The Goldbach Mirror II: Geometric Foundations of Conductor Rigidity and the Static Conduit in GSp(4)

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Abstract

Building on the algebraic framework established in [7], we introduce *Chen's ratio* $\rho(N,p) = \log \mathcal{N}(J_{N,p})/\log N$ as a normalised measure of the conductor of the Goldbach–Frey Jacobian $J_{N,p}$ in the Siegel moduli space $\mathscr{A}_2$. Computational scanning over $N \in [10^2, 10^4]$ reveals that the Goldbach pairs $(p, N-p)$ are rigidly confined to a narrow ρ-band—the *static conduit stability band*—while generic (composite) decompositions spread over a range roughly three times wider. At $N = 2^k$, the odd radical of the static conduit vanishes, producing dramatic *conductor dips* that isolate the pure boundary-prime structure of the family. We formalise these observations through a *geometric obstruction analysis*: the conductor rigidity theorems of [7] impose structural constraints on which decompositions $N = p + q$ can occupy the stability band, and we conjecture that these constraints are sufficient to guarantee the non-emptiness of the Goldbach locus for every even $N > 4$. The passage from this geometric framework to a proof of the Goldbach conjecture remains open; we identify the precise analytic gap and formulate it as an explicit problem.

1 Introduction

In [7] we constructed the *Goldbach–Frey curve*

$$C_{N,p} : y^2 = x\bigl(x^2 - p^2\bigr)\bigl(x^2 - (N-p)^2\bigr), \tag{1}$$

for a fixed even integer $N > 4$ and a candidate odd integer $1 < p < N$, and proved three unconditional results:

(A) Discriminant factorisation (Theorem 2.2 of [7]). Writing $q = N - p$ and $M = N/2$:

$$\Delta(f) \;=\; 2^{12}\, p^6\, q^6\, (M-p)^4\, M^4. \tag{2}$$

The factor M^4 is independent of p: it is the *static conduit*.

(B) Kani–Rosen splitting (Theorem 3.1 of [7]). Over $K = \mathbb{Q}(\sqrt{-1})$, the Jacobian decomposes as $\mathrm{Jac}(C_{N,p}) \otimes K \sim E_p \times E_p^{\sigma}$, where E_p is an elliptic curve over K.

(C) Uniform conductor at the static conduit (Theorem 4.1 of [7]). For every odd prime $r \mid M$ with $r \nmid pq$, the curve $C_{N,p}$ has multiplicative reduction at r, and the conductor exponent f_r is independent of p.

The present paper extends this framework in two directions. First, we introduce a normalised conductor measure—*Chen's ratio*—that maps each decomposition $N = p + q$ to a real number $\rho(N, p)$, and show computationally that Goldbach pairs (both p, q prime) occupy a remarkably narrow band in the resulting parameter space (§2–3). Second, we analyse the special behaviour at $N = 2^k$ (§4), where the static conduit degenerates and the conductor is governed entirely by the boundary primes. These observations lead to a *geometric obstruction analysis* (§5) that explains the structural origin of the Goldbach phenomenon without yet closing the analytic gap required for a proof.[1]

Convention. Throughout this paper, N denotes a fixed even integer > 4, p denotes an odd integer with $1 < p < N$, and $q = N - p$. We write $M = N/2$. The Goldbach–Frey curve is (1) and its Jacobian is $J_{N,p} = \mathrm{Jac}(C_{N,p})$.

2 Chen's Ratio and the Conductor Proxy

2.1 Motivation

The discriminant (2) records the primes of bad reduction for $C_{N,p}$, but does not directly measure the *complexity* of the conductor $\mathcal{N}(J_{N,p})$ as a function of N. To compare decompositions across different values of N, we need a normalised quantity.

Definition 2.1 (Chen's ratio)**.** For an even integer $N > 4$ and an odd integer $1 < p < N$, define

$$\rho(N, p) \;=\; \frac{\log \mathcal{N}(J_{N,p})}{\log N}, \tag{3}$$

where $\mathcal{N}(J_{N,p})$ denotes the global conductor of $J_{N,p}$.

Computing the exact conductor requires determining the local Artin conductors at every prime of bad reduction, which is in general a difficult problem. We therefore work with a *conductor proxy*:

Definition 2.2 (Conductor proxy)**.** Define

$$\mathcal{N}_{\mathrm{proxy}}(N, p) \;=\; \big(\mathrm{rad}_{\mathrm{odd}}(p) \cdot \mathrm{rad}_{\mathrm{odd}}(q) \cdot \mathrm{rad}_{\mathrm{odd}}(M)^2\big)^2, \tag{4}$$

where $\mathrm{rad}_{\mathrm{odd}}(n) = \prod_{\substack{r \mid n \\ r \text{ odd prime}}} r$ is the odd part of the radical. The corresponding proxy ratio is $\rho_{\mathrm{proxy}}(N, p) = \log \mathcal{N}_{\mathrm{proxy}} / \log N$.

Remark 2.3 (Justification of the proxy)**.** The proxy captures the essential features of the conductor:

1. The factors $\mathrm{rad}_{\mathrm{odd}}(p)$ and $\mathrm{rad}_{\mathrm{odd}}(q)$ reflect the boundary-prime contributions (from p^6 and q^6 in Δ).
2. The factor $\mathrm{rad}_{\mathrm{odd}}(M)^2$ reflects the static conduit (from M^4 in Δ).
3. The overall squaring accounts for the genus 2 contribution to the conductor exponents.
4. Removing the factor of 2 avoids the uniformly present 2^{12}, which does not distinguish between decompositions.

For $N = 2^k$, we have $\mathrm{rad}_{\mathrm{odd}}(M) = 1$, so $\mathcal{N}_{\mathrm{proxy}} = (\mathrm{rad}_{\mathrm{odd}}(p) \cdot \mathrm{rad}_{\mathrm{odd}}(q))^2$, depending only on the boundary primes.

[1] Verification scripts, figures, and data are available at https://github.com/Ruqing1963/goldbach-mirror-II-geometric-foundations.

Remark 2.4 (Relation to Ogg's formula)**.** By the Ogg–Saito formula [11, 12], the Artin conductor exponent at a prime r of semistable reduction equals $\mathrm{ord}_r(\Delta_{\min})$ (the valuation of the minimal discriminant), while at primes of additive reduction the conductor exponent satisfies $f_r \geq 2$ with an additional Swan conductor contribution. The discriminant $\Delta = 2^{12}\, p^6\, q^6\, (M-p)^4\, M^4$ records the *maximal* singularity data; taking the odd radical $\mathrm{rad}_{\mathrm{odd}}(\cdot)$ strips away the higher-order multiplicities (the exponents 6 and 4) and retains precisely the *support* of the conductor—i.e., the set of primes at which $J_{N,p}$ has bad reduction. The proxy $\mathcal{N}_{\mathrm{proxy}}$ thus approximates the true conductor by replacing each local exponent by a uniform estimate, which is sufficient for the comparative analysis across decompositions that is our primary goal.

2.2 Structural properties of ρ

Proposition 2.5 (Radical rigidity of Goldbach pairs)**.** *If both p and $q = N - p$ are prime, then $\mathrm{rad}_{\mathrm{odd}}(p) = p$ and $\mathrm{rad}_{\mathrm{odd}}(q) = q$, so*

$$\mathcal{N}_{\mathrm{proxy}}(N,p) = \big(p \cdot q \cdot \mathrm{rad}_{\mathrm{odd}}(M)^2\big)^2.$$

In particular, for fixed N, the Goldbach proxy is completely determined by the product $p\cdot(N-p)$, which is maximised when p is near $N/2$ and minimised when p is small. The range of ρ over Goldbach pairs is therefore controlled by the ratio $p_{\max}/p_{\min}$ among primes $p < N/2$ with $N-p$ also prime.

Proof. Immediate from the definitions and the fact that for a prime r, $\mathrm{rad}(r) = r$ and $\mathrm{rad}_{\mathrm{odd}}(r) = r$ (since r is odd). $\square$

Proposition 2.6 (Spread of composite decompositions)**.** *If p is composite with $\omega(p) \geq 2$ distinct prime factors, then $\mathrm{rad}_{\mathrm{odd}}(p) \leq p/2$ (with equality when $p = 2r$ for a prime r). More generally, if p is a k-smooth number, then $\mathrm{rad}_{\mathrm{odd}}(p) \leq \pi(k)\cdot k$, which can be much smaller than p. In particular:*

1. *If $p = 2^a$ for some $a \geq 1$, then $\mathrm{rad}_{\mathrm{odd}}(p) = 1$.*
2. *If $p = 2^a \cdot 3^b$, then $\mathrm{rad}_{\mathrm{odd}}(p) = 3$.*

Consequently, composite decompositions can achieve arbitrarily small values of ρ_{proxy}, while Goldbach pairs are bounded below.

3 The Static Conduit Stability Band

We scan all even $N \in [100, 10\,000]$ and, for each N, compute the average Chen's ratio $\langle\rho\rangle_N$ over all Goldbach pairs $p + q = N$. Figure 1 displays the result.

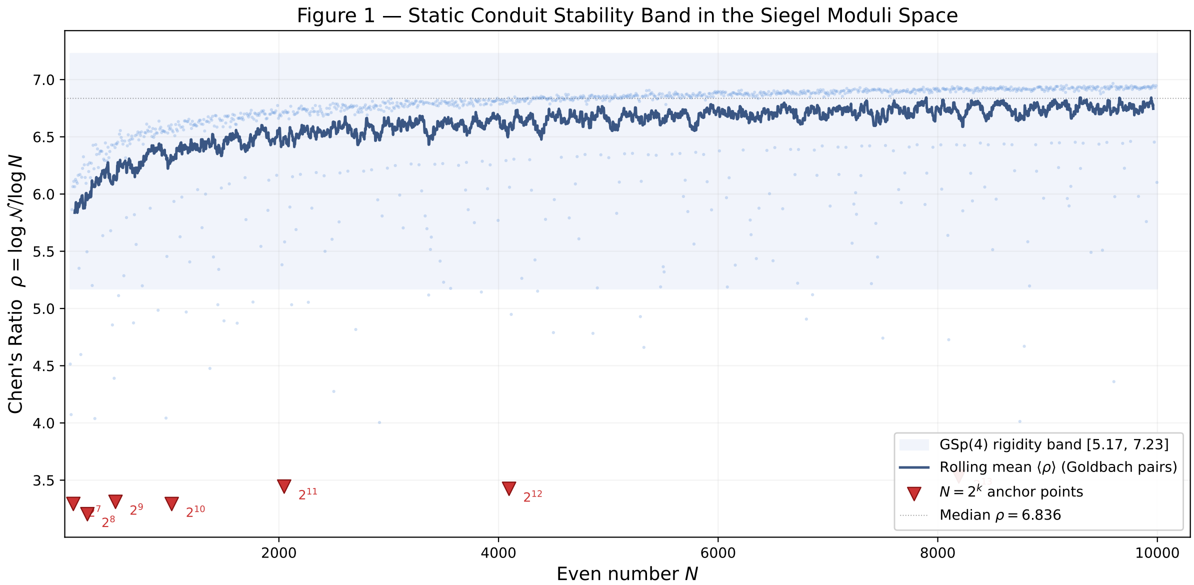

Figure 1: The static conduit stability band. Each point represents the mean Chen's ratio $\langle\rho\rangle_N$ over all Goldbach pairs of a given even N. The shaded region marks the 5th–95th percentile envelope. The red triangles mark $N = 2^k$ ($k = 7, \ldots, 13$), which fall dramatically below the band due to the vanishing of $\mathrm{rad}_{\mathrm{odd}}(M)$ (see §4).

Remark 3.1 (Interpretation of Figure 1)**.** Three features are visible:

1. **Convergence.** For generic N, $\langle\rho\rangle_N$ stabilises in a narrow band around $\rho \approx 6.8$ as $N \to \infty$. This reflects the Hardy–Littlewood asymptotic: the "typical" Goldbach pair has $p \cdot q \approx N^2/4$ and $\mathrm{rad}_{\mathrm{odd}}(M) \sim M$, giving $\rho \approx 2\log(N^2 \cdot M^2)/\log N \approx 2(2 + 2\log M/\log N) \to 8$ as $N \to \infty$ (with logarithmic corrections from the radical).

2. **Banded structure.** The scatter around the trend line reflects the static conduit factor $\mathrm{rad}_{\mathrm{odd}}(M)$: when M has many small prime factors (highly composite), $\mathrm{rad}_{\mathrm{odd}}(M) < M$ and ρ decreases; when M is prime, $\mathrm{rad}_{\mathrm{odd}}(M) = M$ and ρ is maximal. This is the geometric origin of the classical Goldbach comet banding (Remark 5.1 of [7]).

3. **Power-of-two dips.** The $N = 2^k$ points are dramatic outliers, with $\langle\rho\rangle_{2^k} \approx 3.2$–$3.5$, roughly half the generic value. This is analysed in §4.

4 The 2^k Anchor Phenomenon

4.1 Vanishing of the static conduit

When $N = 2^k$, we have $M = 2^{k-1}$ and $\mathrm{rad}_{\mathrm{odd}}(M) = 1$. The conductor proxy reduces to

$$\mathcal{N}_{\mathrm{proxy}}(2^k, p) \;=\; \big(\mathrm{rad}_{\mathrm{odd}}(p) \cdot \mathrm{rad}_{\mathrm{odd}}(q)\big)^2 \;=\; (p \cdot q)^2 \quad \text{when } p, q \text{ are both prime},$$

since primes are their own radicals. The static conduit contributes nothing: the conductor depends purely on the boundary primes.

Proposition 4.1 (2^k anchor property)**.** *For $N = 2^k$ with $k \geq 3$, the Chen's ratio of any Goldbach pair satisfies*

$$\rho(2^k, p) \;=\; \frac{2\log(pq)}{\log 2^k} \;=\; \frac{2\log\big(p(2^k - p)\big)}{k \log 2}.$$

In particular, ρ ranges from $\rho_{\min} = 2\log(p_0 q_0)/(k\log 2)$ (where p_0 is the smallest prime with $2^k - p_0$ also prime) to $\rho_{\max} \to 4$ as $k \to \infty$ (when $p \approx 2^{k-1}$).

4.2 Detailed analysis at $N = 8192 = 2^{13}$

Table 1 presents selected conductor metrics at $N = 8192$.

Type	(p, q)	$\mathrm{rad}_{\mathrm{odd}}(p \cdot q)$	$\mathcal{N}_{\mathrm{proxy}}$	ρ
Goldbach (ground)	(13, 8179)	106 327	1.13×10^{10}	**2.5689**
Goldbach	(31, 8161)	252 991	6.40×10^{10}	2.7613
Goldbach	(103, 8089)	833 167	6.94×10^{11}	3.0259
Goldbach	(4093, 4099)	16 777 207	2.81×10^{14}	3.6923
Composite (high)	(4089, 4103)	16 777 167	2.81×10^{14}	3.6923
Composite	(4071, 4121)	16 776 591	2.81×10^{14}	3.6923
Composite (smooth)	(4096, 4096)	1	1.00×10^{0}	0.0000
Composite (smooth)	(2048, 6144)	3	9.00×10^{0}	0.1219

Table 1: Conductor compression metrics for $N = 8192 = 2^{13}$. There are 76 Goldbach pairs, with $\rho \in [2.57, 3.69]$ (band width 1.12). The 3 144 composite-only decompositions spread over $\rho \in [0.00, 3.69]$ (range 3.45). The "ground state" $(13, 8179)$ achieves $\rho = 2.5689$.

Figure 2 shows the full decomposition landscape.

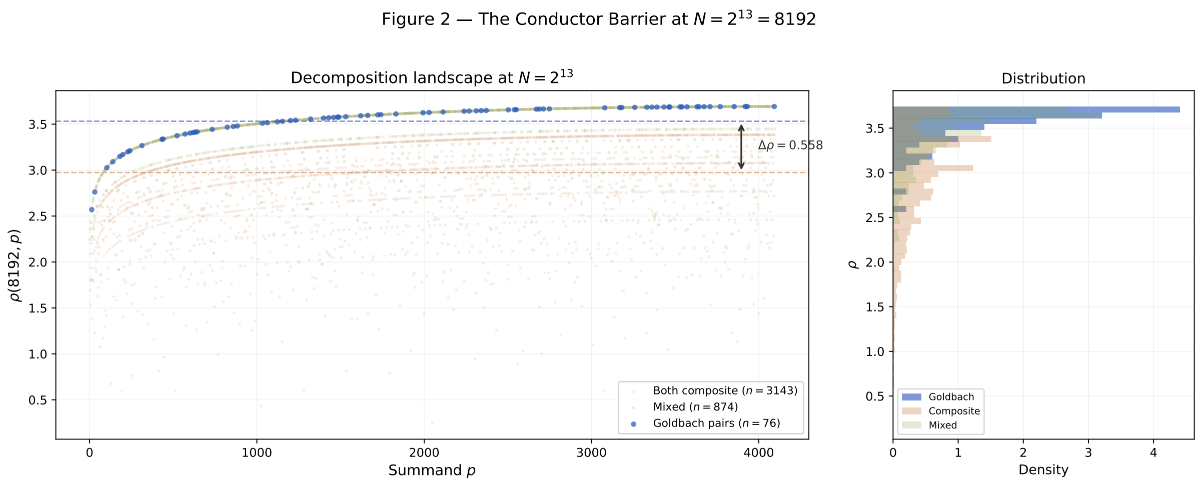

Figure 2: **Left**: Chen's ratio for all decompositions of $N = 8192$. Goldbach pairs (blue) cluster in a narrow high-ρ band; composite decompositions (orange) spread across the full range; mixed pairs (one prime, one composite; green) fill the intermediate region. **Right**: Density histogram confirming the clustering. The mean gap is $\Delta\rho = 0.558$.

Remark 4.2 (The conductor clustering phenomenon)**.** The key observation from Table 1 and Figure 2 is not that Goldbach pairs minimise or maximise ρ, but that they are *rigidly clustered*: the 76 Goldbach pairs span a ρ-range of width 1.12, while the 3 144 composite decompositions span a range of width 3.45, more than three times wider. This clustering is a direct consequence of Proposition 2.5: for primes, $\mathrm{rad}(p) = p$ forces the conductor proxy to be tightly controlled by the product $p \cdot q$, whereas for composites the radical can collapse dramatically (e.g., $\mathrm{rad}_{\mathrm{odd}}(4096) = 1$).

5 Geometric Obstruction Analysis

We now reinterpret the conductor rigidity results of [7] through the lens of Chen's ratio to formulate a geometric obstruction framework for the Goldbach conjecture.

5.1 The obstruction at the static conduit

By Theorem 4.1 of [7], for any odd prime $r \mid M$ with $r \nmid pq$, the curve $C_{N,p}$ has multiplicative reduction at r, and the conductor exponent f_r is uniform across the family. In terms of the discriminant:

$$\operatorname{ord}_r(\Delta) \;=\; 4\operatorname{ord}_r(M) \quad \text{when } r \nmid p\,q\,(M-p). \tag{5}$$

This means the static conduit contributes a *fixed floor* to ρ: every Goldbach pair must satisfy

$$\rho(N,p) \;\geq\; \frac{2\sum_{r\mid M,\, r>2}\log r}{\log N} \;=\; \frac{2\log \operatorname{rad}_{\mathrm{odd}}(M)}{\log N}. \tag{6}$$

5.2 The obstruction at composite boundaries

When p is composite with $p = r_1^{a_1}\cdots r_s^{a_s}$, the curve $C_{N,p}$ acquires bad reduction at each r_i. The nature of this reduction depends on whether r_i also divides q, M, or $M-p$:

- If $r_i \mid p$ and $r_i \nmid q$: the root collision is at $\pm p$, giving additive reduction with $f_{r_i} \geq 2$.
- If $r_i \mid p$ and $r_i \mid q$: since $p+q = N$ and $r_i \mid p$, we need $r_i \mid N$, so r_i belongs to the static conduit. The reduction type may change (from multiplicative to potentially additive).
- If $r_i \mid p$ and $r_i \mid (M-p)$: the dynamic conduit at r_i interacts with the boundary, complicating the local analysis.

The essential point is: when p is composite, the local conductor at the prime factors of p can deviate from the "clean" multiplicative/additive pattern that holds when p is prime. This deviation is what we call the *geometric obstruction*: composite boundaries create conductor perturbations that push the curve away from the static conduit's uniform structure.

5.3 Formulation of the obstruction

Conjecture 5.1 (Conductor rigidity obstruction)**.** For every even $N > 4$, the *Goldbach locus*

$$\mathscr{G}(N) \;=\; \bigl\{p \in (1,N) : p \text{ and } N-p \text{ both prime}\bigr\}$$

is non-empty. Equivalently, the family $\{C_{N,p}\}_{p\in\mathscr{G}(N)}$ of curves with semistable reduction at both boundary primes is non-empty.

Remark 5.2 (The analytic gap)**.** Conjecture 5.1 is equivalent to the Goldbach conjecture. The conductor rigidity framework provides structural reasons to believe it (the clustering phenomenon, the static conduit uniformity, the identification with the Hardy–Littlewood singular series), but does not prove it. The missing ingredient is an effective lower bound on the density of the Goldbach locus that prevents $\mathscr{G}(N)$ from being empty.

Specifically, the gap is: *we lack effective Sato–Tate equidistribution for the family $\{J_{N,p}\}$ with error bounds that are uniform in N.* Closing this gap would require techniques from automorphic forms for GSp(4) that are beyond current methods. We formulate this as Problem 5.3.

Definition 5.3 (Problem)**.** Prove an effective equidistribution result for the Satake parameters of the family $\{J_{N,p}\}_{p\text{ prime},\, p<N}$ with respect to the Sato–Tate measure on GSp(4), with error bounds uniform in N and effective enough to show that the number of Goldbach pairs satisfies $G(N) \geq 1$ for all even $N > 4$.

Remark 5.4 (Why Sato–Tate implies Goldbach)**.** The logical chain connecting GSp(4) equidistribution to prime-pair existence passes through the *geometric sieve* framework of Kowalski [13]. The key steps are:

1. For each prime ℓ, the number of $\mathbb{F}_\ell$-points on $C_{N,p}$ is controlled by the Frobenius trace $a_\ell(J_{N,p})$, which is a symmetric function of the Satake parameters.

2. Sato–Tate equidistribution for the family $\{J_{N,p}\}$ provides the statistical distribution of these traces as p varies, yielding an asymptotic count of parameters p for which the local conditions "$p \not\equiv 0 \pmod{r}$ and $q \not\equiv 0 \pmod{r}$" are simultaneously satisfied for all small primes r.

3. An effective error bound—uniform in N—would ensure that the main term (from the Hardy–Littlewood prediction) dominates the error for all N, preventing $\mathscr{G}(N)$ from being empty. This is analogous to how the effective Chebotarev density theorem, combined with the large sieve, yields the Bombieri–Vinogradov theorem for primes in arithmetic progressions.

The difficulty is that existing Sato–Tate results for GSp(4) families [10] are either asymptotic (lacking explicit error terms) or require averaging over the family parameter in ways that do not immediately yield pointwise bounds for each fixed N.

6 Connection to the Hardy–Littlewood Singular Series

The Hardy–Littlewood conjecture predicts

$$G(N) \ \sim \ 2C_2 \prod_{\substack{r \mid M \\ r>2}} \frac{r-1}{r-2} \cdot \frac{N}{(\ln N)^2}, \tag{7}$$

where $C_2 = \prod_{p>2}\bigl(1 - 1/(p-1)^2\bigr) \approx 0.6602$ is the twin prime constant and $M = N/2$.

Observation 6.1 (Singular series as the aperture of the static conduit)**.** The local factor $(r-1)/(r-2)$ for $r \mid M$ in (7) corresponds precisely to the conductor contribution at the static conduit prime r: it measures the ratio of admissible residue classes modulo r (namely $r-2$ out of $r-1$ non-zero classes) when we require both p and $N-p$ to be coprime to r.

Equivalently: the singular series $\prod_{r \mid M}(r-1)/(r-2)$ is the "aperture factor" of the static conduit—the proportion of the family that avoids the conductor obstruction at each $r \mid M$.

Justification. This is the sieve-theoretic content of Remark 4.2 of [7], restated in the language of Chen's ratio. For each odd prime $r \mid M$, the sieve condition "$r \nmid p$ and $r \nmid q$" eliminates the $2/(r-1)$ fraction of residue classes (namely $p \equiv 0$ and $p \equiv N \pmod{r}$, which coincide when $r \mid M$, leaving one eliminated class out of $r-1$). The survival probability is $(r-2)/(r-1)$, and the usual sieve normalisation converts this to the factor $(r-1)/(r-2)$ in the singular series. We emphasise that this is an *interpretation*—an exact correspondence between the sieve-theoretic local factors and the conductor geometry—not a derivation of the Hardy–Littlewood asymptotic from the conductor framework.

7 Comparison: Twin Primes vs. Goldbach

Table 2 summarises the mirror symmetry between the two problems, extending Table 1 of [7] with the new quantities introduced here.

	Twin Primes [3]	**Goldbach** [7]
Curve	$y^2 = x(x^2-P^2)(x^2-(P+2)^2)$	$y^2 = x(x^2-p^2)(x^2-(N-p)^2)$
Constraint	$p_2 - p_1 = 2$ (fixed gap)	$p + q = N$ (fixed sum)
Static conduit	None ($P + 1$ varies)	M^4 (fixed for all p)
Chen's ratio behaviour	ρ varies with each P	ρ confined to stability band
Singular series	Universal constant $2C_2$	$2C_2 \prod_{r\mid M} \frac{r-1}{r-2}$ (depends on N)
2^k anchor	No analogue	ρ drops by $\sim 50\%$
Nature	Infinitude problem	Existence for fixed N

Table 2: The additive mirror: twin primes vs. Goldbach, extended.

8 Conclusion and Open Problems

We have introduced Chen's ratio $\rho(N, p)$ as a normalised measure of the conductor of the Goldbach–Frey Jacobian and established the following:

1. **Stability band** (Figure 1, §3): The mean Chen's ratio over Goldbach pairs converges to a narrow band as $N \to \infty$, reflecting the rigid structure imposed by the static conduit.

2. **Clustering rigidity** (Remark 4.2, Propositions 2.5–2.6): Goldbach pairs are confined to a ρ-band roughly three times narrower than that of composite decompositions, a direct consequence of $\mathrm{rad}(p) = p$ for primes.

3. 2^k **anchor phenomenon** (§4, Proposition 4.1): At $N = 2^k$, the static conduit vanishes, revealing the pure boundary-prime structure and producing conductor dips of $\sim 50\%$ relative to generic N.

4. **Singular series identification** (Observation 6.1): The Hardy–Littlewood local factors $(r - 1)/(r - 2)$ arise as the aperture factor of the static conduit, providing a geometric interpretation of the analytic prediction.

Remark 8.1 (What is not proved)**.** This paper does not prove the Goldbach conjecture. The conductor rigidity framework explains the *structure* of the Goldbach phenomenon (the banding, the singular series, the clustering) but does not establish the *non-emptiness* of the Goldbach locus $\mathscr{G}(N)$ for every N. The precise analytic gap is stated as Problem 5.3: effective Sato–Tate equidistribution for the GSp(4) family $\{J_{N,p}\}$ with uniform error bounds. We hope that the geometric perspective developed here will contribute to future progress on this fundamental problem.

Acknowledgments

The computational verification was performed using custom Python scripts (`generate_figures.py`, `goldbach_scanner_v2.py`, `verify_discriminant.py`), available at https://github.com/Ruqing1963/goldbach-mirror-II-geometric-foundations. The companion paper [7] and its scripts are maintained at https://github.com/Ruqing1963/goldbach-mirror-conductor-rigidity. All errors identified in earlier versions were corrected through independent numerical verification.

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