SE 276C/MAE 232C Nonlinear and Advanced Finite Element Methods **Spring 2019**

Final Project, Due June 12, 2019
Option 2

A cantilever beam subjected to a shear load in Fig. 1 is to be modeled by a 2-dimensional nonlinear finite element method. As shown in the Figure 1 below, the dimension of the beam is L=50.0 *in*, and D=5.0 *in*, and the shear load is $P = 1.0\times10^2$ *lb*. The beam is made of a hyperelastic material characterized with following Saint Venant-Kirchhoff strain energy density function:

$$W = \frac{1}{2} C^s_{ijkl} E_{ij} E_{kl}$$

where

$$C^s_{ijkl} = \lambda \delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

$$\mu = \frac{E}{2(1+\nu)}$$

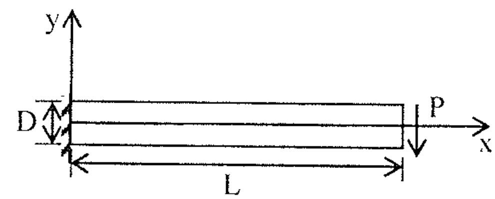

Figure 1. Beam subjected to a tip shear load

The material constants are $E = 30.0\ ksi$, $\nu = 0.4999$.

Consider using the finite element discretization with 2x20, 4x40, 8x80, and 16x160 elements to study the solution accuracy due to mesh refinement.

Solve the beam problem using Q4-FI, Q4-RI, and Q4-SRI to obtain the following solution:

(1) Load (P) - deflection (u_y) response at the centroid of the loading end.

(2) Displacement solution u_y along $y = 0$ at $P = 1.0\times10^2$ *lb*.

(3) Stress solutions σ_{xx} and σ_{xy} along $x = L/2$ at $P = 1.0\times10^2$ *lb*.

1. Discuss how Q4-SRI can avoid incompressible locking in Q4-FI element, and how Q4-SRI can stabilize hourglass instability in Q4-RI element.

2. Compare the displacement and stress solution accuracies between Q4-FI and Q4-RI, and Q4-SRI at various mesh refinements.

Final report should contain the problem statement, numerical formulation, numerical procedures, numerical results, discussions, and a copy of your program should be attached.

Note: The reference solutions of this problem will be posted on the class website.