WORK, POWER and ENERGY

Work, Energy

CORE IDEA

Work, Energy

LEARNING OUTCOMES

1. Calculate the dot or scalar product of vectors
2. Determine the work done by a force (not necessarily constant) acting on a system
3. Define work as a scalar or dot product of force and displacement
4. Interpret the work done a force in one-dimension as an area under a Force vs. Position curve
5. Relate the work done by a constant force to the change in kinetic energy of a system
6. Apply the work-energy theorem to obtain quantitative and qualitative conclusions regarding the work done, initial and final velocities, mass and kinetic energy of a system
7. Represent the work-energy theorem graphically
8. Relate power to work, energy, force, and velocity
9. Relate the gravitational potential energy of a system or object to the configuration of the system
10. Relate the elastic potential energy of a system or object to the configuration of the system
11. Use potential energy diagrams to infer force; stable, unstable, and neutral equilibria; and turning points
12. Solve problems involving work, energy, and power in contexts such as, but not limited to, bungee jumping, design of roller coasters, number pf people required to build structures such as the Great Pyramids and the rice terraces; power and energy requirements of human activities such as sleeping vs. sitting vs. standing, running vs. walking. (Conversion of joules to calories should be emphasized at this point.)

CODES

STEM_GP12WE-If (40-43), STEM_GP12WE-Ig (44-49, 53), STEM_GP12WE-Ih-i (55)

These are our learning outcomes.

Energy

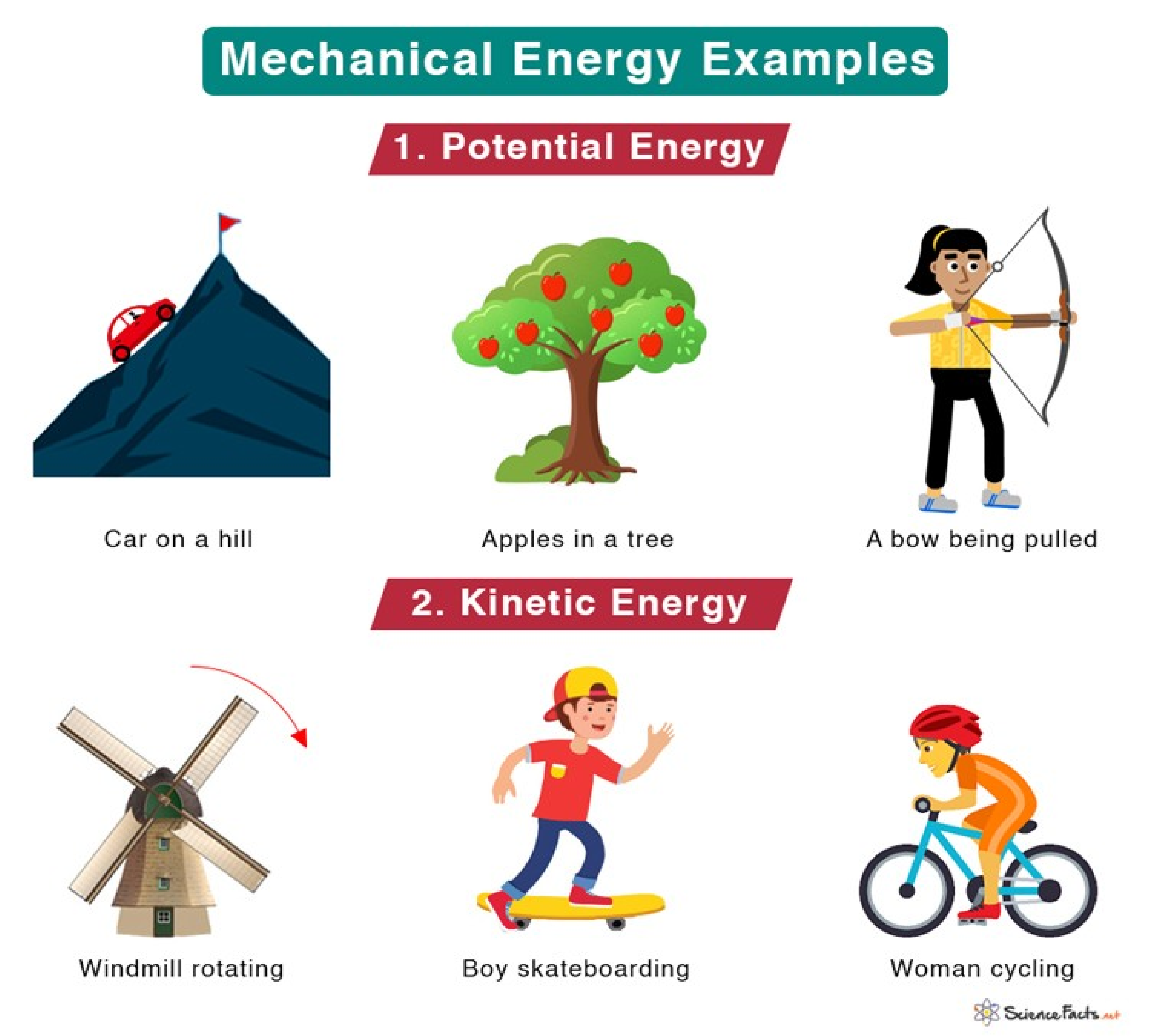
Introduction

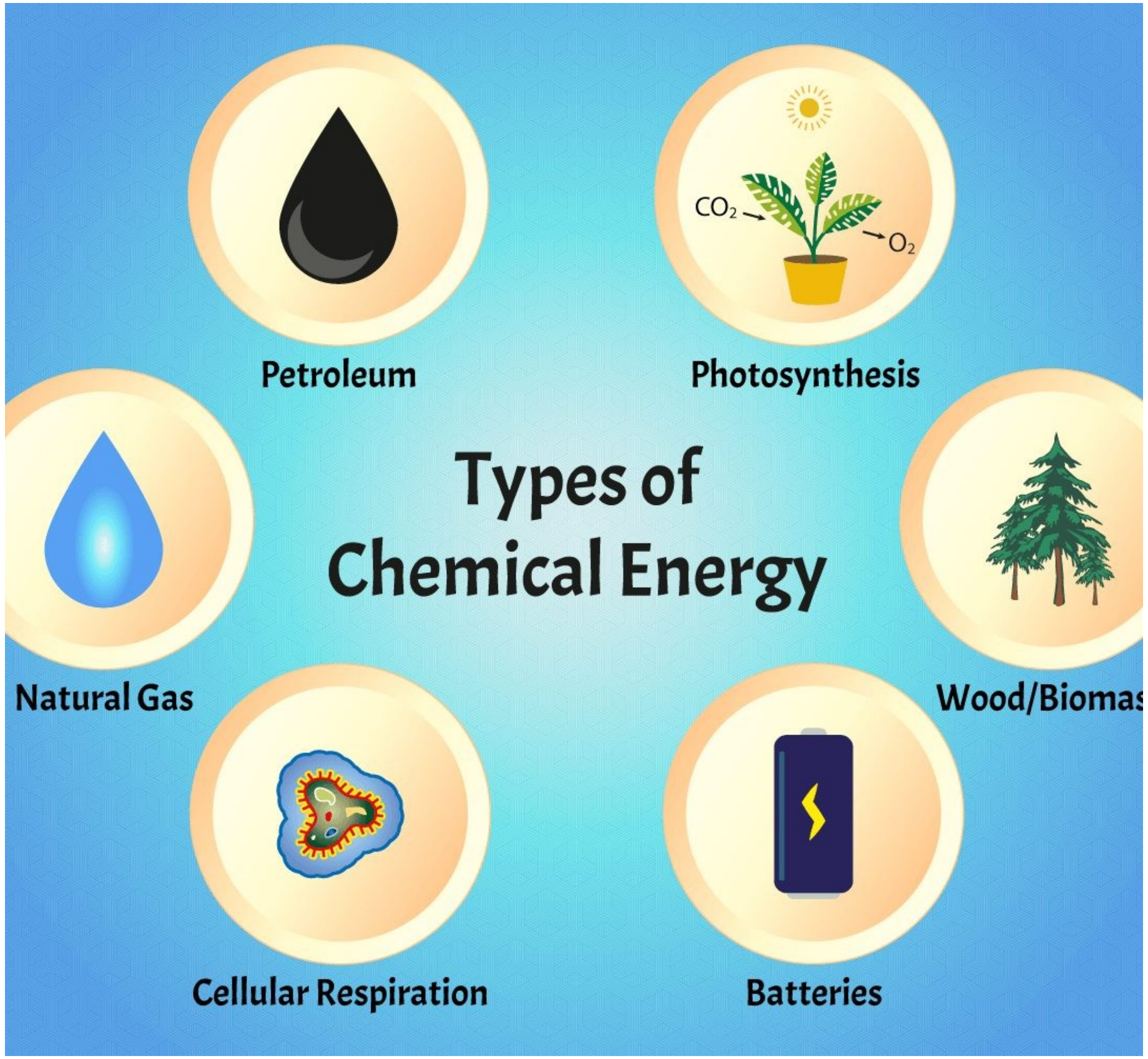
Some common forms of Energy

- Mechanical energy
- Chemical energy
- Radiant energy
- Electrical energy
- Thermal energy

Energy can transform from one form to another, but the total amount of energy in the universe does not change.

Energy comes in different forms. Energy can transform from one form to another, but the total amount of energy in the universe does not change.


Mechanical Energy Examples
1. Potential Energy
Car on a hill
Apples in a tree
A bow being pulled
2. Kinetic Energy
Windmill rotating
Boy skateboarding
Woman cycling
ScienceFacts.net



Chemical energy is energy stored in the bonds of atoms and molecules.

Law of Conservation of Energy Examples:

1. In a torch, the chemical energy of the batteries is converted into electrical energy, which is converted into light and heat energy.
2. In hydroelectric power plants, waterfalls on the turbines from a height. This, in turn, rotates the turbines and generates electricity. Hence, the potential energy of water is converted into the kinetic energy of the turbine, which is further converted into electrical energy.
3. In a loudspeaker, electrical energy is converted into sound energy.
4. In a microphone, sound energy is converted into electrical energy.
5. In a generator, mechanical energy is converted into electrical energy.
6. When fuels are burnt, chemical energy is converted into heat and light energy.
7. Chemical energy from food is converted to thermal energy when it is broken down in the body and is used to keep it warm.

Energy

Definition

Energy is a quantitative property of a physical system. It is transferred to another system to do work, thus, a system's energy is the amount of work the system can do.

Energy is a derived, scalar quantity
SI Unit: joule, J (kg m^2/s^2)
Symbol: E (general), depends on the form

Energy is a quantitative property of a physical system. It is transferred to another system to do work, thus, a system's energy is the amount of work the system can do.

Work

Definition

Work is the amount of energy transferred by a force.

Work is a derived, scalar quantity
SI Unit: joule, J (Nm – Newton meter)
Symbol: W

Work is the amount of energy transferred by a force.

When a force acts upon an object to cause a displacement of the object, it is said that **work** was done upon the object. There are three key *ingredients* to work - force, displacement, and cause. In order for a force to qualify as having done *work* on an object, there must be a displacement and the force must *cause* the displacement.

Read the following five statements and determine whether or not they represent examples of work.

	Is there any work done?
A teacher applies a force to a wall and becomes exhausted.	
A book falls off a table and free falls to the ground.	downward direction
A waiter carries a tray full of meals above his head by one arm straight across the room at constant speed.	does not cause the displacement
A rocket accelerates through space.	through space.

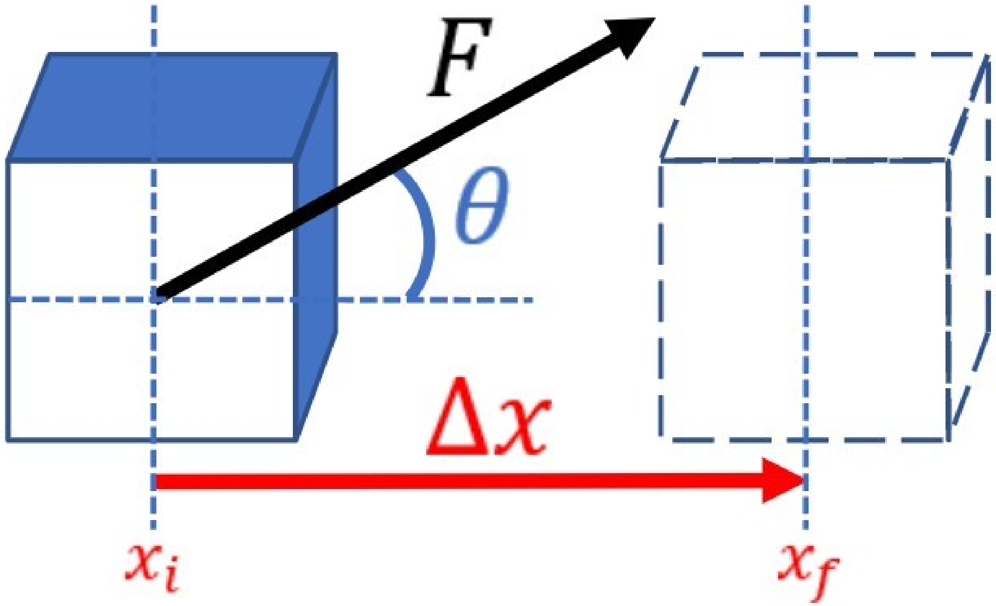
Work

Definition

Work relates force and energy.

Mathematically, it is the dot product of force and displacement.

$$W = F \cdot \Delta x$$

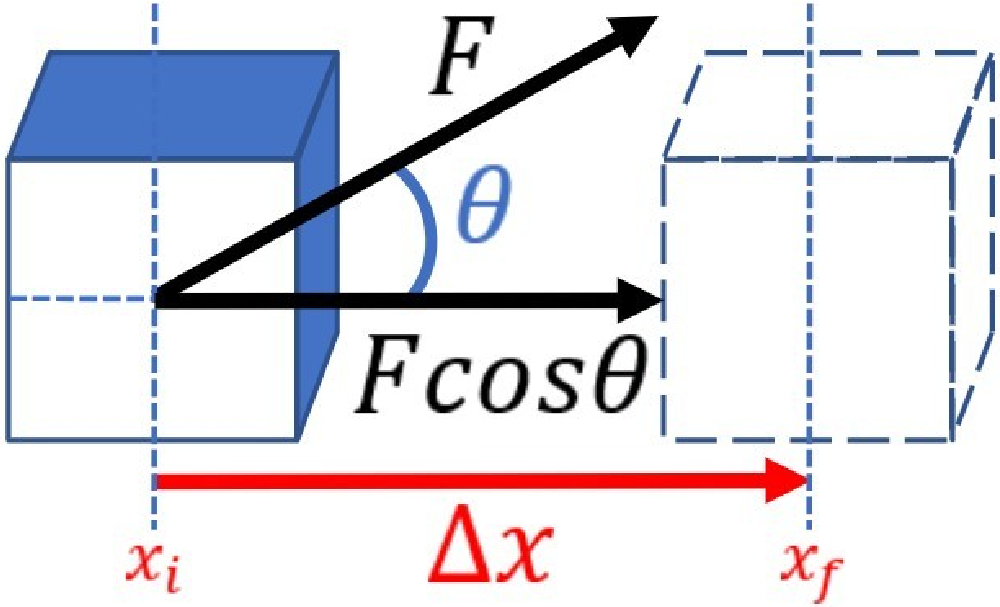


Work relates force and energy.

Work

Definition

$$W = F \cdot \Delta x$$
$$|W| = F\Delta x cos\theta$$
$$|W| = W \quad W = (Fcos\theta)\Delta x$$

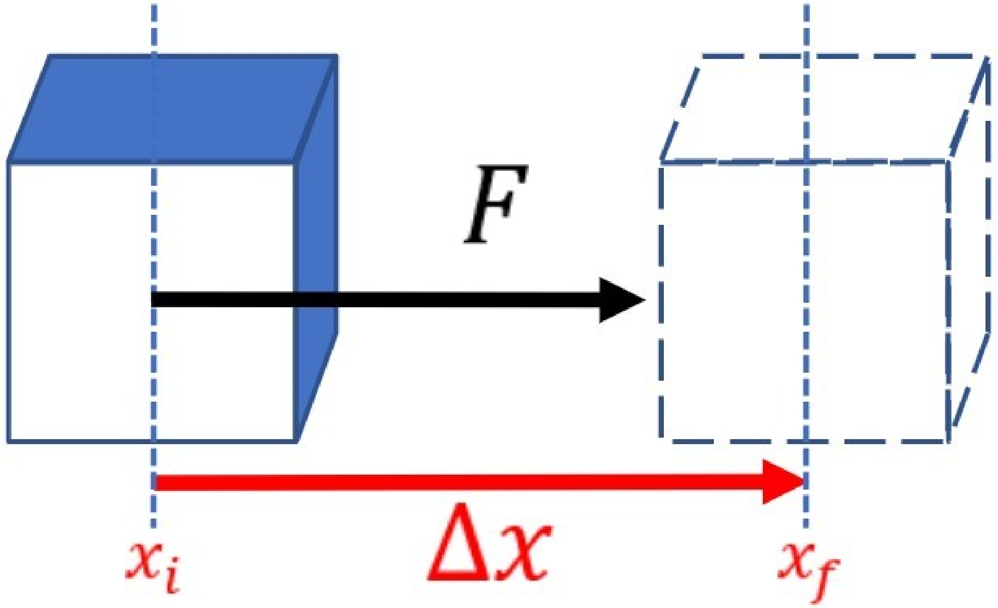


F and Δx can only have a non-zero dot product if F has a component parallel to Δx, otherwise, their dot product is zero. You may notice that Δx solely has an x-component, therefore, as long as F has an x-component (Fcosθ≠0), they will have a non-zero dot product.

This is the mathematical definition of work.

Work

Special Case: θ=0° (F is Parallel to Δx)



$$W = (F\cos\theta)\Delta x$$

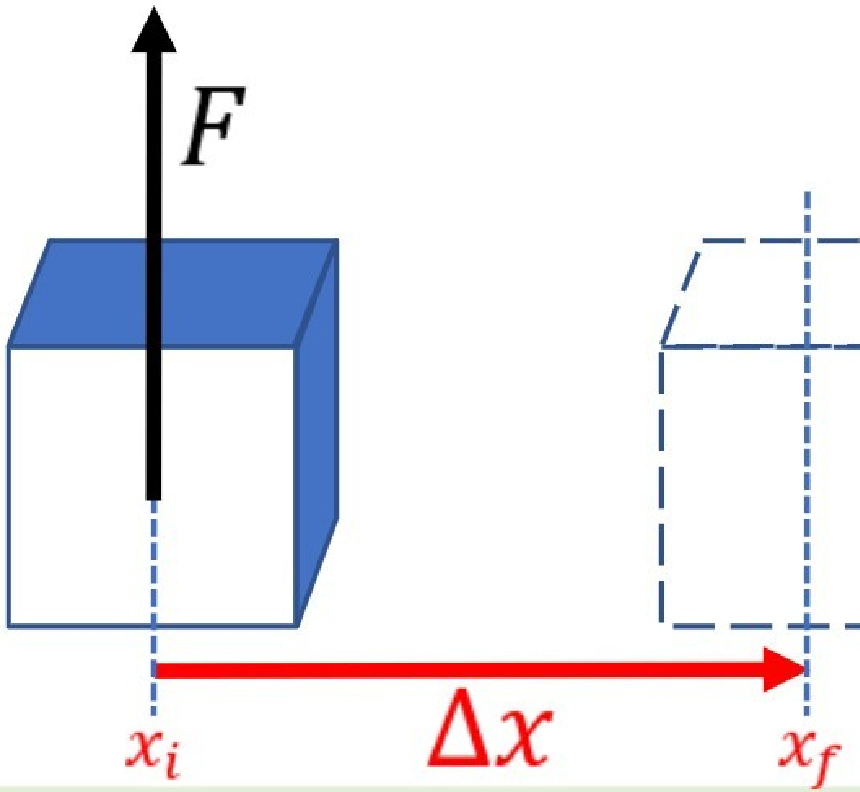
$$\theta = 0°; \cos 0 = 1$$

$$W = F\Delta x$$

Definition of work.

Work

Special Case: θ=90° (F is Perpendicular to Δx)



$$W = (F\cos\theta)\Delta x$$

$\theta = 90°; \cos 90 = 0$

$$W = 0$$

The force has done no work.

Definition of work.

Work

Example: How much work is done by waiters with the way they transport food?

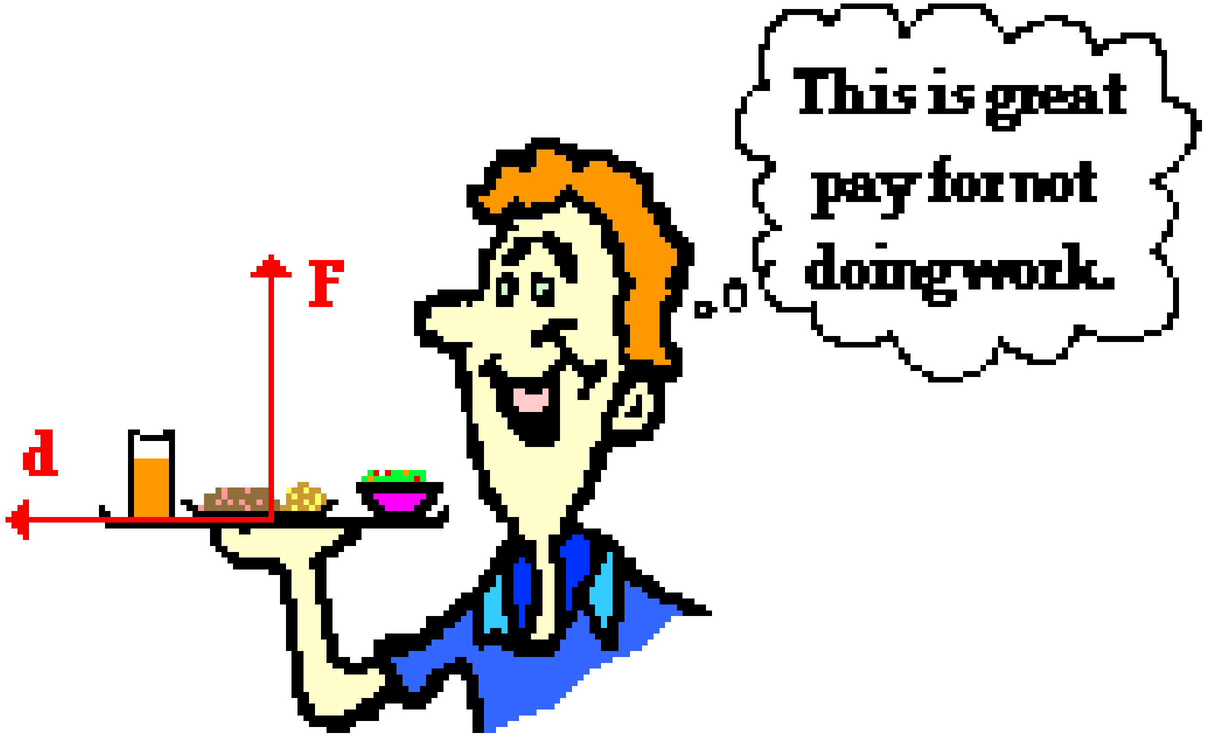
$$W = (F cos\theta)\Delta x$$

$$\theta = 90°; cos90 = 0$$

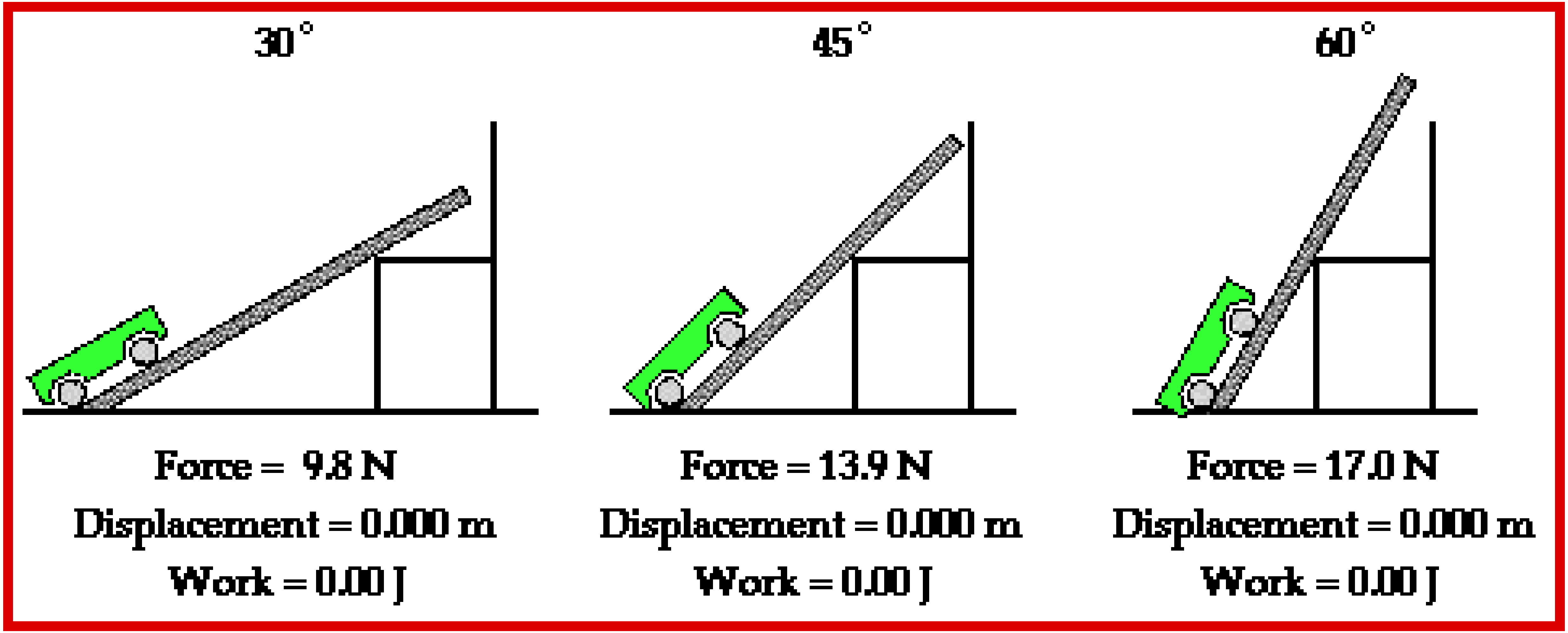
$$W = 0$$

There is no work done with the way waiters transport food

Lets analyze this situation.



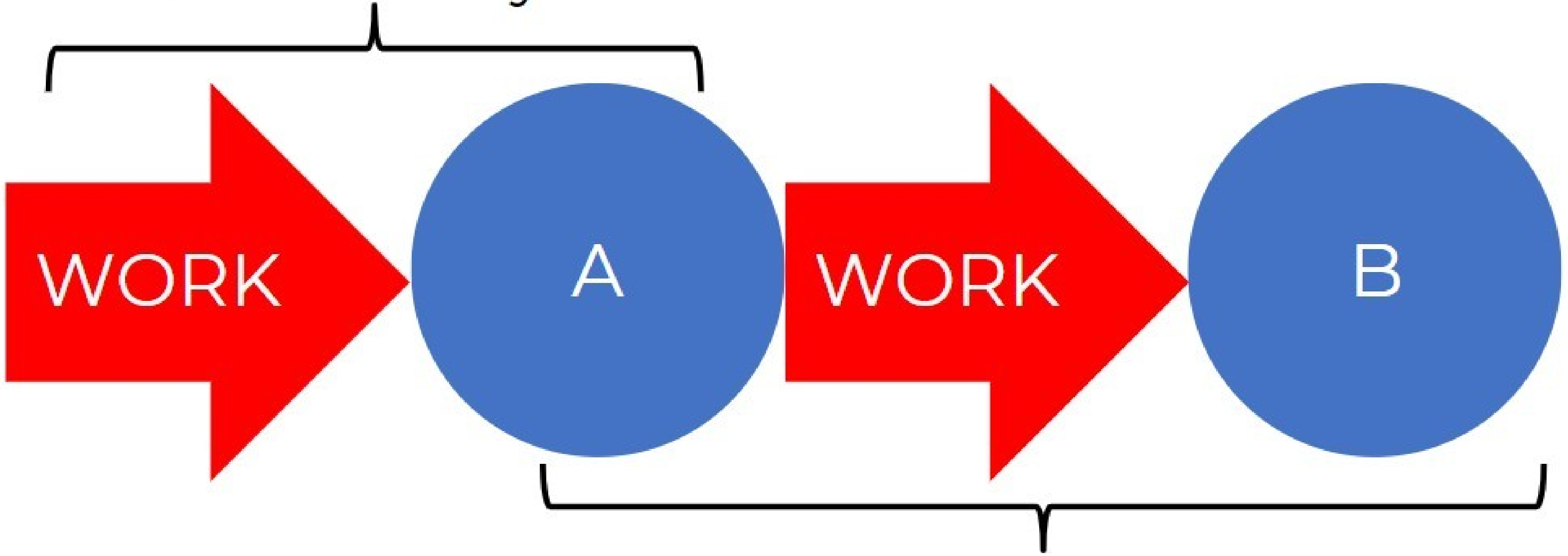
A vertical force can never cause a horizontal displacement; thus, a vertical force does not do work on a horizontally displaced object!!


30°
45°
60°
Force = 9.8 N
Displacement = 0.000 m
Work = 0.00 J
Force = 13.9 N
Displacement = 0.000 m
Work = 0.00 J
Force = 17.0 N
Displacement = 0.000 m
Work = 0.00 J

Work

Signs

Work is POSITIVE if work is done on the system.



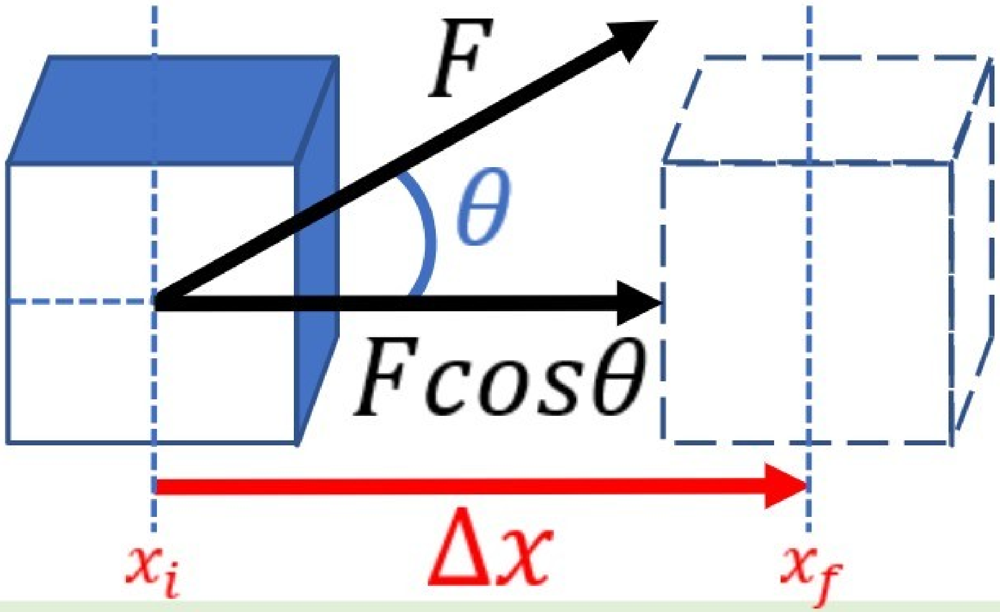
Work is NEGATIVE if the system does work to another system.

Work is POSITIVE if work is done on the system. Work is NEGATIVE if the system does work to another system.

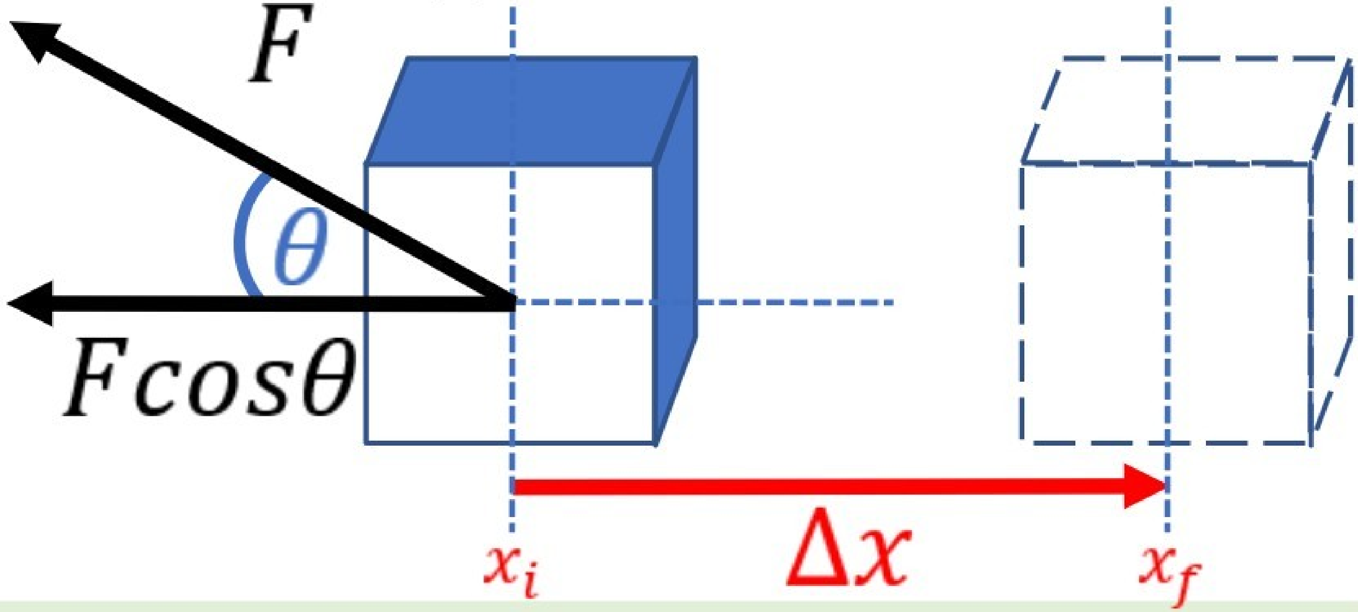
Work

Signs

Work is POSITIVE if the x-component of force has the same direction with Δx



Work is NEGATIVE if the x-component of force has the opposite direction with Δx

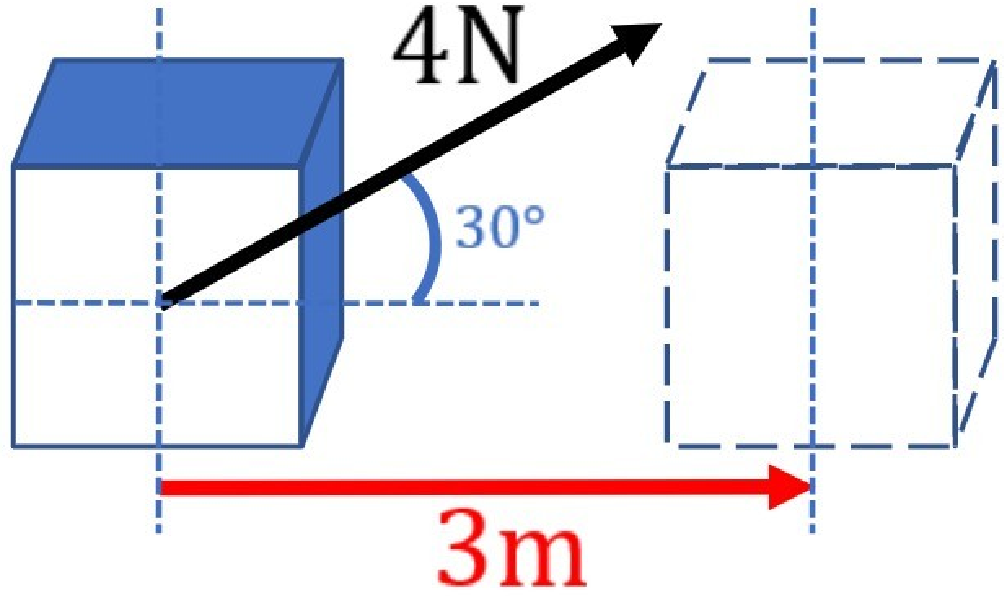


Work can be positive or negative.

Work

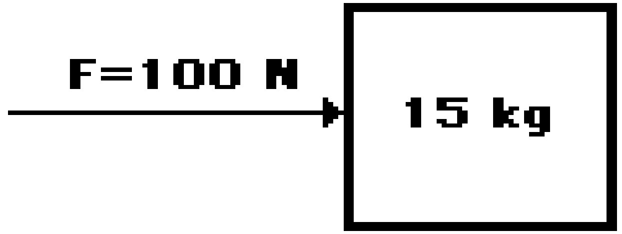
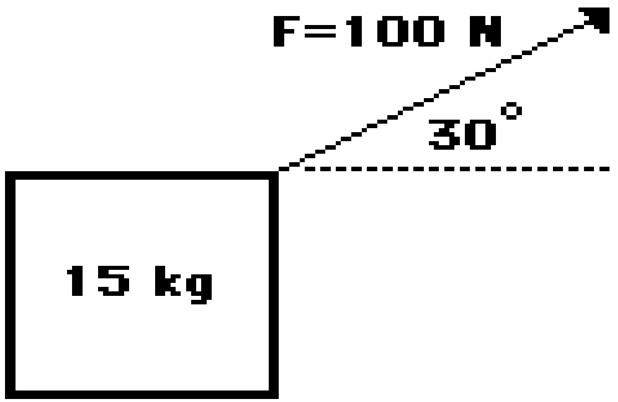
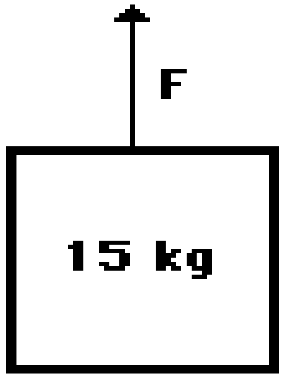
Seatwork

A force of 4 N is applied to a box at an angle of 30° which causes the box to have a displacement of 3 m. How much work is done by this force?



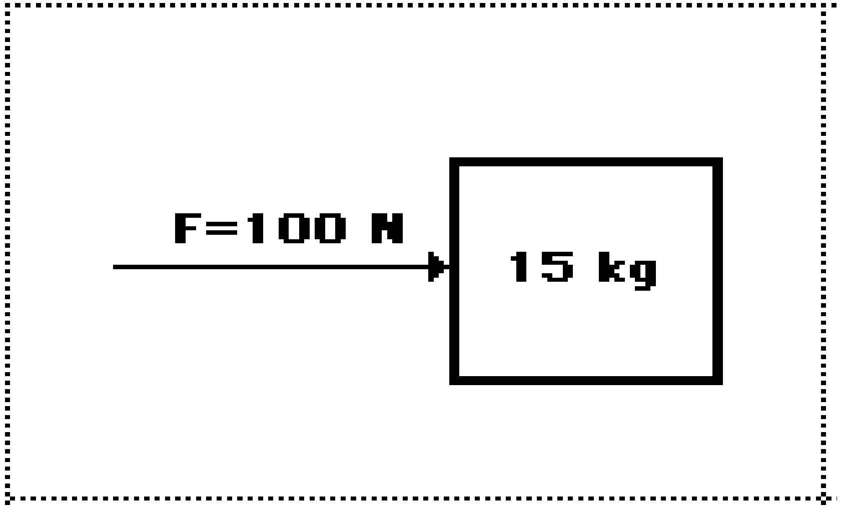
Lets try to solve this problem.

Calculating the Amount of Work Done by Forces

Diagram A	Diagram B	Diagram C
		
A 100 N force is applied to move a 15 kg object a horizontal distance of 5 meters at constant speed.	A 100 N force is applied at an angle of 30° to the horizontal to move a 15 kg object at a constant speed for a horizontal distance of 5 m.	An upward force is applied to lift a 15 kg object to a height of 5 meters at constant speed.

Calculating the Amount of Work Done by Forces

Diagram A



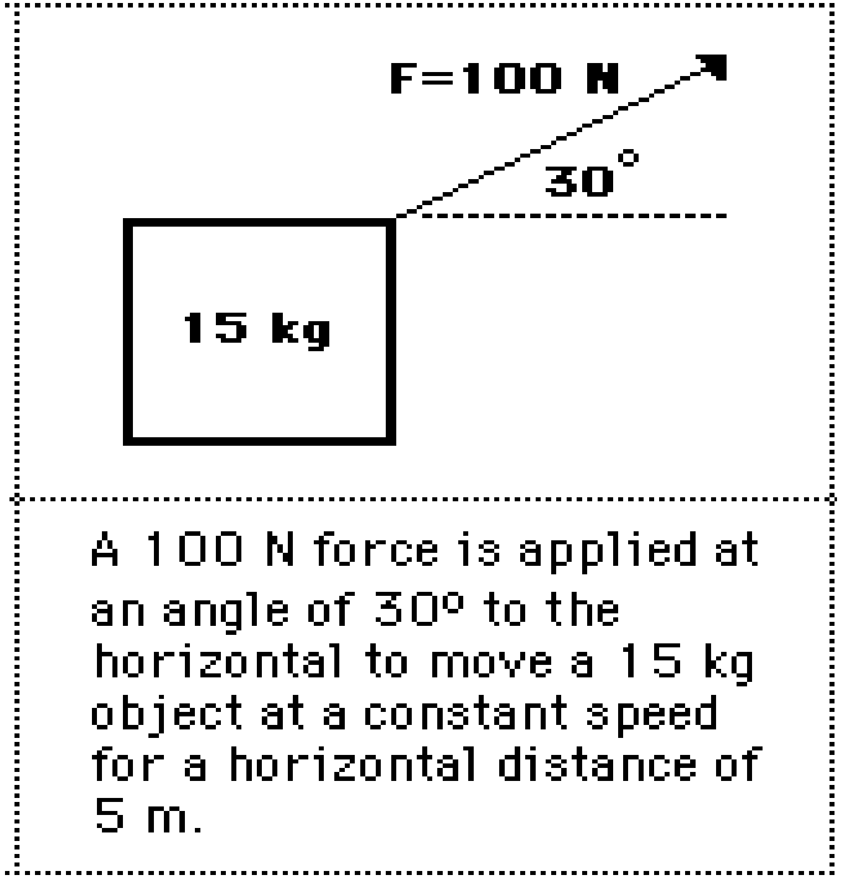
A 100 N force is applied to move a 15 kg object a horizontal distance of 5 meters at constant speed.

W = (100 N) * (5 m)* cos(0 degrees) = 500 J

The force and the displacement are given in the problem statement. It is said (or shown or implied) that the force and the displacement are both rightward. Since F and d are in the same direction,the angle is 0 degrees.

Calculating the Amount of Work Done by Forces

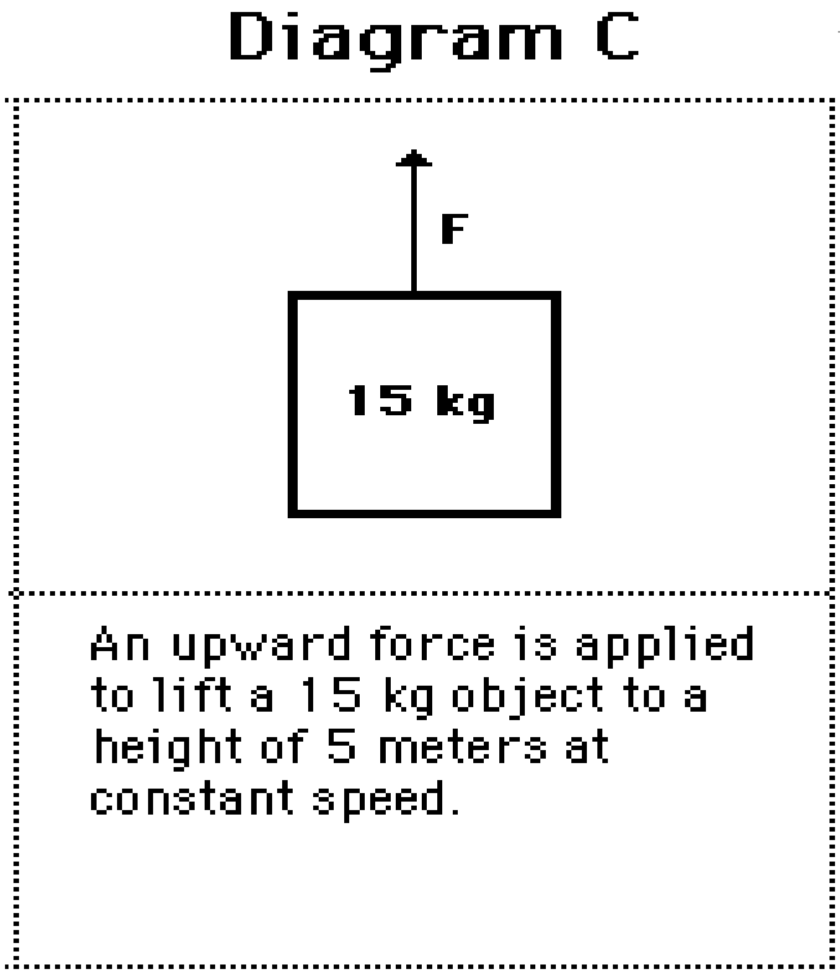
Diagram B



W = (100 N) * (5 m) * cos(30 degrees) = 433 J

The force and the displacement are given in the problem statement. It is said that the displacement is rightward. It is shown that the force is 30 degrees above the horizontal. Thus, the angle between F and d is 30 degrees.

Calculating the Amount of Work Done by Forces



W = (147 N) * (5 m) * cos(0 degrees) = 735 J

The displacement is given in the problem statement. The applied force must be 147 N since the 15-kg mass (Fgrav=147 N) is lifted at constant speed. Since F and d are in the same direction, the angle is 0 degrees.

Work

Work Done By Multiple Forces

- The total work done by different forces acting on an object at different instances is the sum of all the work done by the individual forces.

$$W = F_1\Delta x_1 + F_2\Delta x_2 + F_3\Delta x_3 + \cdots + F_n\Delta x_n$$

- If the forces are acting on an object at the same instance, then

$$\Delta x_1 = \Delta x_2 = \Delta x_3 = \cdots = \Delta x_n = \Delta x$$

And the work done by all the forces is

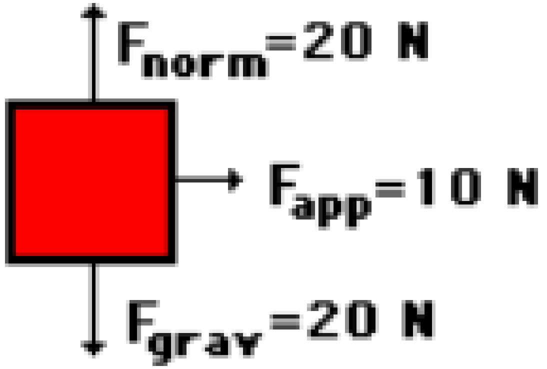
$$W = (F_1 + F_2 + F_3 + \cdots + F_n)\,\Delta x$$
$$W = F_{net}\Delta x$$

The total work done by different forces acting on an object at different instances is the sum of all the work done by the individual forces.

Calculating the Amount of Work Done by Multiple Forces

Free-Body Diagram

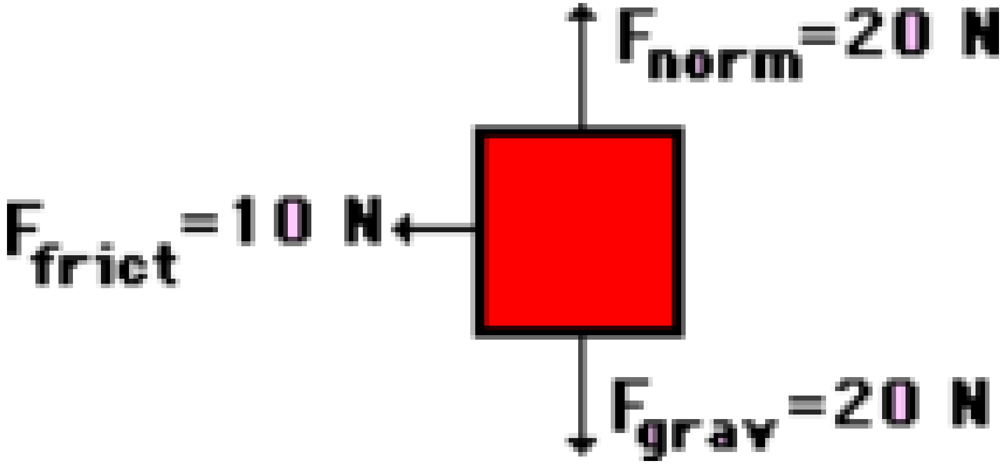
A 10-N force is applied to push a block across a friction free surface for a displacement of 5.0 m to the right.



W_{app} = (10 N) * (5 m) *cos (0 degrees) = **+50 Joules**

Calculating the Amount of Work Done by Multiple Forces

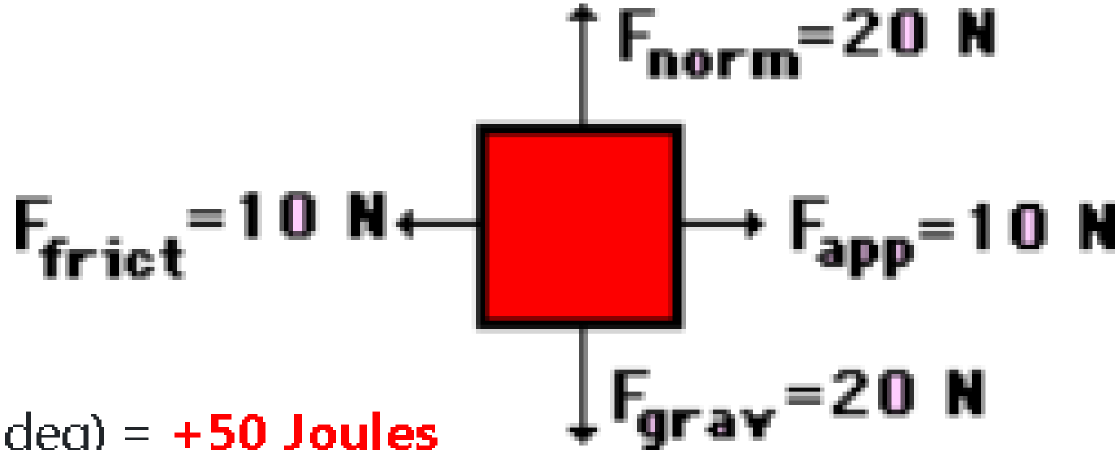
A 10-N frictional force slows a moving block to a stop after a displacement of 5.0 m to the right.



W_{frict} = (10 N) * (5 m) * cos (180 degrees) = **-50 Joules**

Calculating the Amount of Work Done by Multiple Forces

A 10-N force is applied to push a block across a frictional surface at constant speed for a displacement of 5.0 m to the right.

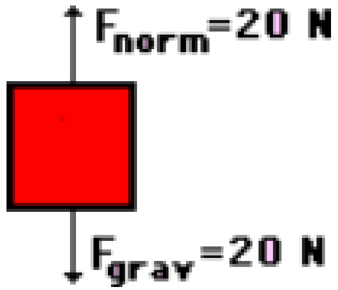


W_{app} = (10 N) * (5 m) * cos (0 deg) = **+50 Joules**

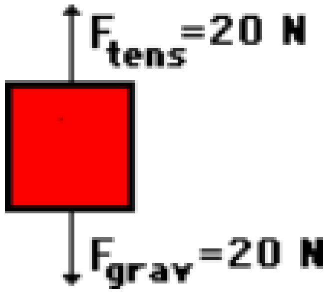
W_{frict} = (10 N) * (5 m) * cos (180 deg) = **-50 Joules**

Calculating the Amount of Work Done by Multiple Forces

An approximately 2-kg object is sliding at constant speed across a friction free surface for a displacement of 5 m to the right.



An approximately 2-kg object is pulled upward at constant speed by a 20-N force for a vertical displacement of 5 m.



Ben Travlun carries a 200-N suitcase up three flights of stairs (a height of 10.0 m) and then pushes it with a horizontal force of 50.0 N at a constant speed of 0.5 m/s for a horizontal distance of 35.0 meters. How much work does Ben do on his suitcase during this <u>entire</u> motion?

The motion has two parts: pulling vertically to displace the suitcase vertically (angle = 0 degrees) and pushing horizontally to displace the suitcase horizontally (angle = 0 degrees).

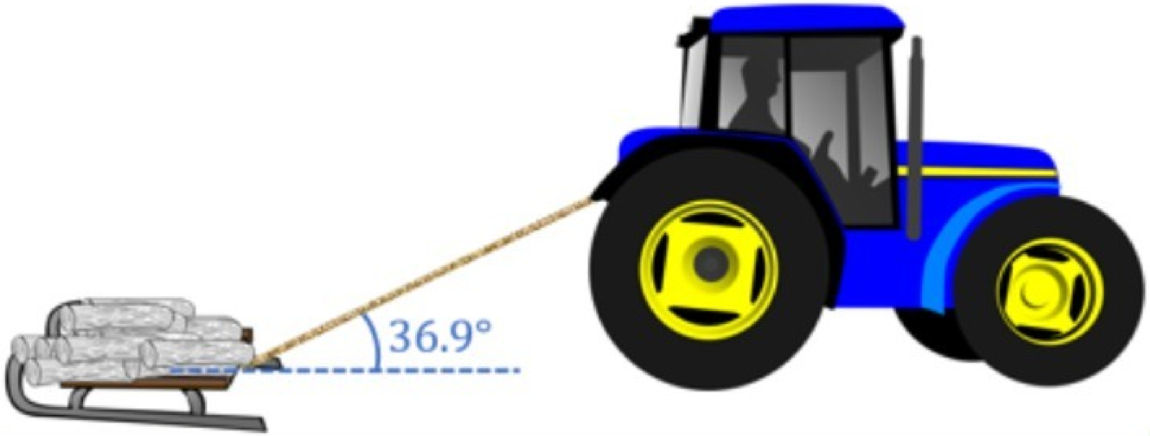
For the vertical part, W = (200 N) * (10 m) * cos (0 deg) = **2000 J.**

For the horizontal part, W = (50 N) * (35 m) * cos (0 deg) = **1750 J.**

The total work done is **3750 J** (the sum of the two parts).

Work

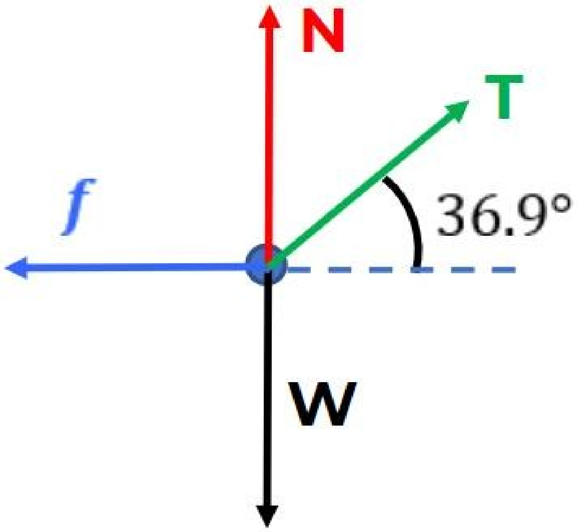
Example: A sled loaded with firewood was pulled by a tractor to a distance of 20 m. The tractor was exerting a constant force of 5000 N at an angle of 36.9° with the horizontal. The sled with the firewood was weighing 14000 N in total and a friction of 3500 N was existing between the sled and the ground. Find the individual work done by each forces and the total work done by all the forces.



Lets try to solve this problem.

Work

Draw the FBD of the sled (with wood)!

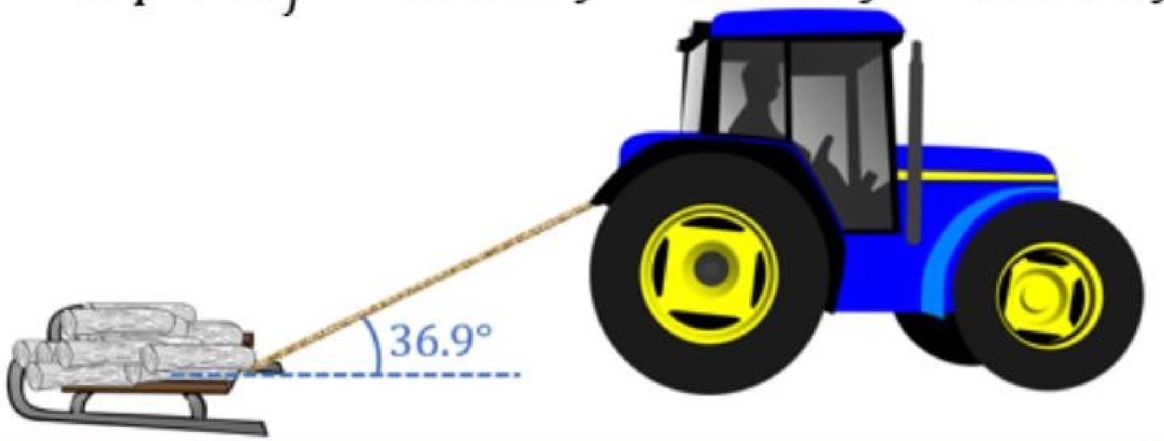


$$W_T = T\Delta x cos 36.9 = (5000\ N)(20\ m)(cos 36.9) \approx 8.0 \times 10^4\ J$$

$$W_f = f\Delta x cos 180 = (3500\ N)(20\ m)(cos 180) = -7.0 \times 10^4\ J$$

$$W_{total} = W_T + W_f = 80000\ J - 70000\ J = 10000\ J$$

Only the forces with x-components have non-zero work done.

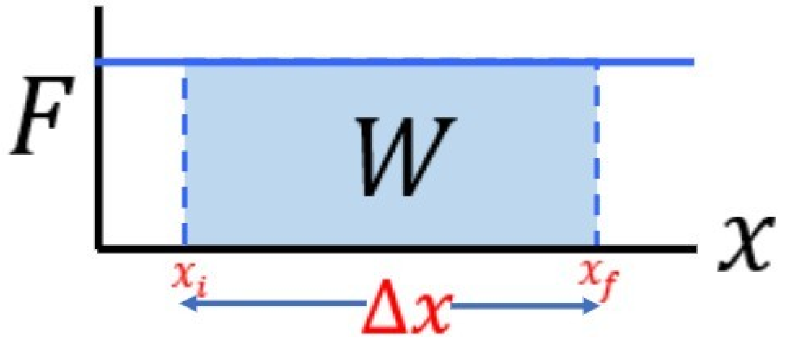


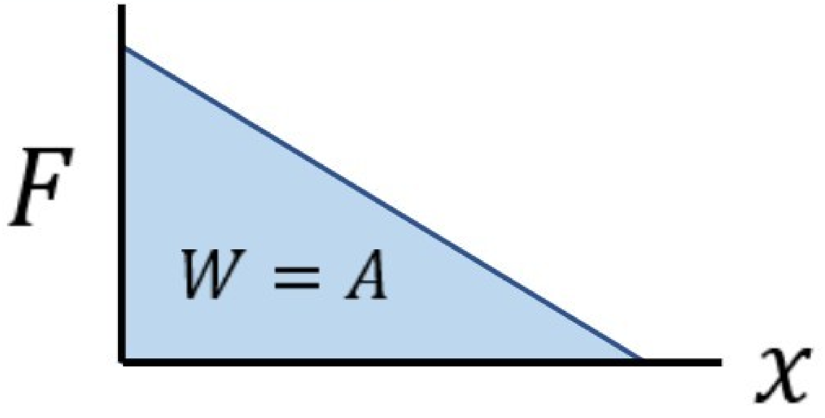
Work done by multiple forces.

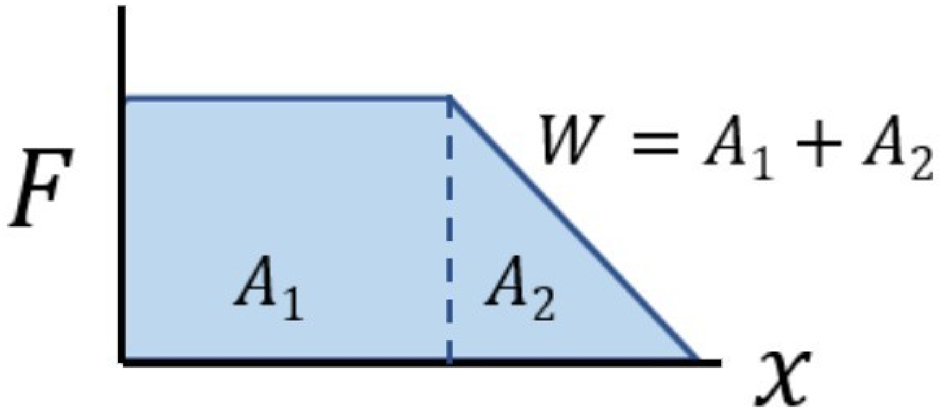
Work

Work Done By A Non-constant Force

Work done by non-constant forces can be determined by constructing a force vs. position graph. The area under the f vs. x curve is equivalent to the work done by the force.



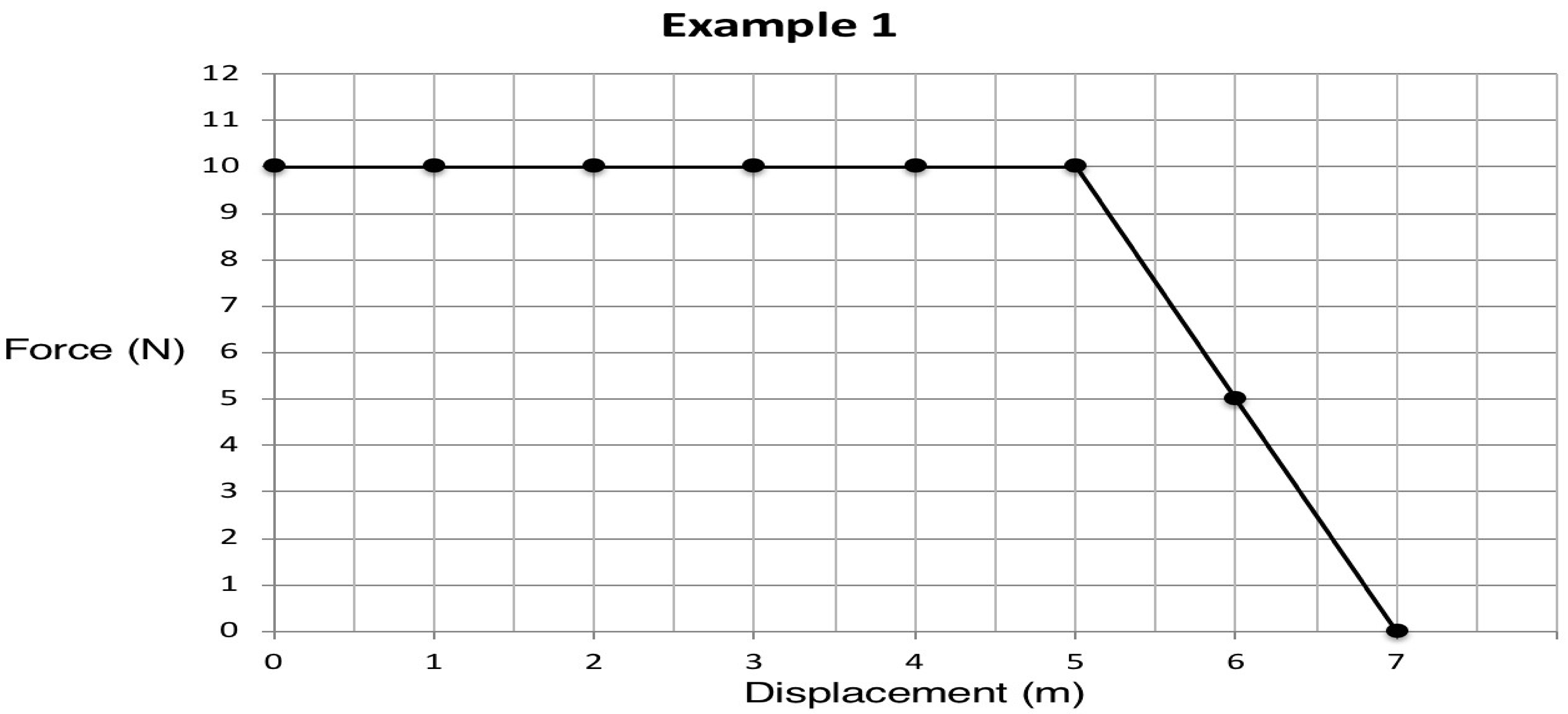




Work done by non-constant forces can be determined by constructing a force vs. position graph. The area under the f vs. x curve is equivalent to the work done by the force.

Example 1:

The graph below shows the force applied to an object as it was moved a distance of 10m. Answer the following:



Example 1:

The graph below shows the force applied to an object as it was moved a distance of 10m. Answer the following:

a) When was a constant force applied to the object?

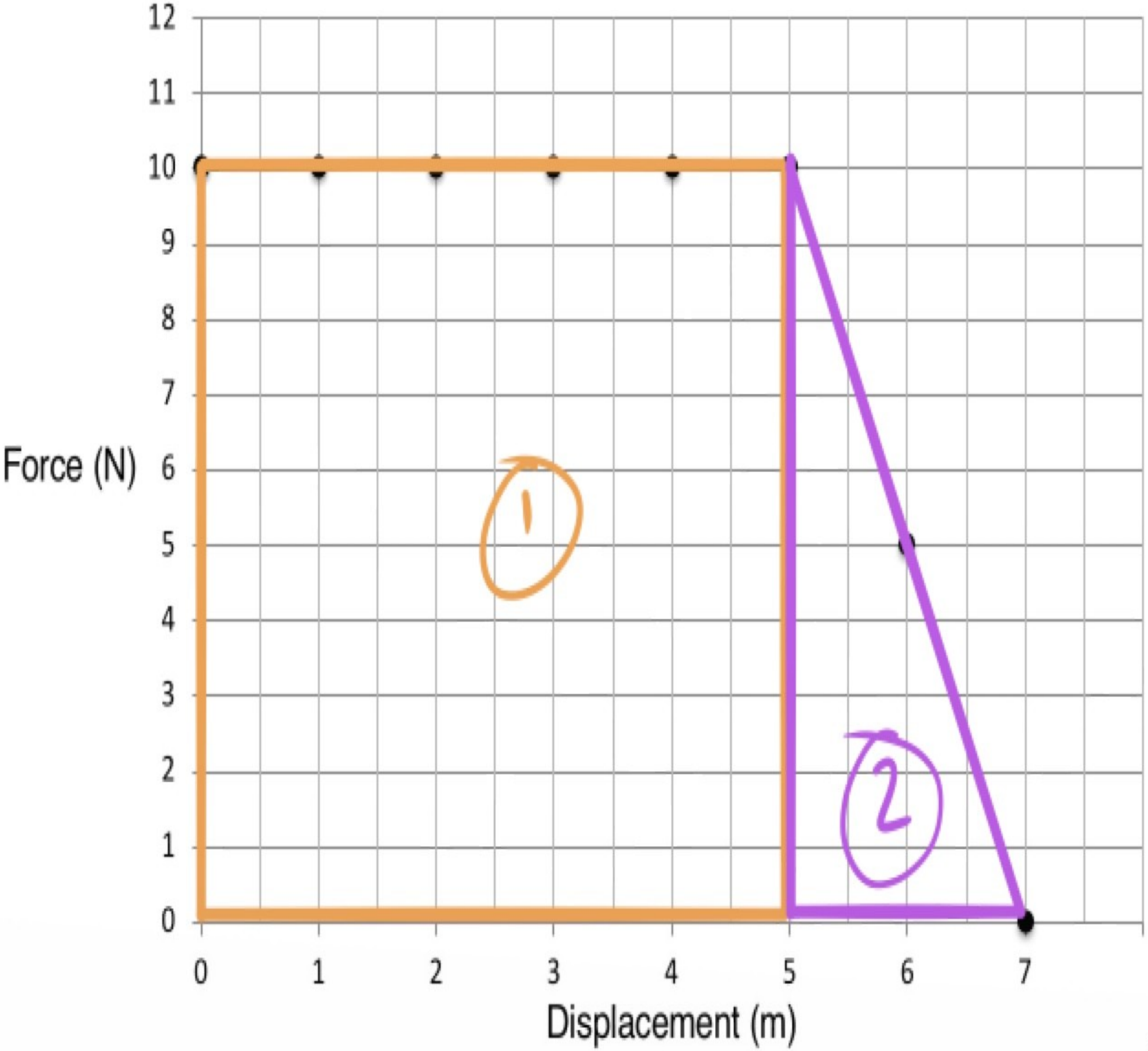
b) What force was applied at a displacement of 6m?

c) How much work was done on the object?

a) Reading directly from the graph, a constant force of 10N was applied between a displacement of 0-5m.

b) At a displacement of 6m, a force of 5N was applied to the object

Example 1

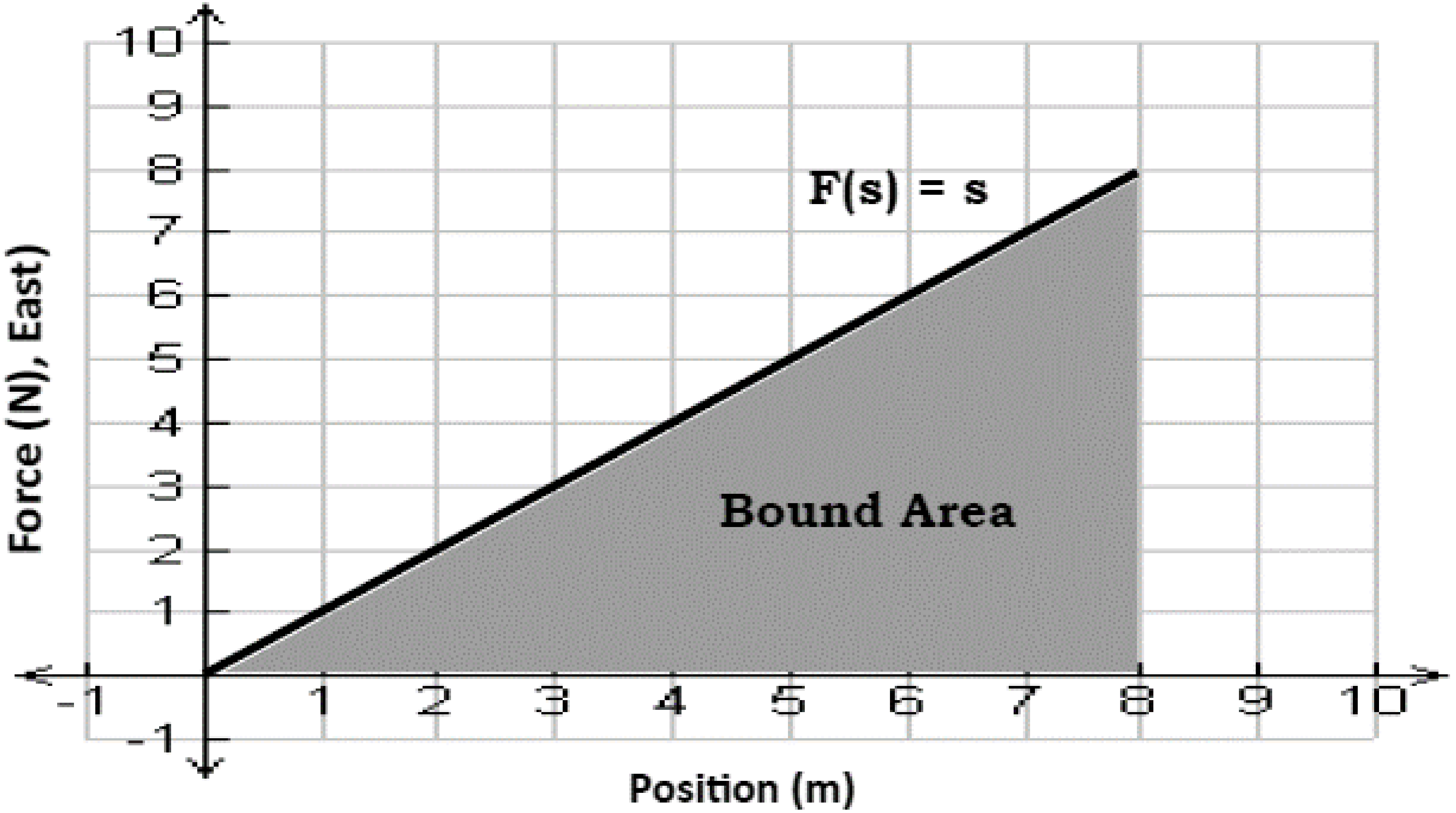


- Area 1 $= 10 \times 5 = 50$
- Area 2 $= \dfrac{1}{2} \times 2 \times 10 = 10$

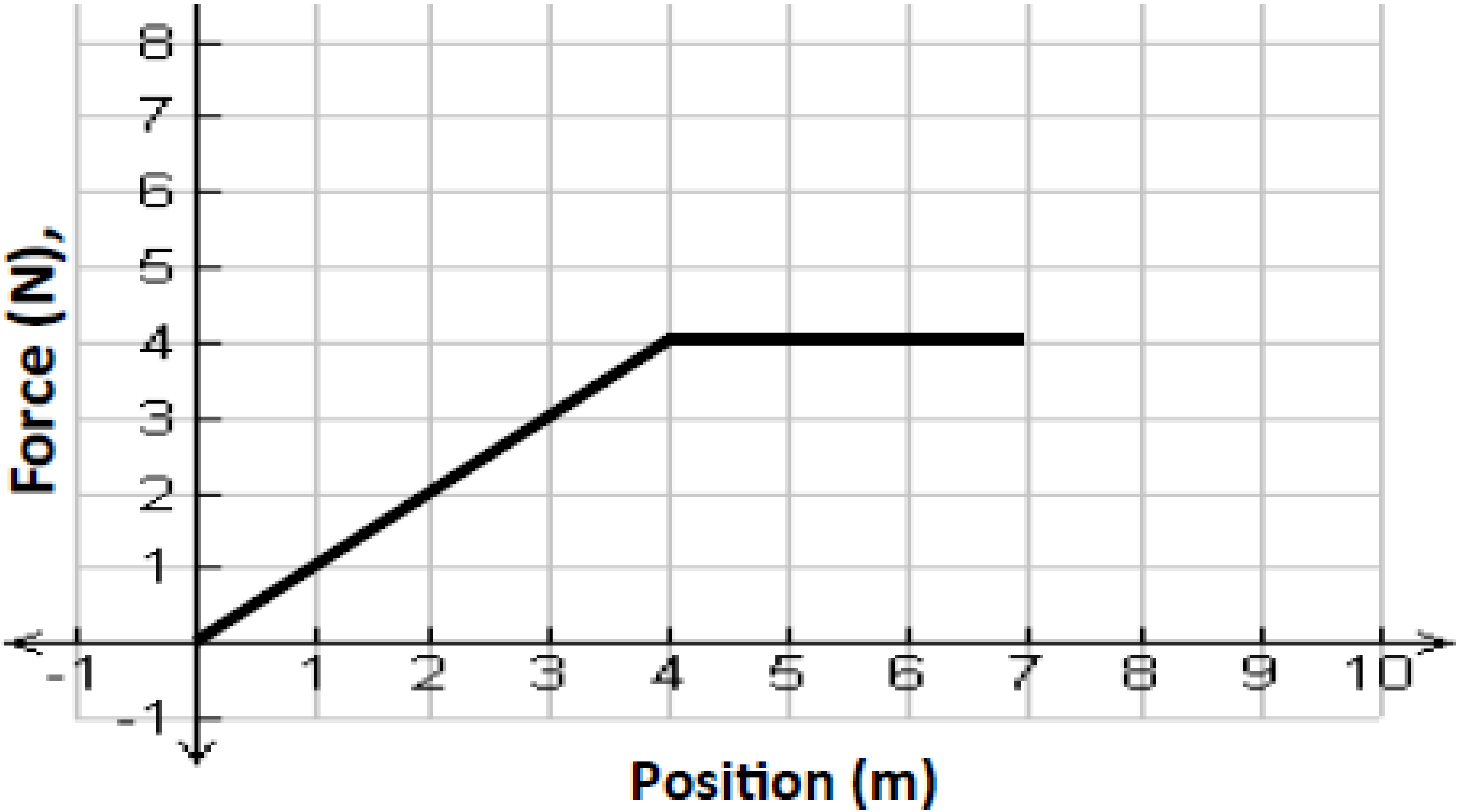
Total area $= 50 + 10 = 60$

Therefore $W = 60 \quad J$

Find the work done from 0 to 8m:



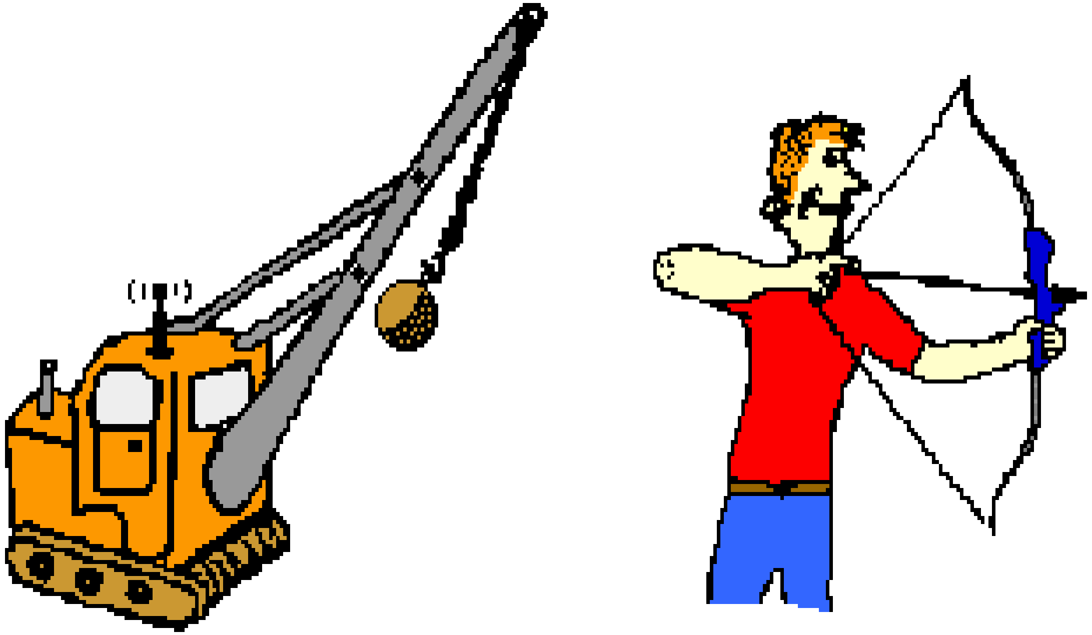
Find the work done:



POTENTIAL ENERGY

POTENTIAL ENERGY

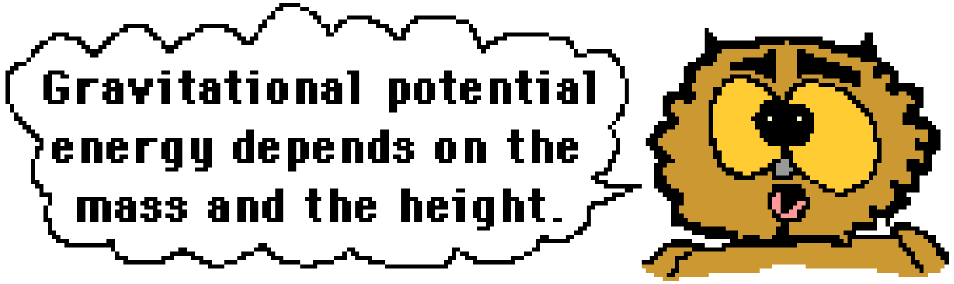
An object can store energy as the result of its position. Potential energy is the stored energy of position possessed by an object.



The massive ball of a demolition machine and the stretched bow possesses stored energy of position - potential energy.

GRAVITATIONAL POTENTIAL ENERGY

Gravitational potential energy is the energy stored in an object as the result of its vertical position or height. The energy is stored as the result of the gravitational attraction of the Earth for the object.



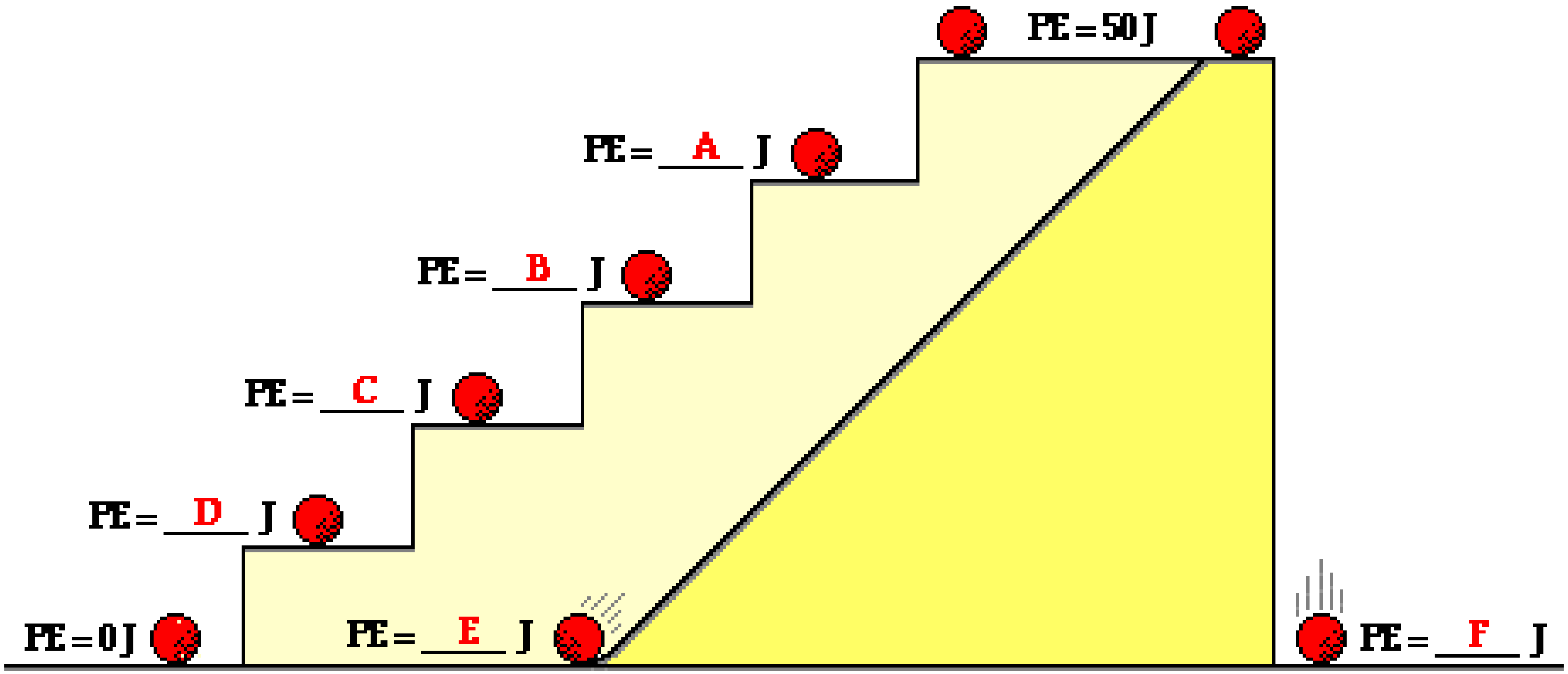
GRAVITATIONAL POTENTIAL ENERGY

There is also a direct relation between gravitational potential energy and the height of an object.

$$PE_{grav} = mass \bullet g \bullet height$$

$$PE_{grav} = m \,{}^{*}\!\bullet g \bullet h$$

GRAVITATIONAL POTENTIAL ENERGY



GRAVITATIONAL POTENTIAL ENERGY

A: PE = 40 J (since the same mass is elevated to 4/5-ths height of the top stair)

B: PE = 30 J (since the same mass is elevated to 3/5-ths height of the top stair)

C: PE = 20 J (since the same mass is elevated to 2/5-ths height of the top stair)

D: PE = 10 J (since the same mass is elevated to 1/5-ths height of the top stair)

E and F: PE = 0 J (since the same mass is at the same zero height position as shown for the bottom stair).

ELASTIC POTENTIAL ENERGY

Elastic potential energy is the energy stored in elastic materials as the result of their stretching or compressing. Elastic potential energy can be stored in rubber bands, bungee chords, trampolines, springs, an arrow drawn into a bow, etc. The amount of elastic potential energy stored in such a device is related to the amount of stretch of the device - the more stretch, the more stored energy.

ELASTIC POTENTIAL ENERGY

A force is required to compress a spring; the more compression there is, the more force that is required to compress it further.

$$F_{spring} = k \bullet x$$

the amount of force is directly proportional to the amount of stretch or compression (x); the constant of proportionality is known as the spring constant (k).

ELASTIC POTENTIAL ENERGY

There is a special equation for springs that relates the amount of elastic potential energy to the amount of stretch (or compression) and the spring constant.

$$PE_{spring} = 0.5 \bullet k \bullet x^2$$

where k = spring constant

x = amount of compression
(relative to equilibrium position)

KINETIC ENERGY

KINETIC ENERGY

Kinetic energy is the energy of motion. An object that has motion - whether it is vertical or horizontal motion - has kinetic energy.

$$KE = 0.5 \bullet m \bullet v^2$$

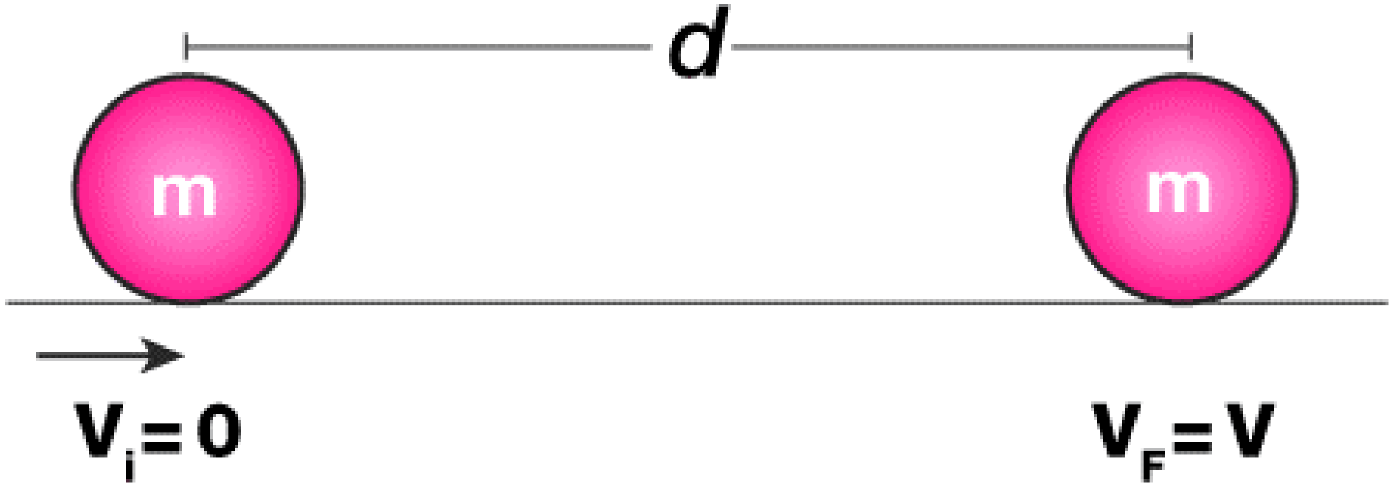
where **m** = mass of object

v = speed of object

Work-Energy Theorem

Work-Energy Theorem argues the net work done on a particle equals the change in the particle's kinetic energy. According to this theorem, when an object slows down, its final kinetic energy is less than its initial kinetic energy, the change in its kinetic energy is negative, and so is the net work done on it. If an object speeds up, the net work done on it is positive.

Work-Energy Theorem



$$KE = \frac{1}{2} m.v^2$$

1 Joule = 1 kg • m²/s²

KE - Kinetic Energy | m - Mass of object | v - Velocity

Check your understanding

1. Determine the kinetic energy of a 625-kg roller coaster car that is moving with a speed of 18.3 m/s.

$$KE = 0.5 * m * v^2$$

$$KE = (0.5) * (625\ kg) * (18.3\ m/s)^2$$

$KE = 1.05 \times 10^5$ Joules

Check your understanding

2. Missy Diwater, the former platform diver for the Ringling Brother's Circus, had a kinetic energy of 12 000 J just prior to hitting the bucket of water. If Missy's mass is 40 kg, then what is her speed?

$$KE = 0.5 * m * v^2$$

$$12\,000\ J = (0.5) * (40\ kg) * v^2$$

$$300\ J = (0.5) * v^2$$

$$600\ J = v^2$$

v = 24.5 m/s

Check your understanding

3. A 900-kg compact car moving at 60 mi/hr has approximately 320 000 Joules of kinetic energy. Estimate its new kinetic energy if it is moving at 30 mi/hr.

Answer:

The KE is directly related to the square of the speed. If the speed is reduced by a factor of 2 (as in from 60 mi/hr to 30 mi/hr) then the KE will be reduced by a factor of 4. Thus, the new KE is (320 000 J)/4 or 80 000 J.

POWER

POWER

Power is the rate at which work is done. Mathematically, it is computed using the following equation.

Power = Work / time

or

$P = W / t$

The standard metric unit of power is the **Watt**. Watt is equivalent to a Joule/second.

POWER

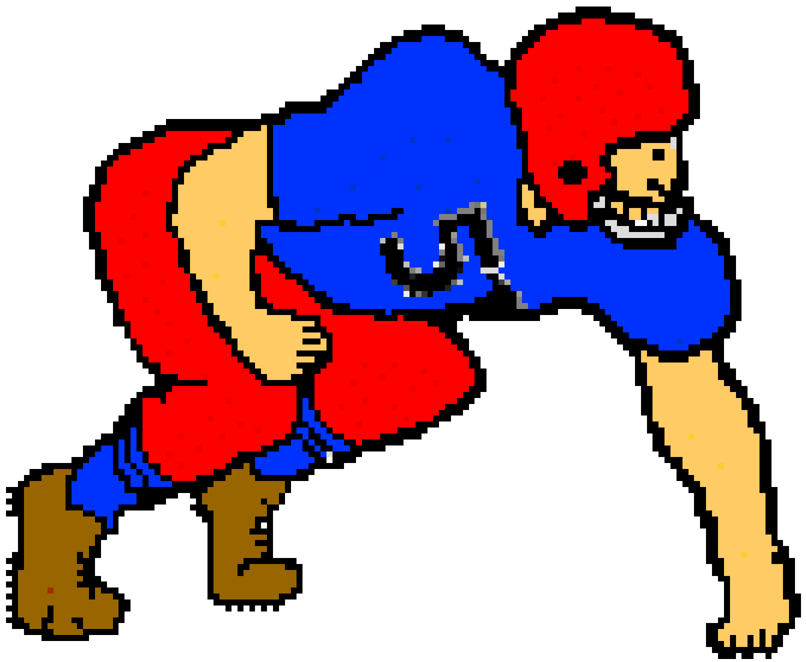
$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{\text{Force} \cdot \text{Displacement}}{\text{Time}}$$

$$\text{Power} = \text{Force} \cdot \frac{\text{Displacement}}{\text{Time}}$$

$$\boxed{\text{Power} = \text{Force} \cdot \text{Velocity}}$$

This new equation for power reveals that a powerful machine is both strong (big force) and fast (big velocity). A powerful car engine is strong and fast. A powerful piece of farm equipment is strong and fast. A powerful weightlifter is strong and fast. A powerful lineman on a football team is strong and fast. A machine that is strong enough to apply a big force to cause a displacement in a small mount of time (i.e., a big velocity) is a powerful machine.

POWER



A powerful lineman is both STRONG (applies a big force) and FAST (displaces objects in small times).

POWER

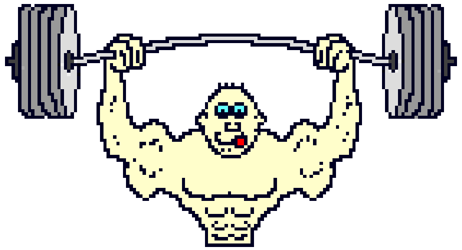
Suppose that Ben Pumpiniron elevates his 80-kg body up the 2.0-meter stairwell in 1.8 seconds.

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{784\ \text{N} \cdot 2.0\ \text{m}}{1.8\ \text{seconds}}$$

$$\text{Power} = 871\ \text{Watts}$$

Check your understanding

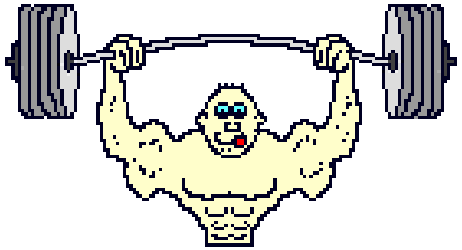
1. Two physics students, Will N. Andable and Ben Pumpiniron, are in the weightlifting room. Will lifts the 100-pound barbell over his head 10 times in one minute; Ben lifts the 100-pound barbell over his head 10 times in 10 seconds. Which student does the most work? ____________________ Which student delivers the most power? ____________________ Explain your answers.



Check your understanding

ANSWER: Ben and Will do the same amount of work. They apply the same force to lift the same barbell the same distance above their heads.

Yet, Ben is the most "power-full" since he does the same work in less time. Power and time are inversely proportional.



Check your understanding

2. During a physics lab, Jack and Jill ran up a hill. Jack is twice as massive as Jill; yet Jill ascends the same distance in half the time. Who did the most work? ____________________ Who delivered the most power? ____________________ Explain your answers.

Check your understanding

ANSWER: Jack does more work than Jill. Jack must apply twice the force to lift his twice-as-massive body up the same flight of stairs. Yet, Jill is just as "power-full" as Jack. Jill does one-half the work yet does it one-half the time. The reduction in work done is compensated for by the reduction in time.

Check your understanding

3. An escalator is used to move 20 passengers every minute from the first floor of a department store to the second. The second floor is located 5.20 meters above the first floor. The average passenger's mass is 54.9 kg. Determine the power requirement of the escalator in order to move this number of passengers in this amount of time.

Check your understanding

$$W_{1\ passenger} = F \cdot d \cdot \cos(0\ deg)$$

$$W_{1\ passenger} = (54.9\ kg \cdot 9.8\ m/s^2) \cdot 5.20\ m = 2798\ J\ (rounded)$$

$$W_{20\ passengers} = 55954\ J\ (rounded)$$

$$P = W_{20\ passengers} / time = (55954\ J) / (60\ s)$$

P = 933 W

POWER