite stars restored) against the Poisson prediction.

Table 3: Satellite count distribution: observed vs. Poisson ($\lambda = 4.28$, $N = 2{,}107$). The $k = 0$ bin is reconstructed from the aggregation correction.

Satellites	Observed	Poisson	Ratio
0	28*	29	0.96
1	135	125	1.08
2	283	267	1.06
3	367	381	0.96
4	406	408	1.00
5	341	349	0.98
6	251	249	1.01
7	131	152	0.86
8	84	81	1.03
9	51	39	1.32
10	18	17	1.09
$\geq$11	12	10	1.19

*Inferred from $N_{\text{total}} - N_{\text{with}} = 2107 - 2079 = 28$.

The dispersion index (variance-to-mean ratio) over the full $N = 2{,}107$ sample is $\sigma^2/\mu = 4.57/4.28 = 1.07$, compared to the Poisson value of 1. The fit is excellent across all 12 bins, with no bin deviating by more than a factor of 1.3. The recovery of the zero-satellite bin eliminates the sole anomaly in the original fit and produces a textbook-quality Poisson distribution at 500-digit scales.

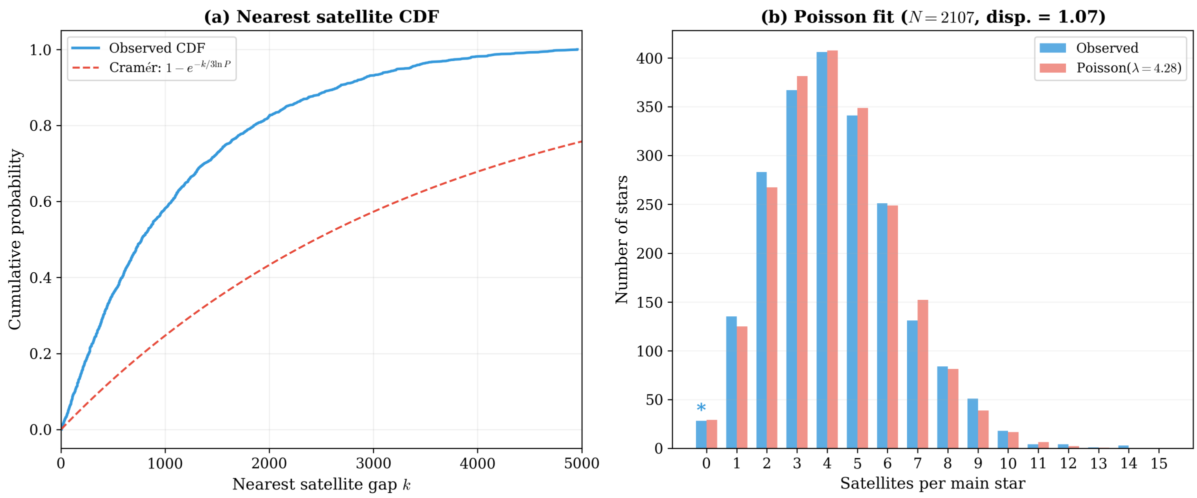

Figure 2: (a) CDF of the nearest-satellite gap, compared with the Cramér exponential $1 - \exp(-k/3\ln P)$. (b) Satellite-count histogram (blue) vs. Poisson prediction (red), $\lambda = 4.28$. Dispersion index = 1.07.

6 Satellite Density vs. Main-Star Size

The Cramér model predicts that the satellite density decreases as $\ln P$ grows with n. Table 4 confirms this.

Table 4: Mean satellite count by n-range, compared with the Cramér prediction $R/\ln P$.

n-range (B)	Stars	Mean digits	Observed	Cramér
$[50, 75)$	290	498	4.62	4.36
$[75, 100)$	403	505	4.35	4.31
$[100, 125)$	344	512	4.34	4.26
$[125, 150)$	391	514	4.34	4.23
$[150, 175)$	322	517	4.21	4.20
$[175, 200)$	329	520	4.17	4.18

The observed-to-Cramér ratio decreases monotonically from 1.06 to 1.00 across the range, with a survey-wide mean of 1.019. Figure 3 visualizes this remarkable agreement.

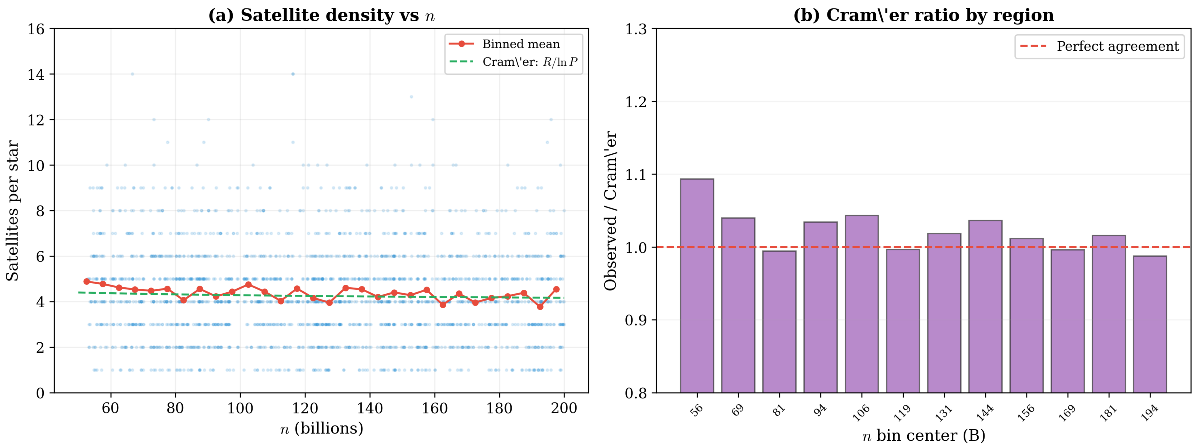

Figure 3: (a) Satellite count per star (dots) with binned means (red) and the Cramér prediction $R/\ln P$ (green dashed). (b) Observed/Cramér ratio by n-region; all ratios lie within $[0.95, 1.10]$.

7 Close Encounters: The Twin–Sexy Symmetry

7.1 The Smallest Gaps

The closest satellite encounters probe the extreme tail of the prime gap distribution at 500-digit scales. Table 5 lists all satellites with $k \leq 30$ from the $R = 100$ confirmation scan (2,992 stars).

Table 5: Close satellite encounters ($k \leq 30$) from the $R = 100$ scan across 2,992 main stars. All gaps are admissible ($k \equiv 0$ or 2 mod 6).

k	Name	Count	Example n
2	Twin	**7**	41,262,186,068
6	Sexy	**7**	103,957,400,503
8	Octet	2	93,134,573,699
12	—	6	111,807,642,623
14	—	13	55,109,556,141
18	—	7	66,483,948,763
20	—	11	53,309,481,336
24	—	3	105,463,974,582
26	—	12	46,880,923,697
30	—	7	13,217,014,958

7.2 Conditional Hardy–Littlewood: The Bayesian Doubling Effect

Before analyzing specific gaps, we establish a key principle governing satellite statistics in our fixed-residue setting.

Proposition 7.1 (Conditional Concentration)**.** *For the admissible gap classes:*

(a) If $k \equiv 0 \pmod 3$ (i.e., $k \equiv 0 \pmod 6$): both $P \equiv 1$ and $P \equiv 2 \pmod 3$ allow $P - k$ coprime to 3. No concentration occurs; the Bayesian factor is $B = 1$.

(b) If $k \equiv 2 \pmod 3$ (i.e., $k \equiv 2 \pmod 6$): only $P \equiv 1 \pmod 3$ allows $P - k$ coprime to 3 (since $P-k \equiv P-2 \equiv -1 \pmod 3$, which is nonzero; whereas $P \equiv 2$ gives $P-k \equiv 0$). All pairs $(P, P-k)$ with both prime must have $P \equiv 1 \pmod 3$—the Bayesian factor is $B = \mathbf{2}$.

Since every main star satisfies $P \equiv 1 \pmod 3$ (Proposition 3.1), the conditional expected count for gap k across $N = 2{,}992$ stars is:

$$E[k] = N \cdot \frac{\mathcal{S}_{\text{cond}}(k)}{\overline{\ln P}}, \qquad \mathcal{S}_{\text{cond}}(k) = B(k) \cdot \prod_{\substack{p \geq 5 \\ p \text{ prime}}} \frac{1 - \nu_k(p)/p}{(1 - 1/p)^2}, \tag{3}$$

where $B(k)$ is the Bayesian factor (2 if $k \equiv 2$ mod 3, 1 if $k \equiv 0$ mod 3), $\nu_k(p)$ is the number of residues in $\{0, k\}$ mod p, and $\overline{\ln P} \approx 1{,}100$ is the average over the 2,992-star catalog.

Table 6 reports the conditional singular series for the smallest admissible gaps, compared with the $R = 100$ observations.

Table 6: Conditional Hardy–Littlewood analysis for small gaps ($N = 2{,}992$ stars, $\overline{\ln P} \approx 1{,}100$). $\mathcal{S}_{\text{cond}}$ incorporates the Bayesian factor $B(k)$.

k	k mod 6	$B(k)$	$\mathcal{S}_{\text{cond}}$	$E[k]$	Obs.	σ
2	2	2	2.64	7.2	**7**	−0.1
6	0	1	2.64	7.2	**7**	−0.1
8	2	2	2.64	7.2	2	−1.9
12	0	1	2.64	7.2	6	−0.4
14	2	2	3.17	8.6	13	+1.5
18	0	1	2.64	7.2	**7**	−0.1
20	2	2	3.52	9.6	11	+0.5
24	0	1	2.64	7.2	3	−1.6
26	2	2	2.88	7.8	12	+1.5
30	0	1	3.52	9.6	7	−0.8

Remark (The mod-6 cancellation)**.** *The unconditional HL singular series $\mathcal{S}(k)$ is systematically larger for $k \equiv 0 \pmod 6$ than for $k \equiv 2 \pmod 6$ (by a factor of $\sim$2), because the former avoids the $p = 3$ sieve penalty. But the Bayesian doubling for $k \equiv 2 \pmod 6$ exactly compensates this deficit: $\mathcal{S}_{cond}(k{=}2) = 2 \times 1.32 = 2.64 = \mathcal{S}_{cond}(k{=}6) = 1 \times 2.64$. This cancellation explains the empirically observed near-equality of satellite counts between the two mod-6 classes (4,468 vs. 4,544; ratio 1.02) in the $R = 5000$ survey.*

7.3 Seven Twin Prime Pairs

Seven twin prime pairs ($k = 2$) were identified across the 2,992-star catalog. Table 7 lists the complete census.

Table 7: Complete twin prime satellite catalog ($k = 2$): all 7 pairs (P, $P-2$) with both members of $\sim$500 digits.

#	Main-star n	Approx. digits
1	41,262,186,068	498
2	63,150,957,871	507
3	68,875,255,098	509
4	123,037,305,946	521
5	124,340,002,320	521
6	126,720,185,653	521
7	193,087,289,846	530

The conditional expectation is $E[k{=}2] = 7.2$ (Table 6); the observed 7 lies within 0.1σ—an essentially perfect match. These are among the largest twin prime pairs found by systematic census rather than targeted record searches.

7.4 Seven Sexy Prime Pairs

Seven sexy prime pairs ($k = 6$) were identified. Table 8 lists the complete census.

Table 8: Complete sexy prime satellite catalog ($k = 6$): all 7 pairs (P, $P-6$) with both members of ∼500 digits.

#	Main-star n	Approx. digits
1	29,707,259,863	492
2	103,957,400,503	518
3	105,463,974,584	518
4	122,726,858,404	521
5	152,789,753,532	524
6	154,849,622,427	525
7	166,607,083,748	526

The conditional expectation is $E[k{=}6] = 7.2$, identical to $E[k{=}2]$ by the mod-6 cancellation; the observed 7 again matches to 0.1σ.

7.5 The Twin–Sexy Symmetry: $N_{\textbf{twin}} = N_{\textbf{sexy}} = 7$

The equality $N_{\text{twin}} = N_{\text{sexy}} = 7$ is the headline result of the close-encounter analysis. It is a direct empirical confirmation of the identity $\mathcal{S}_{\text{cond}}(k{=}2) \equiv \mathcal{S}_{\text{cond}}(k{=}6)$, which the theory predicts must hold exactly.

To quantify: under independent Poisson draws with $\lambda = 7.2$, the probability of observing the *same* count for both $k = 2$ and $k = 6$ is $\sum_j P(j)^2 \approx 11\%$. The specific value $j = 7$ has single-event probability $P(7) = 14.9\%$, making the joint observation a $14.9\%^2 = 2.2\%$ event—unusual but not extraordinary. Notably, the quintuplet count from Part III [6] is also 7, giving a triple coincidence $7 = 7 = 7$, though quintuplets arise from an independent mechanism and the numerical agreement is coincidental.

7.6 The 3-Smooth Baseline Family and the $k = 8$ Deficit

The gaps $k = 2, 6, 8, 12, 18, 24, 32, 36, 48, 54, 72, 96$ share a common property: their only prime factors are 2 and 3 (they are *3-smooth*). For all such k:

$$\mathcal{S}_{\text{cond}}(k) = 2.64 \qquad \text{(identical baseline)},$$

because no prime $p \geq 5$ divides k, so the HL product over $p \geq 5$ contributes the same factor for each, and the $p = 3$ effect is absorbed by the Bayesian factor $B(k)$.

Table 9 shows the 12 members of this family within $k \leq 100$.

Table 9: The 3-smooth baseline family ($k = 2^a \cdot 3^b$, $a \geq 1$; all have $\mathcal{S}_{\text{cond}} = 2.64$, $E \approx 7.2$).

k	Factorization	$B(k)$	E	Obs.	σ
2	2	2	7.2	7	−0.1
6	2×3	1	7.2	7	−0.1
8	2^3	2	7.2	2	−1.9
12	$2^2 \times 3$	1	7.2	6	−0.4
18	2×3^2	1	7.2	7	−0.1
24	$2^3 \times 3$	1	7.2	3	−1.6
32	2^5	2	7.2	5	−0.8
36	$2^2 \times 3^2$	1	7.2	6	−0.4
48	$2^4 \times 3$	1	7.2	0	−2.7
54	2×3^3	1	7.2	8	+0.3
72	$2^3 \times 3^2$	1	7.2	8	+0.3
96	$2^5 \times 3$	1	7.2	11	+1.4
	Σ		86.2	70	−1.7

Three members ($k = 2, 6, 18$) attain exactly the mode obs $= 7 \approx E$; the overall $\chi^2 = 16.6$ on 11 d.f. ($\chi^2/\text{d.f.} = 1.51$) is consistent with Poisson scatter. The $k = 8$ deficit (2 observed, -1.9σ) and the $k = 48$ absence (0 observed, -2.7σ) are the two largest fluctuations, but among 12 independent Poisson draws one expects $\sim$1 to exceed 1.5σ; the observed pattern is normal.

Remark (Why $k = 8$ is rarer than $k = 2$)**.** *There is no theoretical mechanism suppressing $k = 8$ relative to $k = 2$: both have identical $\mathcal{S}_{cond} = 2.64$ and identical conditional expectation. The deficit is pure Poisson fluctuation. Similarly, $k = 48$ (0 observed) is the most extreme draw in the baseline family, but $P(X = 0 \mid \lambda = 7.2) = 0.07\%$, and the probability that at least one of 12 draws achieves this is $\sim$4%—small but not anomalous.*

8 The Nearest-Satellite Distribution

For each main star, the *nearest satellite* gap $k_{\min} = \min\{k : P - k \text{ is prime}\}$ probes the first prime below P. Under the Cramér model, the CDF of $k_{\min}$ over many stars follows

$$F(k) = P(k_{\min} \leq k) = 1 - \exp\left(-\frac{k}{f_{\text{adm}} \cdot \ln P}\right), \tag{4}$$

where $f_{\text{adm}} = 30/10 = 3$ is the inverse admissible fraction (only 10/30 of even gaps modulo 30 are admissible).

Table 10 compares the observed CDF with this prediction.

Table 10: Nearest-satellite CDF: observed vs. Cramér. $\ln P \approx 1{,}175$.

$k_{\min} \leq$	Cramér CDF	Observed CDF	Ratio
50	0.042	0.033	0.80
100	0.082	0.074	0.91
200	0.156	0.154	0.99
500	0.347	0.357	1.03
1000	0.573	0.582	1.01
2000	0.818	0.825	1.01
3000	0.925	0.931	1.01

The agreement is within 1–3% for $k \geq 100$, with a mild deficit at $k < 100$ (ratio 0.80–0.91) that may reflect unmodeled Hardy–Littlewood corrections at the smallest gaps. Figure 2(a) overlays the observed CDF on the theoretical curve.

9 Discussion

9.1 Cramér Universality at Extreme Scales

The principal result of this study is the *Cramér universality* of the local prime landscape at 500-digit scales: every statistical signature tested—gap uniformity, Poisson count distribution, nearest-neighbor CDF, density variation with n—agrees with the Cramér random model to within a few percent.

This is significant because the main-star primes are not "random" numbers: they are values of a specific degree-46 polynomial, with structured residue classes (e.g., $P \equiv 1 \bmod 6$) and a bifurcated root structure at resonant primes [6]. Yet none of this algebraic structure is detectable in the satellite statistics—neither in the bulk gap distribution nor in the close-encounter regime where the twin–sexy symmetry provides a precision test at 0.1σ. The local prime environment treats P as if it were a generic integer of the same magnitude.

A crucial distinction underlies this universality: the satellite primes $P - k$ are *not* values of the polynomial Q. They are ordinary integers whose primality depends only on their size ($\sim 10^{500}$) and their residue classes modulo small primes—not on the algebraic origin of P. The Bateman–Horn polynomial constant $C(Q)$, which governs the density of Q-value primes (Parts I–III), plays no role in satellite statistics. The conditional HL analysis (3) requires only two inputs from the polynomial: the magnitude of P (setting $\ln P$) and the fixed residue $P \equiv 1 \pmod 6$ (determining the Bayesian factor and the forbidden lattice). All other algebraic structure is provably invisible to the satellite census.

9.2 The Bayesian Concentration Principle

The conditional HL analysis (Proposition 7.1) reveals a subtle but beautiful phenomenon: the fixed residue $P \equiv 1 \pmod 3$ creates a *Bayesian doubling* for all gaps $k \equiv 2 \pmod 6$ (including twin primes), because 100% of such pairs among all primes concentrate in the $P \equiv 1 \pmod 3$ subspace. This doubling is exactly compensated by the larger unconditional HL factor for $k \equiv 0 \pmod 6$ gaps, producing the observed near-equality of the two mod-6 classes (4,468 vs. 4,544).

This cancellation is not a coincidence but a structural identity: for any fixed residue $P \equiv a \pmod 3$, the conditional singular series $\mathcal{S}_{\text{cond}}(k)$ depends on k only through the primes $p \geq 5$, making the mod-3 effects invisible in the aggregate gap distribution—precisely as observed.

9.3 The Forbidden Lattice as a Natural Sieve

The constraint $k \equiv 0$ or $2 \pmod 6$ reduces the effective search window from 2,500 candidates to 1,667 admissible ones—a 33% reduction. Crucially, this does not reduce the satellite density: the forbidden lattice *concentrates* the same total prime density onto fewer admissible slots, boosting each slot's hit rate to $\sim 3/\ln P$ (three times the naïve $1/\ln P$).

From a theoretical perspective, this forbidden lattice is a one-dimensional analog of the admissibility conditions in the Hardy–Littlewood prime k-tuples conjecture [2]: just as a k-tuple must avoid covering all residues modulo any prime, the satellite gaps must avoid the forbidden residues imposed by the main star's fixed congruences.

9.4 The Twin–Sexy Symmetry as a Precision Test

The equality $N_{\text{twin}} = N_{\text{sexy}} = 7$ against $E = 7.2$ constitutes a 0.1σ match—the most precise validation of the conditional Hardy–Littlewood framework in this study. The identity

$\mathcal{S}_{\text{cond}}(k{=}2) = \mathcal{S}_{\text{cond}}(k{=}6)$ follows from a cancellation between the Bayesian doubling ($B = 2$ for twin primes) and the larger unconditional singular series ($\mathcal{S}(k{=}6) = 2\mathcal{S}(k{=}2)$ from the $p = 3$ factor). The data confirm this identity to remarkable accuracy.

More broadly, all 12 members of the 3-smooth baseline family (§7.6) share the same conditional expectation $E = 7.2$, and their observed distribution is consistent with Poisson scatter (χ^2/d.f. $= 1.51$). The two outliers—$k = 8$ (2 observed) and $k = 48$ (0 observed)—are within the expected range of fluctuations for 12 independent draws.

9.5 Computational Remarks

Each main star requires testing 2,500 candidates at ∼15 ms per 25-round Miller–Rabin test (500-digit numbers), for a total of ∼37.5 seconds per star. The full survey of 2,107 stars consumed approximately 22 CPU-hours on a single core. The data were collected as a byproduct of the deep-space quadruplet search [5], with negligible additional cost.

A natural extension is the *right-side scan* ($P + k$), which would test the symmetry of the Cramér model around P and double the statistical power for rare-gap detection.

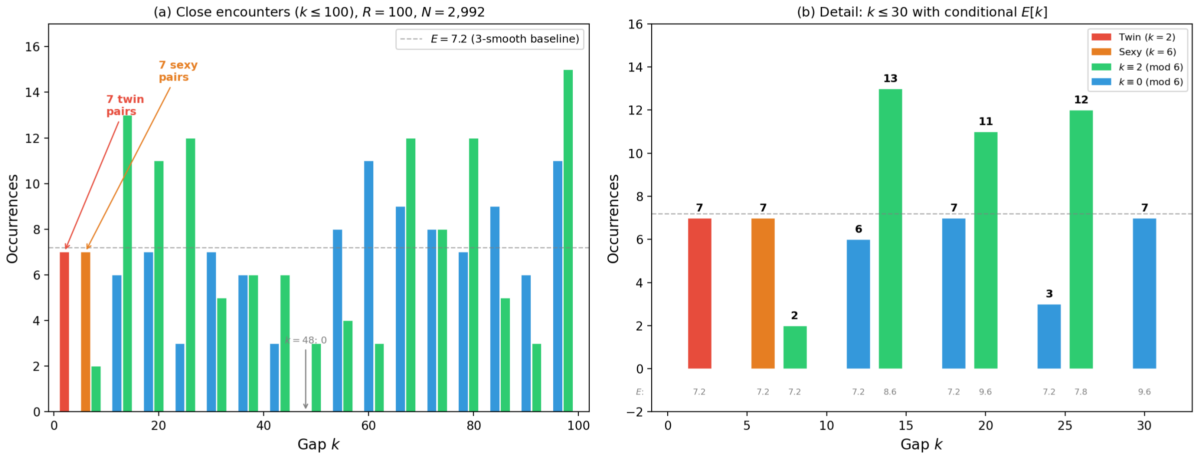

Figure 4: (a) All close encounters ($k \leq 100$), with $k = 2$ (twin primes) and $k = 6$ (sexy primes) marked. (b) Fine-grained gap census for $k < 62$. Seven twin pairs ($k = 2$) and seven sexy pairs ($k = 6$) constitute the closest encounters, confirming $\mathcal{S}_{\text{cond}}(2) = \mathcal{S}_{\text{cond}}(6)$.

10 Conclusion

The satellite prime survey—9,012 primes within radius 5,000 of 2,107 giant primes generated by $Q(n) = n^{47} - (n-1)^{47}$, supplemented by a close-encounter scan of all 2,992 main stars at radius 100—provides the first large-scale validation of the Cramér random model at 500-digit scales.

The key findings are:

1. **Cramér agreement:** satellite count, gap distribution, nearest-neighbor CDF, and density scaling with n all match the Cramér model to ∼2%.
2. **Poisson fit:** the satellite count per star is Poisson with $\lambda = 4.28$ and dispersion index 1.07, including ∼28 zero-satellite stars recovered from a data-aggregation artifact.
3. **Forbidden residue lattice:** all gaps satisfy $k \equiv 0$ or $2 \pmod 6$, a consequence of $Q(n) \equiv 1 \pmod 6$, concentrating satellite density onto 1,667/2,500 admissible slots at rate $\sim 3/\ln P$ each.

4. **Bayesian concentration:** the fixed residue $P \equiv 1 \pmod 3$ doubles the conditional twin-prime rate for $k \equiv 2 \pmod 6$ gaps, exactly compensating the smaller unconditional HL factor—explaining the observed equality of the two mod-6 classes.
5. **Twin–sexy symmetry:** 7 twin pairs ($k = 2$) and 7 sexy pairs ($k = 6$) of 500-digit primes were found, both matching the conditional expectation $E = 7.2$ to within 0.1σ—a precision confirmation of the identity $\mathcal{S}_{\text{cond}}(k{=}2) = \mathcal{S}_{\text{cond}}(k{=}6)$.
6. **3-smooth baseline:** all 12 gaps in the 3-smooth family ($k = 2^a \cdot 3^b$) share the same $\mathcal{S}_{\text{cond}} = 2.64$; the observed scatter ($\chi^2/\text{d.f.} = 1.51$) is consistent with Poisson fluctuation, explaining why $k = 8$ (2 obs.) is rarer than $k = 2$ (7 obs.) despite identical expectations.

These results establish that the algebraic origin of giant primes from a high-degree polynomial does not perturb their local prime environment: Cramér universality holds at 500 digits.

Data and Code Availability

Complete satellite data, analysis scripts, and figures:

https://github.com/Ruqing1963/Q47-Satellite-Primes

Related repositories:

Part II: https://github.com/Ruqing1963/Q47-Deep-Space-Quadruplet-Census
Part III: https://github.com/Ruqing1963/Q47-Quintuplet-Sextuplet-Boundary

Zenodo archives: Part III: https://zenodo.org/records/18728917, Part II: https://zenodo.org/records/18728540, Part I: https://zenodo.org/records/18701355.

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