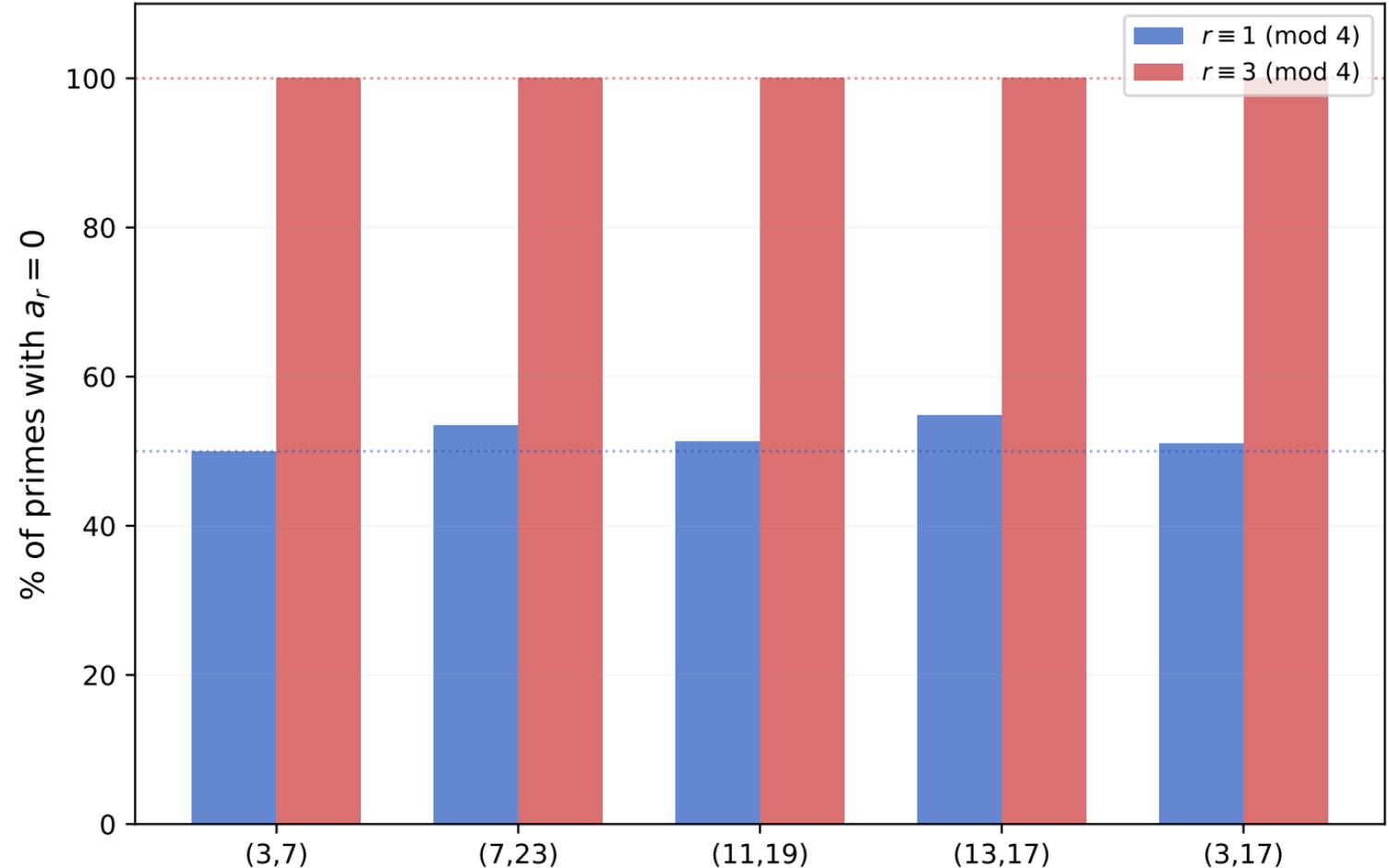


(a) Vanishing rate across 5 curves



(b) Complete local arithmetic at all primes

Local $L$ -factor classification			
Case	Condition	Reduction	Local factor
I / II	$r = p \text{ or } q$	1 cusp ( $A_2$ )	$(1 - a_r r^{-s} + r^{1-2s})^{-1}$
III / IV	$r \mid N \text{ or } r \mid (p-q)$	2 nodes ( $A_1$ )	$(1 - r^{-s})^{-2}$
Good, $r \equiv 3 \pmod{4}$	$r \nmid \Delta$	Smooth	$a_1 = 0 \text{ (proved)}$
Good, $r \equiv 1 \pmod{4}$	$r \nmid \Delta$	Smooth	$a_1 \text{ generic}$