PIVOT 4A LEARNER'S MATERIAL

QUARTER 2

Mathematics

G10

MELC based SLM

DepEd CALABARZON
Curriculum and Learning Management Division

This module was carefully examined and revised in accordance with the standards prescribed by the DepEd Regional Office 4A and CLMD CALABARZON. All parts and sections of the module are assured not to have violated any rules stated in the Intellectual Property Rights for learning standards.

The Editors

PIVOT 4A Learner's Material
Quarter 2
First Edition, 2020

Mathematics
Grade 10

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Guide in Using PIVOT 4A Learner's Material

For the Parents/Guardians

This module aims to assist you, dear parents, guardians, or siblings of the learners, to understand how materials and activities are used in the new normal. It is designed to provide information, activities, and new learning that learners need to work on.

Activities presented in this module are based on the Most Essential Learning Competencies (MELCs) in **Mathematics** as prescribed by the Department of Education.

Further, this learning resource hopes to engage the learners in guided and independent learning activities at their own pace. Furthermore, this also aims to help learners acquire the essential 21st century skills while taking into consideration their needs and circumstances.

You are expected to assist the children in the tasks and ensure the learner's mastery of the subject matter. Be reminded that learners have to answer all the activities in their own answer sheet.

For the Learners

The module is designed to suit your needs and interests using the IDEA instructional process. This will help you attain the prescribed grade-level knowledge, skills, attitude, and values at your own pace outside the normal classroom setting.

The module is composed of different types of activities that are arranged according to graduated levels of difficulty—from simple to complex. You are expected to :

a. answer all activities on separate sheets of paper;

b. accomplish the **PIVOT Assessment Card for Learners on page 38** by providing the appropriate symbols that correspond to your personal assessment of your performance; and

c. submit the outputs to your respective teachers on the time and date agreed upon.

Parts of PIVOT 4A Learner’s Material

	K to 12 Learning Delivery Process	Descriptions
Introduction	What I need to know	This part presents the MELC/s and the desired learning outcomes for the day or week, purpose of the lesson, core content and relevant samples. This maximizes awareness of his/her own knowledge as regards content and skills required for the lesson.
	What is new	
Development	What I know	This part presents activities, tasks and contents of value and interest to learner. This exposes him/her on what he/she knew, what he/she does not know and what he/she wants to know and learn. Most of the activities and tasks simply and directly revolve around the concepts of developing mastery of the target skills or MELC/s.
	What is in	
	What is it	
Engagement	What is more	In this part, the learner engages in various tasks and opportunities in building his/her knowledge, skills and attitude/values (KSAVs) to meaningfully connect his/her concepts after doing the tasks in the D part. This also exposes him/her to real life situations/tasks that shall: ignite his/ her interests to meet the expectation; make his/her performance satisfactory; and/or produce a product or performance which will help him/her fully understand the target skills and concepts .
	What I can do	
	What else I can do	
Assimilation	What I have learned	This part brings the learner to a process where he/she shall demonstrate ideas, interpretation, mindset or values and create pieces of information that will form part of his/her knowledge in reflecting, relating or using them effectively in any situation or context. Also, this part encourages him/her in creating conceptual structures giving him/her the avenue to integrate new and old learnings.
	What I can achieve	

This module is a guide and a resource of information in understanding the Most Essential Learning Competencies (MELCs). Understanding the target contents and skills can be further enriched thru the K to 12 Learning Materials and other supplementary materials such as Worktexts and Textbooks provided by schools and/or Schools Division Offices, and thru other learning delivery modalities, including radio-based instruction (RBI) and TV-based instruction (TVI).

WEEKS 1-2

Polynomial Functions

I Lesson

After going through this lesson, you are expected to illustrate polynomial functions, understand, describe and interpret the graph of polynomial functions; and solve problems involving polynomial functions.

Do you remember when an expression is a polynomial? Which of the given expressions are polynomial? What makes the expressions not a polynomial?

In this lesson, you need to revisit the lessons and your knowledge on evaluating and factoring polynomials necessary in graphing linear and quadratic functions. Your knowledge and familiarity of these topics will help you sketch the graph of polynomial functions manually. The use of graphing utilities/tools might help you to view a clearer graph that will lead you to understand, describe and interpret the properties of the graph.

D

Learning Task 1: Write and answer the following in your answer sheet.

1. Evaluate the following polynomials in your answer sheet.

a. $f(x) = x^4 - 7x^2 + 2x - 6$ at $x = 1$

b. $f(x) = 10x^3 + 4x^2 - 5$ at $x = -3$

c. $f(x) = x^4 + 5x^3 - 2x + 3$ find $(f(2))^2$.

2. Factor each polynomial completely using any method

a. $(x+1)(x^2 - 5x + 6)$

b. $(x^2 - x - 6)(x^2 + 6x + 9)$

c. $x^3 + 3x^2 - 4x - 12$

Remember that the graph of a polynomial function whose degree is 0 or 1 is a line. The graph of a polynomial function is a parabola if it is a quadratic function.

How does the value of *b* in $y = bx + c$ affect the behavior of the line?

How does the value of *a* in $y = ax^2 + bx + c$ affect the end behavior of the parabola?

You are expected in this lesson to focus on polynomial functions of degree higher than 2. What is a polynomial function? A polynomial of degree n is a function of the form $P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$, where n is a nonnegative integer, the a's such as, a_n, a_{n-1}, … a_2, a_1, a_0 are real numbers called the coefficients. $a_n x^n$ is the leading term, a_n is the **leading coefficient** and a_0 is the **constant term**.

The terms of a polynomial may be written in any order. However, if they are written in decreasing powers of x, we say that the polynomial is in **standard form.**

Other than P(x), a polynomial function can also be denoted by f(x). Sometimes, a polynomial function is represented by a set of P of ordered pairs (x, y). Thus, a polynomial function can be written in different ways, like the following.

$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$

$y = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$

Although this general formula might look quite complicated, particular examples are much simpler.

Illustrative Example 1

$f(x) = 4x^3 - 3x^2 + 2$ is a polynomial of degree 3, as 3 is the highest power of x in the formula. This is called a cubic polynomial, or just a cubic. The leading term is $4x^3$, so, the leading coefficient is 4. The term with no variable is 2, so 2 is the constant term.

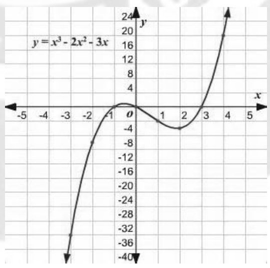
Illustrative Example 2

1. $f(x) = 4x + \sqrt{x} - 1$ is **not** a polynomial function since there is a variable x inside the radical sign which can be written as $f(x) = 4x + x^{\frac{1}{2}} - 1$.

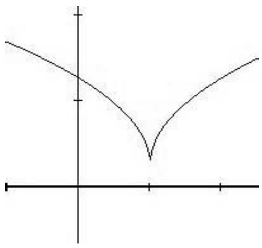
2. $f(x) = 5x^4 - 2x^2 + \frac{3}{x}$ is **not** a polynomial as it contains a variable x in the denominator. The term $\frac{3}{x}$ can also be expressed as $3x^{-1}$, as we learned in the laws of exponent.

Polynomials may also be written in factored form and as a product of irreducible factors, that is, a factor that can no longer be factored using coefficients that are real numbers.

Graph of P(x)



This is not graph of P(x)



1. The graph of any type of function must pass the vertical line test.
2. Every polynomial function with real coefficients has the set of real numbers as its domain; hence it is continuous function. This means that the graph of a polynomial function has no breaks, holes or gaps.
3. A polynomial equation of the nth degree cannot have more than n roots. This only means that the graph cannot intersect the x-axis more than n times.
4. A graph of a polynomial function has only smooth, rounded turns. A polynomial function cannot have a sharp turn.

Zeros of Polynomial Function

It can be shown that for a polynomial function of degree ***n***, the following statements are true:

The function has, at most, *n* real zeros.

The graph has, at most, *n – 1* turning points.

Turning points (relative maximum or relative minimum) are points at which the graph changes from increasing to decreasing or vice versa.

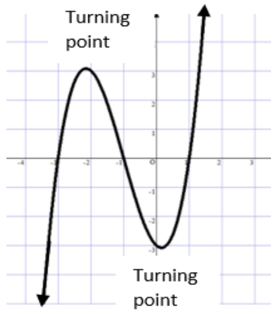
The zeros of a polynomial function are the values of x which make $f(x) = 0$. These values are the **roots**, or **solutions** of the polynomial equation when y = 0. All real roots are the x-intercepts of the graph.

Illustrative Example 3

How many turning points does $f(x) = x^3 + 3x^2 - x - 3$ have?

Find all the zeros of... $f(x) = x^3 + 3x^2 - x - 3$

Set up the equation: $x^3 + 3x^2 - x - 3 = 0$ and solve.



Using synthetic division

-1	1	3	-1	-3
		-1	-2	3
1	1	2	-3	0
		1	-3	
	1	3	0	

Therefore, the zeros are $-3, -1,$ and 1. These are the x-intercepts. And the y-intercept is -3.

The degree of the function is 3, the graph has 2 turning points.

Multiplicities of Zeroes

The multiplicity of a zero is the number of times the real root of a polynomial functions results in $f(x) = 0$. Suppose r is a zero of even multiplicity, then, the graph touches the x-axis at r and bounces at r or is tangent to point (r, 0). Suppose r is a zero of odd multiplicity. Then the graph crosses the x-axis at r. Regardless of whether a multiplicity is even or odd, the graph tends to flatten out near zeros with a multiplicity greater than one.

Illustrative Example 4

Solve for the zeroes of $f(x) = x^2(x-2)^2$, then show the graph.

$x^2(x-2)^2 = 0$

By Zero Product Property

$x^2 = 0$ or $(x-2)^2 = 0$

$x^2 = 0$ therefore $x = 0$ to the multiplicity of 2

$(x-2)^2 = 0$ therefore $x = 2$ to the multiplicity of 2

The exponent tells us the multiplicity.

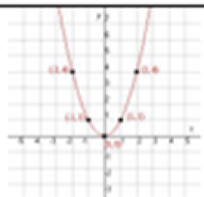
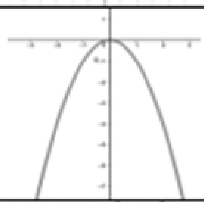
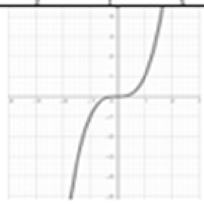
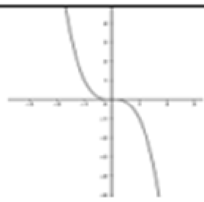
The graph of the polynomial function $f(x) = x^2(x-2)^2$ as shown at the right is tangent to the x-axis at points (0, 0) and (2, 0)

Behavior of Polynomial functions

The behavior of the graph of a function to the far left and far right is called its **end behavior**.

Although the graph of a polynomial function may have intervals where it increases or decreases, the graph will eventually rise or fall without bound as it moves far to the left or far to the right.

How can we determine the end behavior of a polynomial function? Using the table on the next page, observe how the end behavior of the graph changes in relation to the leading coefficient and degree of the polynomial functions.

						Rising or Falling	
	$a_n > 0$	$a_n < 0$	even	odd		Left-hand	Right-hand
$f(x) = x^2$	$a_n > 0$		even			rising	rising
$f(x) = -x^2$		$a_n < 0$	even			falling	falling
$f(x) = x^3$	$a_n > 0$			odd		falling	rising
$f(x) = -x^3$		$a_n < 0$		odd		rising	falling

Solving Word Problems Involving Polynomial Functions

Polynomial functions have varied applications in real life situations. Consider the problem below.

Illustrative Example 5

Find the length of the edge of a cube, if an edge is increased by 3 *dm*, another edge has a 6 *dm* increase and the third one decreases by 2 *dm*, results to 100% increase of its original volume.

Representation:

$x + 3$ – first edge

$x + 6$ – Second edge

$x - 2$ – third edge

The volume of cube is equal to x^3, and it increases 100% or it is doubled.

$(x + 3)(x + 6)(x - 2) = 2x^3$

$(x + 3)(x^2 + 4x - 12) = 2x^3$

$x^3 + 4x^2 - 12x + 3x^2 + 12x - 36 = 2x^3$

$x^3 + 7x^2 - 36 = 2x^3$

$2x^3 - x^3 - 7x^2 + 36 = 0$

$x^3 - 7x^2 + 36 = 0$

Use synthetic division to find x.

3	1	-7	0	36
		3	-12	36
	1	-4	-12	0

The factors are $(x-3)(x^2-4x-12)$ but $(x^2-4x-12)$ is still factorable.
So, the complete factors are $(x-3)(x-6)(x+2)$.

Equating the factors to zero, we have
$(x-3)(x-6)(x+2) = 0$
Then, by Zero product property

$x-3=0$	$x-6=0$	$x+2=0$
$x = 3$	$x = 6$	$x = -2$

-2 is not acceptable solution because when substituted will result to negative dimension. This is considered as extraneous root. Substituting $x=3$ to the edges, the dimensions will be 6 dm x 9 dm x 1dm or if we use $x=6$, dimensions will be 9 dm x 12 dm x 4 dm.

E

Learning Task 2: In your answer sheet, copy and answer the following.
1. Describe the properties of the graph of the given polynomial functions.

A.) $F(x) = x^4 - x^3 - 2x^2$ and
B.) $f(x) = (x-1)^2(x+1)$ as to the following:
   a. Standard form
   b. Leading term
   c. x-intercepts and its multiplicities
   d. y-intercepts
   e. number of turning points
   f. possible graph with end behavior
2. Sketch the graph of the polynomial functions given in Part A.
3. Find two numbers whose difference is 16 and the product is 720.

Learning Task 3: In your answer sheet, write the letter of the correct answer.
1. Which is a polynomial function?

A. $P(x) = 2x^3 + 3x^2 - 4x + 4$
C. c. $G(x) = 2x^{-3} + 5$
B. $H(x) = 4x^{1/2} + 3x - 4$
D. $F(x) = \dfrac{2x^3 - 5x + 3}{3x^2}$

2. What are the zeroes of $P(x) = x^3 - 6x^2 - x + 6$?

A. $-6, -1, 1$
C. $6, -1, 1$
B. $-6, -1, -1$
D. $6, 1, 1$

3. How many turning points does the graph $P(x) = (x-7)(x+4)(x-2)$ have?

A. 1 B. 2 C. 3 D. 4

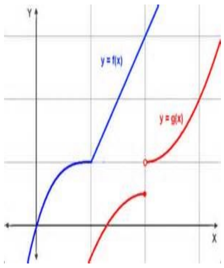
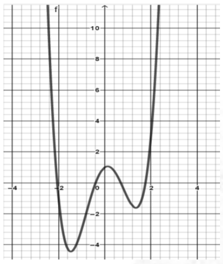
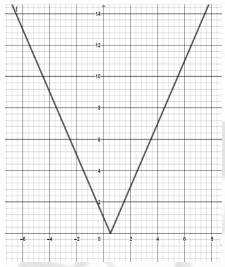
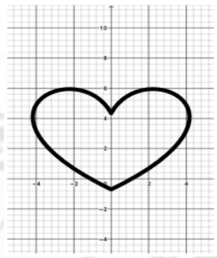
4. What are the three consecutive positive integers such that the sum of their squares is 149?

A. 5,6,7 B. 6,7, 8 C. 7,8,9 D. 8,9,10

5. What is the y-intercept in $P(x) = x^3 - 2x^2 - 5x + 6$?

A. 0 B. 2 C. 4 D. 6

6. Which of the following are graphs of polynomial functions.

A.  B.  C.  D. 

For numbers 7-16, identify the value of the leading coefficient and the degree of the polynomial function. Then, describe the end behavior of the graph of the following polynomials.

7-11. $f(x) = x^4 - 4x^2$

7. a_n ($a_n > 0$ or $a_n < 0$) ____________________
8. degree (odd or even) ____________________
9. end behavior of the graph ____________________
10-11. Sketch the graph

12-16. $f(x) = -2x^3 + 3x^2 - x - 3$

12. a_n ($a_n > 0$ or $a_n < 0$) ____________________
13. degree (odd or even) ____________________
14. end behavior of the graph ____________________
15-16. sketch the graph

For numbers 17-20, answer the problem below:
Trizia Mae, a gymnast, dismounts the uneven parallel bars. Her height, , depends h, on the time, t, that she is in the air as $h = -16t^2 + 8t + 8$.

a) How long will it take Trizia Mae to reach the ground?

b) When will Trizia Mae be 8 feet above the ground?

Learning Task 4. Read, analyze and do the problem. Use a sheet of old cardboard or old pieces of plywood (or any available similar material) as sample materials in preparing an open-topped box with the dimensions mentioned below to help Cassidy in arranging her cactus. The bottom of the box should be closed. Use the rubric below in doing the task. Then, answer the question in your answer sheet.

Cassidy need an open-topped box to display her cactus collections. The open-topped box can be created by cutting squares from the four corners of a 20cm by 30cm piece of cardboard.

What dimensions of the box will create an open-topped box with a volume of 1008 cm^3?

Point	Solution Process	The Conclusion/Answer
4	A complete and appropriate solution	Accurate conclusion, supported by valid evidence and reasons, appropriate to this prob-
3	An appropriate solution process that is almost complete	Inaccurate but logical conclusion, supported by evidence and reasoning but incorrect due to a minor factual error (in details of problem, in computation, recall a formula etc.) or minor
2	An appropriate process that is partially complete	Inaccurate but logical conclusion that overlooks, or get wrong significant facts (about the problem, the rule, computation, etc.)
1	An inappropriate process or no evidence of a process	Inappropriate conclusion, not supported by facts and logic, or there is no conclusion

Criteria for Rating the Output

The box has the needed dimensions and parts.
The box is properly labeled with the required length of parts.
The box is durable.
The box is neat and presentable

Points to be given:
4 points – if all items in the criteria are evident
3 points – if any three of the items are evident
2 points – if any of the two of the items are evident
1 point if any of the items are evident

A

In your journal notebook, reflect and answer the questions.

1. How can we determine the end behavior of a polynomial function?
2. How would you describe the end behavior of the graph?
3. What new strategies do you discover in graphing polynomial functions?

Circles and other Related Terms

Lesson

I

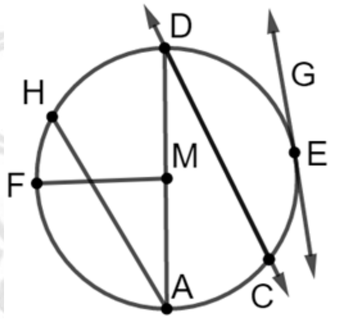
After going through this lesson, you are expected to derive inductively the relations among chords, arcs, central angles and inscribed angles; and prove theorems related to chords, arcs, central angle and inscribed angles.

A circle is an important and useful topic in the study of Grade 10 Math. The circle and its properties, the parts of a circle and the terms related to it form the core of these lessons. Relationships among the measures of tangent lines, secant lines and angles in a circle are also shown through various problems incorporated in the lessons.

D

Learning Task 1: In your answer sheet, use the figure at the right to identify the following:

1. Center
2. Chords
3. Radii
4. Diameter
5. Tangent Lines
6. Secant Lines
7. Central Angles
8. Semi-Circle
9. Intercepted arcs
10. Inscribed Angles



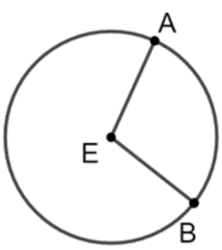
A **circle** is a set of points on a given plane, which is equidistant from a fixed point called the center.

The Central Angle -Intercepted Arc

A central angle is an angle whose vertex is the center and whose sides contain two radii.

The measure of a central angle of a circle is equal to the measure of its intercepted arc.

Illustrative Example 1:



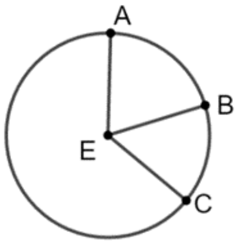
$$m\angle BEA = m\,\widehat{AB}$$

The Arc Addition

The measure of the arc formed by two adjacent arcs is the sum of the measure of the two arcs.

Illustrative Example 2:

$$m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$$

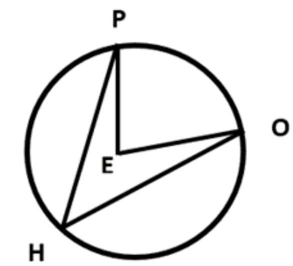


Inscribed Angle and Its Intercepted Arc

An inscribed angle is an angle whose vertex is on a circle and whose sides contain chords of the circle. The arc that lies in the interior of an inscribed angle and has endpoints on the angle is called the intercepted arc.

The measure of inscribed angle is equal to one-half of intercepted arc.

Illustrative Example 3:



If the measure of ∠PEO is 80^0, what is the measure of ∠PHO?

Solution: Since ∠PEO is a central angle

m ∠PEO = m $\overset{\frown}{PO}$

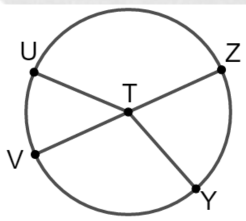
m ∠PHO = $\frac{1}{2}$ m ∠PEO

m ∠PHO = $\frac{1}{2}(80)$

m ∠PHO = 40

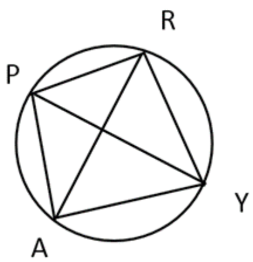
Learning Task 2: In your answer sheet, copy and answer.

If m ∠UTV = 70, m∠VTY = 92 and m YZ= 88. Answer the following:



1. m UV = ___________
2. m∠VTZ = __________
3. m ∠UTY = _________
4. m VY = ____________
5. m∠UTZ = __________

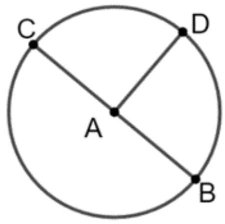
In the figure, m AY =105, mRY = 85 and m ∠PAR = 35, find:



1. m AP = ___________
2. m∠YRA = __________
3. m ∠ AYP = _________
4. m PR = ____________
5. m∠RPA = __________

Theorems Related To Chords, Arcs, Central Angles, and Inscribed Angles

1. In a circle or in congruent circles, two minor arcs are congruent if and ony if their corresponding central angles are congruent.



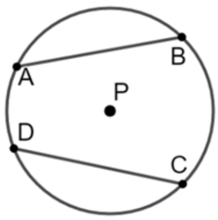
Illustrative Example 4

Given: ΔCAD and ΔBAD are right triangles at A

Prove: $\overset{\frown}{CD} \cong \overset{\frown}{BD}$

STATEMENTS	REASONS
1. ΔCAD and ΔBAD are right triangles at A	Given
2. $\angle CAD$ and $\angle BAD$ are right angles	From 1, definition of right triangle
3. $\angle CAD \cong \angle BAD$	All right angles are congruent
4. $m\angle CAD \cong m\angle BAD$	From 3, definition of congruent angles
5. $m\angle CAD = m\overset{\frown}{CD}$ $m\angle BAD = m\overset{\frown}{BD}$	The central angle and the intercepted arc have the equal measure
6. $m\overset{\frown}{CD} = m\overset{\frown}{BD}$	From 4 and 5, by substitution
7. $\overset{\frown}{CD} \cong \overset{\frown}{BD}$	From 6, definition of congruent arcs

2. In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent



Illustrative Example 5

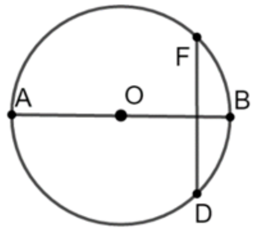
Given: $\overline{AB} \cong \overline{CD}$

Prove: $\overset{\frown}{AB} \cong \overset{\frown}{CD}$

STATEMENTS	REASONS
1. $\overline{AB} \cong \overline{CD}$	Given
2. Construct ΔAPB and ΔDPC	By construction and definition of triangles
3. $\overline{AP}$, $\overline{BP}$, $\overline{DP}$, $\overline{CP}$ are radii of $\odot P$	From 2 and definition of radius
4. $\overline{AP}$, $\overline{BP}$, $\overline{DP}$, $\overline{CP}$ are congruent	Radii of the same circle are congruent
5. $\Delta APB \cong \Delta CPD$	By SSS Congruence Postulate
6. $\angle APB \cong \angle CPD$	CPCTC– Corresponding Parts of Congruent Triangles are also Congruent
7. $m\angle APB = m\angle CPD$	Definition of congruent angles
8. $m\angle APB = m\overset{\frown}{AB}$ $m\angle CPD = m\overset{\frown}{CD}$	The measure of central is equal to the measure of its intercepted arc.
9. $m\overset{\frown}{AB} = m\overset{\frown}{CD}$	From 7 and 8, substitution
10. $\overset{\frown}{AB} \cong \overset{\frown}{CD}$	Definition of congruent arcs

3. In a circle, a diameter bisects a chord and an arc with the same endpoints if and only if it is perpendiular to the chord.

Illustrative Example 6



Given: $\overline{EG} \perp \overline{FD}$

Prove : $\overline{EG}$ bisects $\overline{FD}$

$\overline{EG}$ bisects $\overset{\frown}{FD}$

STATEMENTS	REASON
1. $\overline{EG} \perp \overline{FD}$	Given
2. Construct radii $\overline{EF}$ and $\overline{OD}$	By construction and definition of radi-us
3. $\overline{EF} \cong \overline{OD}$	Radii of the same circle are congruent
4. $\overline{OH} \cong \overline{OH}$	Reflexive Property
5. $\angle OHF$ and $\angle$ OHD are right angles	Definition of perpendicular segments
6. ΔOHF and Δ OHD are right trian-	Definition of right triangles
7. $\Delta OHF \cong \Delta$ OHD	HL Congruence Postulate
8. $\overline{FH} \cong \overline{HD}$	CPCTC
9. $\overline{EG}$ bisects $\overline{FD}$	Definition of segment bisector
10. $\angle HOD \cong \angle$ HOF	CPCTC
11. $\overset{\frown}{FG} \cong \overset{\frown}{DG}$	Congruent central angles intercept congruent arcs
12. $\overline{EG}$ bisects $\overset{\frown}{FD}$	From 11, definition of arc bisector

4. If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

Theorems on Angles Formed By Tangents, Secants, and Chords

5. The Tangent-Secant Theorem

Given an angle with its vertex on a circle, formed by a secant ray and a tangent ray, the measure of the angle is half the measure of the intercepted arc.

Illustrative Example 7

Find the following measures.

a. m $\overset{\frown}{PR}$ b. m $\angle PRS$

Solution: a. $\overset{\frown}{PR}$ is intercepted by $\angle PQR$.

So, $m\angle PQR = 80^0 = \frac{1}{2} m\widehat{PR}$

$2(80^0) = m\widehat{PR}$

$m\widehat{PR} = 160^0$

b. $\widehat{PR}$ is intercepted by $\angle PRS$, which is formed by a secant ray and a tangent ray.

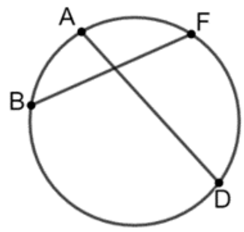
So, $m\angle PRS = \frac{1}{2} m\widehat{PR}$

$= \frac{1}{2}(160^0)$

$m\angle PRS = 80^0$

6. If two chords intersect *within* a circle, then the measure of the angle formed is equal to half the *sum* of the measures of the intercepted arcs.

Illustrative Example 8



Given: Chords $\overline{AD}$ and $\overline{BF}$ intersecting within $\odot C$

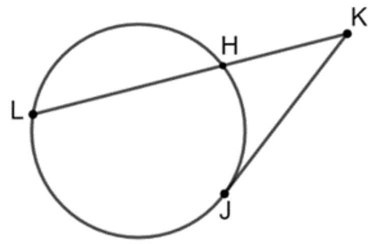
Prove: $m\angle FED = \frac{1}{2}(m\widehat{AB} + m\widehat{FD})$

STATEMENTS	REASON
1. Chords $\overline{AD}$ and $\overline{BF}$ intersecting within $\odot C$	Given
2. Construct chords $\overline{AF}$ and $\overline{BD}$	By construction and definition of chords
3. $\angle FED$ is an exterior angle of ΔEAF; $\angle EAF$ and $\angle AFE$ are its remote interior	Definition of an exterior angle and its remote interior angles
4. $m\angle FED = m\angle EAF + m\angle AFE$	The remote interior theorem
5. $\angle EAF$ and $\angle AFE$ are inscribed angles	Definition of inscribed angle
6. $m\angle EAF = \frac{1}{2}(m\widehat{AB})$	The measure of the inscribed angle is half its intercepted arc.
7. $m\angle FED = \frac{1}{2}(m\widehat{AB}) + \frac{1}{2}(m\widehat{FD})$	From 4 and 6, Substitution
8. $m\angle FED = \frac{1}{2}(m\widehat{AB} + m\widehat{FD})$	By factoring

7. If a tangent and a secant, two secants, or two tangents intersect in a point in the *exterior* of a circle, then the measure of the angle formed is equal to one-half the *difference* of the measures of the intersected arcs.

Illustrative Example 9

Find the missing measure.



a. m $\widehat{HJ} = 110^0$ and m $\widehat{LJ} = 150^0$

m $\angle K =$ ____

b. m $\widehat{LJ} = 160^0$ and m $\angle K = 30^0$

m $\widehat{HJ} =$ ____

Solution:

a. m $\angle K = \frac{1}{2}(m\widehat{LJ} - m\widehat{HJ})$

$= \frac{1}{2}(150^0 - 110^0)$

$= \frac{1}{2}(40^0)$

m $\angle K = 20^0$

b. m $\angle K = \frac{1}{2}(m\widehat{LJ} - m\widehat{HJ})$

$30^0 = \frac{1}{2}(160^0 - x)$

$60^0 = 160^0 - x$

$x = 100^0$

E

Learning Task 3: In your answer sheet, copy and answer.

Given : ΔABC is inscribed in $\odot$ D

Prove : $\angle ABC$ is a right angle

STATEMENT	REASON
1. (1)	Given
2. m $\widehat{AEC}$ + m $\widehat{ABC}$ = 360^0	(2)
3. $\overline{CA}$ is diameter of $\odot$ D	(3)
3. m $\widehat{AEC}$ = m $\widehat{ABC}$ = 180^0	The diameter divides the circle into semicircle
4. m$\angle ABC = \frac{1}{2}$ m $\widehat{AEC}$	(4)
5. m$\angle ABC = \frac{1}{2}(180^0)$	From 3 and 4, substitution.
6. $\angle ABC$ is a right angle	(5)

Learning Task 4: In your answer sheet, write TRUE if the statement is correct and write FALSE if it is not.

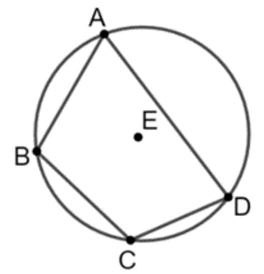
________ 1. Two angles intercepting the same arc are congruent.
________ 2. An inscribed square in a circle separates the circle into four equal arcs.
________ 3. The measure of a central angle is twice the measure of an inscribed angle intercepting the same arc.
________ 4. An arc has only one central angle intercepting it but several intercepting inscribed angles.
________ 5. The vertex of an inscribed angle is the center of the circle.
________ 6. When two chords intersect, they intersect at the center of the circle.
________ 7. When two diameters intersect, they intersect at the center of the circle.
________ 8. When two chords intersect at a point on the circle, an inscribed angle is formed.
________ 9. When two chords intersect, the point of intersection is in the interior of the circle.
________ 10. When two tangents intersect at the exterior point of the circle, the intercepted arcs complete a circle.

Learning Task 5: In your answer sheet, prove the theorem, using the given below.

"If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary."

Given: Quadriateral ABCD is inscribed in $\odot E$

Prove: $\angle ADC$ and $\angle ABC$ are supplementary
$\angle DAB$ and $\angle DCB$ are supplementary



A

In your journal notebook, reflect and answer the questions.

1. When two chords of a circle are parallel, are the arcs they intercept be congruent? How about the arcs they cut off? Explain.

2. How do you determine the measure of the angle formed by the intersection of two chords? two secant segments intersecting at the point in the exterior of the circle?

WEEKS 5-6

Secants, Tangents, Segments, and Sectors of a Circle

Lesson

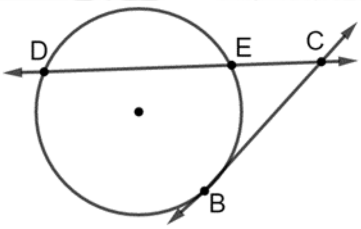
I

After going through this lesson, you are expected to illustrate secants, tangents, segments, and sector of a circle, prove theorems on secant and tangent segments of a circle; and solve problems on circles.

This lesson will add more terminologies to what you have previously learned terms related to circles and explore the theorems on intersecting chords, tangent lines and secant lines.

D

Learning Task 1: Use the figure below to match the definition in Column A with its description in Column B. Copy and answer this in your answer sheet.

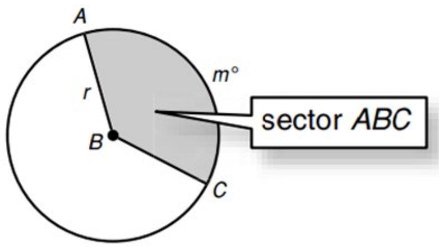


Column A	Column B
1. A line segment, a line a ray that intersects a circle at	A. $\overline{BC}$
2. The point of tangency between a tangent and the circle is called point of tangency.	B. $\overline{DC}$
3. A line segment, a line a ray that intersects a circle at exactly two points is called a secant. Every secant	C. $\overline{EC}$
4. A secant segment is a segment with one endpoint on a circle, one endpoint outside the circle, and one	D. $\overrightarrow{BC}$
5. An external secant segment is the part of a secant segment that is outside a circle.	E. point B
6. The tangent segment is a segment whose endpoints are the point of tangency and the fixed point outside	F. $\overleftrightarrow{DC}$

Sector of a Circle

A **sector of a circle** is the region bounded by two radii and their intercepted arc. To find the area of a sector of a circle, get the product of $\frac{\text{measure of the central angle}}{360}$ and the area of the circle.

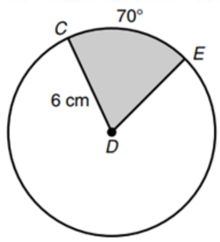
Formula: Area of sector = $\frac{\pi r^2 c}{360}$



where: c is the central angle in degrees
r is the radius of the circle
π is Pi, approximately 3.14

Illustrative Example 1

Find the area of the sector of radius 6 cm and its central angle is 70^0. Note that the measure of the central angle is equal to its intercepted arc.



Solution: Area of sector = $\frac{\pi r^2 c}{360}$

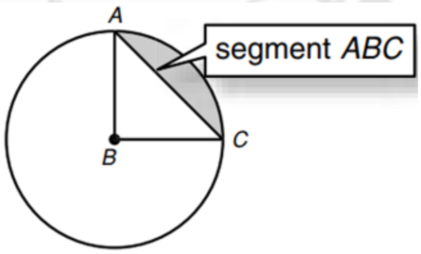
$= \frac{(3.14)(6)^2(70)}{360}$ sq. in.

= 21.99 sq. cm.

Segment of a Circle

A **segment of a circle** is the region bounded by an arc and its chord.

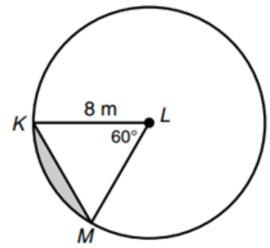
To find the area of the shaded segment, subtract the area of triangle from the area of a sector.



Illustrative Example 2

Solve the area of the shaded region.

Solution:



$$\textit{Area of the segment} = \textit{Area of the sector} - \textit{Area of the triangle}$$

$$\textit{Area of the segment} == \frac{\pi r^2 c}{360} - \frac{1}{2} r^2 \sin C$$

$$\textit{Area of the segment} = \frac{(3.14)(8^2)60}{360} - \frac{1}{2}(8^2)\sin 60$$

= 5.78 sq. m.

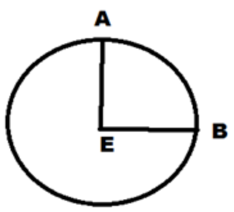
Arc Length

The length of an arc can be determined by using the proportion

$$\frac{\textit{measure of the central angle}}{360} = \frac{\textit{arc length}}{2\pi r}$$

In the given proportion, 360 is the degree measure of the whole circle, while $2\pi r$ is the circumference.

Illustrative Example 3



If $\angle BEA = 90^0$ and the radius is 6 cm, what is the length of arc intercepted by the angle.

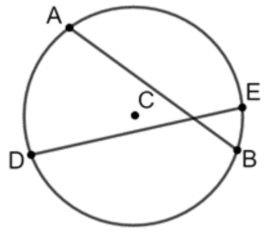
Solution:

$$\textit{Arc length} = \frac{\textit{measure of the central angle}}{360} * 2(3.14)(6)$$

= 9.42 cm.

Theorems on Secant and Tangent Segments of A Circle
Illustrative Example 4
Theorem on Two Intersecting Chords

If two chords of a circle intersect, then the product of the measures of the segments of one chord is equal to the product of the measures of the segments of the other chord.



Given: $\overline{AB}$ and $\overline{DE}$ are chords of $\odot C$ intersecting at M.

Prove: $AM \cdot BM = DM \cdot E$

To prove: Draw $\overline{AE}$ and $\overline{BD}$ so that we formed two similar triangles. Then show the proof using the Two-Column Proof.

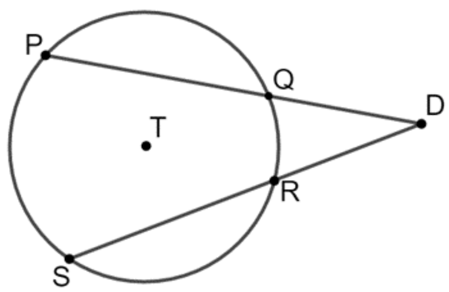
Proof:

Statement	Reason
$m\angle BAE = \frac{1}{2}(m\overset{\frown}{BE})$ and $m\angle BDE = \frac{1}{2}(m\overset{\frown}{BE})$	1. The measure of an inscribed angle is one-half the measure of its intercepted arc.
$\angle BAE \cong \angle BDE$	2. Inscribed angles intercepting the
$\Delta AME \sim \Delta DMB$	3. AA Similarity Theorem
$\frac{EM}{AM} = \frac{BM}{DM}$	4. Lengths of sides of similar trian-
$AM \cdot BM = DM \cdot EM$	5. Multiplication Property

Illustrative Example 5

Theorem on Two Secant Segments

If two secant segments are drawn to a circle from an exterior point, then the product of the lengths of one secant segment and its external secant segment is equal to the product of the lengths of the other secant segment and its external secant segment.



Given: $\overline{DP}$ and $\overline{DS}$ are secant segments of $\odot T$ drawn from exterior point D.

Prove: $DP \cdot DQ = DS \cdot DR$

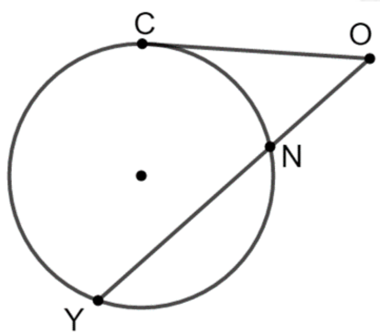
To prove: Draw $\overline{PR}$ and $\overline{QS}$ so that we formed two similar triangles. Then show the proof using the Two-Column Proof.

Proof:

Statement	Reason
∠QPR ≅ ∠ RSQ and ∠PQS	1. Inscribed angles intercepting the same arc are congruent.
∠DQS ≅ ∠ DRP	2. Supplements of congruent angles
Δ DQS ~Δ DRP	3. AA Similarity Theorem
$\frac{DP}{DR} = \frac{DS}{DQ}$	4. Lengths of sides of similar triangles are proportional.
DP · DQ = DS · DR	5. Multiplication Property

Theorem a Tangent Segment and a Secant Segment

If a tangent segment and a secant segment are drawn to a circle from an exterior, then the square of the length of the tangent segment is equal to the product of the lengths of the secant segment and its external secant segment.

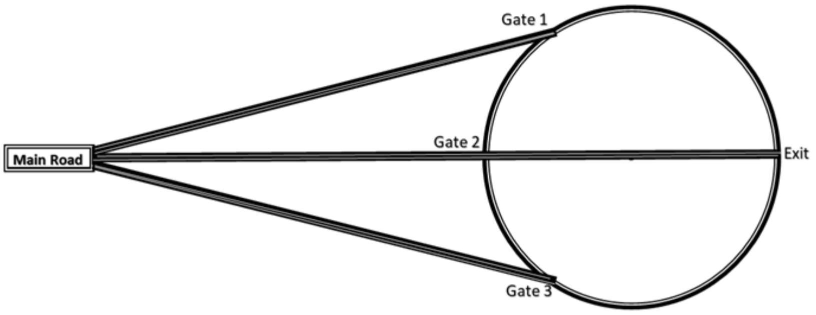


CO^2 = YO * NO

Word Problems Involving Circles

Illustrative Example 6

The figure below shows a sketch of a circular children's park and the different pathways from the main road. If the distance from the main road(M) to gate 2 (G2) is 75 m and the length of the pathway from Gate 2 (G2) to Exit (E) is 60 m, about how far from the main road(M) is gate 1 (G1)?



Using the Theorem on a Tangent Segment and a Secant Segment,

$(MG1)^2 = (MG2 + G2E) \cdot MG2$

$(MG1)^2 = (70 + 60) \cdot 70$

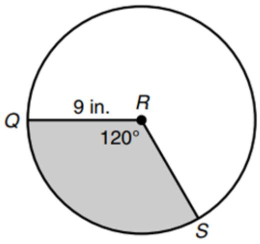
$(MG1)^2 = 130 \cdot 70$

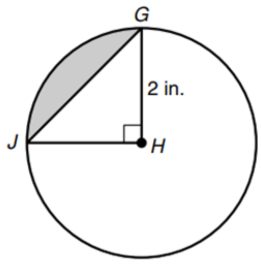
$(MG1)^2 = 9100$

$\sqrt{(MG1)^2} = \sqrt{9100}$

MG = 95.39

Learning Task 2: In your answer sheet, copy and find the area of the shaded region. Use π = 3.14.

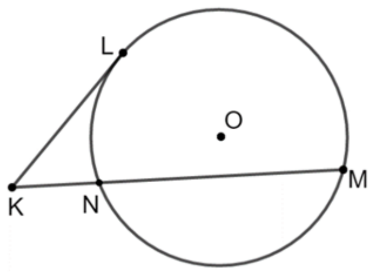




Learning Task 3: In your answer sheet, copy and answer.

Given: $\overline{KL}$ and $\overline{KM}$ are tangent and secant segments, respectively, of ʘO drawn from exterior point K.

$\overline{KM}$ intersects ʘO at N.



Prove: $(KL)^2 = KM \cdot KN$

Draw $\overline{LM}$ and $\overline{LN}$ so that we formed two similar triangles. Then show the proof using the Two-Column Proof.

Proof:

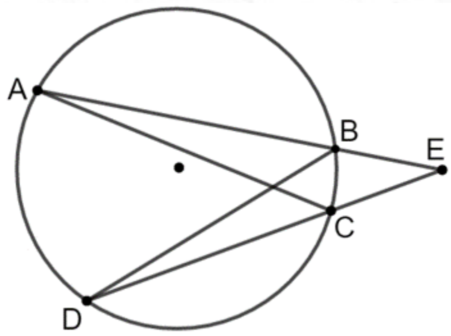
Statement	Reason
$m\angle NLK = \frac{1}{2}(m\overset{\frown}{LN})$ and $m\angle LMN = \frac{1}{2}(m\overset{\frown}{LN})$	(1)
$m\angle NLK = m\angle LMN$	(2)
$\angle NLK \cong \angle LMN$	(3)
$m\angle LNK = m\angle NLM + m\angle LMN$	(4)
$m\angle LNK = m\angle NLM + m\angle NLK$	(5)
$m\angle KLM = m\angle NLM + m\angle NLK$	(6)
$m\angle LNK = m\angle KLM$	(7)
$\angle LNK \cong \angle KLM$	(8)
$\Delta MKL \sim \Delta LNM$	(9)
$\frac{KM}{KL} = \frac{KL}{KN}$	10. Lengths of sides of similar triangles are
$KM \cdot KN = \lvert LK \rvert^2$	(10)

Learning Task 4.

1. Solve the problem.
Different animals have different fields of view. Humans can generally see a 180° arc in front of them. Horses can see a 215° arc. A horse and rider are in
Baguio City which is experiencing heavy fog, so they can see for only 10 yards in any direction
   a. Find the area of the rider's field of view.
   b. Find the area of the horse's field of view.

2. Given: $\overline{AE}$ and $\overline{DE}$ intersect at E.

Prove: $AE * BE = DE * CE$



A

In your journal notebook, reflect and answer the questions.

1. How can you determine the lengths of the segments formed by intersecting two chords?
2. How are the segments formed by intersecting two secants at an external point related? A secant and a tangent at an external point related?

WEEKS 7-8

Using the Distance Formula in Proving Geometric Properties

Lesson

I

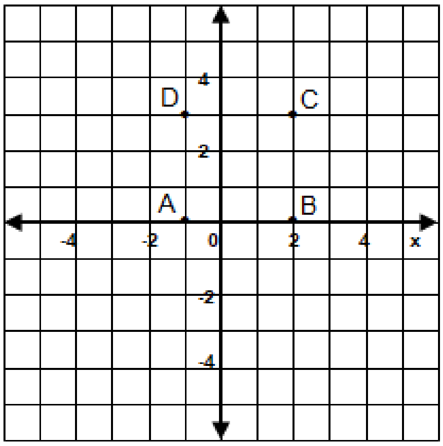
In this lesson, you are expected to applies the distance formula in proving some geometric properties.

Recall that, in a coordinate plane, each point corresponds to exactly one ordered pair of numbers.

If you are given two points on the coordinate plane, how do you get the distance between the two points? Observe and try to do the next activity.

D

Learning Task 1. In your answer sheet, copy and answer.



1. What is the distance of A from B? D from C?
2. What is the distance of A from D? B from C?
3. Compare the distances.
4. What is the figure formed?

Let P (x_1, y_1) and Q (x_2, y_2) be two points. The distance *d* between these points can be determined using the distance formula $d = \sqrt{(x^2 - x^1)^2 + (y^2 - y^1)^2}$ or PQ = $\sqrt{(x^2 - x^1)^2 + (y^2 - y^1)^2}$.

Illustrative Examples:
Find the distance between the points with coordinates (-3, 4) and (5, -2).

Solution:

Using the distance formula, we get

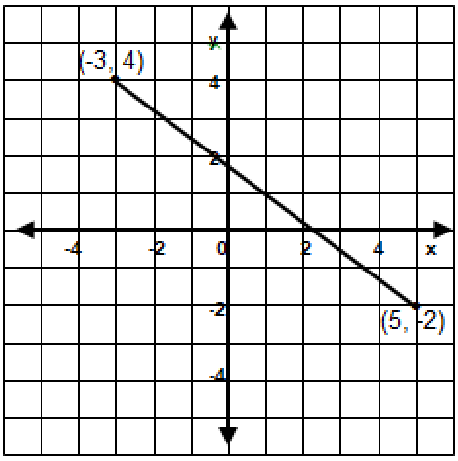
$$d = \sqrt{(x^2 - x^1)^2 + (y^2 - y^1)^2}$$

$$= \sqrt{[5 - (-3)]^2 + [-2 - 4]^2}$$

$$= \sqrt{(8)^2 + (-6)^2}$$

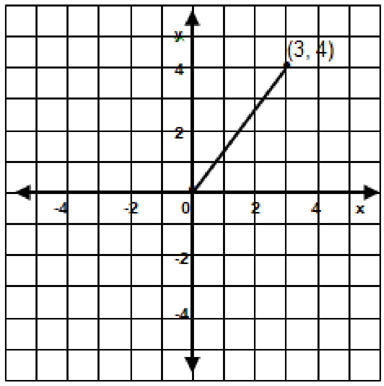
$$= \sqrt{100}$$

$$= 10$$



The distance formula is used even if two points are on the same horizontal or vertical line. Take note that if one point is the origin, the formula for distance is simply

$$d = \sqrt{x^2 + y^2}$$



2. Find the distance between the points with coordinates (3, 4) and (0, 0).

Solution:

Using the distance formula, we have

$$d = \sqrt{x^2 + y^2}$$

$$= \sqrt{3^2 + 4^2}$$

$$= \sqrt{25}$$

$$= \mathbf{5}$$

Many geometric properties can be proven by using a coordinate plane. A proof that uses figures on a coordinate plane to prove geometric properties is called a **coordinate proof**.

To prove geometric properties using the methods of coordinate geometry, consider the following guidelines for placing figures on a coordinate plane.

Use the origin as vertex or center of a figure.

Place at least one side of a polygon on an axis.

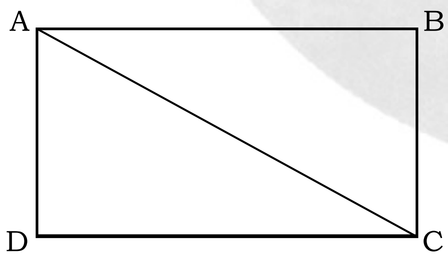
If possible, keep the figure within the first quadrant.

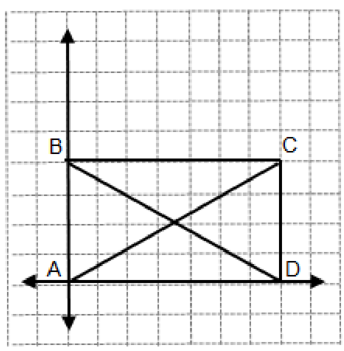
Use coordinates that make computations simple and easy. Sometimes, using coordinates that are multiples of two would make the computation easier.

In some coordinate proofs, the Distance Formula is applied.

Example: Prove that the diagonals of a rectangle are congruent using the methods of coordinate geometry.

Solution:





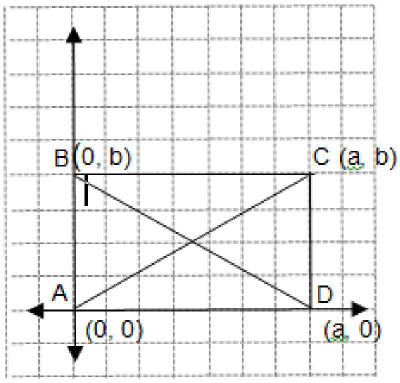
Given: ☐ABCD with diagonals AC and BD.

Prove: AC ≅ BD

To prove:

1. Place ☐ABCD on a coordinate plane.

2. Label the coordinates as shown below.



a. Find the distance between A and C. Given: A (0, 0) and C (a, b)

$$AC = \sqrt{(a-0)^2 + (b-0)^2}$$

$$AC = \sqrt{a^2 + b^2}$$

b. Find the distance between B and D.
c. Given: b (0, b)
and D (a, 0)

$$BD = \sqrt{(a-0)^2 + (0-b)^2}$$

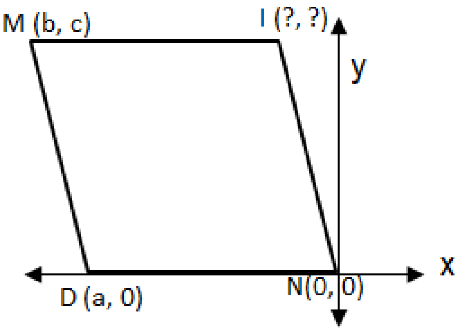
$$BD = \sqrt{a^2 + b^2}$$

Since AC = BD, then AC ≅ BD by substitution.
Therefore, AC ≅ BD. The diagonals of a rectangle are congruent.

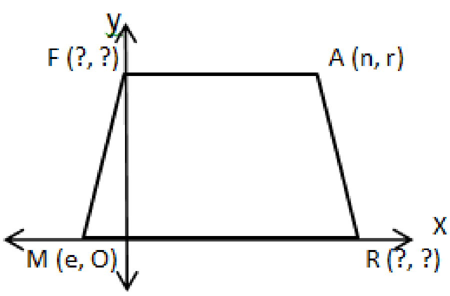
E

Learning Task 2. In your answer sheet, name the missing coordinates in terms of the given variables.

1. MIND is a parallelogram.



2. FARM is an isosceles



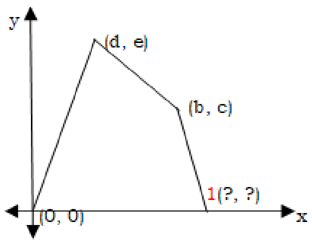
Learning Task 3: In your answer sheet, copy and answer.

A. Find the distance between the two given points.

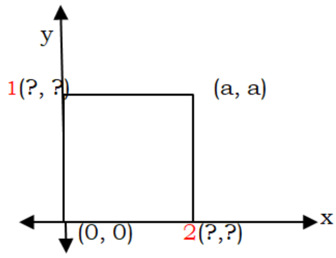
1. S (-1, 6) and T (-1, 14)
2. H (-9, 3) and G (6, 3)
3. M (0, 0) and N (4, 6)
4. U (-5, 2) and P (-5, 7)
5. A (-3, 2) and M (9, 7)

B. Name the missing coordinates in terms of the given variables.

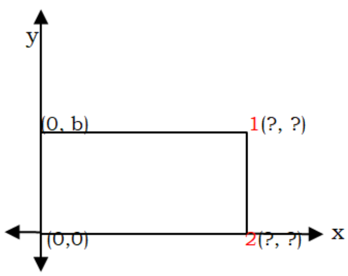
1. Trapezium



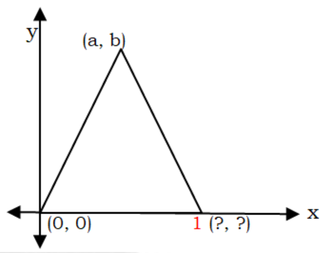
2. Square



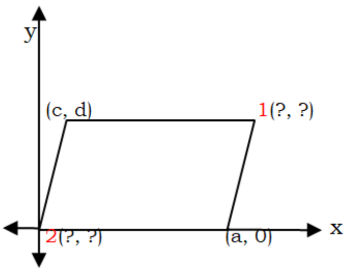
3. Rectangle



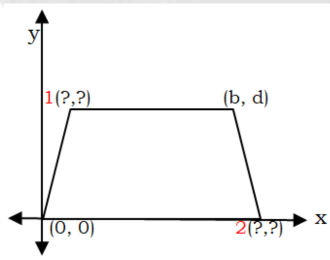
4. Isosceles Triangle



5. Parallelogram



6. Trapezoid



A

In your journal notebook, reflect and answer the questions.

How do you apply the distance formula in proving some geometric properties?

Center-Radius form of the Equation of the Circle

I

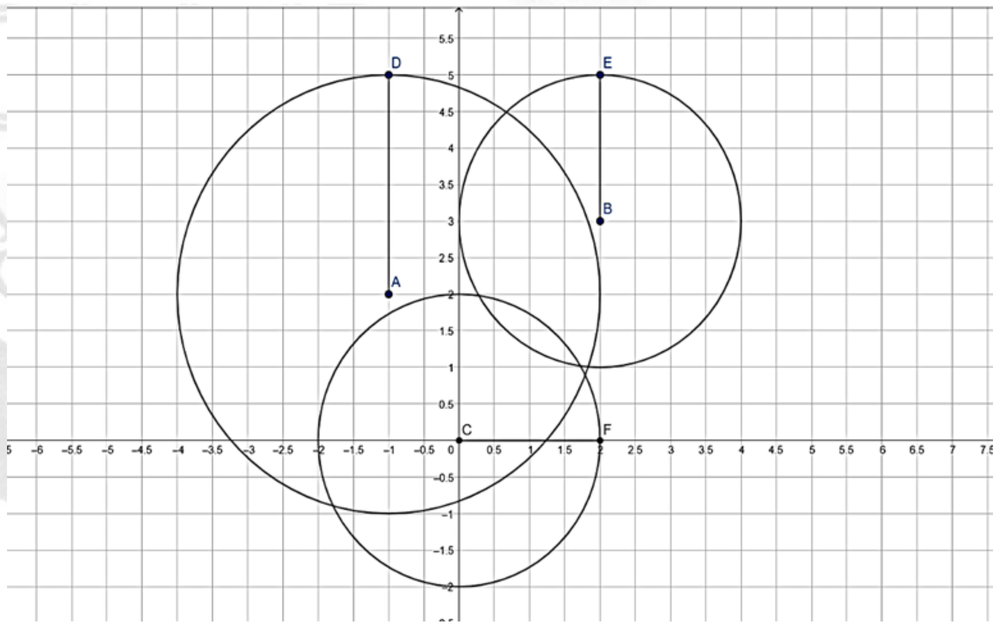
Lesson

After going through this lesson, you are expected to illustrate center-radius form of the equation of the circle; determine the center-radius form of the equation of a circle given its equation and vice versa and; graph a circle and other geometric figures on the coordinate plane.

Circle is set of all points (x, y) in a plane that are equidistant from a fixed center point. Let us test your learning by doing the first activity.

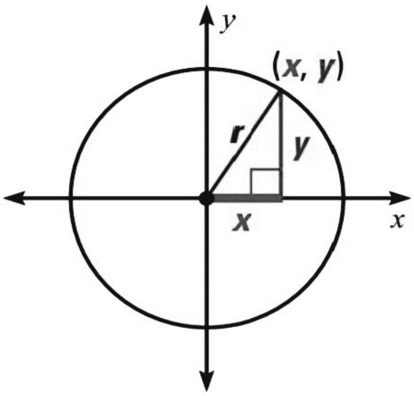
D

Learning Task 1: In the plane below, identify the coordinates of the center of each circle and the measure of its radii. Answer this on your activity sheet.



Circle	Coordinates of the center	Name 1 radius of each circle	Measure of radii
A			
B			
C			

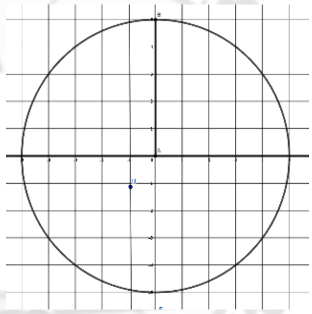
In the circle below, let point (x, y) represent any point on the circle whose center is the origin. Let r represent the radius of the circle.



In a right triangle, r represent the hypotenuse, x and y represent the length of the legs. By Pythagorean Theorem, you can write $x^2 + y^2 = r^2$. This is the equation of the circle with center at the origin.

Illustrative Example 1

Write the equation of the circle whose radius is 5 and the center is at the origin.

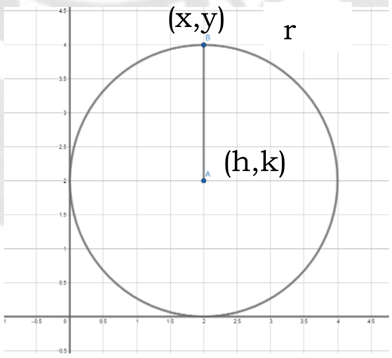


$x^2 + y^2 = r^2$	*Write an equation of a circle with center at the origin*
$x^2 + y^2 = 5^2$	*Substitute 5 for r*
$x^2 + y^2 = 25$	*Simplify*

An equation of the circle is $x^2 + y^2 = 25$.

Center-Radius Form of the Equation of the Circle

In the coordinate plane, the center-radius form of the equation of the circle with center at (h, k) and radius r

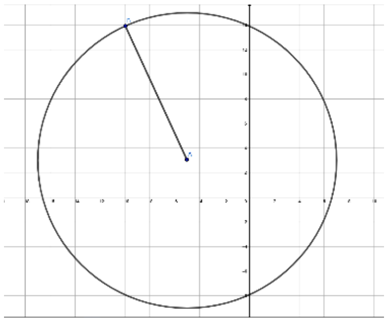


$$(x - h)^2 + (y - k)^2 = r^2$$

x coordinate of *y coordinate of*

Illustrative Example 2

Write the center-radius form of the equation of the circle with center at (–5,3) and a radius of 12 units.



Use the center-radius form $(x - h)^2 + (y - k)^2 = r^2$

Substitute the of h as x- coordinate of the center and k as y-coordinate of the center and the value of radius

$(x + 5)^2 + (y - 3)^2 = 12^2$

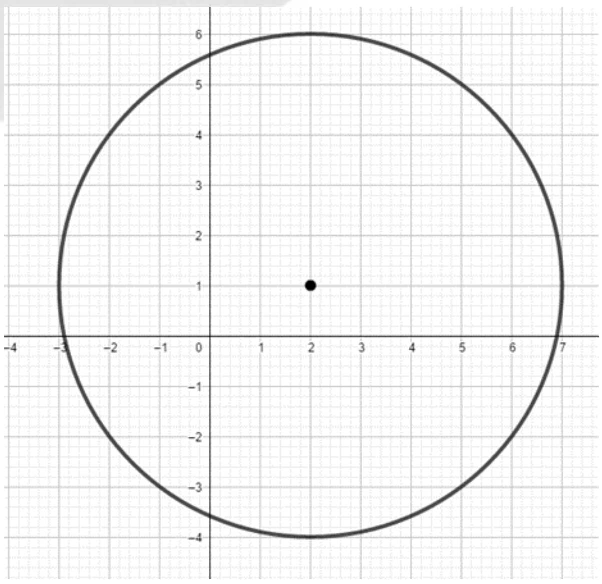
Simplify,

$(x + 5)^2 + (y - 3)^2 = 144$

Illustrative Example 3

For the equation $(x-2)^2 + (y-1)^2 = 5^2$, where the center is h=2 and k=1. These are coordinates of the center point (2,1). Remember, if h and k are confusing, the number with x is the x coordinate. The number in parentheses with y is the y coordinate of the center.

The other important part of this graph is the radius. In this case, the radius r = 5. To draw this graph, we will start at the center point and use the radius to mark points up, down, left, and right. In this case, we start at the point (2,1) and move up 5 units. Mark that point. Go back to the center and move down 5 units. Do the same by starting at the center and going left and right 5 units. Use these 4 points, seen below, as a guide as you draw your circle.



Illustrative Example 4

Determine the coordinates of the center and radius of the given circle described by the equation $x^2 + y^2 - 4x - 8y - 5 = 0$.

Solution: The equation of the circle $x^2 + y^2 - 4x - 8y - 5 = 0$ is written in general form. To determine its center and radius, write the equation in standard form $(x - h)^2 + (y - k)^2 = r^2$.

$x^2 + y^2 - 4x - 8y - 5 = 0$	
$x^2 + y^2 - 4x - 8y - 5 + 5 = 5$	add 5 to both sides of the equation
$(x^2 - 4x) + (y^2 - 8y) = 5$	group them by its common variable
$x^2 - 4x + 4 + y^2 - 8y + 16 = 5 + 4 + 16$	then use completing the square
$(x^2 - 4x + 4) + (y^2 - 8y + 16) = 25$	rewrite the perfect square trinomial into
$(x - 2)^2 + (y - 4)^2 = 25$	square of binomial
$(x - 2)^2 + (y - 4)^2 = 5^2$	

Learning Task 2: In your answer sheet, copy and answer.

1.Write the standard form of the equation of a circle with center at the origin and radius 9. ____

2. Write the standard form of the equation of a circle with center at (-5,6) and is tangent to the x-axis. ____

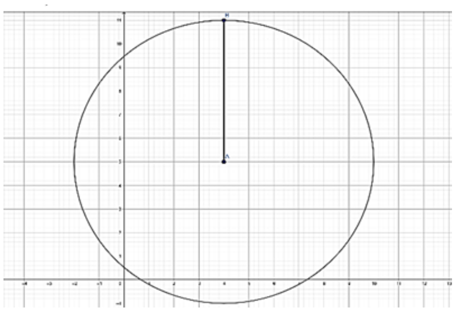
3. What is the center and radius of the circle defined by the equation $(x+1)^2 + (y+3)^2 = 16$? center __________, radius _______

4. Change the equation of the circle $(x+1)^2 + (y+3)^2 = 16$ into general form.

5. What is the center and radius of the circle whose equation is $x^2 + y^2 - 16x + 8y + 31 = 0$?

Center_________, radius________

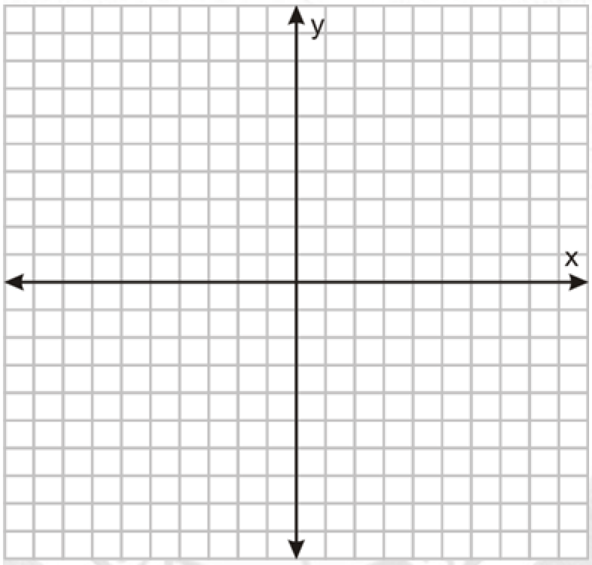
6. Determine the equation for the circle in the given figure.



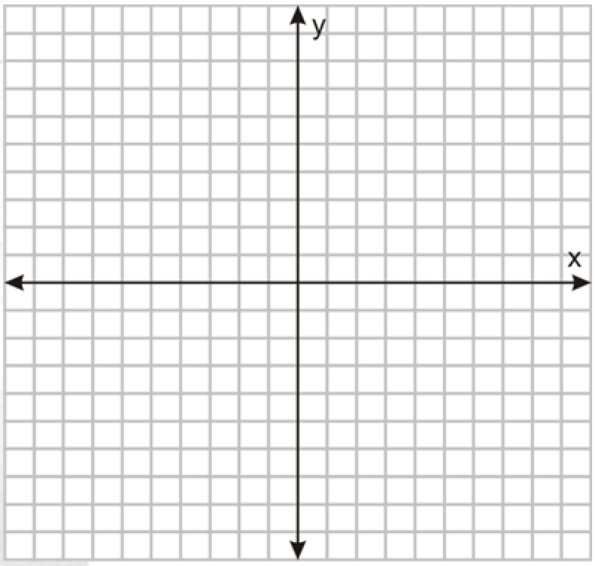
7. A cellular phone network uses towers to transmit calls. If the equation of the circular area transmits by the tower is represented by $(\square - 5)^2 + (\square - 1)^2 = 22$. Can you identify if you are inside or outside of the circular area, if your coordinate location is (6,0)? How about if your location is (8,2)?

Learning Task 3 : In your answer sheet, copy and answer. Identify the center and radius of each equation. Then sketch a graph.

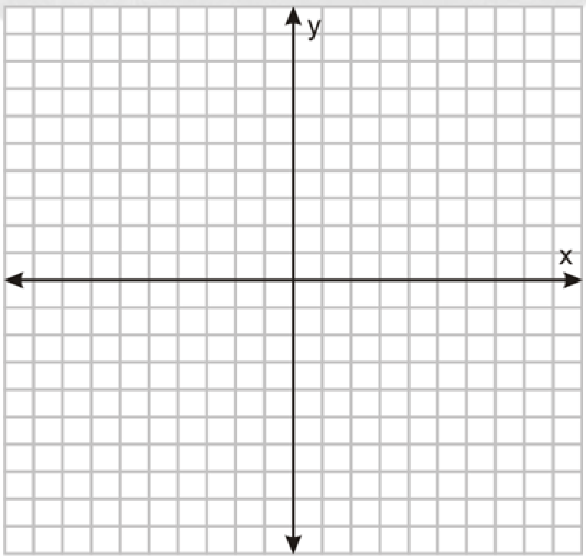
1. $(x + 1)^2 + (y - 2)^2 = 9$



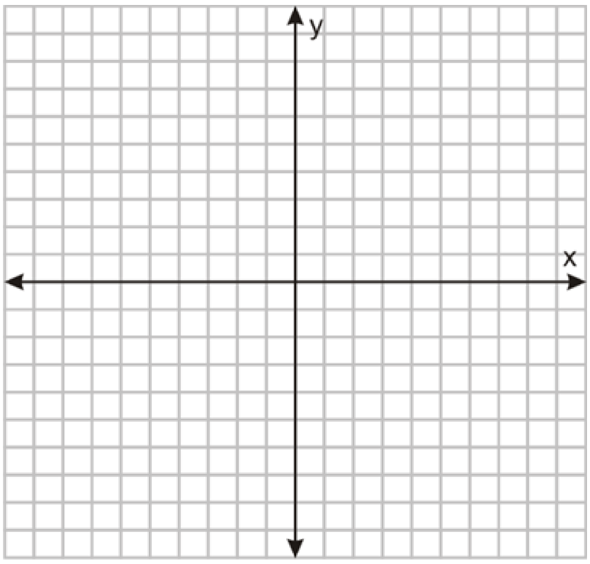
4. $x^2 + y^2 + 8x - 6y - 39 = 0$



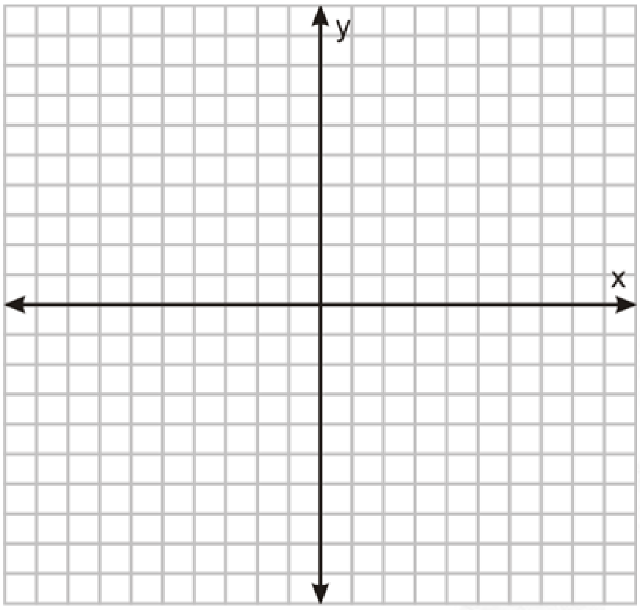
3. $(x + 2)^2 + (y + 3)^2 = 4$



5. $x^2 + y^2 - 10x + 16y - 32 = 0$



3. $(x + 1)^2 + (y + 2)^2 = 25$



Learning Task 4 : In your answer sheet, copy and answer.

1. Transform $x^2+y^2 + 8x - 2y - 64 = 0$ in center-radius form of the equation of the circle. (2 points)
2. Find the center and radius of a circle represented by the equation below.

   a. $x^2 + y^2 = 49$ (2 points)

   b. $(x + 7)^2 + (y + 8)^2 = 64$ (2 points)

3. Write the standard equation of a circle with the given center and radius.

   a. (0, 0); radius = 7 (2 points)

   b. (-4, 4); radius = 5 (2 points)

4. Graph the circle whose equation is $x^2 + y^2 - 6x + 4y - 3 = 0$ (5 points)

Learning Task 4: In your answer sheet, copy and answer.

Mr. Robert Garcia, a municipal hall gardener in San Pedro, Laguna wants the four bushes of Sampaguita in the garden to be watered by a rotating water sprinkler. Mr. Garcia draws a diagram of the garden using a grid in which each unit represents 1 foot. The bushes of sampaguita are at (1, 2), (2, 9), (9, 8) and (8, 1). He wants to position the sprinkler at a point equidistant from each bush of Sampaguita. Let us help Mr. Garcia by answering the following questions.

1. Where should the gardener place the sprinkler?
2. Draw the possible appearance of the garden.
3. Write the equation in standard form that describes the boundary of the circular region to be covered by the sprinkler?

A

In your journal notebook, reflect and answer the questions.

1. How will you determine the center and radius given the equation of the circle in standard form? in general form?
2. How will you determine the equation of the circle given the center and radius?

Key to Correction

Weeks 1 - 2

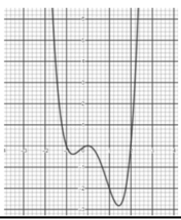
Learning Task 1

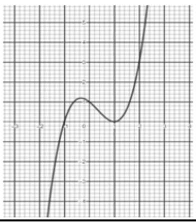
1. a. -10
b. - 239
c. 3125
2. a. (x+1)(x-3)(x-2)
b. (x-3)(x+2)(x+3)(x+3)
c. (x+2)(x-2)(x+3)

Learning Task 2

A1. $F(x) = x^4 - x^3 - 2x^2$
a. Standard form: $F(x) = x^4 - x^3 - 2x^2$
b. Leading term: x^4
c. x-intercepts and its multiplicities:
0 (multiplicity 2), -1, 2
d. y-intercepts: 0
e. number of turning points:3
f. end behavior: Rising in both side

2. $f(x) = (x-1)^2(x+1)$
a. Standard form: $f(x) = x^3 - x^2 - x + 1$
b. Leading term: x^3
c. x-intercepts and its multiplicities:
1 (multiplicity 2), -1
d. y-intercepts: 1
e. number of turning points: 2
f. possible graph with end behavior:
Falling in the left

B.1.


2


Weeks 3 - 4

Learning Task 1

1. pt. M
2. $\overline{AD}$, $\overline{HI}$, $\overline{DC}$
3. $\overline{FM}$, $\overline{DM}$, $\overline{AM}$
4. $\overline{DA}$
5. $\overleftrightarrow{DE}$
6. $\overleftrightarrow{DC}$
7. ∠FMD, ∠FMA
8. $\overset{\frown}{AFD}$ $\overset{\frown}{DEA}$
9. $\overset{\frown}{FD}$ $\overset{\frown}{FA}$ $\overset{\frown}{AC}$
10. ∠HAD, ∠ADC

Learning Task 2

A. 1.70	B. 1.100
2. 180	2. 52.5
3. 198	3. 85
4. 92	4. 70
5. 110	5. 190

Learning Task 3

1. ΔABC is inscribed in ⊙D
2. Arc addition postulate
3. Given
4. The measure of the inscribed angle is half its intercepted arc
5. Definition of right angle

Weeks 5 - 6

Learning Task 1

1. D	4. B
2. E	5. C
3. F	6. A

Learning Task 2

1. 84.78 sq.in.
2. 1.14 sq.in.

Week 7

Learning Task 1

1. A from B = 3 units
D from C = 3 units
2. A from D = 3 units
B from C = 3 units
3. They are all equal
4. Square

Learning Task 2

1. I (a+b, c)
2. F (0, r), R (e+n, 0)

Weeks 8 - 9

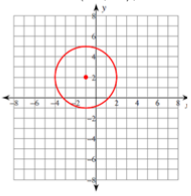
Learning Task 1

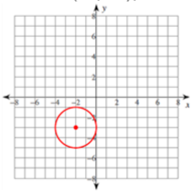
Circle	Coordinates of the center	Name 1 radius of each circle	Measure of radii
A	(-1,2)	$\overline{AD}$	6
B	(2, 3)	$\overline{BE}$	4
C	(0, 0)	$\overline{CF}$	2

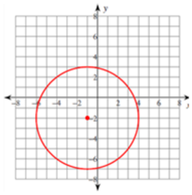
Learning Task 2

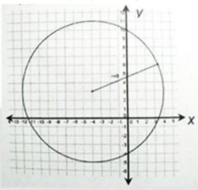
1. $x^2 + y^2 = 81$
2. $(x+5)^2 + (y-6)^2 = 36$
3. center: (-1, -3), r: 4
4. $x^2 + y^2 + 2x + 6y - 6 = 0$
5. center: (8, -4)
r = 7
6. (center at (4, 5)
r = 6

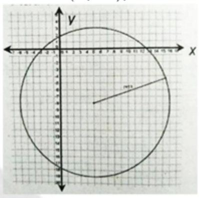
Learning Task 3

1. Center: (-1, 2); Radius =


2. Center: (-2, -3); Radius = 2


3. Center: (-1, -2); Radius = 5


4. Center: (-4, 3); Radius = 8


5. Center: (5, -8); Radius = 11


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Department of Education Region 4A CALABARZON. (2020). *PIVOT 4A Budget of Work in all Learning Areas in Key Stages 1-4: Version 2.0.* Cainta, Rizal: Department of Education Region 4A CALABARZON.

Department of Education. *Mathematics Grade 10 Learner's Module (First Edition, 2015).*Pasig City: Department of Education.

PIVOT Assessment Card for Learners

Personal Assessment on Learner's Level of Performance

Using the symbols below, choose one which best describes your experience in working on each given task. Draw it in the column for Level of Performance (LP). Be guided by the descriptions below.

☆ - I was able to do/perform the task without any difficulty. The task helped me in understanding the target content/lesson.

✓ - I was able to do/perform the task. It was quite challenging but it still helped me in understanding the target content/lesson.

? - I was not able to do/perform the task. It was extremely difficult. I need additional enrichment activities to be able to do/perform this task.

Distribution of Learning Tasks Per Week for Quarter 2

Week 1	LP	Week 2	LP	Week 3	LP	Week 4	LP
Learning Task 1		Learning Task 1		Learning Task 1		Learning Task 1	
Learning Task 2		Learning Task 2		Learning Task 2		Learning Task 2	
Learning Task 3		Learning Task 3		Learning Task 3		Learning Task 3	
Learning Task 4		Learning Task 4		Learning Task 4		Learning Task 4	
Learning Task 5		Learning Task 5		Learning Task 5		Learning Task 5	
Learning Task 6		Learning Task 6		Learning Task 6		Learning Task 6	
Learning Task 7		Learning Task 7		Learning Task 7		Learning Task 7	
Learning Task 8		Learning Task 8		Learning Task 8		Learning Task 8	

Week 5	LP	Week 6	LP	Week 7	LP	Week 8	LP
Learning Task 1		Learning Task 1		Learning Task 1		Learning Task 1	
Learning Task 2		Learning Task 2		Learning Task 2		Learning Task 2	
Learning Task 3		Learning Task 3		Learning Task 3		Learning Task 3	
Learning Task 4		Learning Task 4		Learning Task 4		Learning Task 4	
Learning Task 5		Learning Task 5		Learning Task 5		Learning Task 5	
Learning Task 6		Learning Task 6		Learning Task 6		Learning Task 6	
Learning Task 7		Learning Task 7		Learning Task 7		Learning Task 7	
Learning Task 8		Learning Task 8		Learning Task 8		Learning Task 8	

Note: If the lesson is designed for two or more weeks as shown in the eartag, just copy your personal evaluation indicated in the first Level of Performance in the second column up to the succeeding columns, ie. If the lesson is designed for weeks 4-6, just copy your personal evaluation indicated in the LP column for week 4, week 5 and week 6.

For inquiries or feedback, please write or call:

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