

Mathematics

Class 10

Government of Nepal

Ministry of Education, Science and Technology

Curriculum Development Centre

Sanothimi, Bhaktapur

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Preface

School education is the foundation for preparing the citizen who are loyal to the nation and nationality, committed to the norms and values of federal democratic republic, self-reliant and respecting the social and cultural diversity. It is also remarkable for developing a good moral character with the practical know-how of the use of ICT along with the application of scientific concept and positive thinking. It is also expected to prepare the citizens who are moral and ethical, disciplined, social and human value sensitive with the consciousness about the environmental conversation and sustainable development. Moreover, it should be helpful for developing the skills for solving the real life problems. This textbook 'Mathematics, Grade 10' is fully aligned with the intent carried out by the National Curriculum Framework for School Education, 2076 and is developed fully in accordance with the new Secondary Level Curriculum, Grade 9-10, 2078.

This textbook is initially written by Mr. Narahari Acharya, Mr. Shakti Prasad Acharya, Mr. Sushila Khanal, Mr. Jagannath Adhikari and Mr. Ram Chandra Dhakal. It has been translated by Mr. Madan Kumar Shrestha, Mr. Prem K.C., Mr. Shakti Prasad Acharya and Mr. Ram Chandra Dhakal. The contribution made by Director General Mr. Baikuntha Prasad Aryal, Prof. Dr. Hari Prasad Upadhyay, Mr. Gyanendra Ban, Mrs. Anupama Sharma, Mr. Navin Poudel, Mr. Satya Narayan Maharjan, Mrs. Pramila Bhakati and Mrs. Ritu Shrestha is remarkable in bringing the book in this form. The language of the book has been edited by Mr. Matrika Subedi. Art editing of this book was done by Mr. Shreehari Shrestha by making it four colour. The Curriculum Development Centre extends sincere gratitude to all of them.

The textbook is a primary resource for classroom teaching. Considerable efforts have been made to make the book helpful in achieving the expected competencies of the curriculum. Curriculum Development Centre always welcomes constructive feedback for further betterment of its publications.

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Lesson 1

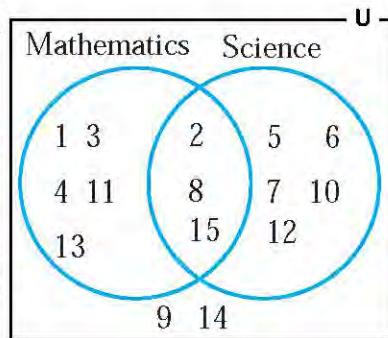
Sets

1.0 Review

Students from roll no. 1 to 15 in grade 10 are surveyed about whether they like Mathematics or Science. The information from the survey is presented in a Venn-diagram.

Discuss on the following questions based on the Venn-diagram alongside:

- Write the set of students who like Mathematics by listing method. Write the cardinal number of the set.
- Write the set of students who like Mathematics only by listing method. Write the cardinal number of the set.
- Write the set of students who like Science by listing method. Write the cardinal number of the set.
- Write the set of students who like Science only by listing method. Write the cardinal number of the set.
- Write the set of students who like both Mathematics and Science by listing method. Write the cardinal number of the set.
- Write down the set of students who do not like either Mathematics or Science by listing method. Write the cardinal number of the set.
- How many students were surveyed?



1.1 Cardinality of the Two Sets

Activity 1

An information obtained from the question asked among the students of grade 10 about whether they like coffee or tea. The following information was obtained.

- The number of students who like coffee is 15.
- The number of students who like tea is 10.
- The number of students who like both is 6.
- The number of students who dislike both is 5.

Discuss the following questions based on the above information:

- How shall the given information be shown in a Venn-diagram?
- How many students like coffee only?
- How many students like tea only?
- How many students were there in the class?

Here, $n(C)$ denotes the number of students who like coffee and $n(T)$ denotes the number of students who like tea. Similarly, $n_0(T)$ and $n_0(C)$ respectively denote the number of students who like tea only and coffee only.

Now, showing the information in a Venn-diagram,

First write the value of $n(C \cap T)$ in venn-diagram since the number of students who like both is denoted by it.

Then, since the number of students who like coffee is $n_0(C) + n(C \cap T)$,

The number of students who like coffee only $n_0(C) = n(C) - n(C \cap T)$.

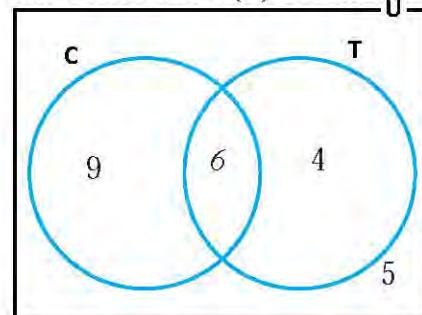
$$\text{i.e. } n_0(C) = 15 - 6 = 9$$

Likewise, in case of the number of students who like tea

$$\begin{aligned} \text{Let us insert the number of students who like tea only } n_0(T) &= n(T) - n(C \cap T) \\ &= 10 - 6 = 4. \end{aligned}$$

Now, let's insert the number of students who dislike tea or coffee $n(\overline{C \cup T}) = 5$

In this way, the total number of students in the class $n(U) = 9 + 6 + 4 + 5 = 24$



If A and B are overlapping sets,

- Total number of elements of both the sets, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
- The number of elements in the set A only, $n_0(A) = n(A) - n(A \cap B)$.
- The number of elements in the set B only, $n_0(B) = n(B) - n(A \cap B)$.
- Total number of elements of both the sets, $n_0(A) + n_0(B) + n(A \cap B)$.
- If A and B are disjoint sets, then $n(A \cup B) = n(A) + n(B)$
- If there are elements of A and B only in U, then $n(U) = n(A \cup B)$
- If there are elements other than A and B in U, then $n(U) = n(A \cup B) + n(\overline{A \cup B})$

Other terminologies:

At least one: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Or, $n(A \cup B) = n_0(A) + n_0(B) + n(A \cap B)$.

At most one: $n(\overline{A \cap B}) = n(U) - n(A \cap B)$.

Exactly one or only one: $n_0(A) + n_0(B) = n(A) + n(B) - 2 \times n(A \cap B)$.

Example 1

In a survey of 300 people of a community, it was found that 175 liked cricket and 150 liked football but 25 liked neither of them. Based on this, answer the following questions:

- Represent the above information in the Venn-diagram.
- Find the number of people who like both the games.
- Find the number of people who like exactly one game.

Solution

Let, C and F respectively denote the set of people who like cricket and football. Likewise, U denotes the set of total people.

According to the question,

$$n(U) = 300, n(C) = 175, n(F) = 150 \text{ and } n(\overline{C \cup F}) = 25$$

Let, $n(C \cap F) = x$

- The information is represented in the Venn-diagram alongside.
- From the Venn-diagram, it can be written as

$$\begin{aligned}n(U) &= n_o(C) + n(C \cap F) + n_o(F) + n(\overline{C \cup F}) \\300 &= (175 - x) + x + (150 - x) + 25\end{aligned}$$

$$\text{or, } 300 = 175 - x + x + 150 - x + 25$$

$$\text{or, } 300 = 350 - x$$

$$\text{or, } x = 350 - 300$$

$$\therefore x = 50$$

$$\text{or, } n(C \cap F) = 50$$

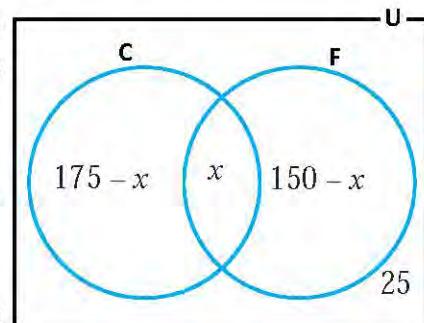
∴ The number of people who like both the games is 50.

Again,

The number of people who liked only cricket, $n_o(C) = 175 - 50 = 125$

The number of people who liked only football, $n_o(F) = 150 - 50 = 100$

∴ The number of people who liked exactly one game, $n_o(C) + n_o(F) = 125 + 100 = 225$



Example 2

The result of a survey among 120 students of grade 10 is as follows:

30 like only Mathematics.

40 like only English.

10 like neither Mathematics nor English.

Based on this information, answer the following questions:

- Represent the above information in a Venn-diagram.
- Find the number of students who like both the subjects.
- Find the number of students who like at least one subject.

Solution

Let, M and E denote the set of students who like Mathematics and English respectively.

Likewise, U denotes the total students.

According to the question,

$$n(U) = 120, n(M) = 30, n(E) = 40 \text{ and } n(M \cup E) = 10$$

Let, $n(M \cap E) = x$

a) The information is represented in a Venn-diagram alongside.

b) From the Venn-diagram,

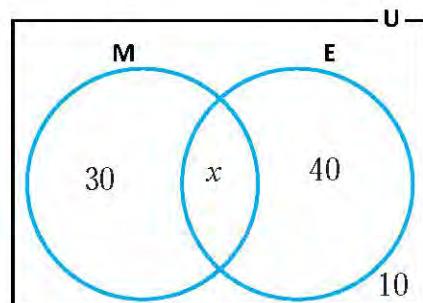
$$30 + x + 40 + 10 = 120$$

$$\text{or, } 80 + x = 120$$

$$\text{or, } x = 120 - 80 = 40$$

$$\therefore x = 40$$

$$\text{or, } n(M \cap E) = 40$$



∴ The number of students who like both Mathematics and English is 40.

c) The number of students who like at least one subject $n(M \cup E) = 30 + 40 + 40 = 110$

Example 3

According to a survey among the SEE appeared students from a school, 75% were interested in studying science and 55% were interested in studying staff nurse but 5% denied to give information whilst 21 students were interested to study both science and staff nurse. Based on this information, answer the following questions:

- Show the above information in the Venn-diagram.
- Find the total number of students inquired in the survey.
- Find the number of students who were interested in studying only staff nurse.

Solution

Let, S and N denote the set of students interested to study science and staff nurse respectively.

Likewise, U denotes the total students.

According to the question,

Let,

$$n(U) = x,$$

$$n(S) = 75\% \text{ of } x = 0.75x,$$

$$n(N) = 55\% \text{ of } x = 0.55x,$$

$$n(S \cap N) = 21 \text{ and } n(S \cup N) = 5\% \text{ of } x = 0.05x,$$

- The information is represented in a Venn-diagram alongside.
- From the Venn-diagram,

$$(0.75x - 21) + 21 + (0.55x - 21) + 0.05x = x$$

$$\text{or, } 1.35x - 21 = x$$

$$\text{or, } 1.35x - x = 21$$

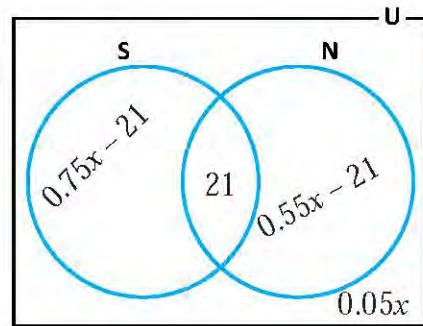
$$\text{or, } 0.35x = 21$$

$$\text{or, } x = \frac{21}{0.35} = 60$$

$$\therefore x = 60$$

$$\text{or, } n(U) = 60$$

\therefore The total number of students surveyed was 60.



- c) From the Venn-diagram,

The number of students who were interested in studying only staff nurse

$$= 0.55x - 21$$

$$= 0.55 \times 60 - 21$$

$$= 33 - 21$$

$$= 12$$

\therefore The number of students who were interested in studying only staff nurse was 12.

Example 4

In a survey of 300 foreign tourists visiting to Nepal, it was found that the ratio of the number of tourists who visited Pokhara and Lumbini was 2:3. Among them, 90 visited both the places and 60 visited neither Pokhara nor Lumbini. Based on this information, answer the following questions:

- Show the above information in a Venn-diagram.
- Determine the number of tourists who visited only one place.
- Find the number of tourists who visited at least one of the places.

Solution

Let, P and L denote the set of tourists who visited Pokhara and Lumbini respectively.

Likewise, U denotes the total tourists.

According to the question,

$$n(U) = 300, n(P \cap L) = 90 \text{ and } n(P \cup L) = 60$$

$$\text{Let, } n(P) = 2x, n(L) = 3x$$

- The information is represented in a Venn-diagram alongside.

- From the Venn-diagram,

$$(2x - 90) + 90 + (3x - 90) + 60 = 300$$

$$\text{or, } 5x - 30 = 300$$

$$\text{or, } 5x = 300 + 30$$

$$\text{or, } 5x = 330$$

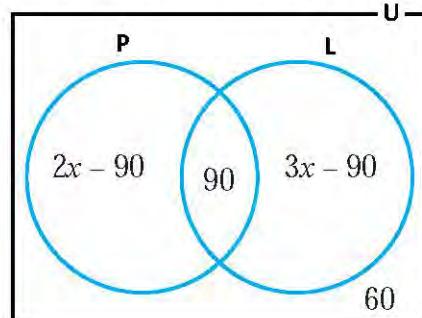
$$\text{or, } x = \frac{330}{5}$$

$$\therefore x = 66$$

$$\text{Thus, } n_o(P) = 2 \times 66 - 90 = 42 \text{ and } n_o(L) = 3 \times 66 - 90 = 108$$

\therefore The number of tourists who visited only one place = $42 + 108 = 150$

c) The number of tourists who visited at least one place = $300 - 60 = 240$



Example 5

In a survey among the 200 students studying in grade 10, it was found that the ratio of the number of students who likes Mathematics and English was 2:3. Among them, 30% like both of them but 15% like neither Mathematics nor English. Based on this information, answer the following questions:

- Represent the above information in a Venn-diagram.
- What is the difference between the number of students who like Mathematics and the number of students who like English? Find it.

Solution

Let, M and E denote the set of students who likes Mathematics and English respectively. Likewise, U denotes the total students.

According to the question,

$$n(U) = 200, n(M \cap E) = 30\% \text{ of } 200 = 60 \text{ and}$$

$$n(\overline{M \cup E}) = 15\% \text{ of } 200 = 30$$

$$\text{Let, } n_o(M) = 2x, n_o(E) = 3x$$

- The information is represented in a Venn-diagram alongside.
- From the Venn-diagram,

$$2x + 60 + 3x + 30 = 200$$

$$\text{or, } 90 + 5x = 200$$

$$\text{or, } 5x = 200 - 90$$

$$\text{or, } 5x = 110$$

$$\text{or, } x = \frac{110}{5}$$

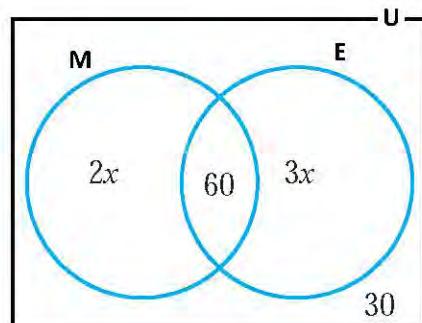
$$\therefore x = 22$$

Thus,

$$\text{The number of students who likes Mathematics, } n(M) = 2x + 60 = 2 \times 22 + 60 = 104$$

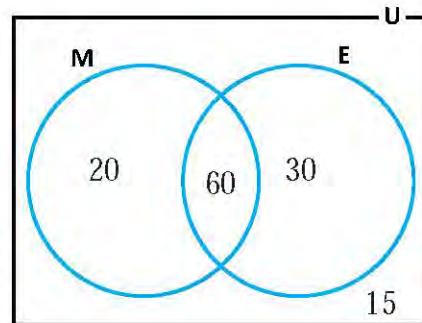
$$\text{The number of students who likes English, } n(E) = 3x + 60 = 3 \times 22 + 60 = 126$$

\therefore The difference between the number of students who like Mathematics and the number students who like English = $126 - 104 = 22$



Exercise 1.1

1.
 - a) Present the cardinality of sets with examples and show it to your teacher.
 - b) For two sets A and B, $A \subset B$, find the values of $n(A \cup B)$ and $n(A \cap B)$.
 - c) If A and B are overlapping sets, state the formula for $n(A \cup B)$.
 - d) There are 12 and 8 elements in the sets A and B respectively. Find the minimum number of elements that would be in the set $n(A \cup B)$.
2. In the given Venn-diagram, 80 people are in set M, 90 people are in set E and 15 people are not in both the sets. Determine the cardinality of following sets.
 - (a) $n_o(M)$
 - (b) $n_o(E)$
 - (c) $n(M)$
 - (d) $n(E)$
 - (e) $n(M \cup E)$
 - (f) $n(M \cap E)$
 - (g) $n(\overline{M \cup E})$
 - (h) $n(U)$
3.
 - a) If $n(U) = 200$, $n_o(M) = 2x$, $n_o(E) = 3x$, $n(M \cap E) = 60$ and $n(\overline{M \cup E}) = 40$ find the value of x .
 - b) If $n(U) = 350$, $n(A) = 200$, $n(B) = 220$ and $n(A \cap B) = 120$, then find $n(A \cup B)$ and $n(\overline{A \cup B})$.
 - c) If $n(A) = 35$ and $n(\overline{A}) = 25$, then find the value of $n(U)$.
 - d) Out of two sets P and Q, there are 40 elements in P, 60 elements in $(P \cup Q)$ and 10 elements in $(P \cap Q)$. How many elements are there in Q? Find.
4.
 - a) In a survey of 180 students of a school, 45 like Nepali only and 60 like English only but 15 like none of the subjects. Based on this information, answer the following questions:
 - i) Show the above information in a Venn-diagram.
 - ii) Find the number of students who like both the subjects.
 - iii) Find the number of students who like at least one subject.
 - b) In a survey among the 1200 students of a school, 100 like Mathematics only and 200 like Science only but 700 like neither of the subjects. Based on the information, answer the following questions:
 - i) Show the above information in a Venn-diagram.
 - ii) Find the number of students who like both the subjects.



- iii) Find the number of students who likes at least one subject.
- c) In a survey among 60 students, 10 play football only and 20 play volleyball only but 12 play neither of the games. Based on the information, answer the following questions:
- Show the above information in a Venn-diagram.
 - Find the number of students who play both the games.
 - Find the number of students who play at least one game.
5. a) A survey was carried out among 900 people of a community. According to the survey, 525 read Madhupark, 450 read Yubamanch but 75 didn't read either of the newspapers. Using the information, answer the following questions:
- Show the information in a Venn-diagram
 - Find the number of people who read both the newspapers.
 - Find the number of people who read only one newspaper.
- b) According to a survey among 150 people, 90 like modern songs, 70 like folk songs but 30 do not like either of the songs. Using the information, answer the following questions:
- Show the information in a Venn-diagram
 - Find the number of people who like both the songs.
 - Find the number of people who like only modern songs.
- c) According to a survey among 360 players, 210 liked to play volleyball, 180 liked to play football but 30 liked to play neither of the games. Using the information, answer the following questions:
- Show the information in a Venn-diagram
 - Find the number of players who like to play both the games.
 - Find the number of people who like to play only one game.
6. a) Out of the students who participated in an examination, 70% passed English, 60% passed Mathematics but 20% failed both the subjects and 550 students passed both the subjects. Based on the information, answer the following questions:
- Show the above information in a Venn-diagram.
 - Find the total number of students participated in the examination.
 - Find how many students passed English only.

- b) According to a survey of students who have appeared the examination of grade 10, 60% are interested to study Science, 70% are interested to study Management but 10% rejected in the interest to study both Science and Management whilst 400 students are interested to study both Science and Management. Based on this information, answer the following questions:
- Show the above information in a Venn-diagram.
 - Find how many students were participated in the survey.
 - Find the number of students who are interested to study Science only.
- c) In a survey among people of a community, 65% ride motorcycle, 35% ride scooter but 20% ride both whereas 200 people ride both motorcycle and scooter. Using this information, answer the following questions:
- Show the above information in a Venn-diagram.
 - Find how many people were participated in the survey.
 - Find the number of people who ride motorcycles only.
7. a) Among 95 people of a community, it was surveyed that the ratio of the number of people who drink tea and coffee is 4:5, whereas 10 people drink tea but 15 do not drink either tea or coffee. Based on the information, answer the following questions:
- Show the information in a Venn-diagram.
 - Find the number of people who drinks exactly one of tea or coffee.
 - Find the number of people who drinks at least one; either tea or coffee.
- b) In a survey of 64 students of a class, the ratio of number of students who like milk only and curd only is 2:1 whereas 16 like both based on this information, Answer the following questions:
- Show the above information in a Venn-diagram.
 - Find the number of students who like milk.
 - Find the number of students who like only one kind of drink.

- c) In a conference of 320 participants, it was surveyed that 60 participants only sing and 100 only dance. If the number of people who do not do both is three times the number of people who do both. With the help of this information, answer the following questions:
- Show the above information in a Venn-diagram.
 - Find how many people do not do both genres.
 - Find the number of people who do one genre at most.
8. According to a survey of 200 people of a community, it was found that the ratio of the number of people who use laptop only and mobile only is 2:3, among them, 30% use both but 15% does not use both the gadgets. Based on this information, answer the following questions:
- Show the above information in a Venn-diagram.
 - Find the number of people who uses laptop.
 - Find how many people use one gadget at most.
9. Out of 300 players in a survey, one-third players play volleyball only. 60% of the remaining players play football only. But 60 players do not play both. Then, find the ratio of the number of players who play volleyball and football by using the Venn-diagram.
10. Among 65 players participated in a survey, 11 play volleyball only and 33 play cricket only. If the number of players who play cricket is the double of the number of players who play volleyball, find the number of players who play both and the number of players who does not play both by using Venn-diagram.
11. In a survey of 80 people, 60 like orange and 10 like both orange and apple. The number of people who likes orange is 5 times the number of people who likes apple. By using the Venn-diagram find the number of people who likes apples only and those who do not like both the fruits.

Project Work

Form group of five students each and go to different classes in your school. Find answers to the following questions:

Which of the following game do you like? (a) Cricket (b) Football (c) Cricket and football both (d) Others

Upon getting the informations, present it in the Venn-diagram and discuss it in the class.

Answers

1. (a) Show to your teacher. (b) $n(B)$, $n(A)$
(c) $n(A) + n(B) - n(A \cap B)$ or $n_0(A) + n_0(B) + n(A \cap B)$ (d) 12
2. (a) 20 (b) 30 (c) 80 (d) 90 (e) 110 (f) 60 (g) 15 (h) 125
3. (a) 20 (b) 300, 50 (c) 60 (d) 30
4. (a) (i) 60, (ii) 165 (b) (i) 200 (ii) 500 (c) (i) 18 (ii) 48
5. (a) (i) 150 (ii) 675 (b) (i) 40 (ii) 50 (c) (i) 60 (ii) 270
6. (a) (i) 1100 (ii) 220 (b) (i) 1000
(ii) 200 (c) (i) 1000 (ii) 450
7. (a) (i) 70 (ii) 80 (b) (i) 48 (ii) 48
(c) (i) 120 (ii) 280
8. (i) 104 (ii) 140
9. 6:7 10. 11, 10
11. 4, 6

1.2 Cardinality of Three Sets

Activity 2

The elements of three sets A, B and C are shown in the following two Venn-diagrams. Based on this, discuss the following questions:

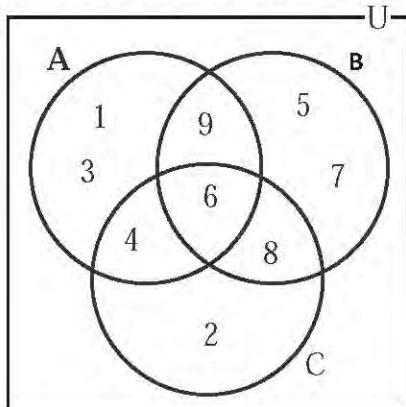


Figure 1

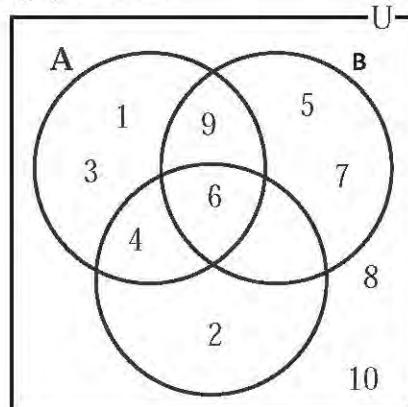


Figure 2

Observing the figure no. 1 and figure no. 2, write the elements of the sets A, B and C by listing method.

What are the cardinalities of each sets A, B and C in figure no. 1?

What are the cardinalities of each sets A, B and C in figure no. 2?

What are the values of $n(A \cup B \cup C)$ and $n(U)$ in figure no. 1?

What are the values of $n(A \cup B \cup C)$ and $n(U)$ in figure no. 2?

The conclusion from the discussion with friends can be shown below:

Figure 1	Figure 2
$A = \{1, 3, 4, 6, 9\} \therefore n(A) = 5$	$A = \{1, 3, 4, 6, 9\} \therefore n(A) = 5$
$B = \{5, 6, 7, 8, 9\} \therefore n(B) = 5$	$B = \{5, 6, 7, 9\} \therefore n(B) = 4$
$C = \{2, 4, 6, 8\} \therefore n(C) = 4$	$C = \{2, 4, 6\} \therefore n(C) = 3$
$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ $\therefore n(A \cup B \cup C) = 9$	$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 9\}$ $\therefore n(A \cup B \cup C) = 8$
$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \therefore n(U) = 9$	$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \therefore n(U) = 10$

From the above table, what is the relation between $n(A \cup B \cup C)$ and $n(U)$ in the figure no.1 and figure no. 2? And why?

Activity 3

In a survey of students of a classroom, 40 students like orange, 35 like mango and 50 like banana. Among them, 15 like orange and mango, 20 like mango and banana, 25 like orange and banana, 5 like all the three fruits and 30 does not like either of the fruits. How shall the number of participants of the survey be found by using the Venn-diagram?

Here, O, M and B denote the set of students who like orange, mango and banana respectively.

At first, let us insert the number of students who likes all the three fruits $n(O \cap M \cap B) = 5$ and the number of students who does not like either of the fruits $n(\overline{O \cup M \cup B}) = 30$

After that, insert the number of students who likes exactly two fruits,

The number of students who likes orange and mango is $n(O \cap M) = 15$. Since 5 has already come in the number of students who likes all the three, insert the number of students who likes orange and mango only $n_0(O \cap M) = 15 - 5 = 10$. Again, since 5 has already come in the number of students who likes all the three, insert the number of students who likes mango and banana only, $n_0(M \cap B) = 20 - 5 = 15$ and the number of students who likes orange and banana only $n_0(O \cap B) = 25 - 5 = 20$.

Similarly,

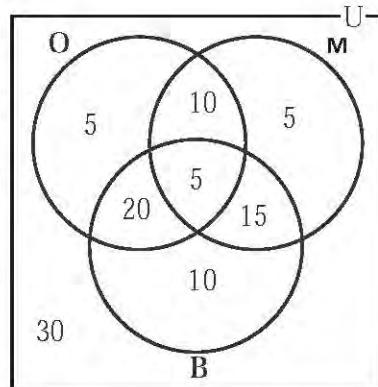
40 students like orange but 5 like all the three fruits; orange, mango and banana. Similarly, 10 like orange and mango only as well as 20 like orange and banana only. Thus, insert the number of students who likes orange only, $n_0(O) = 40 - (5 + 10 + 20) = 5$

35 students like mango but 5 like all the three fruits orange, mango and banana. Similarly, 10 likes orange and mango only as well as 15 like mango and banana only. Thus, insert the number of students who likes mango only, $n_0(M) = 35 - (5 + 10 + 15) = 5$

50 students likes banana but 5 likes all three fruits orange, mango and banana, 15 likes banana and mango only as well as 20 like orange and banana only. Thus, insert the number of students who like banana only, $n_0(B) = 50 - (5 + 15 + 20) = 10$.

Now, the total students $n(U) = 5 + 10 + 5 + 5 + 20 + 15 + 10 + 30 = 100$

\therefore The total number of participants is 100.

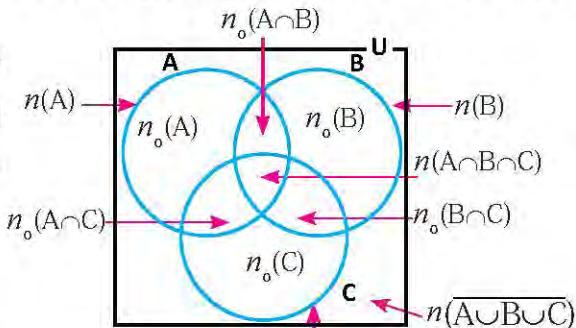


If A, B and C are overlapping sets, the following relations can be written from the Venn-diagram given alongside:

$$(a) n_o(A \cap B) = n(A \cap B) - n(A \cap B \cap C)$$

$$(b) n_o(A \cap C) = n(A \cap C) - n(A \cap B \cap C)$$

$$(c) n_o(B \cap C) = n(B \cap C) - n(A \cap B \cap C)$$



$$(d) n(A) = n_o(A) + n_o(A \cap B) + n_o(A \cap C) + n(A \cap B \cap C)$$

$$(e) n(B) = n_o(B) + n_o(A \cap B) + n_o(B \cap C) + n(A \cap B \cap C)$$

$$(f) n(C) = n_o(C) + n_o(A \cap C) + n_o(B \cap C) + n(A \cap B \cap C)$$

$$(g) n(U) = n_o(A) + n_o(B) + n_o(C) + n_o(A \cap B) + n_o(B \cap C) + n_o(A \cap C) + n(A \cap B \cap C) + n(\overline{A \cup B \cup C})$$

or, $n(U) = n(A \cup B \cup C) + n(\overline{A \cup B \cup C})$ where,

$$n(A \cup B \cup C) = n_o(A) + n_o(B) + n_o(C) + n_o(A \cap B) + n_o(B \cap C) + n_o(A \cap C) + n(A \cap B \cap C)$$

From the Venn-diagram with three sets, the following relations can also be written:

If A, B and C are overlapping sets, then,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\text{Thus, } n(A \cup B \cup C) = n\{(A \cup B) \cup C\}$$

$$= n(A \cup B) + n(C) - n\{(A \cup B) \cap C\}$$

$$= n(A) + n(B) - n(A \cap B) + n(C) - n\{(A \cap C) \cup (B \cap C)\}$$

$$[\because (A \cup B) \cap C = (A \cap C) \cup (B \cap C)]$$

$$= n(A) + n(B) - n(A \cap B) + n(C) - [n(A \cap C) + n(B \cap C) - n\{(A \cap C) \cap (B \cap C)\}]$$

$$= n(A) + n(B) - n(A \cap B) + n(C) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

$$[\because (A \cup C) \cap (B \cap C) = (A \cap B \cap C)]$$

$$= n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

$$\therefore n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

If the sets are disjoint sets then

$$n(A \cup B \cup C) = n(A) + n(B) + n(C)$$

Example 1

If $n(U) = 120$, $n(A) = 48$, $n(B) = 51$, $n(C) = 40$, $n(A \cap B) = 11$, $n(B \cap C) = 10$, $n(A \cap C) = 9$, and $n(A \cap B \cap C) = 4$ then find $n(A \cup B \cup C)$ and $n(\overline{A \cup B \cup C})$. Also show the information in the Venn-diagram.

Solution

Here, given that

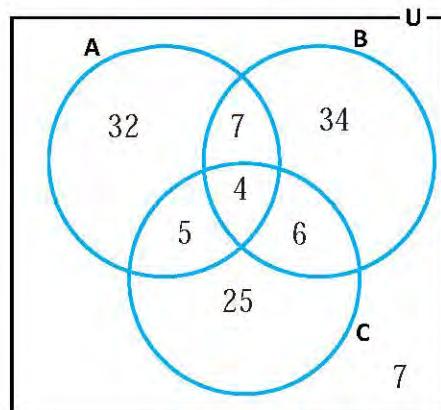
$$n(U) = 120, n(A) = 48, n(B) = 51, n(C) = 40, n(A \cap B) = 11, n(B \cap C) = 10, n(A \cap C) = 9, \text{ and } n(A \cap B \cap C) = 4$$

We know that

$$\begin{aligned} n(A \cup B \cup C) &= n(A) + n(B) + n(C) - n(A \cap B) \\ &\quad - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C) \\ &= 48 + 51 + 40 - 11 - 10 - 9 + 4 \\ &= 113 \end{aligned}$$

Again,

$$\begin{aligned} n(U) &= n(A \cup B \cup C) + n(\overline{A \cup B \cup C}) \\ \text{or, } 120 &= 113 + n(\overline{A \cup B \cup C}) \\ \text{or, } n(\overline{A \cup B \cup C}) &= 120 - 113 \\ \therefore n(\overline{A \cup B \cup C}) &= 7 \end{aligned}$$



The obtained information is shown in the Venn-diagram alongside.

Example 2

Among the 180 students who participated in SLC examination in 2071 from Nepal Madhyamik Vidhyaalaya, 86 passed in Science, 80 passed Maths and 76 passed in Nepali. Out of them, 26 passed in Science and Maths, 36 passed in Maths and Nepali as well as 32 passed in Science and Nepali but 20 did not pass all the subjects. Then,

- Show the given information in the Venn-diagram.
- Find the number of students who passed in all three subjects.

Solution

Here, $n(U)$, $n(M)$, $n(S)$ and $n(N)$ respectively denote the total number of students, the number of students who passed in Maths, the number of students who passed in Science and the number of students who passed in Nepali.

Here,

Total number of students $n(U) = 180$

The number of students who passed in Science, $n(S) = 86$

The number of students who passed in Maths, $n(M) = 80$

The number of students who passed in Nepali, $n(N) = 76$

The number of students who passed in Science and Maths, $n(S \cap M) = 26$

The number of students who passed in Maths and Nepali, $n(M \cap N) = 36$

The number of students who passed in Science and Nepali, $n(S \cap N) = 32$

The number of students who did not pass in any subject, $n(\overline{M \cup N \cup S}) = 20$

a) We know that,

$$n(U) = n(S) + n(M) + n(N) - n(S \cap M) - n(M \cap N) - n(S \cap N) + n(S \cap M \cap N) + n(\overline{S \cup M \cup N})$$

$$\text{or, } 180 = 86 + 80 + 76 - 26 - 36 - 32 + 20 + n(S \cap M \cap N)$$

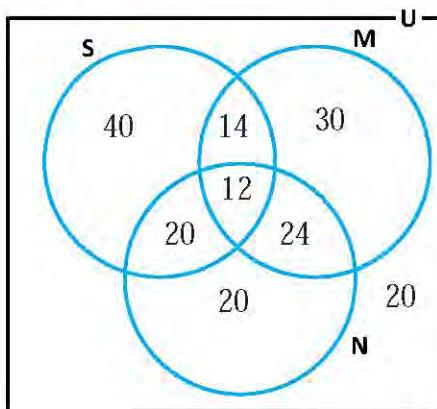
$$\text{or, } 180 = 168 + n(S \cap M \cap N)$$

$$\text{or, } n(S \cap M \cap N) = 180 - 168$$

$$\text{or, } n(S \cap M \cap N) = 12$$

Thus, the number of students who passed in all three subjects is 12.

b) Illustrating in the Venn-diagram,



Alternative Method

Suppose the number of students who passed in all three subjects $n(M \cap N \cap S) = x$

The information is shown in the Venn-diagram.

From venn-diagram,

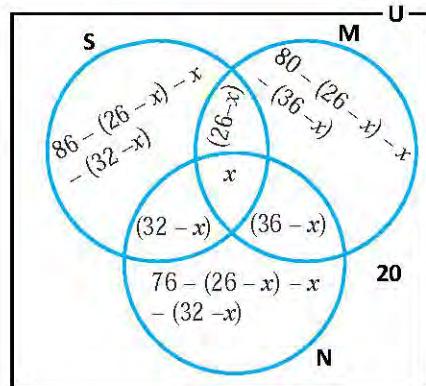
$$\{86 - (26 - x) - x - (32 - x)\} + (26 - x) + (32 - x) \\ + x + (36 - x) + \{80 - (26 - x) - x - (36 - x)\} + \\ \{76 - (36 - x) - x - (32 - x)\} + 20 = 180$$

$$\text{or, } (28 + x) + 94 - 2x + (18 + x) + (8 + x) + 20 \\ = 180$$

$$\text{or, } 168 + x = 180$$

$$\text{or, } x = 180 - 168$$

$$\therefore x = 12$$



Thus the number of students who passed in all three subjects is 12.

Example 3

A school distributed medals for the students in different events of a competition. 36 got medals in dance, 12 in drama and 18 in music. If only 45 students got medals and 4 students got medals in all three events, then find the number of students who got medals in exactly two events.

Solution

Here, $n(A)$, $n(B)$ and $n(C)$ denote the number of students who got medals in dance, drama and music respectively. Then, we have

The number of students who got medals in dance, $n(A) = 36$

The number of students who got medals in drama, $n(B) = 12$

The number of students who got medals in music, $n(C) = 18$

The number of students who got medals in at least one event, $n(A \cup B \cup C) = 45$

The number of students who got medals in all three events, $n(A \cap B \cap C) = 4$

The number of students who got medals in exactly two events

$$n_o(A \cap B) + n_o(B \cap C) + n_o(A \cap C) = ?$$

We know that,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$\text{or, } 45 = 36 + 12 + 18 - n(A \cap B) - n(B \cap C) - n(A \cap C) + 4$$

$$\text{or, } 45 = 70 - n(A \cap B) - n(B \cap C) - n(A \cap C)$$

$$\text{or, } n(A \cap B) + n(B \cap C) + n(A \cap C) = 70 - 45$$

$$\therefore n(A \cap B) + n(B \cap C) + n(A \cap C) = 25$$

Now,

$$\begin{aligned} n_o(A \cap B) + n_o(B \cap C) + n_o(A \cap C) \\ &= n(A \cap B) - n(A \cap B \cap C) + n(B \cap C) - n(A \cap B \cap C) + n(A \cap C) - n(A \cap B \cap C) \\ &= n(A \cap B) + n(B \cap C) + n(A \cap C) - 4 - 4 - 4 \\ &= 25 - 12 \\ &= 13 \end{aligned}$$

Alternatively,

$$\text{Let, } n_0(A \cap B) = a, n_0(B \cap C) = b, n_0(A \cap C) = c$$

We know that $a + b + c = ?$

The information is shown in the Venn-diagram alongside:

From the Venn-diagram, we have:

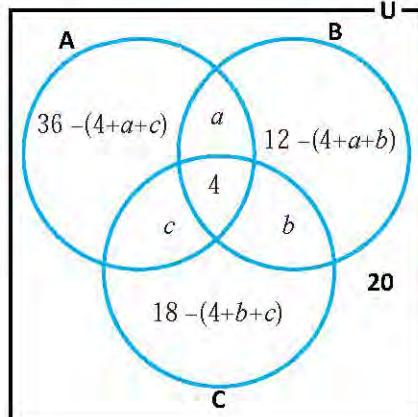
$$\{36 - (4 + a + c)\} + a + 4 + b + c + \{12 - (4 + a + b)\} + \{18 - (4 + b + c)\} = 45$$

$$\text{or, } (32 - a - c) + 4 + a + b + c + (8 - a - b) + (14 - b - c) = 45$$

$$\text{or, } 58 - a - b - c = 45$$

$$\text{or, } a + b + c = 58 - 45$$

$$\therefore a + b + c = 13$$



Hence, the number of students who got medals in only two events is 13.

Exercise 1.2

- 1.** In the given Venn-diagram, the elements of sets P, Q and R are illustrated. Based on this, find the values of the following sets.

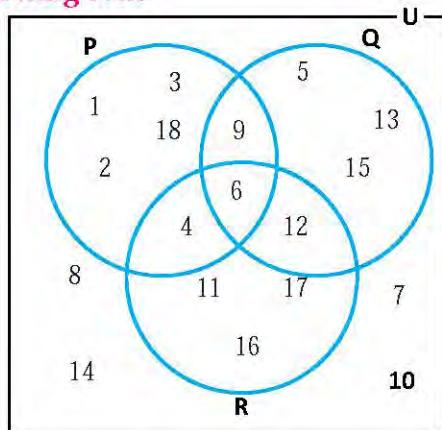
- (a) $n(P)$
- (b) $n(Q)$
- (c) $n(P \cup Q \cup R)$
- (d) $n_o(P)$
- (e) $n_o(R)$
- (f) $n(P \cap R)$
- (g) $n(\overline{P \cup Q \cup R})$
- (h) $n_o(P \cap Q)$
- (i) $n(P \cap Q \cap R)$

- 2.** If $U = \{\text{positive integers less than } 30\}$,

$P = \{\text{multiples of 2 less than } 30\}$,

$Q = \{\text{multiples of 3 less than } 30\}$ and

$R = \{\text{multiples of 5 less than } 30\}$ then



Show the relation between the sets P, Q and R in a Venn-diagram and verify the following relations:

- (a) $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$
 - (b) $n(P \cup Q \cup R) = n(P) + n(Q) + n(R) - n(P \cap Q) - n(Q \cap R) - n(R \cap P) + n(P \cap Q \cap R)$
 - (c) $n(P \cup Q \cup R) = n(P - Q) + n(Q - R) + n(R - P) + n(P \cap Q \cap R)$
- 3.**
- (a) If $n(U) = 100$, $n(M) = 45$, $n(E) = 50$, $n(S) = 35$, $n(M \cap E) = 20$, $n(E \cap S) = 20$, $n(S \cap M) = 15$ and $n(M \cap E \cap S) = 5$, then find $n(\overline{M \cup E \cup S})$
 - (b) If $n(U) = 105$, $n(A) = 40$, $n(B) = 35$, $n(C) = 30$, $n(A \cap B) = 15$, $n(B \cap C) = 12$, $n(A \cap B \cap C) = 6$ and $n(\overline{A \cup B \cup C}) = 30$, then find $n(A \cap C)$
 - (c) If $n(U) = 120$, $n(M) = 50$, $n(E) = 40$, $n(S) = 45$, $n(M \cap E) = 15$, $n(E \cap S) = 15$, $n(S \cap M) = 15$ and $n(\overline{M \cup E \cup S}) = 15$, then find $n(M \cap E \cap S)$
 - (d) If $n(A \cup B \cup C) = 105$, $n_o(A) = 25$, $n_o(B) = 25$, $n_o(C) = 15$, $n_o(A \cap B) = 15$, $n_o(A \cap C) = 10$ and $n(A \cap B \cap C) = 10$, then find $n_o(B \cap C)$
- 4.**
- a) Out of 90 students who participated in an examination, 43 passed in Science, 40 in Mathematics and 38 in Nepali. Among them, 13 passed in Science and Mathematics, 18 in Mathematics and Nepali as well as 16 passed in Science and Nepali. Using the information, answer the following questions:

- i) Show the information in the Venn-diagram.
- ii) Find the number of students who did not pass in any subject.

- b) In a survey of a group, 60 like tea, 45 like coffee, 30 like milk, 25 like the coffee and tea, 20 like milk and tea, 15 like coffee and milk and 10 like all three drinks. Based on the information, answer the following questions:
- Show the information in a Venn-diagram.
 - Find how many people were surveyed.
- c) In a survey among 60 students, 23 played volleyball, 15 played basketball and 20 played cricket. If 7 played volleyball and basketball, 5 played basketball and cricket, 4 played volleyball and cricket but 15 played neither of the games. Based on this information, answer the following questions:
- Show the information in Venn-diagram.
 - Find how many students played all the three games.
 - How many played only volleyball and cricket.
5. Out of total students who participated in an examination, 40% passed in Science, 45% in Mathematics and 50% in Nepali. Similarly, 10% passed in Science and Mathematics, 20% in Mathematics and Nepali as well as 15% in Science and Nepali but 5% failed in all the three subjects. Based on the information, answer the following questions:
- Find the percentage of students who passed in all the three subjects.
 - Find the percentage of students who passed in only one subject.
 - Find the percentage of students who passed in only two subjects.
 - Find the percentage of students who passed in at least one subject.
 - Show the information in a Venn-diagram
6. The following information was obtained from the survey on a questionnaire whether they read Yubamanch or Madhupark or Muna conducted among some people in a community:
- 30 read Yubamanch, 25 read Madhupark, 15 read both Yubamanch and Muna, 12 read both Yubamanch and Madhupark and 9 read Madhupark only, 11 read Muna only, 5 read Yubamanch and Madhupark only but 10 read neither of the newspapers. Based on this information, answer the following questions:
- Show the information in a Venn-diagram.
 - Find the total number of people who participated in the survey.

- c) Find the number of people who read exactly two newspapers.
d) Find the number of people who read Muna.
7. In a survey among 90 people, who were asked which language film they like, 48 like Nepali, 40 like English, 31 like Hindi, 24 like Nepali and English, 19 like Hindi and English, 6 like of all the three languages and 21 did not like any. Then,
- How many people did like both Nepali and Hindi films?
 - How many people did not like Hindi film?
 - How many people did not like both Nepali and Hindi films?

Project work

It is informed from school administration that your class has to decide the destination of educational excursion to be organized by your school. For this, form groups of all students containing 5 members each. Collect the answers from all students of different classes by asking the following questions:

Which place do you like to go for your educational excursion?

- (a) Pokhara (b) Lumbini (c) Kathmandu (d) Pokhara and Lumbini (e) Lumbini and Kathmandu (f) Pokhara and Kathmandu (g) Pokhara, Lumbini and Kathmandu (h) other places than these

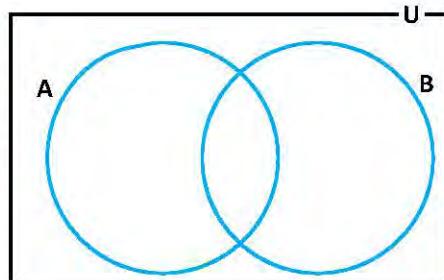
Find the number of students who like to visit only one place by illustrating the information obtained from the students answers in a Venn-diagram. Then, in turn, present the group work in the classroom.

Answers

- | | | | | | |
|----|----------------|----------------|-----------------|----------------|-------|
| 1. | (a) 7
(f) 2 | (b) 6
(g) 4 | (c) 14
(h) 1 | (d) 4
(i) 1 | (e) 3 |
| 3. | (a) 20 | (b) 9 | (c) 15 | (d) 5 | |
| 4. | (a) (ii) 10 | (b) (ii) 85 | (c) (ii) 3 | (iii) 1 | |
| 5. | (a) 5% | (b) 60% | (c) 30%, | (d) 95% | |
| 6. | (b) 64 | (c) 17 | (d) 30 | | |
| 7. | (a) 13 | (b) 50 | (c) 24 | | |

Mixed Exercise

1. There are two overlapping sets A and B shown alongside in a Venn-diagram where $n_o(A) = 16 + x$, $n_o(B) = 5x$, $n(A \cap B) = y$ and $n(\overline{A \cup B}) = x$. Then, answer the following questions:



- Insert the above information by drawing a Venn-diagram.
 - If $n(A) = n(B)$, find the value of $n(\overline{A \cup B})$.
 - If $n(U) = 50$, find the ratio of $n(A \cap B)$ and $n(\overline{A \cup B})$.
2. A and B are the subsets of Universal set U such that $n(U) = 100$, $n(A - B) = 32 + x$, $n(B - A) = 5x$, $n(A \cap B) = x$ & $n(\overline{A \cup B}) = y$.
- Show the above information in a Venn-diagram.
 - If $n(A) = n(B)$, find the value of $n(A \cap B)$.
 - Find the value of $n(\overline{A \cup B})$.
- By what percent $n(A \cap B)$ is more or less than $(\overline{A \cup B})$? Find.
3. According to a survey of 93 women of a community, the number of women engaged in agriculture is 80 and that in sewing is 71 but the number of women engaged in other job is 10.
- Present the information in Venn-diagram by finding the cardinality of sets.
 - Find how many women were engaged in both agriculture and sewing.
 - By how many times the number of women engaged in agriculture only is more than the number of women engaged in sewing only? Calculate it.
4. According to a survey of 1000 farmers in a community, the number of farmers cultivating potatoes was 800 and the number of farmers cultivating tomatoes was 500 but 50 crops other than these.
- Show the information in Venn-diagram by finding the cardinality of sets.
 - Find the number of farmers who cultivate both.
 - Write the number of farmers cultivate potato only and that of tomato only in ratio.

5. In a survey of 400 people of a community, it was found that the ratio of the people having motorcycle license only and car license only was 5:3. Among them, one -fourth of the people had license of vehicles but 60 did not have any license.

- Show the above information in a Venn-diagram.
- From the above information, how many people had license of each vehicle?
- Find the number of people who had license of motorcycle.

6. The information of the students of a school whether they like volleyball, football or cricket is as follows:

- 30 like volleyball and football, 20 like volleyball and cricket as well as 35 like football and cricket.
 - 10 like all the three games; football, volleyball and cricket but 5 like neither of the games.
- Represent the given information in cardinality of sets.
 - Show the information in Venn-diagram.
 - Find the total number of students in the school.
 - What percentage of students like football only?

7. The following information from a survey of 45 people of different lingual group of a community is obtained:

25 speak Nepal Bhasa, 23 speak Tamang and 15 speaks Maithili.

12 speaks Nepal Bhasa and Tamang, 5 speak Nepal Bhasa and Maithili as well as 10 speak Tamang and Maithili. 4 speak all the three languages.

Based on the information, answer the following questions:

- Show the above information in a Venn-diagram.
- Find how many people speak the language other than these languages; Nepal Bhasa, Tamang and Maithili.
- How many people speak only one language? Find.
- How many people speak both Nepal Bhasa and Tamang but do not speak the Maithili language?

Answars

1. (b) 4 (c) 3:2

2. (b) 8 (c) 12 (d) more than 50%

3. (b) 68 (c) 4

4. (b) 350 (c) 3:1

5. (b) 250 and 190 (c) 250

6. (c) 230 (d) 19.57%

7. (b) 5 (c) 21 (d) 8

2.0 Review

Discuss the following questions and present the conclusion in the classroom:

A person borrowed Rs. 10,00,000 for a business from a commercial bank.

- How much interest should he pay to the bank after 2 years at the rate of 8% per annum?
- What total sum of money should he pay after 2 years? What is the sum of money called?
- If he had borrowed the money for 5 years, how much interest should be paid at the same interest rate?

The conclusion drawn from the discussion is presented below:

The sum of money paid to the bank at the rate of 8% p.a. after 2 years is called simple interest.

The total sum of borrowed money and additional interest is called amount.

The following formula is used for the calculation of simple interest: Simple interest (SI) = $\frac{P \times T \times R}{100}$

2.1. Introduction of Compound Interest

2.1.1 Compound Interest Compounded Annually

Activity 1

A teacher borrowed Rs. 5,00,000 for 2 years at the interest rate of 12% per annum from a commercial bank. He could not pay the interest at the end of the first year so that he had to pay interest of interest of the first year in the second year. In this situation, discuss the following questions and present the conclusion in the classroom:

- How much interest to be paid in the first year?
- What is the principal of the second year? Find it.
- How much interest of second year should be paid?

The conclusion from the discussion is mentioned below:

$$\begin{aligned} \text{The interest to be paid in the first year (I}_1\text{)} &= \frac{P \times T \times R}{100} = \frac{5,00,000 \times 1 \times 12}{100} \\ &= \text{Rs. } 60,000 \end{aligned}$$

Since he could not pay the interest in the first year,

The principal for the second year (P_1) = Rs. 5,00,000 + Rs. 60,000 = Rs. 5,60,000

$$\text{Thus, the interest for the second year } (I_2) = \frac{P_1 \times R}{100} = \frac{5,60,000 \times 1 \times 12}{100}$$
$$= \text{Rs. } 67,200$$

Hence, the total interest to be paid by the teacher = Rs. 60,000 + Rs. 67,200 = Rs. 1,27,200

Why is there difference between the interests for the first year and for the second year?

If the interest of a principal after every year or after certain period (yearly, half yearly or terminal) is calculated and added to the principal and again the interest is calculated, then the interest so obtained is called compound interest. The sum of the principal and compound interest is called compound amount.

Activity 2

Bishal and Badri borrowed Rs. 30,000 each from a bank for 3 years at the rate of 10% yearly with the promise that Bishal should pay simple interest and Badri should pay compound interest. Discuss the following questions based on the information:

- How much total interest should Bishal pay?
- How much total interest should Badri pay?
- Who should pay more interest?

Should be paid by Bishal (SI)	Should be paid by Badri (CI)
<p>For the first year, Principal (P_1) = Rs. 30,000 Rate of interest (R) = 10%, Time (T_1) = 1 year $\text{Interest } (I_1) = \frac{30,000 \times 10 \times 1}{100}$ = Rs. 3,000</p>	<p>For the first year, Principal (P_1) = Rs. 30,000 Rate of interest (R) = 10%, Time (T_1) = 1 year $\text{Interest } (I_1) = (I_1) = \frac{30,000 \times 10 \times 1}{100}$ = Rs. 3,000</p>
<p>For the second year, Principal (P_2) = Rs. 30,000 Rate of interest (R) = 10%, Time (T_2) = 1 year $\text{Interest } (I_2) = \frac{30,000 \times 10 \times 1}{100}$ = Rs. 3,000</p>	<p>For the second year, Principal (P_2) = Rs. 30,000 + Rs. 3,000 = Rs. 33,000 Rate of interest (R) = 10%, Time (T_2) = 1 year $\text{Interest } (I_2) = \frac{33,000 \times 10 \times 1}{100}$ = Rs. 33,00</p>

<p>For the third year,</p> <p>Principal (P_3) = Rs. 30,000</p> <p>Rate of interest (R) = 10%, Time (T_3) = = 1 year</p> <p>Interest (I_3) = $\frac{30,000 \times 10 \times 1}{100}$ = Rs. 3,000</p> <p>Total interest (I) = $I_1 + I_2 + I_3$ = $3000 + 3000 + 3000$ = Rs. 9,000</p>	<p>For the third year,</p> <p>Principal (P_3) = Rs. 33,000 + Rs. 3,300 = Rs. 36,300</p> <p>Rate of interest (R) = 10%, Time (T_3) = 1 year</p> <p>Interest (I_3) = $\frac{36,300 \times 10 \times 1}{100}$ = Rs. 3,330</p> <p>v (I) = $I_1 + I_2 + I_3$ = $3000 + 3300 + 3630$ = Rs. 9,930</p>
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Yeh! The compound interest is more than simple interest even at the same rate of interest. In compound interest, the principal of every year is equal to the sum of the interest and the principal of the preceding year. On which, the interest of interest is imposed.



While calculating simple interest, the principal is same for each year. But while calculating compound interest, the principals are different every year (the principal for the second year is the amount of the first year, the principal for the third year is the amount of second year, etc.). The compound interest of the same principal is more at the same rate and time than the simple interest of the same principal.

Activity 3

If a principal (P) is deposited at the rate of R% per annum for T years, then discuss the following questions:

- How much interest is at the end of the first year?
- How much interest is at the end of the second year?
- In this way, how much interest is at the end of the third year?
- How much interest is at the end of T years?

$$\text{The interest at the end of the first year } (I_1) = \frac{P \times T \times R}{100} = \frac{P \times 1 \times R}{100} = \frac{PR}{100}$$

$$\text{The amount at the end of the first year } (A_1) = P_1 + I_1 = P + \frac{PR}{100} = P \left(1 + \frac{R}{100}\right)$$

We know that,

The amount at the end of the first year = Principal for the second year

Thus, the principal of the second year (P_2) = $P \left(1 + \frac{R}{100}\right)$

$$\begin{aligned} \text{The interest of the second year } (I_2) &= \frac{P \left(1 + \frac{R}{100}\right) \times T \times R}{100} \\ &= \frac{P \left(1 + \frac{R}{100}\right) \times 1 \times R}{100} = P \left(1 + \frac{R}{100}\right) \times \frac{R}{100} \\ \text{The amount at the end of the second year } (A_2) &= P \left(1 + \frac{R}{100}\right) + P \left(1 + \frac{R}{100}\right) \times \frac{R}{100} \\ &= P \left(1 + \frac{R}{100}\right) \left(1 + \frac{R}{100}\right) \\ &= P \left(1 + \frac{R}{100}\right)^2 \end{aligned}$$

In this way, the amount at the end of second year = principal for the third year

$$\text{Thus, principal for the third year } (P_3) = P \left(1 + \frac{R}{100}\right)^2$$

$$\text{Interest of the third year } (I_3) = \frac{P_3 \times T \times R}{100} = \frac{P \left(1 + \frac{R}{100}\right)^2 \times 1 \times R}{100}$$

$$= P \left(1 + \frac{R}{100}\right)^2 \times \frac{R}{100}$$

$$\text{The amount at the end of the third year } (A_3) = P_3 + I_3$$

$$= P \left(1 + \frac{R}{100}\right)^2 + \left(1 + \frac{R}{100}\right)^2 \times \frac{R}{100}$$

$$= P \left(1 + \frac{R}{100}\right)^2 \left(1 + \frac{R}{100}\right) = P \left(1 + \frac{R}{100}\right)^3$$

From the above calculation, what will be the compound amount (CA) at the end of T years?

$$\text{Hence, the compound amount at the end of T years } (CA) = P \left(1 + \frac{R}{100}\right)^T$$

Similarly, compound interest (CI) = Amount (CA) – Principal (P)

$$CI = P \left(1 + \frac{R}{100}\right)^T - P, \quad CI = P \left[\left(1 + \frac{R}{100}\right)^T - 1 \right]$$

Activity 4

How shall we calculate compound interest and compound amount of the following conditions?

- a) The rate of interest is different every year.
- b) Time in T years and M months.
- c) If the interest is to be calculated half yearly.
- d) If the interest is to be calculated terminally.

(a) The rate of interest is different every year

How can we calculate compound interest and compound amount in 3 years such that the rate of interest for the first year $R_1\%$, the second year $R_2\%$ and the third year $R_3\%$ respectively?

$$\text{Compound Amount (CA)} = P \left(1 + \frac{R_1}{100}\right) \left(1 + \frac{R_2}{100}\right) \left(1 + \frac{R_3}{100}\right) \text{ and,}$$

$$\text{Compound Interest (CI)} = P \left[\left(1 + \frac{R_1}{100}\right) \left(1 + \frac{R_2}{100}\right) \left(1 + \frac{R_3}{100}\right) - 1 \right]$$

(b) If Time in 'T' years and 'M' months is given, how shall compound interest and compound amount be computed?

$$\text{Compound Amount (CA)} = P \left(1 + \frac{R}{100}\right)^T \left(1 + \frac{MR}{1200}\right)$$

$$\text{Compound Interest (CI)} = P \left[\left(1 + \frac{R}{100}\right)^T \left(1 + \frac{MR}{1200}\right) - 1 \right]$$

(c) If the interest is to be calculated half yearly

If the interest is to be calculated half yearly, the rate of interest $R\%$ per annum being calculated as $\frac{R}{2}\%$ per semi-annual and Time (T) being calculated as $2 \times T$.

$$\text{Now, compound amount (CA)} \text{ is } P \left(1 + \frac{R}{2 \times 100}\right)^{2T} = P \left(1 + \frac{R}{200}\right)^{2T}$$

$$\text{And compound interest CI is } P \left(1 + \frac{R}{2 \times 100}\right)^{2T} - P = P \left[\left(1 + \frac{R}{200}\right)^{2T} - 1 \right].$$

A financial institution releases the interest at the rate of $R\%$ yearly in the account of depositors two times, the first of Magh and the first of Shrawan every year. In this way, the interest of following 6 months is computed at the principal as the sum of the interest of the previous 6 months and the principal. This kind of interest is called half yearly compound interest.

What is the difference between yearly compound interest and half yearly compound interest? Discuss.

(d) If the interest is to be calculated terminally

If the principal and interest in a bank is to be computed terminally, then rate of interest $R\%$ is termed as $\frac{R}{4}\%$ per quarter and time (T) is termed as $4T$ per quarter. In this case,

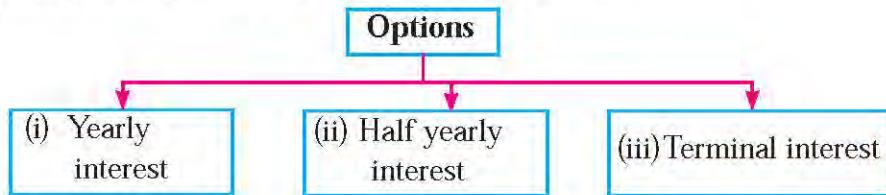
$$\text{Compound amount (CA)} = P \left(1 + \frac{R}{4 \times 100}\right)^{4T} = P \left(1 + \frac{R}{400}\right)^{4T}$$

$$\text{And compound interest (CI)} = P \left(1 + \frac{R}{4 \times 100}\right)^{4T} - P = P \left[\left(1 + \frac{R}{400}\right)^{4T} - 1 \right]$$

Terminal compound interest is also computed as the yearly compound interest and half yearly compound interest.

Activity 5

Your uncle has planned to invest Rs. 1,00,000 for a year at the rate of 15% compound interest per annum. He has been thinking about whether to invest it in yearly interest, half yearly interest or terminal interest to get more benefit. Which alternative would you suggest him to choose? Why? Discuss.



Here, principal (P) = Rs. 1,00,000, the rate of interest (R) = 15%, time (T) = 1 year

a) According to the first option,

$$\begin{aligned}\text{Compound interest (CI)} &= 1,00,000 \left[\left(1 + \frac{15}{100}\right)^1 - 1 \right] \\ &= 1,00,000 \left[\left(\frac{115}{100}\right) - 1 \right] = \text{Rs. } 15,000\end{aligned}$$

b) According to the second option,

$$\text{Compound interest (CI)} = 1,00,000 \left[\left(1 + \frac{15}{200}\right)^{2 \times 1} - 1 \right] = \text{Rs. } 15,562.50$$

c) According to the third option,

$$\text{Compound interest (CI)} = 1,00,000 \left[\left(1 + \frac{15}{400}\right)^{4 \times 1} - 1 \right] = \text{Rs. } 15,865.04$$

Out of these three options, I would suggest him to invest according to the third option because the interest of this option is more than that of other two options. Accordingly, the interest is Rs. 865.04 more than the first option and Rs. 562.50 more than the second option.

While calculating the interest of the same sum at the same rate of interest at the same time, then terminal compound interest is more than half yearly compound interest and half yearly compound interest is greater than yearly compound interest.

Example 1

What will be the compound interest and compound amount of Rs. 2,000 at the interest rate of 12% p.a. in 2 years? Find the compound interest without using formula.

Solution

Here,

Principal (P_1) = Rs. 2,000

Rate of interest (R) = 12% p.a.

Time (T) = 2 years

At the end of the first year, simple interest (I_1) = $\frac{P_1 T R}{100} = \frac{2000 \times 1 \times 12}{100}$ = Rs. 240

Principal for the second year (P_2) = Amount at the end of the first year
= $P_1 + I_1$ = Rs. $(2000 + 240)$

= Rs. 2,240

Again, simple interest for the second year (I_2) = $\frac{2240 \times 1 \times 12}{100}$ = Rs. 268.8

Thus, the compound interest at the end of 2 years (CI) = $I_1 + I_2$ = $240 + 268.8$ = Rs. 508.8

Compound amount (CA) = $P_1 + CI$ = $2000 + 508.8$ = Rs. 2,508.8

Example 2

Find the compound interest and compound amount of the borrowed amount of Rs. 25,000 which is paid in exactly 3 years at the rate of yearly compound interest rate 12%.

Solution

Principal (P) = Rs. 25,000

Rate of interest (R) = 12% per year

Time (T) = 3 year

Compound interest and compound amount = ?

According to the formula,

$$\begin{aligned}\text{Compound interest (CI)} &= \left[\left(1 + \frac{R}{100} \right)^T - 1 \right] \\ &= 25,000 \left[\left(1 + \frac{12}{100} \right)^3 - 1 \right] \\ &= 25,000 \left[\left(\frac{112}{100} \right)^3 - 1 \right] \\ &= 25,000 [1.404928 - 1] \\ &= 25,000 \times 0.404928 \\ &= 10123.20\end{aligned}$$

$$\text{Again, compound amount (CA)} = P + CI$$

$$= \text{Rs. } 25,000 + \text{Rs. } 10,123.20 = \text{Rs. } 35,123.20$$

Example 3

A man borrowed Rs. 32,000 from his friend at the rate of simple interest of 12.5% per annum. He lent the whole sum to a shopkeeper at the same rate of compound interest. How much more money will he get in 3 years? Find.

Solution

Here,

$$\text{Principal (P)} = \text{Rs. } 32,000$$

$$\text{Rate of interest (R)} = 12.5\%$$

$$\text{Time (T)} = 3 \text{ year}$$

$$\text{Simple interest (SI)} = ?$$

$$\text{Compound interest (CI)} = ?$$

$$\text{Case I : simple interest (SI)} = \frac{P T R}{100} = \frac{32000 \times 3 \times 12.5}{100} = \text{Rs. } 12,000$$

$$\begin{aligned}\text{Case II : compound interest (CI)} &= P \left[\left(1 + \frac{R}{100} \right)^T - 1 \right] \\ &= 32,000 \left[\left(1 + \frac{12.5}{100} \right)^3 - 1 \right]\end{aligned}$$

$$\begin{aligned}
 &= 32,000 \left[\left(\frac{112.5}{100} \right)^3 - 1 \right] \\
 &= 32,000 [1.423828125 - 1] \\
 &= 32,000 \times 0.423828125 \\
 &= \text{Rs. } 13562.50
 \end{aligned}$$

\therefore The more money received by the man = CI - SI = 13562.50 - 12000 = Rs. 1562.50

Example 4

Sameer decided to invest Rs. 5,000 at the rate of 8% per annum for 2 years. For this, he has two safe alternative. The first alternative is to get half yearly compound interest and the second alternative is to get yearly compound interest. If you were to suggest him, which alternative would you suggest? Write with reason.

Solution

Here,

Principal (P) = Rs. 5,000

Rate of interest (R) = 8%

Time (T) = 2 year

a) According to the first alternative,

$$\begin{aligned}
 \text{Half yearly compound interest (CI}_1\text{)} &= P \left[\left(1 + \frac{R}{200} \right)^{2T} - 1 \right] \\
 &= 5,000 \left[\left(1 + \frac{8}{200} \right)^{2 \times 2} - 1 \right] \\
 &= 5,000 \left[\left(\frac{208}{200} \right)^{2 \times 2} - 1 \right] \\
 &= 5,000 [1.16985856 - 1] \\
 &= 5,000 \times 0.16985856 \\
 &= \text{Rs. } 849.29
 \end{aligned}$$

- b) According to the second alternative,

$$\begin{aligned}
 \text{Yearly compound interest } (\text{CI}_2) &= P \left[\left(1 + \frac{R}{100} \right)^T - 1 \right] \\
 &= 5000 \left[\left(1 + \frac{8}{100} \right)^2 - 1 \right] \\
 &= 5000 \left[\left(\frac{108}{100} \right)^2 - 1 \right] \\
 &= 5000 [(1.08)^2 - 1] \\
 &= 5000 (1.1664 - 1) \\
 &= 5000 \times 0.1664 \\
 &= \text{Rs. } 832
 \end{aligned}$$

The difference between half yearly compound interest and yearly compound interest is given by $\text{CI}_1 - \text{CI}_2 = \text{Rs. } 849.29 - \text{Rs. } 832 = \text{Rs. } 17.29$

\therefore Since half yearly compound interest is Rs. 17.29 more than yearly compound interest, I would suggest him to invest in first alternative.

Example 5

A twelve-grade student invest Rs. 10,000 for 2 years at the rate of yearly compound interest. If the compound interest in 1 year is Rs. 11,200

(a) Find the rate of yearly compound interest.

(b) Find the yearly compound amount at the end of the second year.

Solution

Here, (a) Principal (P) = Rs. 10,000

Compound amount at the end of the first year (CA) = Rs. 11,200

Time (T_1) = 1 year

$$\begin{aligned}
 \text{Using the formula, } (\text{CA}) &= P \left(1 + \frac{R}{100} \right)^T \\
 \text{or, } 11200 &= 10000 \left(1 + \frac{R}{100} \right)^1 \\
 \text{or, } \frac{11200}{10000} &= 1 + \frac{R}{100} \\
 \text{or, } 1.12 &= 1 + \frac{R}{100} \\
 \text{or, } 0.12 \times 100 &= R \\
 \text{or, } R &= 12\%
 \end{aligned}$$

Thus, the rate of yearly compound interest is 12%.

(b) The compound amount at the end of the second year (CA) = ?

Time (T) = 2 year

$$\begin{aligned}\text{Compound amount (CA)} &= P \left(1 + \frac{R}{100}\right)^T \\ &= 10,000 \left(1 + \frac{12}{100}\right)^2 \\ &= 10,000 \left(\frac{112}{100}\right)^2 \\ &= \text{Rs. } 12,544\end{aligned}$$

Thus, the compound amount at the end of the second year = Rs. 12,544

Example 6

Find the compound interest and compound amount of Rs. 2,00,000 invested for 3 years such that the rate of interest for the first year is 8% p.a., for the second it is 10% p.a. and for the third year it is 12%.

Solution

Here,

Principal (P) = Rs. 2,00,000

Time (T) = 3 year

Rate of interest for the first year (R_1) = 8%

Rate of interest for the second year (R_2) = 10%

Rate of interest for the third year (R_3) = 12%

Compound amount (CA) = ?

Compound interest (CI) = ?

According to the formula, compound amount

$$\begin{aligned}(\text{CA.}) &= P \left(1 + \frac{R_1}{100}\right) \left(1 + \frac{R_2}{100}\right) \left(1 + \frac{R_3}{100}\right) \\ &= 2,00,000 \left(1 + \frac{8}{100}\right) \left(1 + \frac{10}{100}\right) \left(1 + \frac{12}{100}\right) \\ &= 2,00,000 \left(\frac{108}{100}\right) \left(\frac{110}{100}\right) \left(\frac{112}{100}\right) \\ &= \text{Rs. } 2,66,112\end{aligned}$$

Thus, compound interest (CI) = CA - P = Rs. 2,66,1112 - Rs. 2,00,000 = Rs. 66,112

Example 7

A sum amounts to Rs. 14,520 in 2 years and Rs. 15,972 in 3 years at a certain rate of annual compound interest. Then,

- (a) Find the rate of compound interest.
 - (b) Find what is the principal.

Solution

Here,

Rate of interest (R) = R% and Principal (P) = Rs. x

Case I:

Compound amount (CA_p) = Rs. 14,520

Time (T) = 2 years

Case II:

Compound amount (CA_2) = Rs. 15,972

Time (T) = 3 years

Dividing equation (ii) by (i), we get

$$\text{or, } \frac{15,972}{14,520} = \frac{x \left(1 + \frac{R}{100}\right)^3}{x \left(1 + \frac{R}{100}\right)^2}$$

$$\text{or, } 1.10 = 1 + \frac{R}{100}$$

$$\text{or, } 1.10 - 1 = \frac{R}{100}$$

$$\text{or, } 0.10 \times 100 = R$$

or, $R = 10\%$

∴ Rate of interest (R) = 10% p.a.

Again, putting $(R) = 10\%$ in equation (i), we get

$$14520 = x \left(1 + \frac{R}{100}\right)^2$$

$$\text{or, } 14520 = x \left(1 + \frac{10}{100}\right)^2$$

$$\text{or, } 14520 = x \times 1.21$$

$$\text{or, } \frac{14520}{121} = x$$

$$x = 12000$$

∴ Principal (P) = x = Rs. 12,000

Alternatively,

Compound amount of 2 years (CA₁) = Rs. 14,520

$$\text{or, } P \left(1 + \frac{R}{100}\right)^2 = \text{Rs. } 14,520$$

Again, compound amount of 3 years (CA_2) = Rs. 15,972

$$\text{or, } P \left(1 + \frac{R}{100}\right)^3 = \text{Rs. } 15,972$$

From the equation (i) and (ii)

$$\text{or, } x \left(1 + \frac{R}{100}\right)^2 \left(1 + \frac{R}{100}\right) = 15,972$$

$$14,520 \left(1 + \frac{R}{100}\right) = 15,972 \text{ [from the equation (i)]}$$

$$\text{or, } \left(1 + \frac{R}{100}\right) = \frac{15,972}{14,520}$$

$$\text{or, } 1 + \frac{R}{100} = 1.10$$

$$\text{or, } \frac{R}{100} = 1.10 - 1$$

$$\text{or, } R = 0.10 \times 100$$

$$\therefore R = 10\% \text{ p.a.}$$

Now, putting $R = 10\%$ in the equation (i), we get

$$x \left(1 + \frac{10}{100}\right)^2 = \text{Rs. } 14,520$$

$$\text{or, } x \left(\frac{110}{100}\right)^2 = 14,520$$

$$\text{or, } x \times 1.21 = 14,520$$

$$\text{or, } x = \frac{14,520}{1.21}$$

\therefore Principal (P) = Rs. 12,000 and rate of interest (R) = 10% p.a.

Example 8

A person deposited Rs. 2,00,000 in a development bank for 2 years to get the half yearly compound interest at the rate of 10% per annum after deducting the 5% tax on the interest. But right after a year, bank has changed the policy and decided to accomplish the interest terminally at the same rate of interest.

- Find the interest of the first year by deducting the tax.
- What would be the interest of the second year after deducting the tax?
- What is the difference between interests of the first year and second year after deducting the tax? Find.
- After deducting the tax, by what percentage the interest of the first year differ from the interest of the second year?

Solution

Here, principal (P) = ₹. 2,00,000

Rate of interest (R) = 10% p.a.

- For the first year, the half yearly compound interest

$$\begin{aligned} CI_1 &= P \left[\left(1 + \frac{R}{200}\right)^{2T} - 1 \right] \\ &= 200000 \left[\left(1 + \frac{10}{200}\right)^{2 \times T} - 1 \right] \\ &= 2,00,000 \left[\left(\frac{210}{200}\right)^{2 \times 1} - 1 \right] \end{aligned}$$

$$= 2,00,000 [1.1025 - 1] \\ = 2,00,000 \times 0.1025 = \text{Rs. } 20,500$$

After deducting 5% tax, $\text{CI}_1 = \text{Rs. } 20,500 - \text{Rs. } 20,500 \times \frac{5}{100}$
 $= \text{Rs. } 20500 - \text{Rs. } 1025 = \text{Rs. } 19,475$

\therefore Interest of the first year after deducting tax is $\text{Rs. } 19,475$

(b) Compound amount after a year (CA) = $\text{Rs. } 2,00,000 + \text{Rs. } 19,475 = \text{Rs. } 2,19,475$

Now, principal for the second year = compound amount of the first year = $\text{Rs. } 2,19,475$

According to the quarterly compound interest, $(\text{CI}_2) = P \left[\left(1 + \frac{R}{400} \right)^{4T} - 1 \right]$

$$= 2,19,475 \left[\left(1 + \frac{10}{400} \right)^{4 \times 1} - 1 \right] \\ = 2,19,475 [0.1038128906] \\ = 22,784.33 \\ = 22,784.33$$

Again, the interest after deducting 5% tax $\text{CI}_2 = 22,784.33 - 22,784.33 \times \frac{5}{100}$
 $= 22,784.33 - 1,139.21$
 $= \text{Rs. } 21,645.12$

\therefore The interest after deducting 5% tax is $\text{Rs. } 21,645.12$

(c) The difference in interests $= \text{CI}_2 - \text{CI}_1 = 21,645.12 - 19,475$
 $= \text{Rs. } 2170.12$

(d) The difference of interests in percentage $= \frac{\text{CI}_2 - \text{CI}_1}{\text{CI}_1} \times 100\%$
 $= \frac{2170.12}{19475} \times 100\% = 11.14\%$

Thus, the interest of the second year differs by 11.14% than that of the first year.

Example 9

A commercial bank releases a loan of Rs. 52,500 to Babulal and Jibanlal at the rate of yearly 10% compound interest. If the compound amount paid by Babulal in 2 years is the same as the compound amount paid by Jibanlal in 3 years, how much loan did each of them borrow from the bank?

Solution

Let, the loan amount of Babulal (P_1) = Rs. x

The loan amount of Jibanlal (P_2) = Rs. $(52,500 - x)$

Here, the compound amount to be paid by Babulal in 2 years

$$\begin{aligned} CA_1 &= P_1 \left(1 + \frac{R}{100}\right)^T \\ &= x \left(1 + \frac{10}{100}\right)^2 \\ &= x \left(\frac{110}{100}\right)^2 \\ &= 1.21x \end{aligned}$$

Similarly, the compound amount to be paid by Jibanlal in 3 years

$$\begin{aligned} CA_2 &= P_2 \left(1 + \frac{R}{100}\right)^T \\ &= (52,500 - x) \left(1 + \frac{10}{100}\right)^3 \\ &= (52,500 - x) \left(\frac{110}{100}\right)^3 \\ &= (52,500 - x) 1.331 \\ &= 69,877.5 - 1.331x \end{aligned}$$

Now, according to the question,

$$CA_1 = CA_2$$

$$1.21x = 69,877.5 - 1.331x$$

$$\text{or, } 1.21x + 1.331x = 69,877.5$$

$$\text{or, } 2.541x = 69,877.5$$

$$\therefore x = 27,500$$

Thus, Babulal borrowed Rs. 27,500 and Jibanlal borrowed Rs. $(52,500 - 27,500)$ = Rs. 25000.

Exercise 2.1

1. Define:

- (a) Yearly compound interest
- (b) Half yearly compound interest
- (c) Quarterly compound interest

2. (a) According to the yearly compound interest, if principal is P, yearly rate of interest is R and time is T years, write the formula to compute compound amount.

(b) The compound interest CI of a sum P in T years at the rate of yearly compound interest is R%, write the relation between P, T, R and CI.

(c) Employee Provident Fund changes the rate of interest per annum according to the economic liquidity of the state. As per the given condition the compound amount of a sum P at the interest rate of $R_1\%$, $R_2\%$ and $R_3\%$ for the first, second and third years respectively is CA, then write the formula to find CA.

3. Without using formula, find the compound interest and compound amount for the following conditions:

- (a) Principal (P) = Rs. 10,000, Time (T) = 2 years and Rate of interest (R) = 6% p.a.
- (b) Principal (P) = Rs. 64,000, Time (T) = 3 years and Rate of interest (R) = 6% p.a.
- (c) Principal (P) = Rs. 20,000, Time (T) = 2 years, Rate of interest for the first year (R_1) = 10% p.a. and the rate of interest for the second year (R_2) = 12% p.a.

4. (a) At what rate of compound interest per year will the compound interest of Rs. 100 in a year be Rs. 12? Write.

(b) At what rate of compound interest per year will the compound interest of Rs. 200 in 2 years be Rs. 42? Find it.

5. (a) A farmer borrowed Rs. 20,000 from a co-operative to invest in poultry farm for 3 years at the rate of compound interest of 15% per year. Find the compound interest and compound amount of 3 years.

(b) A teacher deposited Rs. 50,000 in a bank at the account of his daughter. If the bank provides yearly 10% interest, what will be the compound interest and compound amount in 3 years? Find.

(c) Sabita deposited Rs. 1,50,000 in a bank. If the bank provides yearly 6% interest after 2 years 6 months,

(i) how much is the compound amount?

(ii) how much is the compound interest?

6. (a) Manisha deposited Rs. 50,000 in a bank at the rate of compound interest 8% p.a. If the bank provides half yearly compound interest, then find the compound interest and compound amount she receives after 2 years.
- (b) A bank provides quarterly compound interest. If Sunil deposited Rs. 50,00,000 for 1 year at the rate of 12% p.a. interest, then find the compound interest and compound amount.
7. (a) Karma Gurung deposited Rs. 80,000 in a bank at the rate of 8% compound interest 8% p.a. Find the difference between simple interest and compound interest of the sum in 2 years.
- (b) Find the difference between simple interest and yearly compound interest of a sum Rs. 7,500 at the rate of 12% p.a. interest in 3 years.
8. (a) Chhiring deposited Rs. 40,000 at the rate of 6% annual compound interest. Find the difference of yearly compound interest and half yearly compound interest of the sum in 2 years.
- (b) What is the difference between semi-annual compound interest and quarterly compound interest of Rs. 18,000 in a year at the rate of 12% compound interest p. a.? Find it.
- (c) You went to deposit Rs. 60,000 in a bank for 2 years. The information in the notice board of the bank is given below:

Notice for the interest of deposit collection

It is notified that two types of fixed deposit accounts are going to be effective from 2079/01/01. Grab the opportunity that of fixed deposit in time.

Fixed deposit (P)	Fixed deposit (Q)
Rate of half yearly compound interest 10% p.a.	Rate of yearly compound interest 12% p.a.

- (i) How much interest will be collected in account (P) after 2 years?
- (ii) How much interest will be collected in account (Q) after 2 years?
- (iii) After knowing the interest rates of both options, by which option will you deposit the money? And why?
9. (a) If the yearly compound interest of a sum in 2 years at the rate of 15% p.a. interest is Rs. 180 more than simple interest, then find the sum.
- (b) If the half yearly compound interest of a sum in a year at the rate of 10% interest p.a. is Rs. 40 more than the yearly compound interest of the same sum for the same period of time at the same rate of interest, then find the sum.

- 10.** (c) Suprim borrowed some money for 2 years at the rate of compound interest of 10% p.a. and immediately he lent the money at the same rate of half yearly compound interest for the same period of time. In this transaction, if he gained Rs. 2019.24, then find how much he borrowed?

10. (a) In how many years, a sum of Rs. 1,00,000 amounts to Rs. 1,21,000 at the rate of compound interest 10% p.a.?

(b) According to the compound interest, in how many years will the compound interest of Rs. 8,00,000 at the rate of 10% p.a. be Rs. 12,61,000? Find it.

(c) At what rate of yearly compound interest, the sum of Rs. 700 amounts to Rs. 847 in 2 years?

(d) At what rate of annual compound interest will the compound interest of Rs. 3,43,000 in 3 years be Rs. 1,13,533? Find.

11. (a) At the rate of yearly compound interest, a sum will be Rs. 6,050 in 2 years and Rs. 6,655 in 3 years respectively then
(i) find the rate of the compound interest. (ii) find the sum.

(b) At the rate of annual compound interest, a sum amounts to Rs. 10,580 in 2 years and Rs. 12,167 in 3 years respectively, then
(i) find the rate of the compound interest.
(ii) find the sum.

(c) The compound interest of a sum at the rate of yearly compound interest in 1 year and 2 years are respectively Rs. 1,800 and Rs. 3,816, then find the rate of the interest and the sum.

12. (a) A person deposited Rs. 5,00,000 interest received from a commercial bank for 2 years to get the half yearly compound interest at the rate of 10% per annum. 5% tax on the interest will be levied. But right after a year, the bank has changed the policy and decided to accomplish the interest quarterly at the same rate of interest.
(i) Find the interest of the first year by deducting the tax.
(ii) What would be the interest of the second year be after deducting the tax?
(iii) What is the difference between the interest of the first year and second year after deducting the tax? Find.
(iv) After deducting the tax, by what percentage the interest of the first year differ from the interest of the second year?

(b) A person deposited Rs. 80,000 in a co-operative limited for 2 years to get the yearly compound interest at the rate of 15% per annum after deducting the 5% tax on the interest.

But right after a year, the bank has changed the policy and decided to accomplish the interest half yearly at the same rate of interest. After deducting tax, by what percentage the interest of the first year differ from the interest of the second year?

- 13.** (a) Ram divided Rs. 41,000 in two parts and deposited in the bank account of two daughters at the rate of annual compound interest 5% for 2 years and 3 years respectively. If the compound amount received by them after 2 years and 3 years respectively are equal, find how much did each of them get?

(b) Divide Rs. 21,000 in two parts in such a way that the compound amount of the first part at the rate of 10% p.a. for 3 years is equal to the compound amount for 2 years. What sums are there in the first part and the second part? Find it.

14. a) According to the yearly compound interest, the compound interest of a sum for 1 year and 2 years are respectively Rs. 450 and Rs. 945, find the rate of interest and the sum.

b) According to the yearly compound interest, the compound interests of a sum for 1 year and 2 years are Rs. 1800 and Rs. 3816 respectively. Find the rate of the interest and the sum.

Project work

Visit a co-operative or bank or financial institution in your surroundings. Get information about various plannings by visiting head of the institution or receptionist or any other who can give information. If you find the brochure of the institution, study the various schemes. If your father or mother or any other family member is interested to deposit a sum of money, which scheme do you prefer? Why? Prepare a report with your reason and present the report in your classroom.

Answer

8. (a) Rs. 76.35 (b) Rs. 34.36 (c) (i) Rs. 12930.37
(ii) Rs. 15264 (iii) Rs. 2333.62 more interest in account Q

9. (a) Rs. 8,000 (b) Rs. 16,000 (c) Rs. 366717.82

10. (a) 2 year (b) 3 years (c) 10% (d) 10%

11. (a) (i) 10% (ii) Rs. 5,000
(b) (i) 15% (ii) Rs. 8,000 (c) 12% Rs. 15,000

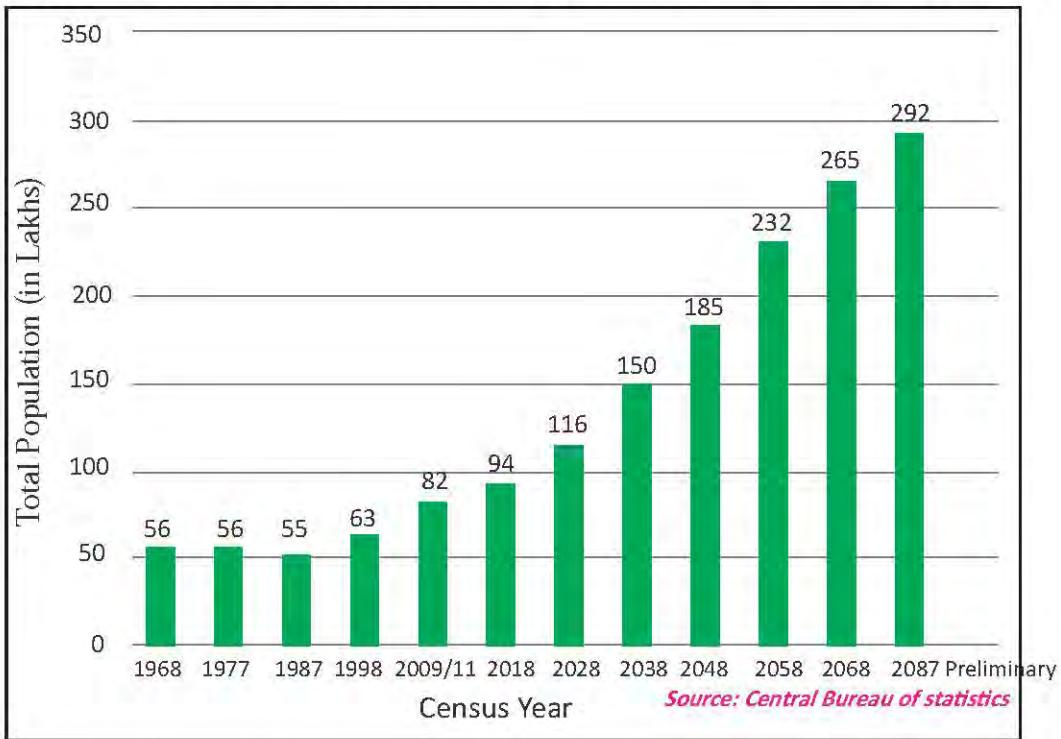
12. (a) (i) Rs. 48,687.5 (ii) 54112.79
(iii) 5425.2936 (iv) 11.14% (b) 18.53%

13. (a) Rs. 21,000 and Rs. 20,000 (b) Rs. 10,000 and Rs. 11,000

14. (a) 10% and Rs. 4,500 (b) 12% and Rs. 1,500

3.0 Review

In Nepal, the census began in 1968 BS (1911) and since then, it has been conducted almost every 10 years. The initial result of the census from 1968 B.S. to 2078 B.S. is presented below. Discuss the following questions based on the table below:



- What was the population of Nepal in 1968 B.S.?
- How much did the population reach in the initial report of the 2078 B.S. census?
- According to the above table, discuss how much the population is increased or decreased in which years.
- With reference to the population of 2068 B.S., by what percentage has the population of 2078 B.S. increased?

- (e) With reference to the population of 1977 B.S., by what percentage was the population of 1987 B.S. decreased?
- (f) By how much was the populations increased or decreased in each census than that of the previous census? Compute.
- (g) In our daily life, what are other additional examples of such increment? Make a list.

3.1.1 Growth

Activity 1

According to the census 2078, the population of a municipality was 25,000. If the population increases by 2% every year,

- (a) What will be the population of the municipality in 2079 B.S.?
- (b) What will be the population of the municipality in 2080 B.S.?
- (c) Is the population growth similar to computing the compound interest?

Here, The population of the municipality in 2078 B.S. = 25,000

The population of the municipality in 2079 B.S. = $25,000 + 2\% \text{ of } 25,000$

$$= 25,000 + 25,000 \times \frac{2}{100} = 25,500$$

Again, the population of the municipality in 2080 B.S. = $25,500 + 2\% \text{ of } 25,500$

$$= 25,500 + 25,500 \times \frac{2}{100} = 26,010$$

The population in 2078 B.S. (P) = 25,000

Population growth rate (R) = 2%

Time (T) = 2 years

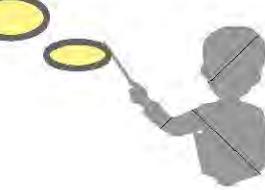
Can we use the formula of compound interest in the above condition?

Population after 2 years (P_2) = ?

$$\text{We know that, } P_T = P \left(1 + \frac{R}{100}\right)^T$$

$$\begin{aligned} \text{or, } P_2 &= 25000 \left(1 + \frac{2}{100}\right)^2 \\ &= 25000 \times \left(\frac{51}{50}\right)^2 \end{aligned}$$

$$= 26,010$$



If external setup does not affect, the population of a place increases at a certain rate. This is called population growth rate and increased population is called growth population. The problem related to growth population can be solved by comparing with compound interest system.

$$\text{Population after } T \text{ years } (P_T) = P \left(1 + \frac{R}{100}\right)^T$$

$$\text{Increased population after } T \text{ years} = P \left[\left(1 + \frac{R}{100}\right)^T - 1 \right]$$

Here, P = The population of the initial year

R = Population growth rate

T = Time

But in compound interest,

P denotes principal, R denotes rate and T denotes time.

Population growth is affected by migration and death.

If the population growth rate differs year by year, then the population after T years,

$$(P_T) = P \left(1 + \frac{R_1}{100}\right) \left(1 + \frac{R_2}{100}\right) \left(1 + \frac{R_3}{100}\right) \dots \left(1 + \frac{R_T}{100}\right). \text{ Where } R_1, R_2, R_3, \dots, R_T$$

R_T respectively represent the population growth rates of the first year, second year, third year T^{th} year's population growth.

Example 1

The population of a city in 2078 B.S. was 50,000. If the annual population growth rate was 2% , what will be the population in 2080 B.S.? Calculate it.

Solution

Here, initial population of the city (P) = 50,000

Population growth rate (R) = 2% p.a.

Time (T) = 2080 B.S. – 2078 B.S. = 2 years

$$\begin{aligned} \text{We know that, } P_T &= P \left(1 + \frac{R}{100}\right)^T \\ &= 50,000 \left(1 + \frac{2}{100}\right)^2 \\ &= 50,000 \left(\frac{102}{100}\right)^2 && = 50,000 \times 1.0404 = 52,020 \end{aligned}$$

Thus, the population in 2080 B.S. will be 52,020.

Example 2

Two years ago, the price of a sack of 25 kg jeera masino rice was Rs. 1,300. If the inflation rate was 5% p.a., by how much is the price increased now? Find it.

Solutions

Here, the initial price of jeera masino rice (P) = Rs. 1300

Increased rate of price (R) = 5% p.a.

Time (T) = 2 years

Increased price = ?

We know that,

$$\begin{aligned}\text{Increased price} &= P \left[\left(1 + \frac{R}{100}\right)^T - 1 \right] \\ &= \text{Rs. } 1300 \left[\left(1 + \frac{5}{100}\right)^2 - 1 \right] \\ &= \text{Rs. } 1300 \left[\left(1 + \frac{105}{100}\right)^2 - 1 \right] \\ &= \text{Rs. } 1300 \times 0.1025 \\ &= \text{Rs. } 133.25\end{aligned}$$

∴ The increased price of jeera masino rice is Rs. 133.25.

Example 3

If the price of a photocopy machine increases from Rs. 1,00,000 to Rs. 1,21,000 in 2 years, find the rate of yearly increment.

Solution

Here, the initial price of a photocopy machine (P) = Rs. 1,00,000

Time (T) = years

The present price of the photocopy machine (P_T) = Rs. 1,21,000

Rate of price increment (R) = ?

We know that $P_T = P \left(1 + \frac{R}{100}\right)^T$

$$\text{or, } \text{Rs. } 1,21,000 = \text{Rs. } 1,00,000 \left(1 + \frac{R}{100}\right)^2$$

$$\text{or, } \frac{121000}{100000} = \left(1 + \frac{R}{100}\right)^2$$

$$\text{or, } \left(\frac{11}{10}\right)^2 = \left(1 + \frac{R}{100}\right)^2$$

$$\text{or, } \frac{11}{10} = 1 + \frac{R}{100}$$

$$\text{or, } \frac{11}{10} - 1 = \frac{R}{100}$$

$$\text{or, } \frac{11 - 10}{10} = \frac{R}{100}$$

$$\text{or, } \frac{1}{10} \times 100 = R$$

$$\therefore R = 10\%$$

Example 4

The number of students of a basic school is 500 now. In how many years will the number be 720 if the number of students increases by 20% p.a. every year?

Solution

Initial number of student (P) = 500

Increase rate (R) = 20% p.a.

Time (T) = T years

The number of students after T years (P_T) = 720

We know that, $P_T = P \left(1 + \frac{R}{100}\right)^T$

$$\text{or, } 720 = 500 \left(1 + \frac{20}{100}\right)^T$$

$$\text{or, } \frac{720}{500} = \left(1 + \frac{20}{100}\right)^T$$

$$\text{or, } \frac{720}{500} = \left(\frac{120}{100}\right)^T$$

$$\text{or, } \frac{36}{25} = \left(\frac{6}{5}\right)^T$$

$$\text{or, } \left(\frac{6}{5}\right)^2 = \left(\frac{6}{5}\right)^T$$

$$\Rightarrow T = 2 \quad [\because \text{indices are equal as the bases are same}]$$

Therefore, after 2 years, the number of students will be 720.

Example 5

The price of a plot of land is Rs. 15,97,200 per aana. If the rate of increase in price is 10% p.a., what was the price of the land per ropani before 3 years?
Find it. (1 ropani = 16 aana)

Solution

Here, the price of a plot of land per aana (P_T) = Rs. 15,97,200

Time (T) = 3 years

Rate of increase in price (R) = 10% p.a.

Price of the land per aana before 3 years (P) = ?

$$\text{We know that, } P_T = P \left(1 + \frac{R}{100}\right)^T$$

$$\text{or, } 15,97,200 = P \left(1 + \frac{10}{100}\right)^3$$

$$\text{or, } 15,97,200 = P \left(\frac{110}{100}\right)^3$$

$$\text{or, } 15,97,200 = P \times 1.331$$

$$\text{or, } \frac{1597200}{1.331} = P$$

$$\therefore P = \text{Rs. } 12,00,000$$

Thus the price of land per aana (P) = Rs. 12,00,000

We know that, 16 aana = 1 ropani

\therefore The price of land per ropani = $16 \times \text{Rs. } 12,00,000 = \text{Rs. } 1,92,00,000$.

Example 6

The number of SEE appeared students from a district in 2076 B.S. was 50,000. If in the coming 3 years, the number increased by 5%, 6% and 4% respectively, find how many students will appear SEE in 2079 B.S.

Solution

Here, the number of the SEE appeared students before 3 years (P) = 50,000

Time (T) = 3 years

Rate of increase in the first year (R_1) = 5% p.a., rate of increase in the second year (R_2) = 6% p.a. and increase in the third year (R_3) = 4% p.a.

Number of the SEE appeared students after 3 years (P_T) = ?

$$\begin{aligned} \text{We know that, } (P_T) &= P \left(1 + \frac{R_1}{100}\right) \left(1 + \frac{R_2}{100}\right) \left(1 + \frac{R_3}{100}\right) \\ &= 50,000 \left(1 + \frac{5}{100}\right) \left(1 + \frac{6}{100}\right) \left(1 + \frac{4}{100}\right) \\ &= 50,000 \left(\frac{105}{100}\right) \left(\frac{106}{100}\right) \left(\frac{104}{100}\right) \\ &= 57,876 \end{aligned}$$

Therefore 57,876 students will appear in the SEE in 2079 B.S.

Example 7

The population of a municipality in 2078 B.S. was 1,00,000. In 2079 B.S., 8000 migrated there from other places and 500 died due to several circumstances. If the population increase rate is 2% p.a. every year, what will be the population in 2081 B.S.? Find it.

Solution

Here, case I,

Population in 2078 B.S. (P) = 1,00,000

Population increase rate (R) = 2% p.a.

Time (T) = 1 year

In-migration population (M_{in}) = 8,000

Death population (D) = 500

The population in 2079 B.S.

Since the population increases by 2% every year,

$$\begin{aligned}\text{We know that, } P_T &= P \left(1 + \frac{R}{100}\right)^T \\ &= 1,00,000 \left(1 + \frac{2}{100}\right)^1 \\ &= 1,00,000 \left(\frac{102}{100}\right)^1 \\ &= 1,02,000\end{aligned}$$

Now, the final population in 2079 B.S. = 1,02,000 + 8,000 + 500 = 1,09,500

Case II

Population in 2079 B.S. (P) = 1,09,500

Time (T) = 2 years

Population increase rate (R) = 2% p.a.

Population in 2081 B.S. (P_T) = ?

$$\begin{aligned}\text{We know that, } P_T &= P \left(1 + \frac{R}{100}\right)^T \\ &= 1,09,500 \left(1 + \frac{2}{100}\right)^2 \\ &= 1,09,500 \left(\frac{102}{100}\right)^2 \\ &= 1,13,924\end{aligned}$$

Therefore, the population of the municipality in 2081 B.S. will be 1,13,924.

Exercise 3.1

1. (a) If the present population of a locality is P , the population after T years is P_T and yearly population growth rate is $R\%$, write the formula to find P_T .
(b) If the number of tigers of Chitwan National Park in 2079 B.S. is x and annual growth rate of tiger is $R\%$, then what will be the number of tigers after N years?
(c) The growth rate of foreign employment from Nepal of the first year, second year and third year are respectively $R_1\%$, $R_2\%$ and $R_3\%$. Write the formula to find the number of employees after 3 years.
2. (a) If death rate is less than birth rate of a country, does the population of the country increase or decrease?
(b) One year ago, the price of a sack of 25 kg rice was Rs. 1500. As the price has increased 10% p.a., how much does the sack of 25 kg rice cost now?
3. (a) According to the census 2021, the population of a city was 5,18,452. If the growth rate was 4.5% p.a., what will be the population of the city after 3 years?
(b) A landlord made an agreement with a businessman to increase the rent 5% per annum. If the rent of a shutter is Rs. 10,000 now, what will be the rent after 3 years?
(c) The growth rate of a bacteria in curd is 10% per hour. If the number of bacteria at 6 a.m. is 4×10^{11} , what will be the number of bacteria after 2 hours? Find.
4. (a) The population of a rural municipality before 2 years was 28,500. If the population growth rate is 2% p.a., then by how much is the population increased in 2 years?
(b) Monthly fee of grade 10 of a school before 3 years was Rs. 6,500. If fees increase every year by 10% according to the rules and regulation of the school, then by how much has the fee increased in 3 years?
(c) A land costs Rs 6,00,000 at present. If the yearly increase rate of the price is 10%, then how much will increase in the price of the land in 2 years? Find.
5. (a) The number of students studying in a university at present is 21,632. 2 years ago, the number of students studying in the university was 20,000. What was the yearly increment rate?

- (b) The population of a rural municipality at the end of 2018 A.D. was 40,000. If at the end of 2020 A.D., the population increased to 44,100, then find the yearly population growth rate.
- (c) 3 years ago, the price per liter of oil was Rs. 125. Now, the price has increased to Rs 216 per liter, then what is inflation rate?
6. (a) The population of a village is 13,310. Whilst the population growth rate of the village is 10% p.a., how many years ago was the population of the village 10,000?
- (b) At the beginning of Baisakh, the papaya plant was 4 meter high. If the growth rate of the plant is 4% per month, in how many months will the height of the plant be 4.3264 meter?
7. (a) The price of a plot of land is Rs. 15,97,200 per aana. If the rate of increase in the price is 10% p.a., what was the price of the land per ropani before 3 years? Find it. (1 ropani = 16 aana)
- (b) The price of a land is Rs. 2,66,200 per aana. If the rate of increase in price is 10% p.a., what was the price of the land per ropani before 3 years? Find it.
8. (a) When urine of a patient was tested in a laboratory at 6 a.m., it was found that the number of bacteria 1×10^5 . After that, it was tested at 7 a.m., 8 a.m. and 9 a.m. and found that the increase rate per hour was 3%, 4% and 5% respectively. Find the total number of bacteria at 9 a.m. .
- (b) 3 years ago, the population of a city was 1,50,000. In the following 3 years, the population increased by 2% in the first year, 4% in the second year and 5% in the third year. Find the present population of the city.
9. (a) In an insurance company established 3 years ago, 1000 agents were trained by all the branches throughout the country. Customers should be increased according as the market competition. So that, from the very beginning, a policy “a group of every 5 agents should add 1 more agent every year” was implemented and the number of agents has been increased. How many agents are there in the company now?
- (b) A financial institution established at the beginning of 2078 has 200 marketers. After the expansion of market of the institution, a policy of a group of every 5 marketers should add 1 more new marketer every year was implemented and the number of marketer has been increased. Find how many marketers will be there at the end of 2079?

10. The population of a rural municipality is increasing by 10% every year. At the end of 2 years, the population reached to 30,000. If 5,800 migrated to the place finally, find the initial population.
11. After continuous inflation of American dollar at the rate of 5% p.a., in two years it becomes \$1 = Rs.120. Before 2 years, how much Nepalese currency was equal to \$1?
12. The population of a village before 2 years was 31,250. The population growth rate is 6% p.a.. One year ago, 625 migrated to other places. Find the number of present populations of the village.
13. A district had 3,75,000 population before 3 years. If 1480 migrated to the village at the end of 2 years and 2,750 died due to natural disaster and yearly population growth rate is 2%, then find the number of present populations of the district.
14. At the beginning of 2075 B.S., the population of a metropolitan city was 5,00,000. At the end of 2077 B.S, the population of the city was 6,65,500.
 - a) What was the population growth rate?
 - b) If the population increases in the same way, what will be the population of the city at the end of 2079?

Project work

By forming appropriate groups, every group should collect the data of present population from the respective ward office. From your municipality or ward office or other books or using brochures, find the increased population of the ward based on the data of census 2078. Prepare a report on how many migrated from other places, how many migrated to other places and how many died. Then present your report to the class.

Answers

- 1 and 2 show to your teacher.
3. (a) 5,91,640 (b) Rs. 15,208.75 (c) 4.84×10^{11}
 4. (a) 1152 (b) Rs. 2151.5 (c) Rs. 1,26,000
 5. (a) 4% (b) 5% (c) 20% 6. (a) 3 years (b) 2 month
 7. (a) Rs. 1,92,00,000 (b) Rs. 32,00,000 8. (a) 1.1248×10^5
(b) 1,67,070 9. (a) 1728 (b) 288 10. 20,000
 11. 1 = Rs. 108.84 12. 34,450 13. 396,658 14. (a) 10% (b) 8,05,255

3.2 Depreciation

Activity 1

By dividing the students in appropriate groups, discuss the following questions:

- If a farmer sells a tractor for Rs. 3,60,000 which was purchased for Rs. 5,50,000 before 2 years, why has the price decreased?
- A cupboard can be bought for Rs. 15,000, less than the price of a new one from a second-hand shop. Why?
- A photocopy machine was bought for Rs. 2,10,000 before some years. After using for sometimes, it costs Rs. 1,00,000 only now. Why?

From the above situations, the price of the tractor diminishes by Rs. 1,90,000 in (a). Similarly, in (b), it is known that the second-hand goods are cheaper than the new ones. In (c), after using the machinery goods, the price depreciates.

A product is prepared for a certain period. Its efficiency decreases in accordance with its increased uses. Thus, after using some machinery items for certain period of time, their cost diminishes at certain rate. This is called depreciation. The price that diminishes at a certain rate at certain rate during a certain period of time is called compound depreciation.

Activity 2

All the students Sit in appropriate groups.

Initial price of a good = V_0 , similarly, yearly depreciation rate = $R\%$, time duration = T

Price of good after T years = V_T

In this condition, how can we find the price after T years? This has been discussed in groups and the conclusion is presented as below:

$$\begin{aligned}\text{Price after 1 year } (V_1) &= V_0 - V_0 \text{ or } R\% \\ &= V_0 - V_0 \times \frac{R}{100} \\ &= V_0 \left(1 - \frac{R}{100}\right)\end{aligned}$$

For 2 years,

$$\begin{aligned}\text{Price after 2 years } (V_2) &= V_0 \left(1 - \frac{R}{100}\right) - R\% \text{ of } V_0 \left(1 - \frac{R}{100}\right) \\&= V_0 \left(1 - \frac{R}{100}\right) - V_0 \left(1 - \frac{R}{100}\right) \times \frac{R}{100} \\&= V_0 \left(1 - \frac{R}{100}\right) \left(1 - \frac{R}{100}\right) \\&= V_0 \left(1 - \frac{R}{100}\right)^2\end{aligned}$$

In this way, price after 3 years $(V_3) = V_0 \left(1 - \frac{R}{100}\right)^3$

Like this,

(a) Price after T years $(V_T) = V_0 \left(1 - \frac{R}{100}\right)^T$.

(b) Again, how can we find the depreciated price?

$$\begin{aligned}\text{Depreciated price} &= V_0 - V_T \\&= V_0 - V_0 \left(1 - \frac{R}{100}\right)^T \\&= V_0 \left[1 - \left(1 - \frac{R}{100}\right)^T\right].\end{aligned}$$

(c) Does the price of a good depreciate at the same rate every year? Discuss.

If $R_1\%$, $R_2\%$ and $R_3\%$ $R_T\%$ are respectively depreciating rate for first year, second year, third year T^{th} year, then price after T years

$$(V_T) = V_0 \left(1 - \frac{R_1}{100}\right) \left(1 - \frac{R_2}{100}\right) \left(1 - \frac{R_3}{100}\right) \dots \left(1 - \frac{R_T}{100}\right).$$

Increases in growth and decreases in depreciation.



Example 1

3 years ago a book was published costing Rs. 200, is being sold in an exhibition at 5% rate of yearly depreciation. What is the price of the book this year?

Solution

Here, the initial price of the book (V_0) = Rs. 200

Rate of depreciation (R) = 5% p.a.

Time (T) = 3 years

Solution

Present price of the book (V_T) = ?

$$\begin{aligned}\text{We know that, } V_T &= V_0 \left(1 - \frac{R}{100}\right)^T \\ &= 200 \left(1 - \frac{5}{100}\right)^3 \\ &= 200 \left(\frac{95}{100}\right)^3 \\ &= 200 \times 0.857375 \\ &= \text{Rs. } 171.47\end{aligned}$$

Therefore, the book costs Rs. 171.47 this year.

Example 2

Seema admitted in BBA. She purchased a computer for Rs. 40,000 for her study. After using it for 2 years, if the price of the computer depreciates by Rs. 7,600, then find the rate of depreciation.

Solution

Here, the initial price of the computer (V_0) = Rs. 40,000

Rate of depreciation (R) = ?

Time (T) = 2 years

Depreciated price = Rs. 7,600

Now, the price after 2 years (V_T) = $V_0 - \text{Rs. } 7,600 = \text{Rs. } 40,000 - \text{Rs. } 7,600 = \text{Rs. } 32,400$

$$\begin{aligned}\text{We know that, } V_T &= V_0 \left(1 - \frac{R}{100}\right)^T \\ \text{or, } 32,400 &= 40,000 \left(1 - \frac{R}{100}\right)^2\end{aligned}$$

$$\text{or, } \frac{32400}{40000} = \left(1 - \frac{R}{100}\right)^2$$

$$\text{or, } \left(\frac{18}{20}\right)^2 = \left(1 - \frac{R}{100}\right)^2$$

$$\text{or, } \frac{18}{20} = 1 - \frac{R}{100}$$

$$\text{or, } \frac{R}{100} = 1 - \frac{18}{20}$$

$$\text{or, } \frac{R}{100} = \frac{2}{20}$$

$$\text{or, } R = \frac{2}{20} \times 100 = 10\%$$

Therefore, the rate of depreciation of the computer is 10% p.a.

Example 3

The price of a house is Rs. 20,00,000 now. If its price decreases by 10% every year, then in how many years will its price be Rs. 14,58,000?

Solution

Here, the present price of the house (V_0) = Rs. 20,00,000

Rate of depreciation (R) = 10%

Price after T years (V_T) = Rs. 14,58,000

Time (T) = ?

$$\text{We know that, } V_T = V_0 \left(1 - \frac{R}{100}\right)^T$$

$$\text{or, } 14,58,000 = 20,00,000 \left(1 - \frac{10}{100}\right)^T$$

$$\text{or, } \frac{1458000}{2000000} = \left(\frac{90}{100}\right)^T$$

$$\text{or, } 0.729 = (0.9)^T$$

$$\text{or, } (0.9)^3 = (0.9)^T$$

$$\Rightarrow T = 3 \text{ years}$$

Thus, 3 years later, the price of the house will be Rs. 14,58,000.

Example 4

A factory established with the capital of Rs. 4 crore gained Rs. 75 Lakhs in 3 years but its cost depreciated by 2.5% p.a. Then company was sold after 3 years. Now, calculate whether the factory is in profit or loss.

Solution

Here, initial price of investment (V_0) = Rs. 4,00,000

Rate of depreciation (R) = 2.5%

Time (T) = 3 years

Price after 3 years (V_3) = ?

$$\text{We know that, } V_T = V_0 \left(1 - \frac{R}{100}\right)^T$$

$$\text{or, } V_3 = 4,00,00,000 \left(1 - \frac{2.5}{100}\right)^3$$

$$= 4,00,00,000 \left(\frac{97.5}{100} \right)^3$$

$$= \text{Rs. } 3,70,74,375$$

Again, the profit gained by the factory after 3 years = Rs. 75,00,000

The total amount gained from the factory = Rs. 3,70,74,375 + Rs. 75,00,000 = Rs. 4,45,74,375

Total investment = Rs. 4,00,00,000

Thus, the profit while selling it in 3 years = Rs. 4,45,74,375 – Rs. 4,00,00,000 = Rs. 45,74,375

Example 5

When the price of a share of a finance company depreciates continuously for 2 years by 10% p.a. and it is sold for Rs. 25,920 then how many shares of Rs. 100 were sold? Find it.

Solution

Here, present value of the share of the finance (V_T) = Rs. 25,920

Depreciation rate (R) = 10%

Time (T) = 2 years

Initial price of shares of the finance (V_0) = ?

$$\text{We know that, } V_T = V_0 \left(1 - \frac{R}{100} \right)^T$$

$$\text{or, } 25,920 = V_0 \left(1 - \frac{10}{100} \right)^2$$

$$\text{or, } 25,920 = V_0 \left(\frac{90}{100} \right)^2$$

$$\text{or, } 25,920 = V_0 \times \frac{81}{100}$$

$$\text{or, } V_0 = \text{Rs. } \frac{25920 \times 100}{81}$$

$$\text{or, } V_0 = \text{Rs. } 32,000$$

Hence, the value of the share before 2 years (V_0) = Rs. 32,000

Two years ago, the price of a share = Rs. 100

$$\text{Total number of shares} = \frac{32000}{100} = 320$$

∴ Two years ago, the finance company sold 320 shares of Rs. 100.

Exercise 3.2

1. a) If the initial price of a good is Rs. P and the rate of depreciation is R% p.a., then write the formula to find the price of the good after T years.
b) What does 'R' in $V_T = V_0 \left(1 - \frac{R}{100}\right)$ represent value after 'T' years?
2. a) If Ram sold a watch which cost Rs. 5,000 after 1 year at 7% depreciation, then what will be the depreciated price?
b) A motorcycle is sold for Rs. 57,000 in 1 year after depreciating at the rate of 5% p.a. At what price was the motorcycle purchased for?
3. a) What will be the price of a cupboard of cost Rs. 16,800 after 2 years at the rate of 15% depreciation p.a. ?
b) The present price of a motorcycle of efficiency 125 c.c. produced in India is Rs. 2,50,000. If its cost depreciates every year at the rate of 4% p.a., what will be the price of motorcycle after using 3 years? Find it.
4. a) Sameer has bought a mobile phone for Rs. 30,000. Due to his household problem he has to sell it after using 2 years at 30% rate of depreciation. Find the depreciated amount.
b) As the laptop is convenient for online class, a mathematics teacher purchased a laptop for Rs. 96,000. It is depreciated at the rate of 15% every year. If he sells it after using it for 3 years, how much money is depreciated? Find it.
5. a) 3 years ago, a plot of land of 4 ropani in hilly region was purchased for Rs. 12,50,000. It can be sold now for Rs. 1,60,000 per ropani. By what percent per annum is the price of land depreciated? calculate it.
b) A man bought a watch for Rs. 5,000 and he sold it after using it for 3 years for Rs. 625 only. Find the annual rate of depreciation.
6. a) A press machine was purchased for Rs. 4,00,000 before some years and it is depreciated to Rs. 1,96,000 by reducing the price every year by 30%. How many years ago was the machine purchased?
b) A car costs Rs. 8,00,000. If its price reduces every year by 10%, in how many years will its price be Rs. 5,83,200?

- 7.** a) After continue devaluation of the American dollar in 2 years by 5% p.a., American dollar \$ 1 = Rs.125 now, how much Nepali rupees was equal to \$ 1 before 2 years? Find it.
- b) 3 years ago, a small group of youths returned back from foreign employment and started a cow farm investing Rs. 2,80,000. Due to political instability of state, if the price of the farm depreciates by 5% p.a. then how much does the farm cost now?
- 8.** a) A photocopy machine costs Rs. 5,00,000 now. If the machine depreciates by 15% in the first year and similarly depreciates by 10% and 5% in the second and third year respectively, then what will be the price of the machine in 3 years?
- b) A lamination machine is sold for Rs. 24,168 after depreciating it by 4% and 5% in the first and second year respectively. Find what the price of the machine before 2 years.
- 9.** An entrepreneur purchased a heavy truck by investing Rs. 48,00,000. He earned Rs. 6,80,000 in 2 years. If it is depreciated by 10% every year and sold in 2 years, then find his loss or profit.
- 10.** A bus owner bought a bus for Rs. 16,00,000 and conducted it in the Kathmandu - Baglung route for 3 years. He earned Rs. 5,10,000 only. If the value is depreciates every year by 5% and he sells it in 3 years, then find his loss or profit.
- 11.** The price of a finance company listed in Nepal share market is falling down its price by 10% in 2 years. Your neighbour having the shares of the finance company sold his shares for Rs. 28,350. How many shares were sold by the company before 2 years at the rate of Rs. 100 per share?
- 12.** **The price of the share of a hydropower company is reducing by 10% p.a. If Sashi sells all his shares now and has the present value Rs. 7,10,775, then**
- a) What was the price of his shares before 2 years?
- b) He purchased the shares at the rate of Rs. 100 per share in IPO, how many shares did he buy before 2 years?
- 13.** **In certain rate of annual depreciation, the price of a good will be Rs. 10,240 and Rs. 8,192 in 2 years and 3 years respectively. Then,**
- a) Find the rate of depreciation.

- b) What was the initial price of the good? Find it.
14. In a certain rate of yearly depreciation, the price of an article will be Rs. 5,41,500 and Rs. 5,14,425 in 2 and 3 years respectively. Then
- What is the rate of depreciation?
 - What was the initial price of the article? Find it.

Project work

Make groups of suitable number of students. Visit the nearest shop or company or other intervention where second hand goods are sold. Ask the cost price of electric items like vehicle, photocopy machine or furniture, etc. After using the goods for a certain time, what is the present value? After purchasing the goods, how much profit or loss will be made? With the answers of the questions, find by what percent are the goods are depreciated? What percent of loss or profit can be made in selling the goods? Prepare a report and present in the classroom.

Answers

- | | | |
|---------------------------|-------------------------|------------------------------------|
| 11. Show to your teacher. | 2. (a) Rs. 350 | (b) Rs. 60,000 |
| 3. (a) Rs. 12,138 | (b) Rs. 2,21,184 | 4. (a) Rs. 15,300 (b) Rs. 37,044 |
| 5. (a) 20% | (b) 50% | 6. (a) 2 years (b) 3 |
| 7. (a) \$1 = Rs. 138.50 | (b) Rs. 2,40,065 | 8. (a) Rs. 3,63,375 (b) Rs. 26,500 |
| 9. Rs. 2,32,000 shares | 10. Rs. 2,81,800 profit | 11. 350 shares |
| 12. (a) Rs. 8,77,500 | (b) 8,775 shares | 13. (a) 20% (b) Rs. 16,000 |
| 14. (a) 5% | (b) Rs. 6,00,000 | |

4.0 Review

Make groups of suitable number of students and discuss the followings:

- You are going to visit abroad for an educational excursion. Can you buy goods and food items by spending Nepali rupees there?
- Sagun is a prestigious businessman. He imported goods from abroad countries. Can he pay the bills in Nepali rupees?
- Nepali people working abroad send money in Nepal. Do their family get the money in Nepali rupees? At what basis do they get the money in Nepali rupees of the foreign currency?

In the above mentioned conditions, wherever we go or conduct our business, we use the currency of the respective country. Family gets the Nepali currency if the money sent from different countries.

4.1 Currency and Exchange Rate

Activity 1

Make groups of suitable number of students. Study the exchange rate of currency of several countries issued by Nepal Rastra Bank on Bhadra 5, 2079, and answer the following questions:

Nepal Rastra Bank
Central Office, Thapathali, Kathmandu
Bhadra 5, 2079 (August 21, 2022)

Country	Currency	Unit	Buying rate (Rs.)	Selling rate (Rs.)	Symbol
India	Indian Rupees	1	160.00	160.15	₹
America	American Dollar	1	127.35	127.95	\$
Countries in European Union	Euro	1	128.13	128.73	€
UK	Pound Sterling	1	150.94	151.65	£

Switzerland	Swiss Frank	1	133.13	133.76	CHF
Australia	Australian Dollar	1	87.82	88.23	AUD\$
Canada	Canadian Dollar	1	98.08	98.54	CAD\$
Singapore	Singapore Dollar	1	91.68	92.11	SGD\$
Japan	Japanese Yen	10	9.32	9.37	¥
China	Chinese Yuan	1	18.70	18.79	¥
Saudi Arabia	Saudi Arabian Riyal	1	33.91	34.07	SAR ₪
Qatar	Qatari Riyal	1	34.82	34.98	QAR ₩
Thailand	Thai Bhat	1	3.57	3.59	THB ₧
United Arab Emirates	United Arab Emirate Dihram	1	34.67	34.83	₾
Malaysia	Malaysian Ringgit	1	28.45	28.58	MR
South Korea	South Korean wan	100	9.58	9.63	₩
Sweden	Swedish Corner	1	12.07	12.12	SEK kr
Denmark	Denish Corner	1	17.23	17.31	DKK kr
Hongkong	Hongkong Dollar	1	16.23	16.31	HKD \$
Kuwait	Kuwaiti Dinar	1	414.23	416.18	KD د.ك
Bahrain	Bahrain Dinar	1	337.80	339.39	BD د.ب

- a) Who determines the exchange rate of currency in our nation?
- b) If the currency of America is called dollar. What is the currency of China called?
- c) According to the exchange rate given above, why is selling rate more than the buying rate?
- d) What are buying rate and selling rate? Discuss.

The exchange rate of currency of a country to another country will be determined by the government or central bank of the country. Such rate is called foreign currency exchange rate. In our country, exchange rate with India is fixed and exchange rate with other countries are instable. In our country, the exchange rate is declared by Nepal Rastra Bank. The exchange rate given by bank or other financial institutions while purchasing the foreign currency is called buying rate. Similarly, the exchange rates fixed by bank or other financial institutions while selling the foreign currency is called selling rate.

Example 1

Based on the exchange rate given above, find the difference between buying and selling price of pound sterling 500.

Solution

According to the buying rate, 1 pound sterling = Rs. 150.94

Now, buying price of 500 pound sterling = $Rs. 150.94 \times 500 = Rs. 75,420$

Again, according to the selling rate, 1 pound sterling = Rs. 151.65

Selling price of 500 pound sterling = $Rs. 151.65 \times 500 = Rs. 75,825$

Hence, the difference between selling price and buying price = $Rs. 75,825 - Rs. 75,420 = Rs. 405$

Example 2

The exchange rate between the American Dollar and Nepali rupees for a specific day is \$ 1 = Rs. 126.35

- How many American Dollars can be exchanged for Rs. 85,500?
- How many rupees can be exchanged for \$ 3,000 ?

Solution

Here, a) $\$ 1 = Rs. 126.35$

Or, $Rs. 126.35 = \$ 1$

Or, $Rs. 1 = \$ \frac{1}{126.35}$

or, $Rs. 85,500 = \$ \frac{1}{126.35} \times 85,500 = \$ 676.69$

b) Again, $\$ 1 = Rs. 126.35$

Or, $\$ 3000 = Rs. 126.35 \times 3000 = Rs. 3,79,050$

Example 3

Based on the above mentioned exchange rate, change the following currencies:

- 1 Canadian Dollar into Japanese Yen
- 250 Australian Dollar into Swiss Frank.

Solution

Here, a) 1 Canadian Dollar = Rs. 98.08

Or, Rs. 98.08 = 1 Canadian Dollar (According to buying rate)

$$\text{Or, Re. } 1 = \frac{1}{98.08} \dots\dots\dots \text{(i)}$$

Again, 10 Japanese Yen = Rs. 9.37

Or, Rs. 9.37 = 10 Japanese Yen (According to the selling rate)

$$\text{Or, Rs. } 1 = \frac{10}{9.37} \dots\dots\dots \text{(ii)}$$

From equation (i) and (ii), we have

$$\frac{1}{98.08} \text{ Canadian Dollar} = \frac{10}{9.37} \text{ Japanese Yen}$$

$$1 \text{ Canadian Dollar} = \frac{10 \times 98.08}{9.37} \text{ Japanese Yen} = 104.67 \text{ Japanese Yen}$$

Hence, 1 Canadian Dollar = 104.67 Japanese Yen.

Alternatively,

Let Canadian \$ 1 = x Japanese Yen

Now, we have,

Canadian Dollar \$1 = x Japanese Yen

10 Japanese Yen = Rs. 9.37

NRs. 98.05 = Canadian \$1

Multiply each values of the left hand side and similarly multiply each values of the right hand side. Then, we have;

$$1 \times 10 \times 98.05 = x \times 9.37 \times 1$$

$$\text{Or, } 10 \times \frac{10 \times 98.05}{9.37} = x$$

$$\text{Or, } x = 104.67$$

Hence, 1 Canadian Dollar = 104.67 Japanese Yen.

Yeh! While exchanging currency of other countries in Nepal, the currency of the country can be got after exchanging in Nepalese currency at first. The banks give Nepalese rupees for foreign currency at buying rate and foreign currency for Nepalese rupees at the selling rate to us.

1. Since starting with Canadian Dollar on the left hand side and taking Japanese Yen on the right hand side of first line, start with Japanese Yen in the second line.
2. Since the Nepali Rupees on right hand side of second line, write Nepali rupees on the left hand side and the Canadian Dollar on the right hand side of the third line because it is started with Canadian dollar.

b) 1 Australian Dollar = Rs. 87.82

Or, Rs. 87.82 = 1 Australian Dollar

Or, Rs. 1 = $\frac{1}{87.82}$ Australian Dollar(i)

Again, 1 Swiss Frank = Rs. 133.76

Or, Rs. 133.76 = 1 Swiss Frank

Or, Rs. 1 = $\frac{1}{133.76}$ Swiss Frank(ii)

From equation (i) and (ii), we get

$\frac{1}{87.82}$ Australian Dollar = $\frac{1}{133.76}$ Swiss Frank

or, 1 Australian Dollar = $\frac{1 \times 87.82}{133.76}$ Swiss Frank

Or, 250 Australian Dollar = $\frac{87.82}{133.76} \times 250$ Swiss Frank = 164.14 Swiss Frank

Hence, 250 Australian Dollar = 164.14 Swiss Frank

Alternatively

Let, 250 Australian Dollar = x Swiss Frank

1 Swiss Frank = Rs. 133.76

Rs. 87.82 = 1 Australian Dollar

$250 \times 1 \times 87.82 = x \times 133.76 \times 1$

Or, $x = \frac{250 \times 87.82}{133.76}$

= 164.14 Swiss Frank

The alternative process of finding the values of unknown variables other than unitary method, ratio and proportion is chain rule.

Let, A, B and C are the currencies of different countries, then

If A = B, B = C and C = A then

A × B × C = B × C × A

Example 4

If American Dollar (\$) 500 = Pound Sterling (£) 390 and Nepali Rupees Rs. 7,547 = Pound sterling (£) 50, find how many American Dollars can be exchanged for Nepali Rupees. 10,308?

Let, American dollar \$ x = Nepali Rupees Rs. 10,308

And then write accordingly,

$$\$ 500 = £ 390$$

$$£ 50 = \text{Rs. } 7,547$$

$$\text{Rs. } 10,308 = \$x$$

Using chain rule, we have

$$\text{or, } 500 \times 50 \times 10,308 = 390 \times 7547 \times x$$

$$\text{or, } \frac{500 \times 50 \times 10308}{7547 \times x} = x$$

$$\text{or, } x = 87.55$$

Hence \$ 87.55 can be exchanged for Nepali Rupees 10,308.

Example 5

A businessman exchanged Nepali Rupees 8,40,000 at the exchange rate of the Pound Sterling (£) 1 = Rs. 150. After 5 days, Nepali currency is deflated by 5% and then he exchanged Nepali currency into Pound Sterling. How much profit or loss does he get?

Solution

A businessman exchanged the Pound Sterling for Rs. 8,40,000

Rate of deflation = 5%

Loss or profit = ?

Now, Rs. 150 = £ 1

Or, Rs. 1 = $\frac{1}{150}$

Or, Rs. 8,40,000 = £ $\frac{1}{150}$ Rs. 8,40,000 = £ $\frac{1}{150} \times 8,40,000$ = £ 5,600

Since Nepali currency is deflated by 5% in 5 days, then new exchange rate is:

£ 1 = Rs. ($150 - 5\% \text{ of } 150$) = $150 - 150 \times \frac{5}{100}$ = Rs. $150 - \text{Rs. } 7.50$ = Rs. 142.50

Again, he exchanged his Pound Sterling into Nepali Rupee.

Thus, £ 5,600 = Rs. $142.50 \times 5,600$ = Rs. 7,98,000

Here, Rs. 7,98,000 < Rs. 8,40,000 so he gets a loss.

Hence, loss amount = Rs 8,40,000 – Rs. 7,98,000 = Rs. 2,000

Example 6

A businessman of Nepali origin from Norway purchased 900 Pasmina shawls at the rate of Rs. 4,000 in Kathmandu. He exported by paying 5% export tax and then at how much Euro should he sell all the shawls at the profit of 20%? (€1 = Rs. 130)

Solution

Here, the price of a Pasmina shawl = Rs. 4,000

The cost of 900 shawls = Rs. $4,000 \times 900$ = Rs. 36,00,000

The cost price of the shawls including 5% export tax = Rs. 36,00,000 + 5% of Rs. 36,00,000

$$\begin{aligned} &= \text{Rs. } 36,00,000 + \text{Rs. } 36,00,000 \times \frac{5}{100} \\ &= \text{Rs. } 37,80,000 \end{aligned}$$

We know that, €1 = Rs. 130

$$\text{Or, } \text{Rs. } 130 = \text{€1}$$

$$\text{Or, } \text{Rs. } 1 = \text{€1/130}$$

$$\text{Or, } \text{Rs. } 37,80,000 = \frac{\text{€1}}{130}$$

$$\text{or, } \text{Rs. } 37,80,000 = \frac{\text{€1}}{130} \times 37,80,000$$

$$= \text{€29076.92}$$

∴ The total cost price in euro = €29076.92

To sell at the profit of 20%

$$\begin{aligned} \text{The total selling price of 900 shawls} &= \text{€29076.92} + \text{€29076.92} \times \frac{20}{100} \\ &= \text{€29076.92} + 5,815.38 \end{aligned}$$

Hence, the selling price of all the shawls = € 34,892.30

Alternatively

The cost of 900 Pasmina shawls with 5% export tax = Rs. 37,80,000

To sell with 20% profit,

The selling price of 900 shawls = Rs. 37,80,000 + 20% of Rs. 37,80,000

$$= \text{Rs. } 37,80,000 + \text{Rs. } 7,56,000$$

$$= \text{Rs. } 45,36,000$$

Now, Rs. 130 = €1

$$\text{Or, Rs. } 1 = \frac{\text{€}1}{130}$$

$$\begin{aligned}\text{Rs. } 4536000 &= \frac{\text{€}1}{130} \times 4536000 \\ &= 34892.30\end{aligned}$$

∴ The selling price of all the shawls = €34892.30

Exercise 4

1.
 - a) What is meant by money exchange?
 - b) Which of the buying rate or selling rate does a Nepali student use to exchange Nepali Rupees into American Dollars while s/he is going for abroad study in America? Write.
2. By using the buying rate money exchange given above, change the following currency of different countries into Nepali currency:

a) Indian Rupees 1425	b) American Dollar 2000
c) Pound Sterling Dollar 4672	d) Australian Dollar 672
e) Saudi Arabian Riyal 1851	f) Qatari Riyal 2225
g) South Korean Wan 58,230	h) Hongkong Dollar 4512
i) Malaysian Ringgit 6725	j) Chinese Yuan 3450
3. By using the selling rate of money exchange, change Rs. 2,00,000 into the following currencies:

a) Australian Dollar	b) American Dollar
c) Euro	d) United Arab Emirates Dirham

- 4.** According to the exchange rate declared by Nepal Rastra Bank on a specific day, the buying and selling rate for 1 American Dollar are Rs. 127.50 and Rs. 128 respectively,
- How many American Dollars can be exchanged for Rs. 81,280?
 - Find how many Nepali Rupees can you exchange with American Dollar 600?
- 5.** According to the exchange rate declared by Nepal Rastra bank on 23 August 2022, the buying and selling rate for Chinese Yuan 1 are Rs. 18.64 and Rs. 18.73 respectively,
- How many Chinese Yuan can be exchanged for Rs. 37,460?
 - How many Nepalese rupees can you exchange with Chinese Yuan 5000?
- 6.** Using the above-mentioned abovementioned, answer the following questions:
- How many United Emirates Dirhams are equal to 1 Euro?
 - How many Malaysian Ringgits are equal to 1 Qatari Riyal?
 - How many Canadian Dollars are equal to 200 American Dollar.
 - How many Thai Bhatts are equal to 200 Pound Sterling?
- 7.** Sobita went to Australia from Nepal for her higher studies. After completing her studies, she has been working as a secondary level teacher. She earns 37 Australian Dollar per hour. She teaches 5 hours a day except Sunday. If exchange rate of the Australian Dollar 1 = Rs. 90,
- How much salary does she have in Nepali Rupees for a week?
 - Find how much monthly salary does she have in Nepali Rupees?
- 8.**
- An American businessman exchanged American Dollar for NRs. 12,40,000 at the exchange rate of American Dollar 1 = NRs. 124. After a week, Nepali currency is inflated by 10%. At that time, he exchanged the dollar into Nepali currency then, how much loss or profit have? Find it.
 - A businessman buys goods from Thailand. For this, he exchanged Nepali currency Rs. 7,20,000 at the exchange rate 1 Thai Bhatt = NRs. 3.60. After a day, Nepali currency is deflated by 5%. Due to the inadequate circumstances of the country, he wishes not to buy the goods from Thailand. So that at that time of deflation, if he again exchanged the Thai Bhatt into Nepali currency, how much loss or profit will he have? Find it.

9. a) A Nepali went to visit the UAE. In that time, he purchased a branded television of 75 inch for 10,000 Dirham. The exchange rate of that day was UAE Dirham 1 = NRs. 34.50. He paid 20% custom tax and 13 VAT. His relative wishes to buy it. At what price should he sell at break-even? Find it.
- b) 20% profit can be gained at a good sold for Indian rupees 21,600. At what Nepalese rupees can it be sold to gain 25% profit?
10. a) A Nepali does business in Australia. He came to Nepal for his household work. He wished to take 500 Pasmina shawls while returning back. He purchased the shawls at the rate of Rs. 3,500 each. He took it in Australia by paying 5% export tax. He paid Australian Dollar 30 for transportation from the airport to his residence. He sold the shawls at the rate of Australian Dollar 80 for each, find the loss or profit percent he got. (Australian dollar 1 = NRs. 88.50)
- b) The fair of air ticket from Kathmandu to Bangkok of Nepal Airlines is Rs. 28,000. Again, if the same ticket can be bought for 8000 Thai Bhat for Thailand, what percentage is it cheaper to buy the ticket from which place? (Thai Bhatt 1 = NRs. 3.50)
11. a) If the American Dollar $176 = 100$ Pound Sterling and Pound Sterling = Nepali Rupees 151, how many Nepali Rupees can be exchanged for 132 American Dollars?
- b) If I.C. $100 = \text{NRs. } 160$ and 1 American Dollar = NRs. 120, how many American Dollars are equal to I.C. 7500?
- c) If Chinese Yuan 1 = NRs. 18.70 and UAE Riyal 1 = NRs. 33.91, then change the Chinese Yuan 5612 into the United Arab Emirates Riyal.

Project work

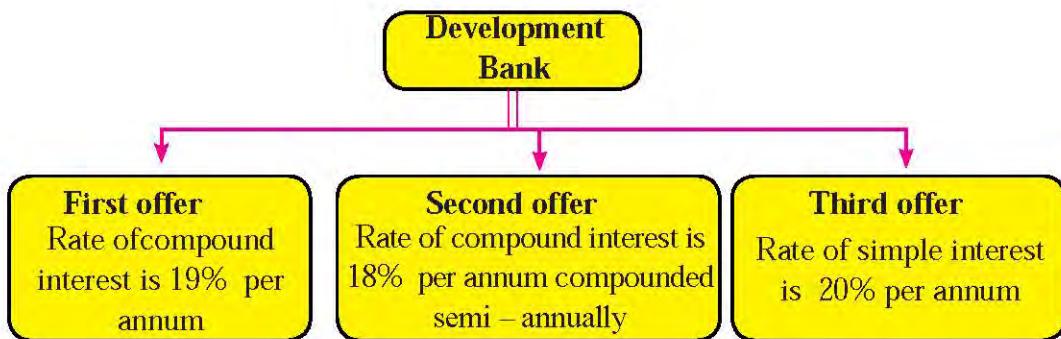
Make groups of suitable number of students. Collect exchange rates of the day searching from newspapers or the internet. Do you find difference in exchange rate of the day from the exchange rate of 2 days before? Besides, how much is the cost of gold per kilogram or per tola or per gram in the international market on that day? How much is in America? How much is in UK? How much is in Australia? How much is in the United Arab Emirates? On that day, in which city is the gold cheaper to buy in which city of which nation? Cheaper by how much. Prepare a report & present it to the class.

Answers

Mixed Exercise

- 1.** A family wants to deposit Rs. 4,00,000 in the account of a child aged 11 who has just started grade 11. The money is intended for his/her study. If the money is deposited with the agreement that the amount with principal and interest would be drawn after he/she passed grade 12 at the age of 19. Then,

 - a) How many times will the interest be calculated in 2 years according to the half yearly compound interest? Write.
 - b) Which of the following alternatives would you suggest to your parents to deposit the money and why? Justify with calculation.



2. A sum of money amounts to Rs. 12,000 and Rs. 13,200 in 2 years and 3 years respectively at a certain rate of interest compounded annually.
 - (a) Write the formula to find the compound interest.
 - (b) What should be the rate of the compound interest?
 - (c) What should be the principal?
3. A man bought a land at Rs. 80,00,000 on 25th Baisakh of 2075 BS and started construction of a house on the same day . The construction of the house completed at the cost of Rs. 2,70,00,000. The price of land increased at the rate of 20% per year and the price of house decreased at the rate of 20% per year.
 - (a) What does R indicate in the price after T years $(P_T) = P \left(1 + \frac{R}{100}\right)^T$?
 - (b) What will be the price of the land after 2 years?
 - (c) What will be the price of the house after 2 years?
 - (d) Will the prices of the land and house be the same after 2 years? If not, in how many years will the prices of the land and house be equal?
4. Ram exchanged some Nepali rupees with American dollars at the exchange rate of \$1=Rs. 110. After 5 days, Nepali currency devalued against American dollars by 10% and he made a profit of Rs. 33,000 by exchanging the same dollars into Nepali currency again.
 - (a) What is meant by exchange rate of currency?
 - (b) How much Nepalese rupees are equal to one American dollar (\$1) after devaluation of the Nepali currency?
 - (c) Find how much Nepali rupees did Ram exchange with American dollars at first?
 - (d) How much profit or loss would be there for him, if the Nepali rupees had revalued by 10% instead of devaluation of 10%?

5. The management committee of Nepal Bank Limited in its regular meeting has decided to change its annual policy slightly. According to the decision, the rate of interest for fixed deposit compounded semi annually is given below.

Depositing period	Rate of interest	Minimum deposit amount
Up to 6 months	6.75 %	Rs. 50,000
From 6 months to 1 year	7.25%	Rs. 50,000
From 1 year to 6 years	7.5%	Rs. 75,000

Sujit Thakur borrowed Rs. 2,00,000 from a cooperative one condition of paying it back in 2 years at the rate of simple interest of 5%. Immediately after borrowing, he deposited the same sum in a fixed deposit account of Nepal Bank Limited for the same duration.

- (a) Which rate of interest should Sujit Thakur deposit the money in the fixed deposit?
- (b) Find the compound amount that Sujit Thakur could receive in 2 years.
- (c) Find the total amount to be paid to cooperative in 2 years.
- (d) How much profit did Sujit Thakur earn in 2 years?

6. Anish has returned back Nepal from United Arab Emirate after 5 years. He earned 60,000 United Arab Emirate Dirhams and exchanged it with the Nepali rupees at the exchange rate of 1 Dirham = Rs. 34.83. He deposited the sum in the fixed deposit account of Nepal Bank for 1 year as per the following rate of interest compounded semi - annually.

Depositing Period	Rate of interest	Minimum deposit amount
Up to 6 months	9 %	Rs. 50,000
From 6 months to 1 year	10%	Rs. 50,000
From 1 year to 5 years	12%	Rs. 75,000

- (a) Which body of government decides the exchange rate of currency in our country?
- (b) How much Nepali rupees did Anish receive by exchanging 60,000 United Arab Emirate Dirhams?
- (c) Which deposit offer of the bank is applicable for Anish? Give reason.
- (d) How much amount will Anish receive from the bank after 1 year?

7. The price of a piece of land in Kathmandu was fixed at Rs. 60,00,000 at the end of 2020 AD. The price increased by 10% as the consequence of increasing buying pressure at beginning of 2021. But the price of land decreased by 4% due to the economic crisis at the end of 2022 AD.

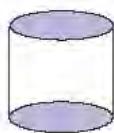
- (a) What does $P \left(1 + \frac{R}{100}\right)^T - P$ as per usual notation indicate?
- (b) What will be the price of the land at the end of 2022?
- (c) How much loss will be there to the person if the price decreased by 5% instead of decreasing by 4% in 2022 AD?
- 8.** **Kul Bahadur decided to go for foreign employment and lent a sum of Rs. 2,50,000 for 2 years at the interest rate of 15% compounded annually from a cooperative bank. After 1 year, he remitted 7,000 Ringgits to his home from the saving.**
- (a) Write the formula to find the compound amount.
- (b) What is the total amount to be paid after 1 year?
- (c) How much Nepali rupees are equal to Malaysian Ringgits 7,000?
- (d) How much total amount should he pay to clear all the debts finally?
- 9.** **A person lent a sum of Rs. 1,50,000 for 2 years at the interest rate of 10% per annum compounded annually. He paid Rs. 85,000 at the end of the first year.**
- (a) How much total amount should he pay to clear all the debts at the end of second year?
- (b) Find the total interest that he paid in two years.
- (c) If he cleared all the debts only after two years, how much more or less interest should have been paid?

Answers

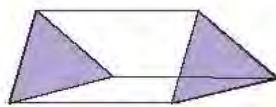
1. (a) 4 times (b) As the first offer
2. (a) $P \left[\left(1 + \frac{R}{100}\right)^T - 1 \right]$ (b) 10% (c) 9917.35
3. (a) Growth rate (b) Rs. 1,15,20,000 (c) Rs. 1,72,80,000 (d) No, 3 year
4. (b) Rs. 121 (c) Rs. 3,30,000 (c) Rs. 33,000 loss
5. (a) 7.5 % (b) Rs. 2,31,125 (c) Rs. 2,20,000 (d) Rs. 31,125
6. (a) Central Bank of Nepal (b) 20,89,800 (c) 10 % (d) Rs. 23,04,004.5
7. (a) increase (b) Rs. 63,36,000 (c) Rs. 66,000 (loss)
8. (b) Rs. 2,87,500 (c) Rs. 2,00,060 (d) Rs. 1,00,556
9. (a) Rs. 88,000 (b) Rs. 23,000 (c) Rs. 8,500 more

5.0 Review

Form different groups of students of proper size and each group should take a solid object as given below. Discuss the following questions after observing the objects and also present it the classroom.



(i)



(ii)



(iii)



(iv)

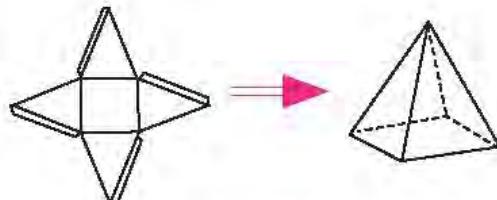
- (a) What are the curved surface area and total surface area of the cylinder of diagram (i), if the radius of base is 7cm and height is 10cm?
- (b) What is the volume of prism of diagram (ii), if the area of base is 24cm^2 and length is 8 cm?
- (c) What is the radius of the sphere of diagram (iii) , if its volume is 616cm^3 ?
- (d) What will be the ratio of the surface area of the sphere of diagram (iii) and the total surface area of hemisphere of diagram (iv) if their radii are equal?

5.1.1 Introduction of Pyramid

Activity 1

A figure of net of square based pyramid is given below. How can a pyramid be formed from a piece of paper?

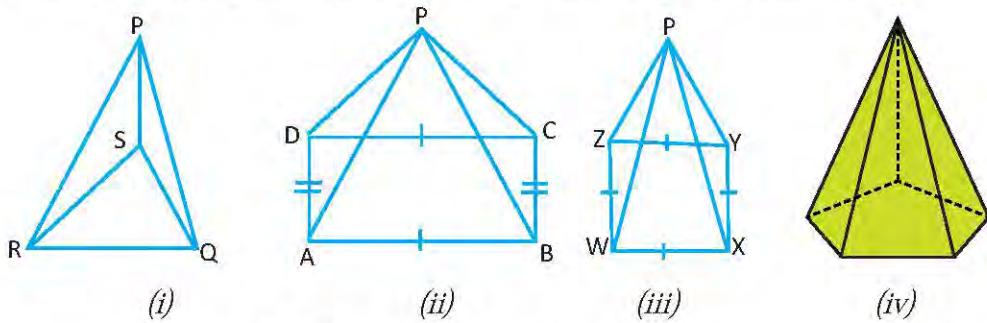
Take a rectangular piece of paper in pairs. Make a net of square based pyramid as given in the figure aside, and cut it with a scissor. Then, prepare a model of square based pyramid by joining all lateral surfaces with glue stick so that they all meet at a common vertex. Touch different parts of the model object and ask your friends the following questions.



- (a) How many surfaces are there?
- (b) What are the shapes of the surfaces?
- (c) How many vertices are there?
- (d) In what way are the positions of surfaces are formed?

Activity 2

Form different groups containing proper number of students. In each group, take one solid object made from a paper or glass or wood. Discuss the following questions by observing the objects and also present it in your classroom.



- (a) What are the shapes of the bases?
- (b) Are there any surfaces other than bases? What are their shapes?
- (c) How many vertices are there?
- (d) Which surfaces are congruent?

The given solid objects are the images of pyramid. All the pyramids have polygonal base and besides this, there are also triangular lateral surfaces. The lateral surfaces of right pyramids with regular polygonal bases are congruent. The lateral surfaces have a common vertex. The vertical height is perpendicular to the surface of the base.

The three dimensional solid object having polygonal base and one common vertex to the lateral surfaces is called pyramid. Pyramids are given different names according to the types of polygonal base. For example: triangular based pyramid, square based pyramid.

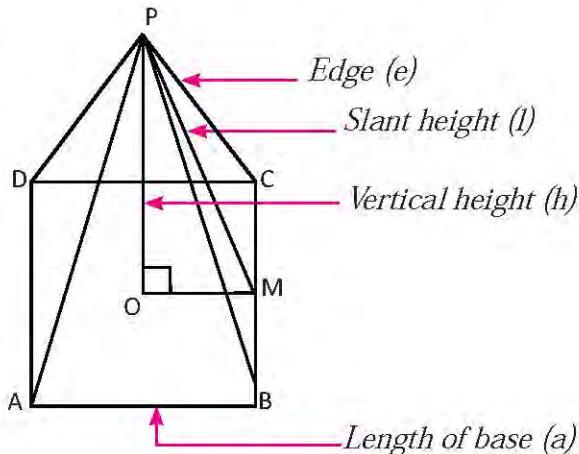
5.1.2 Different parts of pyramid and their relationship

Activity 3

A diagram of square based pyramid is given below. Study the given diagram and discuss the following questions:

Here, 'e', 'l', 'h' and 'a' denote the edge, slant height, vertical height and the length of edge of base respectively.

- What are the different parts of pyramid?
- What does the vertical height of a pyramid mean?
- What is slant height called?
- What is the edge of a pyramid?
- What is the relation between edge, slant height, vertical height and the length of edge of the base?



Now, from the diagram,

In the right angled triangle POM,

$$(PM)^2 = (PO)^2 + (OM)^2 \quad [\because \text{According to the Pythagoras theorem}]$$

$$l^2 = h^2 + \left(\frac{a}{2}\right)^2 \quad [OM = \frac{1}{2} AB = \frac{1}{2} a]$$

Similarly, in the right angled triangle PMC,

$$(PC)^2 = (PM)^2 + (MC)^2 \quad [\because \text{According to the Pythagoras theorem}]$$

$$e^2 = l^2 + \left(\frac{a}{2}\right)^2 \quad [MC = \frac{1}{2} BC = \frac{1}{2} a]$$

5.1.3 Surface Area of Pyramid

Activity 4

A pyramid and also its net are given in the diagram below, where ' l ' is the slant height and ' a ' is the length of the edge of base.

Now, discuss the following questions.

- How many triangular surfaces are there in the pyramid?
- Do all triangular surfaces have equal area?
- What are the shapes of surfaces in the pyramid and what are their area?

Since, the base of the pyramid is square.

$$\begin{aligned}\text{So, the area of base} &= \text{area of square} \\ &= (\text{length of side})^2 = l^2\end{aligned}$$

In the pyramid, the lateral surfaces are triangular.

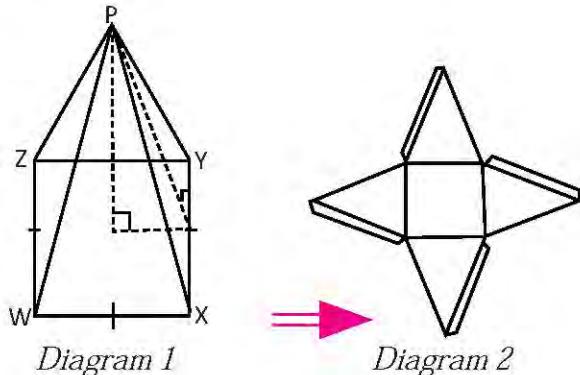
$$\begin{aligned}\text{Therefore, the area of one triangular surface} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times a \times l \quad [\because \text{The slant height of pyramid is the vertical height of the triangle}]\end{aligned}$$

There are 4 triangular lateral surfaces with equal area in the pyramid.

$$\begin{aligned}\text{So, the area of lateral surface area of pyramid} &= 4 \left(\frac{1}{2} \times a \times l \right) \\ &= 2al\end{aligned}$$

Lateral surface area of square based pyramid (LSA) = $2al$

$$\begin{aligned}\text{Total surface area (TSA)} &= \text{Area of base} + \text{Area of lateral surface} \\ &= (a^2 + 2al) = a(a + 2l)\end{aligned}$$



Example 1

Find the area of lateral and total surfaces of a square based pyramid if the length of side (a) = 8 cm and slant height (l) = 3 cm.

Solution

Here, the length of the side of base (a) = 8 cm

Slant height (l) = 3 cm

We know that the area of base = $a^2 = (8 \text{ cm})^2 = 64 \text{ cm}^2$

Lateral surface area (LSA) = $2al = 2 \times 8 \times 3 = 48 \text{ cm}^2$

Again, the total surface area = Area of base + Lateral surface area

$$= 64 \text{ cm}^2 + 48 \text{ cm}^2 = 112 \text{ cm}^2$$

Hence, the surface area of the pyramid = 112 cm^2 .

Example 2

Find the total surface area of a square based pyramid with the length of the side of base is 10 cm and the length of the edge is 13 cm.

Solution

In the square based pyramid,

Length of the side of base (a) = 10 cm

Length of the edge (e) = 13 cm

Total surface area = ?

We know, $e^2 = l^2 + \left(\frac{a}{2}\right)^2$

$$\text{or, } (13)^2 = l^2 + \left(\frac{10}{2}\right)^2$$

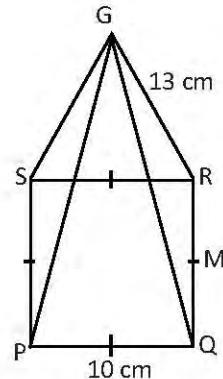
$$\text{or, } 169 = l^2 + 25$$

$$\text{or, } 169 - 25 = l^2$$

$$\text{or, } 144 = l^2$$

$$\therefore l = 12 \text{ cm}$$

$$\therefore \text{The slant height } (l) = 12 \text{ cm}$$



Again, we know that the area of base (A) = $a^2 = (10)^2 = 100 \text{ cm}^2$

Lateral surface area (LSA) = $2al = 2 \times 10 \times 12 = 240 \text{ cm}^2$

Then, the total surface area = Area of base (A) + Lateral surface area

$$= 100 + 240 = 340 \text{ cm}^2$$

Hence, the total surface of the pyramid is 340 cm^2 .

Example 3

Find the vertical height and the length of the edge of a square based pyramid if its total surface area is 144 cm^2 and slant height is 5 cm.

Solution

Here, in the square based pyramid,

Total surface area (TSA) = 144 cm^2

Slant height (l) = 5 cm

Vertical height (h) = ?

Length of the edge (e) = ?

We know that,

Total surface area = $a^2 + 2al$

$$\text{or, } 144 = a^2 + 2a \times 5$$

$$\text{or, } 144 = a^2 + 10a$$

$$\text{or, } a^2 + 10a - 144 = 0$$

$$\text{or, } a^2 + (18 - 8)a - 144 = 0$$

$$\text{or, } a^2 + 18a - 8a - 144 = 0$$

$$\text{or, } a(a + 18) - 8(a + 18) = 0$$

$$\text{or, } (a + 18)(a - 8) = 0$$

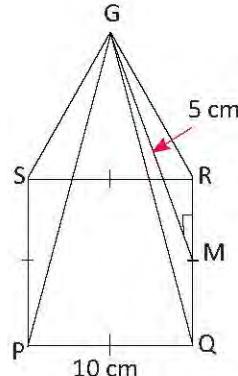
$$\text{either, } a + 18 = 0 \therefore a = -18$$

[\because length cannot be negative.

$$\text{or, } a - 8 = 0 \therefore a = 8$$

So, a cannot be equal to -18]

Therefore length of side of base (a) = 8 cm



$$\text{Again, } l^2 = h^2 + \left(\frac{a}{2}\right)^2$$

$$\text{or, } 5^2 = h^2 + \left(\frac{8}{2}\right)^2$$

$$\text{or, } 25 = h^2 + 16$$

$$\text{or, } 25 - 16 = h^2$$

$$\text{or, } 9 = h^2$$

$$h = 3 \text{ cm}$$

∴ The vertical height (h) = 3 cm

$$\text{And, } e^2 = l^2 + \left(\frac{a}{2}\right)^2$$

$$\text{or, } e^2 = 5^2 + \left(\frac{8}{2}\right)^2$$

$$\text{or, } e^2 = 25 + 16$$

$$\text{or, } e^2 = 41$$

$$\therefore e = \sqrt{41} \text{ cm}$$

Hence, the vertical height (h) and the length of the edge of the pyramid are 3 cm and $\sqrt{41}$ cm respectively.

Example 4

The figure given alongside is a square based pyramid. The length of base is 12 cm and the lateral surface area is 240 cm^2 . Find the slant height and vertical height.

Solution

Here, length of the base (a) = 12 cm

Lateral surface area (LSA) = 240 cm^2

Vertical height (h) = ?

Slant height (l) = ?

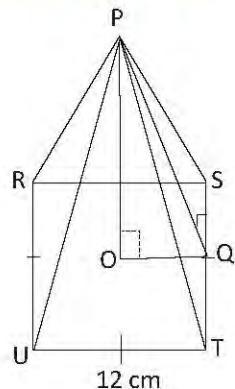
We know,

Lateral surface area = $2al$

$$\text{or, } 240 = 2 \times 12 \times l$$

$$\text{or, } \frac{240}{2 \times 12} = l$$

$$\therefore l = 10 \text{ cm}$$



$$\text{Now, } l^2 = h^2 + \left(\frac{a}{2}\right)^2$$

$$\text{or, } (10)^2 = h^2 + \left(\frac{12}{2}\right)^2$$

$$\text{or, } 100 = h^2 + 36$$

$$\text{or, } 100 - 36 = h^2$$

$$\text{or, } 64 = h^2$$

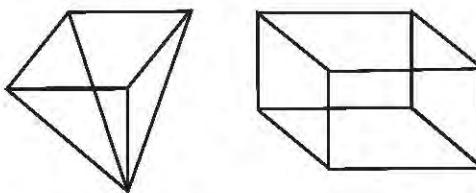
$$\therefore h = 8 \text{ cm}$$

Hence, the slant height (l) = 10 cm and the vertical height (h) = 8 cm.

5.1.4 Volume of Pyramid

Activity 1

Using card paper, construct a square based pyramid and a cube having equal base area and height. Construct the models so that we could keep sand or flour in them. Estimate how many times it should be poured from the pyramid into the cube to fill it up completely. Test whether your estimation is correct or not and also present the conclusion drawn to your classroom.



Here, we notice that the cube is filled with sand or flour after pouring 3 times from the pyramid.

Hence, $3 \times$ Volume of the square based pyramid = volume of cube

Or, volume of the square based pyramid = $\frac{1}{3}$ volume of cone

= $\frac{1}{3}$ area of base \times height

If 'a' is the length of base and 'h' is the height of the square based pyramid, then
its volume = $\frac{1}{3}$ area of base \times height

$$= \frac{1}{3} \times A \times h = \frac{1}{3} \times a^2 \times h$$

Example 5

Find the volume of square based pyramid in the given figure alongside.

Solution

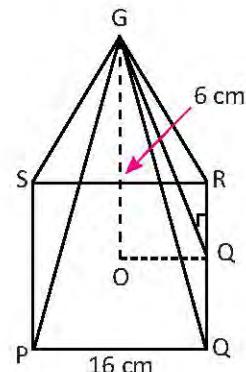
Length of the side of base (a) = 16 cm

Vertical height (h) = 6 cm

Volume of pyramid (V) = ?

We know, area of base (A) = $a^2 = (16)^2 = 256 \text{ cm}^2$

Now, volume of pyramid (V) = $\frac{1}{3} \times A \times h = \frac{1}{3} \times 256 \times 6 = 512 \text{ cm}^3$



Example 6

The volume of a square based pyramid is 384 cm^3 and the length of the side of base is 12 cm. Find the lateral surface area of the pyramid.

Solution

In the square based pyramid,

The volume of pyramid (V) = 384 cm^3

Length of the side of base (a) = 12 cm

Lateral surface area = ?

We know that, volume of the pyramid (V) = $\frac{1}{3} \times a^2 \times h$

$$\text{or, } 384 = \frac{1}{3} \times (12)^2 \times h$$

$$\text{or, } 384 = \frac{1}{3} \times 144 \times h$$

$$\text{or, } \frac{384}{144} \times 3 = h$$

$$\therefore h = 8 \text{ cm}$$

Again, $P = h^2 + \left(\frac{a}{2}\right)^2$

$$\text{or, } P = 8^2 + \left(\frac{12}{2}\right)^2$$

$$\text{or, } P = 64 + 36$$

$$\text{or, } P = 100 \text{ cm}^2$$

$$\therefore I = 10 \text{ cm}$$

Then, lateral surface area (LSA) = $2aI = 2 \times 12 \times 10 = 240 \text{ cm}^2$.

Example 7

Find the volume of a square based pyramid if its total surface area is 96 cm^2 and the length of the side of base is 6 cm.

Solution

In the square based pyramid,

$$\text{Total surface area (TSA)} = 96 \text{ cm}^2$$

$$\text{Length of the side of base (a)} = 6 \text{ cm}$$

$$\text{Volume of pyramid (V)} = ?$$

According to the formula,

$$\text{Total surface area (TSA)} = a^2 + 2al$$

$$\text{or, } 96 = 6^2 + 2 \times 6 \times l$$

$$\text{or, } 96 - 36 = 12 \times l$$

$$\text{or, } 60 = 12 \times l$$

$$\text{or, } l = \frac{60}{12}$$

$$\therefore l = 5 \text{ cm}$$

$$\text{And, } l^2 = h^2 + \left(\frac{a}{2}\right)^2$$

$$\text{or, } 5^2 = h^2 + \left(\frac{6}{2}\right)^2$$

$$\text{or, } 25 = h^2 + 9$$

$$\text{or, } 25 - 9 = h^2$$

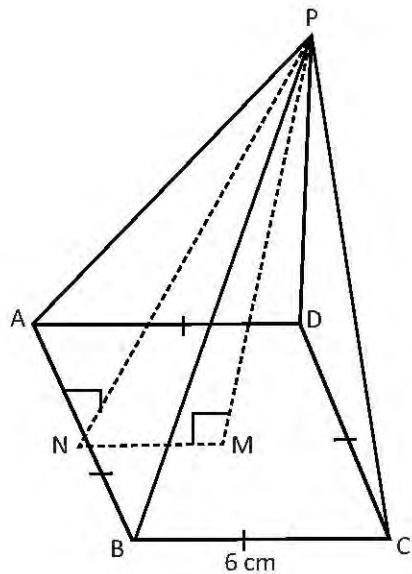
$$h^2 = 16$$

$$\therefore h = 4 \text{ cm}$$

Then, the volume of pyramid (V) = $\frac{1}{3}$ area of base \times height

$$= \frac{1}{3} \times 6^2 \times 4 = 48 \text{ cm}^3$$

Hence, volume of the pyramid is (V) = 48 cm^3 .

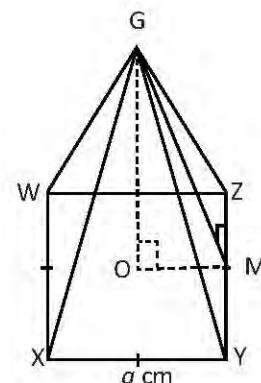


Exercise 5.1

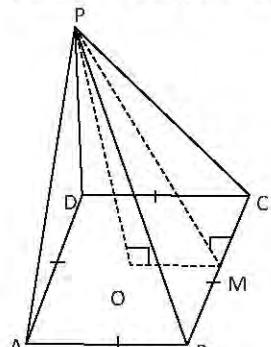
1. Study the given diagram of a square based pyramid and answer the following questions.

- (a) What do GO and GM indicate in the pyramid?
- (b) If $XY = a \text{ cm}$, then what is the length of OM ?
- (c) If the vertical height = h , slant height = l and the side of base = a , write their mathematical relation.
- (d) What do the length GZ , GY , GX and GW represent? Are they equal to one another?

2. (a) Write the total surface area of a square based pyramid in which the side of base is ' m ' cm and slant height is ' n ' cm.
- (b) What is the lateral surface area of a square based pyramid if the length of the side of base is ' p ' cm and slant height is ' q ' cm?
- (c) Write the volume of square based pyramid in terms of r and h , if the length of the side of base is ' r ' cm and the vertical height is ' h ' cm.
- (d) Find out the area of triangular surfaces of a square based pyramid, if its area of base is ' z ' square unit and total surface area is ' y ' square unit.
3. (a) Find the volume of a square based pyramid in which the area of base is 64 cm^2 and height is 15 cm.
- (b) The total surface area of a square based pyramid is 285 cm^2 and the area of its triangular surfaces is 192 cm^2 . Find the area of base of the pyramid.
- (c) The diagrams of square based pyramids are given below:

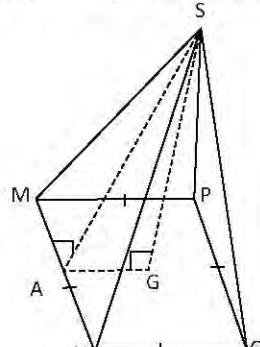


(i) If $PO = 8 \text{ cm}$ and $OM = 12 \text{ cm}$,



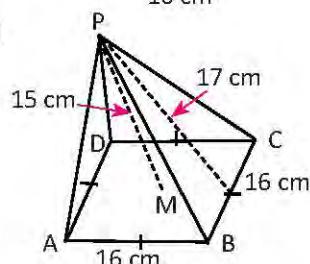
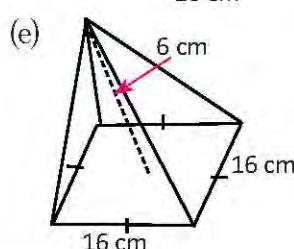
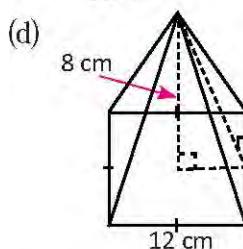
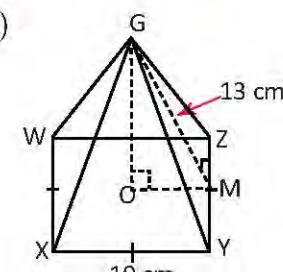
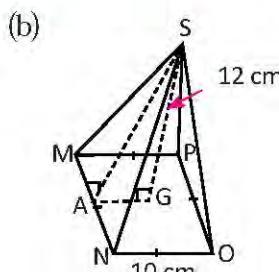
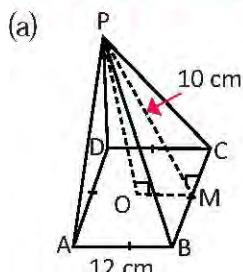
What are values of slant height and length of edge?

(ii) If $SA = 13 \text{ cm}$ and $NO = 10 \text{ cm}$,



What are the values of vertical height and length of the edge?

4. Find the total surface area and volume of the square based pyramid from the given measurements.



5. Sophiya has a pot with the shape of square based pyramid. She has filled it with water. Find how much water it can contain, if its height is 25 cm and the length of base is 30 cm.
6. The total surface area of a square based pyramid is 800 cm^2 and its side of base is 16 cm. Find the area of triangular surfaces and volume of the pyramid.
7. The total surface area of an aquarium with the shape of square based pyramid is 400 sq.inch. and its slant height is 15 inch. Then, find the volume of the aquarium.

8. A group of tourists visiting Egypt reached at a place where a pyramid was located. Their guide described that the area of the base of the pyramid is 3600 m^2 and its height is 50 m. On the basis of this information, a mathematics expert in the group was able to find its total surface area. Find the value that he obtained.
9. **A hotel is planning to manage a tent with a shape of a square based pyramid for a group of tourists from or foreign country in which, the length of each side of base is 32 m and height is 50 m.**
- How much clothes in square meter is required to construct the tent?
 - What will be the total cost to construct the tent if the cost of cloth per square metre is Rs. 500?
10. **The figure given alongside is a telephone tower of Nepal Telecommunication Corporation (NTC) built in Asharam's field. The base of the tower is square shaped . Its slant height is 50 ft. and vertical height is 40 ft.**
- (a) What is the area of the land occupied by the tower?
(b) If the rent of the field is Rs. 50 per sq.ft, find the total amount of rent for 20 years to be paid by NTC.
11. The figure given aside is a chocolate in the shape of a pyramid with a square base. The total surface area of the chocolate is 1920 cm^2 and length of the base is 30 cm. Find its vertical height.



Project work

Using chart paper, prepare the models of square based pyramids with different measurements. Measure the various parts of the pyramids and find out the value of following.

- Area of base
- Area of triangular surfaces
- Total surface area
- Volume

Also, present the results in the classroom.

Answers

1. Show to your teacher.
2. (a) $(m^2 + 2mn)$ cm² (b) $2pq$ cm²
(b) $\frac{1}{3} \pi r^2 h$ cm³ (c) $(y - z)$ sq. unit
3. (a) 320 cm³ (d) 93 cm²
(c) (i) $4\sqrt{13}$ cm and $4\sqrt{22}$ cm (d) 12 cm, $\sqrt{194}$
4. (a) 384 cm² and 384 cm³ (b) 360 cm² and 400 cm³
(c) 360 cm² and 400 cm³ (d) 384 cm² and 384 cm³
(e) 576 cm² and 512 cm³ (f) 800 cm² and 1280 cm³
5. 7.5 l
6. 544 cm² and 1280 cm³
7. 471.40 inch³
8. 10597.14 cm²
9. 3359.84 cm² and Rs. 1679923.81
10. (a) 3600 sq.ft (b) Rs. 36,00,000 11. 8 cm

5.2.1 Introduction to cone

Activity 6

There is a cone of ice cream and a birthday cap in the figure.



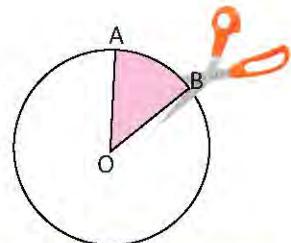
Do they have same shapes? What are the shapes of these solid objects?

Cut the birthday cap with scissors as shown in the figure. Then, spread it on the table. Which shape that changed into?

The curved surface part of cone changes into sector after spreading it on the flat surface.

Activity 7

- Draw a circle on a rectangular piece of paper.
- Draw an angle $\angle AOB$ at the centre and cut the sector AOB as shown as in the figure.
- Now, bend the sector and join AO and BO.



Here, a cone with the circular base is formed in which the circumference of circular base is equal to arc AB.

Activity 8

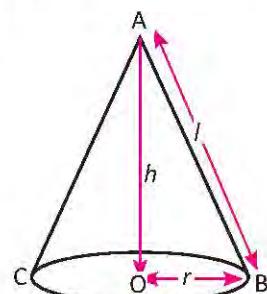
A cone is shown in the figure given below where, the radius of the circular base is OB = r cm. The line AO, joining the vertex A and the center O of circular base, is the height (h). And, the lines AC and AB are known as slant height (l).

Now, discuss the relationship between the radius, height and slant height.

In the right angled triangle AOB,

$$(AB)^2 = (AO)^2 + (OB)^2 \quad [\because \text{From Pythagorus theorem}]$$

$$\begin{aligned} l^2 &= h^2 + r^2 \\ \therefore l &= \sqrt{h^2 + r^2} \end{aligned}$$



The slant height is equal to the square root of the sum of squares of height and radius of circular base, i.e. $l = \sqrt{h^2 + r^2}$.

Surface Area of Cone

Activity 9

Take a cone made from a paper. Cut it with scissor and spread it as shown in the figure (i). Now, the curved surface of cone is transformed into flat form as shown in the figure, which is a sector. And the length of this sector is equal to the circumference ($2\pi r$) of the circular base of the cone.

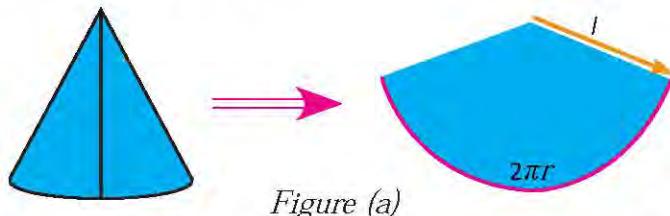


Figure (a)

Now, cut the sector into 4 equal pieces as shown in the figure (ii) and color two pieces with red.

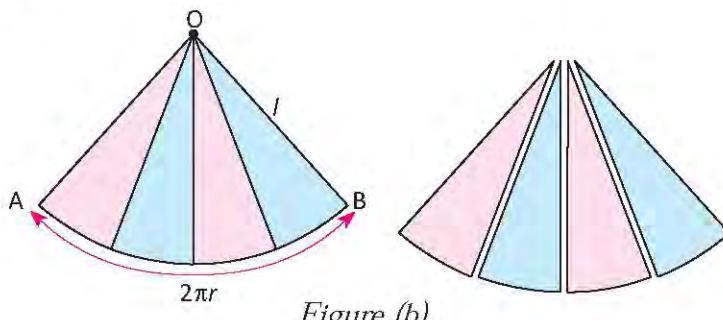


Figure (b)

Then, arrange two small sectors in the same directions and remaining two in opposite directions as shown in figure (iii).

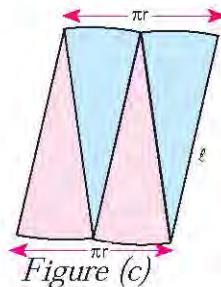


Figure (c)

Here,

If the radius of base of the cone is ' r ', slant height is ' l ' and the vertical height is ' h ', then,

$$\begin{aligned}
 \text{Curved surface area of the cone (LSA)} &= \text{Area of parallelogram} \\
 &= \text{length} \times \text{breadth} \\
 &= \pi r \times l = \pi r l
 \end{aligned}$$

$$\begin{aligned}
 \text{Total surface area of the cone (TSA)} &= \text{Area of base} + \text{Curved surface area (CSA)} \\
 &= \pi r^2 + \pi r l \\
 &= \pi r(r + l)
 \end{aligned}$$

If the radius of base of the cone is 'r', slant height is 'h' and the vertical height is 'h', then

$$\text{Curved surface area (CSA)} = \pi r \times l = \pi r l$$

$$\text{Total surface area (TSA)} = \pi r^2 + \pi r l = \pi r(r + l)$$

Example 1

Find the area of base and curved surface area of the cone given below.

Solution

Here, the vertical height of the cone (h) = 12 cm

Slant height of the cone (l) = 13 cm

Area of base of the cone = ?

Curved surface area of the cone = ?

Radius of base = r cm

In the figure,

From right angled triangle POQ,

$$PQ^2 = PO^2 + OQ^2 \quad [\because h^2 = p^2 + b^2]$$

$$\text{or, } 13^2 = 12^2 + r^2$$

$$\text{or, } 169 - 144 = r^2$$

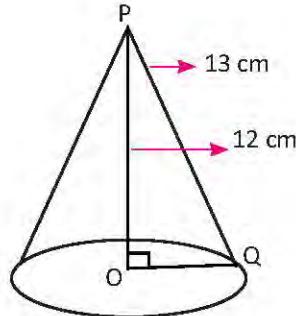
$$\text{or, } r^2 = 25$$

$$\therefore r = 5 \text{ cm}$$

We know that,

$$\text{The area of base} = \pi r^2 = \frac{22}{7} \times (5)^2 = 78.57 \text{ cm}^2$$

$$\text{Curved surface area of the cone} = \pi r l = \frac{22}{7} \times 5 \times 13 = 204.28 \text{ cm}^2$$



Example 2

Find the total surface area of a cone, if the diameter of base is 12 cm and the height is 8 cm.

Solution

Here, diameter of the base of the cone (d) = 12 cm

$$\text{Radius of the base of the cone } (r) = \frac{d}{2} = \frac{12}{2} = 6 \text{ cm}$$

Vertical height of the cone (h) = 8 cm

Total surface area of the cone = ?

From the figure,

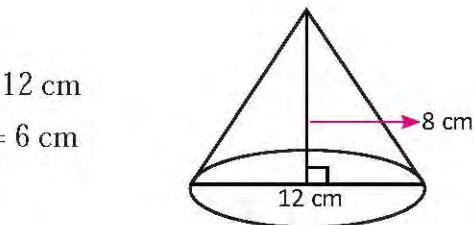
$$l^2 = h^2 + r^2$$

$$\text{or, } l^2 = 8^2 + 6^2$$

$$\text{or, } l^2 = 64 + 36$$

$$\text{or, } l^2 = 100 \quad \therefore l = 10 \text{ cm}$$

Now, the total surface area of the cone



$$\begin{aligned} &= \pi r(r + l) \\ &= \frac{22}{7} \times 6 \times (6 + 10) \\ &= 301.71 \text{ cm}^2 \end{aligned}$$

Example 3

The sum of radius of base and slant height of a cone is 64 cm. If its total surface area is 2816 cm^2 , find its curved surface area.

Solution

Here, total surface area (TSA) = 2816 cm^2

By the question,

Radius of base (r) + slant height (l) = 64 cm

$$\therefore r + l = 64$$

Curved surface area of the cone (CSA) = ?

We know, total surface area of the a cone = $\pi r(r + l)$

$$\text{or, } 2816 = \frac{22}{7} \times r \times 64$$

$$\text{or, } 2816 \times 7 = r \times 22 \times 64$$

$$\text{or, } r = \frac{2816 \times 7}{22 \times 64}$$

$$\therefore r = 14 \text{ cm}$$

Putting the value of r in equation (i),

$$14 + l = 64$$

$$\text{or, } l = 64 - 14 = 50 \text{ cm}$$

Now, the curved surface area of the cone $= \pi r l = \frac{22}{7} \times 14 \times 50 = 2200 \text{ cm}^2$

Example 4

Find the length of circumference of base of a cone, if its total surface area and curved surface area are 1320 cm^2 and 704 cm^2 .

Solution

Here, total surface area (TSA) $= 1320 \text{ cm}^2$

Curved surface area (CSA) $= 704 \text{ cm}^2$

Length of circumference of base (C) $= ?$

We know,

Total surface area of a cone $=$ Area of base $+$ curved surface area

$$\text{or, } 1320 = \pi r^2 + 704$$

$$\text{or, } 1320 - 704 = \frac{22}{7} r^2$$

$$\text{or, } \frac{616}{22} \times 7 = r^2$$

$$\text{or, } r^2 = 196$$

$$\therefore r = 14 \text{ cm}$$

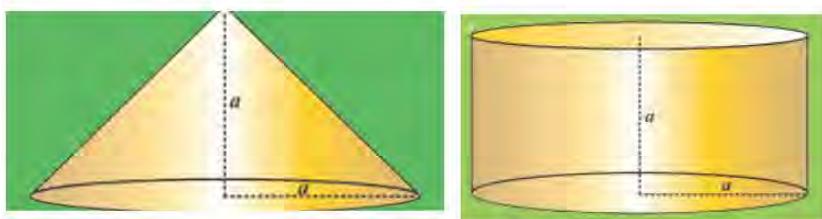
Now, the length of circumference (C) $= 2\pi r = 2 \times \frac{22}{7} \times 14 = 88 \text{ cm}$

Therefore, the length of circumference of the base of the cone is 88 cm.

5.2.2 Volume of Cone

Activity 10

Form different group of student of proper size. Using chart paper, make a cylinder with equal radius of base and height (suppose) in each group. Also, make a cone with the equal height and radius of the base as that of a cylinder.



Fill the cone with sand or dust of soil and pour it into the cylinder until it is filled up completely.

- How many times it should be poured from the cone to fill up the cylinder?
- Is it filled in three times?

Definitely, the cylinder will be filled up after pouring three times from the cone.

$$\begin{aligned}\text{So, volume of the cone } (V) &= \frac{1}{3} \text{ volume of cylinder} \\ &= \frac{1}{3} \times A \times h = \frac{1}{3} \pi r^2 h \quad [\text{Volume of a cylinder} = \pi r^2 h]\end{aligned}$$

Example 5

Find the volume of cone given aside.

Solution

Here, vertical height of cone (h) = 24 cm

Slant height of cone (l) = 26 cm

Volume of cone (V) = ?

In the figure,

Since, triangle MAN is a right angled triangle, from Pythagoras theorem,

$$AN = \sqrt{MN^2 - MA^2}$$

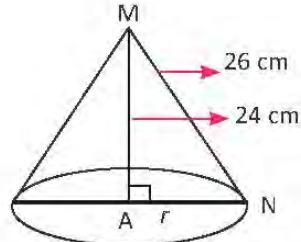
$$\text{or, } r = \sqrt{26^2 - 24^2}$$

$$\text{or, } r = \sqrt{676 - 576}$$

$$\therefore r = 10 \text{ cm}$$

We know,

$$\text{Volume of cone } (V) = \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (10)^2 \times 24 = 2514.28 \text{ cm}^3$$



Example 6

If the curved surface area of a cone is 1158.3 cm^2 and slant height is 19.5 cm , find its volume.

Solution

Here, curved surface area (CSA) = 1158.3 cm^2

Slant height (l) = 19.5 cm

Volume of cone (V) = ?

We know,

Curved surface area of cone = $\pi r l$

$$\text{or, } 1158.3 \text{ cm}^2 = \frac{22}{7} \times r \times 19.5 \text{ cm}$$

$$\text{or, } \frac{1158.3}{22 \times 19.5} \times 7 \text{ cm} = r$$

$$\text{or, } r = 18.5 \text{ cm}$$

$$\text{Now, height (h)} = \sqrt{l^2 - r^2} = \sqrt{(19.5)^2 - (18.5)^2} = 4.8 \text{ cm}$$

Again, we know that,

$$\text{Volume of cone is (V)} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (18.5)^2 \times 4.8 = 1796.25 \text{ cm}^3$$

Hence, the volume of cone (V) = 1796.25 cm^3

Example 7

The ratio of radius of base and height of a cone with volume 314.86 cm^3 is 5:12. Find its curved surface area and total surface area.

Solution

Here, the ratio of radius of base and height is 5:12.

Let radius (r) = $5x$ and height (h) = $12x$

Volume (V) = 314.86 cm^3

Curved surface area = ?

Total surface area = ?

We know,

$$\text{Volume of cone } (V) = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (5x)^2 \times 12x$$

$$\text{or, } 314.86 \times 21 \text{ cm}^3 = 22 \times 25x^2 \times 12x$$

$$\text{or, } \frac{314.86 \times 21}{22 \times 25 \times 12} = x^3$$

$$\therefore x = 1.00060 \text{ cm} = 1 \text{ cm}$$

So, radius (r) = $5 \times 1 \text{ cm} = 5 \text{ cm}$ and height (h) = $12 \times 1 \text{ cm} = 12 \text{ cm}$

$$\begin{aligned}\text{Slant height } (l) &= \sqrt{h^2 + r^2} = \sqrt{(12)^2 + (5)^2} \\ &= \sqrt{144 + 25} = \sqrt{169} = 13 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{We know that the curved surface area (CSA)} &= \pi r l = \frac{22}{7} \times 5 \times 13 \\ &= 204.28 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Total surface area of cone (TSA)} &= \pi r(r + l) = \frac{22}{7} \times 5(5 + 13) \\ &= 282.85 \text{ cm}^2\end{aligned}$$

Exercise 5.2

1. What is the shape of the surface of base in given figure? Answer the following questions on the basis of information given in the figure:

(a) What do the symbols l , h and r represent?

(b) Write the mathematical relationship of l , h and r .

(c) What is the value of h , if $l = 13 \text{ cm}$ and $r = 5 \text{ cm}$?

2. (a) Write the curved surface area of a cone, if the radius of base is ' r ' cm and slant height is ' l ' cm.

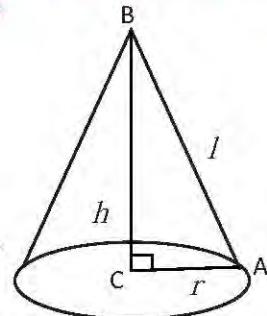
(b) Write the total surface area of cone in terms of x and y , if the radius of base is ' x ' cm and slant height is ' y ' cm.

(c) What is the volume of a cone, if the diameter of base is ' p ' cm and vertical height is ' q ' cm.

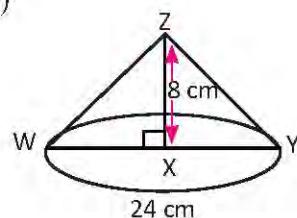
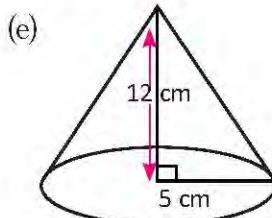
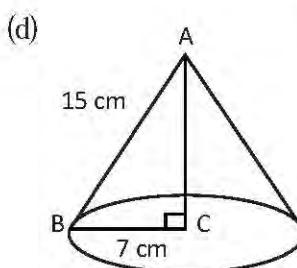
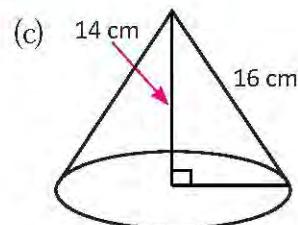
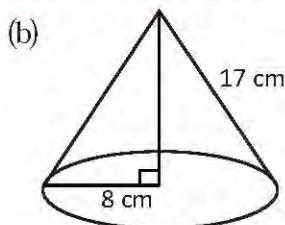
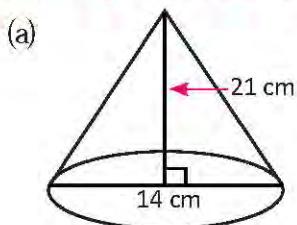
3. (a) Find the volume of a cone, if its curved surface area is 81 cm^2 and height is 15 cm .

(b) The total surface area of a cone is 250 cm^2 and the area of base is 118 cm^2 . Find its curved surface area.

(c) Find the volume of a cone if its slant height is 5 cm and radius of base is 4 cm .



- 4. Find the curved surface area, total surface area and volume of the following cones on the basis of given information.**



5. If the volume of a right circular cone is $100\pi \text{ cm}^3$ and height is 12 cm, find its slant height.
6. Find the radius of base and height of conical shaped tent if its volume is 1232 cm^3 and the area of base is 154 cm^2 .
7. The amount of plastic required to construct a conical shaped tent with slant height 14 cm is 77 m^2 . Find the area and perimeter of the base of tent.
8. Find the volume, curved surface area and total surface area of a cone with the diameter of base 8 cm and vertical height 21 cm.
9. The vertical height of a solid cone is three times the diameter of the base. Find its total surface area if the volume is $54\pi \text{ cm}^3$.
10. A student studying in class 10 has prepared a cone made of wood as a task of his project work. The area of base and height of the cone are 154 m^2 and 14 m respectively, how much does it cost to color the surface of the cone excluding the base, if the cost per square meter is Rs. 1.50?
11. A cone shaped pot is filled with water. The height of the pot is 21 cm and the diameter of base is 14 cm. When two solid cones with equal dimensions are immersed into the pot, one third of water flows outside from the pot. If so, what is the volume of each cone immersed in the pot?

12. Three solid cones having the same height and of radii 3 cm, 4 cm and 5 cm respectively are melted together and a big single solid cone is made. Find the diameter of the cone if its height is equal to the height of the small one.
13. A toy of conical shape is made from a wood and the diameter of its base is 10 cm long. It costs Rs. 880 to color its whole surface at the rate of Rs. 4 per square cm. Find the volume of the pot.

Project work

Using papers of different sizes, make cones of different dimensions. Measure the dimensions of the cones and find the area of the base, curved surface area, total surface area and volume of the cones. Also, present it in the classroom.

Answers

- | | | | |
|-----|---|---------------------------------|---|
| 1. | (a) $l^2 = h^2 + r^2$ | (b) $h = 12 \text{ cm}$ | |
| 2. | (a) $\pi mn \text{ cm}^2$ | (b) $\pi x(x + y) \text{ cm}^2$ | (c) $\frac{1}{12} \pi p^2 q \text{ cm}^3$ |
| 3. | (a) 405 cm^3 | (b) 132 cm^2 | (c) $16\pi \text{ cm}^3$ |
| 4. | (a) $486.99 \text{ cm}^2, 640.99 \text{ cm}^2, 1078 \text{ cm}^3$ | | |
| | (b) $136\pi \text{ cm}^2, 200\pi \text{ cm}^2, 320\pi \text{ cm}^3$ | | |
| | (c) $123.94\pi \text{ cm}^2, 183.94\pi \text{ cm}^2, 280\pi \text{ cm}^3$ | | |
| | (d) $330 \text{ cm}^2, 484\pi \text{ cm}^2, 681.01 \text{ cm}^3$ | | |
| | (e) $65\pi \text{ cm}^2, 90\pi \text{ cm}^2, 100\pi \text{ cm}^3$ | | |
| | (f) $156\pi \text{ cm}^2, 300\pi \text{ cm}^2, 240\pi \text{ cm}^3$ | | |
| 5. | 13 cm | 6. 7 cm, 24 cm | 7. $9.62 \text{ m}^2, 11\text{m}$ |
| 8. | $352 \text{ cm}^3, 268.54 \text{ cm}^2, 318.80 \text{ cm}^2$ | 9. 200.34 cm^2 | 10. Rs. 516.53 |
| 11. | 179.67 cm^3 | 12. $10\sqrt{2} \text{ cm}$ | 13. 195.99 cm^3 |

5.3.1 Combined Solid Object

Activity 11

Observe the following figures and discuss the questions given below.



(i)



(ii)



(iii)

(a) What shapes do the given figures contain?

(b) What types of solid objects are these?

Here, (i) the tent contains a cone and a cylinder.



Tent



Cone

Cylinder

(ii) The ice cream contains a cone and a hemisphere.



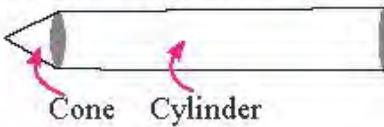
Ice cream



Hemisphere

Cone

(iii) The pencil contains a cone and a cylinder.



The combined solid objects contain two or more three dimensional objects.

5.3.2 Area and Volume of Combined Solid Objects

Activity 12

Discuss the formula to find the area and volume of combined solid objects in your groups.

1. Cylinder and Cone



The solid object combined with cone and cylinder is given in the figure. If the radius of the base is 'r', the height of the cylinder is ' h_1 ', the height of the cone is ' h_2 ' and slant height of the cone is ' l ', then,

$$\text{The base area of the cone} = \text{area of circle} = \pi r^2$$

$$\text{Curved surface area of the cylinder} = 2\pi r h_1$$

$$\text{Curved surface area of the cone} = \pi r l$$

- (a) Total surface area = area of base + curved surface area of cylinder + curved surface area of cone

$$= \pi r^2 + 2\pi r h_1 + \pi r l$$

- (b) Volume (V) = volume of cylinder + volume of cone

$$= \pi r^2 h_1 + \frac{1}{3} \pi r^2 h_2$$

$$= \pi r^2 (h_1 + \frac{1}{3} h_2)$$

2. Cone and Hemisphere

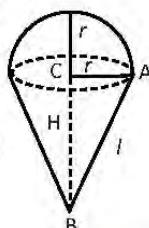
A combined solid object containing the cone and hemisphere is shown in the figure. The vertical height of the conical part is 'h', the slant height is 'l' and the radius is 'r'.

- (a) Total surface area (TSA) = curved surface area of cone + curved surface area of hemisphere

$$= \pi r l + 2\pi r^2 = \pi r (l + 2r)$$

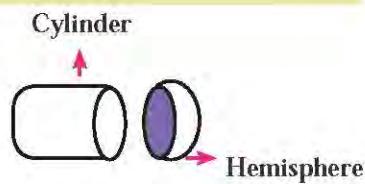
- (b) Volume (V) = volume of cone + volume of hemisphere

$$= \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3 = \frac{1}{3} \pi r^2 (h + 2r)$$



3. Cylinder and Hemisphere

A figure of solid object containing a cylinder and a hemisphere is given aside. The radius of the base of the object is ‘ r ’ and the height of the cylindrical part is ‘ h ’.



- (a) Total surface area (TSA) = area of base + curved surface area of cylinder + curved surface area of hemisphere
 $= \pi r^2 + 2\pi r h + 2\pi r^2 = 2\pi r h + 3\pi r^2$

- (b) Total volume = volume of cylinder + volume of hemisphere
 $= \pi r^2 h + \frac{2}{3} \pi r^3 = \pi r^2 (h + \frac{2}{3} r)$

4. Cone and Cone

A figure of combined solid object containing two cones is given aside. Where,
Common radius of base = r

Vertical height of upper cone = h_1

Slant height of upper cone = ℓ_1

Vertical height of lower cone = h_2

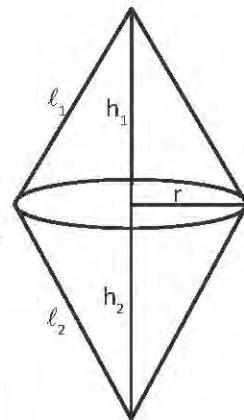
Slant height of lower cone = ℓ_2

- (a) Total surface area (TSA) = curved surface area of upper cone + curved surface area of lower cone

$$= \pi r \ell_1 + \pi r \ell_2 = \pi r (\ell_1 + \ell_2)$$

- (b) Volume (V) = volume of upper cone + volume of lower cone

$$\begin{aligned} &= \frac{1}{3} \pi r^2 h_1 + \frac{1}{3} \pi r^2 h_2 \\ &= \frac{1}{3} \pi r^2 (h_1 + h_2) \end{aligned}$$



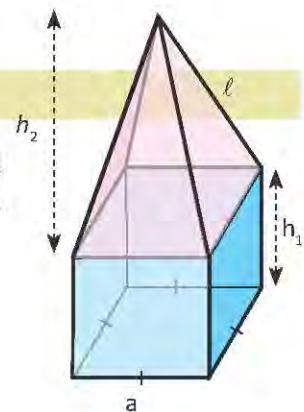
5. Prism and Pyramid

A figure of combined solid object containing a prism and a pyramid is given aside. The shape of the base of prism is square.

The different dimensions of solid are as follows:

Length of base = a

Height of prism = h_1



Height of pyramid = h_2

Slant height of pyramid = l

Area of base (A) = area of square = a^2

- (a) Total surface area (TSA) = area of base + lateral surface area of prism + lateral surface area of pyramid

$$= a^2 + p \times h_1 + 2al \quad (\text{Where, } p \text{ is the perimeter of the base of prism})$$

- (b) Total volume (V) = volume of prism + volume of pyramid

$$= A \times h_1 + \frac{1}{3} Ah_2 = a^2 h_1 + \frac{1}{3} a^2 h_2 = a^2 (h_1 + \frac{1}{3} h_2)$$

Think!

Are the total surface and lateral surface area equal for all types of combined solid objects?



Example 1

A figure of a pencil is given aside. Find the total surface area and volume of the pencil.

Solution

Here, radius of the base (r) = 7 cm

Height of the cylinder (h_1) = 39 cm

Height of the cone (h_2) = 24 cm

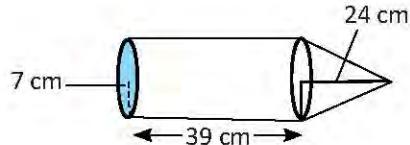
Total surface area = ?

Volume = ?

We know, $P = l^2 + r^2 = (24)^2 + (7)^2 = 576 + 49 = 625 \text{ cm}^2$

Therefore, the slant height of the cone (l) = 25 cm

$$\begin{aligned}\text{Again, total surface area} &= \pi r^2 + 2\pi r h_1 + \pi r l \\ &= \pi r (r + 2h_1 + l) \\ &= \frac{22}{7} \times 7 (7 + 2 \times 39 + 25) = 22 \times 110 = 2420 \text{ cm}^2\end{aligned}$$



Volume (V) = volume of cylinder + volume of cone

$$\begin{aligned}&= \pi r^2 h_1 + \frac{1}{3} \pi r^2 h_2 \\ &= \pi r^2 (h_1 + \frac{1}{3} h_2) = \frac{22}{7} \times (7)^2 (39 + \frac{1}{3} \times 24) = 7238 \text{ cm}^3\end{aligned}$$

Example 2

A figure of ice cream is given aside. If the radius of the circular base is 21 cm and the volume of ice cream is 32340 cm³,

- Find the height of conical part.
- Find the total surface area.

Solution

Here, radius of the base (r) = 21 cm

Volume of the cone with ice cream (V) = 32340 cm³

- We know,

$$\text{Volume of cone with ice cream (V)} = \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3$$

$$\text{or, } 32340 = \frac{1}{3} \pi r^2 (h + 2r)$$

$$\text{or, } 32340 = \frac{1}{3} \times \frac{22}{7} \times (21)^2 (h + 2 \times 21)$$

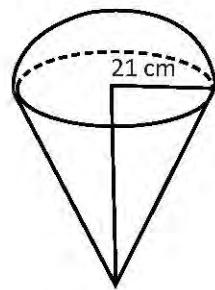
$$\text{or, } \frac{32340 \times 21}{22 \times 441} = (h + 42)$$

$$\text{or, } 70 - 42 = h$$

$$\therefore h = 28 \text{ cm}$$

$$\begin{aligned} \text{(b) Slant height } (l) &= \sqrt{h^2 + r^2} \\ &= \sqrt{(28)^2 + (21)^2} \\ &= \sqrt{1225} \\ &= 35 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Total surface area (TSA)} &= \pi r l + 2\pi r^2 \\ &= \pi r (l + 2r) \\ &= \frac{22}{7} \times 21 (35 + 2 \times 21) \\ &= 66 (35 + 42) = 66 \times 77 \\ &= 5082 \text{ cm}^2 \end{aligned}$$



Example 3

The solid object in the given figure contains two cones. On the basis of the given measurements, find its volume.

Solution

Here, the common diameter to the base to the both cones $(d) = 6 \text{ cm}$

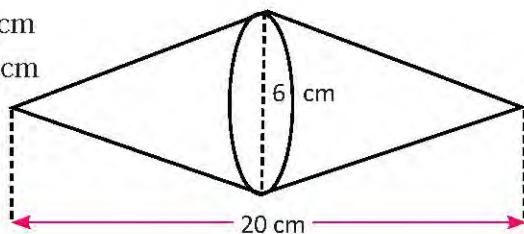
Total height of the solid $= 20 \text{ cm}$

Let the height of cone of the left part $= h_1 \text{ cm}$
and the height of cone of the right part $= h_2 \text{ cm}$

$$\therefore h_1 + h_2 = 20 \text{ cm}$$

(a) Radius of base $(r) = \frac{d}{2} = \frac{6}{2} = 3 \text{ cm}$

$$\begin{aligned}\text{Volume of solid } (V_1) &= \frac{1}{3} \pi r^2 (h_1 + h_2) \\ &= \frac{1}{3} \times \frac{22}{7} \times (3)^2 (20) = 188.57 \text{ cm}^3\end{aligned}$$



Example 4

A figure of stupa is given aside. The shape of its lower part is square based prism and the upper part is square based pyramid. Then, find its

- (a) Volume
- (b) Total surface area.

Solution

Here, total height of the stupa $= 5.5 \text{ m}$

Height of the prism $(h_1) = 5 \text{ m}$

Height of the pyramid $(h_2) = 5.5 - 5 = 0.5 \text{ m}$

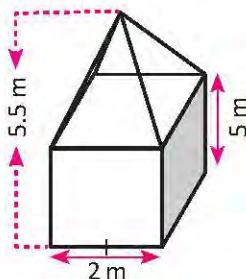
Length of the base of stupa (a) $= 2 \text{ m}$

(a) Area of base $(A_1) = a^2 = 2^2 = 4 \text{ m}^2$

$$\text{Volume of the prism shaped part } (V_1) = A_1 \times h_1 = 4 \times 5 = 20 \text{ m}^3$$

$$\text{Volume of the pyramid shaped part } (V_2) = \frac{1}{3} A_1 \times h_2 = \frac{1}{3} \times 4 \times 0.5 = \frac{2}{3} \text{ m}^3$$

$$\text{Volume of the stupa } (V) = V_1 + V_2 = 20 + \frac{2}{3} = 20.67 \text{ m}^3$$



(b) Perimeter of base (P) = $4a = 4 \times 2 = 8$ m

$$\begin{aligned}\text{Slant height of the pyramidal part } (l) &= \sqrt{(h_2)^2 + \left(\frac{a}{2}\right)^2} \\ &= \sqrt{(0.5)^2 + \left(\frac{2}{2}\right)^2} = \sqrt{0.25 + 1} = \sqrt{1.25} \text{ m}\end{aligned}$$

$$\text{Lateral surface area of the prism shaped part } (A_2) = p \times h_1 = 8 \times 5 = 40 \text{ m}^2$$

$$\text{Lateral surface area of the pyramid shaped part } (A_3) = 2al = 2 \times 2 \times \sqrt{1.25} = 4.47 \text{ m}^2$$

$$\begin{aligned}\text{Total surface area of the stupa } (A) &= A_1 + A_2 + A_3 \\ &= 4 + 40 + 4.47 \\ &= 48.47 \text{ m}^2\end{aligned}$$

Hence, the total surface area of the stupa is 48.47 m^2

Example 5

The given figure contains two square based pyramids with equal height. If the length of the base of each pyramid is 6 cm and the total volume is 96 cm^3 , find the height of each pyramid.

Solution

Here, the length of the base of each pyramid (a) = 6 cm

Total volume of pyramids (V) = 96 cm^3

Suppose, the height of each pyramid = h

Total volume of pyramids (V_2) = $\frac{1}{3}a^2h + \frac{1}{3}a^2h$

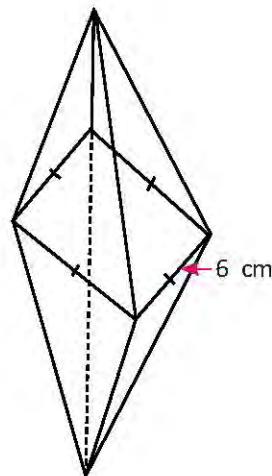
$$\text{or, } 96 = \frac{2}{3}a^2h$$

$$\text{or, } \frac{96 \times 3}{2} = (6)^2 \times h$$

$$\text{or, } \frac{96 \times 3}{2 \times 36} = h$$

$$\therefore h = 4 \text{ cm}$$

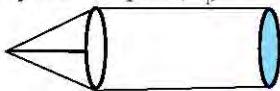
Hence, the height of each pyramid (h) is 4 cm.



Exercise 5.3

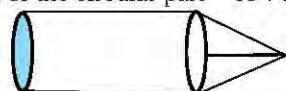
1. Answer the following questions on the basis of given information corresponding to the combined solid objects.

- (a) Total volume of combined solid (V) = 1050 cm^3
 Volume of cylindrical part (V_1) = 748 cm^3



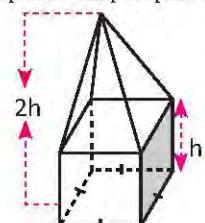
$$\text{Volume of conical part } (V_2) = ?$$

- (b) Curved surface area of the conical part = 252 cm^2
 Curved surface area of the cylindrical part = 272 cm^2
 Area of the circular part = 154 cm^2



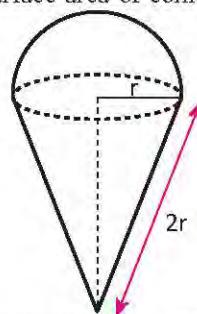
$$\text{Total surface area} = ?$$

- (c) Area of the base = 36 cm^2
 Volume of the prism shaped part (V_1) = 144 cm^3



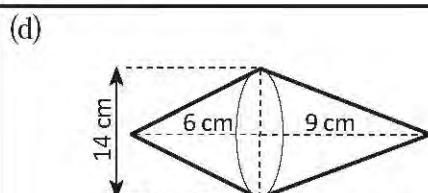
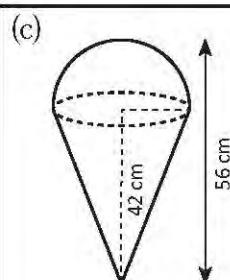
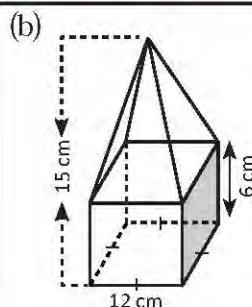
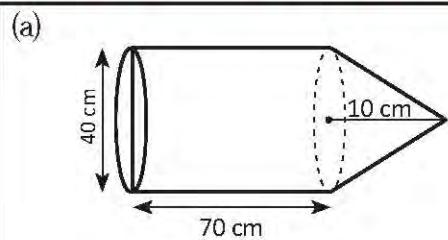
$$\text{Volume of the pyramid shaped part} = ?$$

- (d) Curved surface area of conical shaped part = 308 cm^2

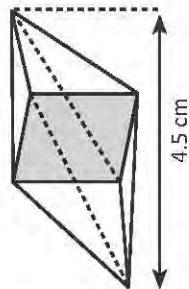


$$\text{Curved surface area of the hemispherical part} = ?$$

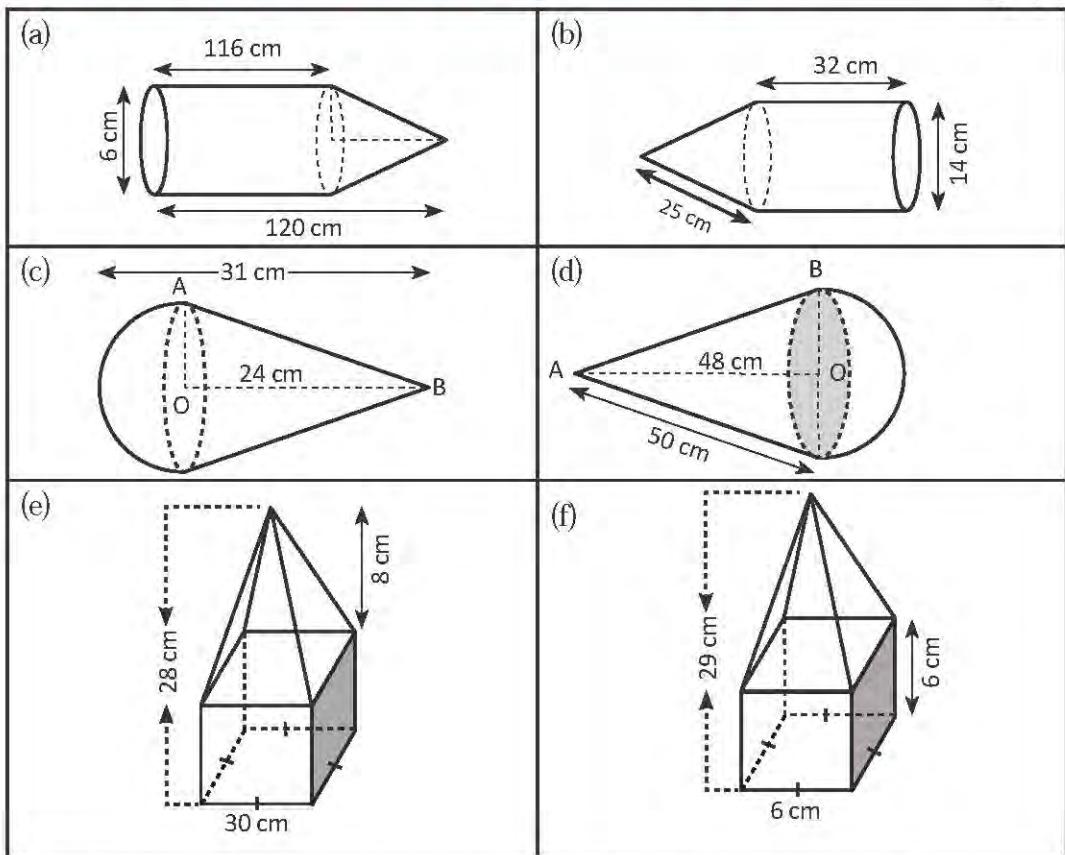
2. Find the volume of solid objects with given dimensions.



3. In the given figure of a crystal, the shaded part is square shaped of length 2.5 cm. The total height of the object is 4.5 cm. If the height of upper and lower pyramids are equal, find the total surface area and volume of the crystal.

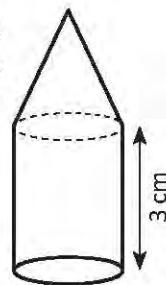


4. Find the total surface area of the given combined solids.

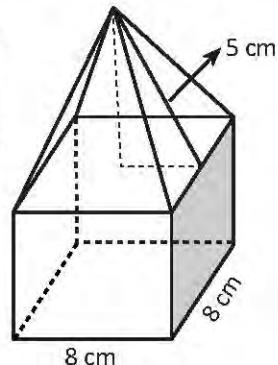


5. A toy contains conical and hemispherical parts of radius 14 cm. Find its total surface area if its total height is 49 cm.
6. A pyramid of height 12 cm is placed on the top of a solid of square based cuboid. If the base area of cuboid shaped solid is 100 cm^2 and its height is 10 cm, find
 (a) total volume and (b) total surface area of the combined solid.
7. A pyramid with vertical height 8 cm is placed on the top of a cubical solid. Find the total volume of the combined solid if the length of the base of cube is 12 cm.

8. (a) The solid object given in the figure contains a cone and a cylinder. The area of the base of cylinder is 100 cm^2 and its height is 3 cm. Find the total height of the solid if its total volume is 600 cm^3 .

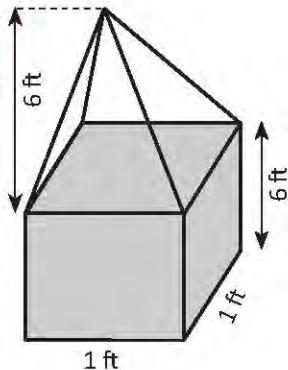


- (b) The upper part of a solid given aside is a pyramid with the slant height 5 cm. The lower part is a square based prism whose length of base is 8 cm. If the volume of solid is 448 cm^3 , find the height of the prism shaped portion.



9. In the adjoining figure, a square based pyramid is placed on the top of a tower. The height of tower and pyramid are 6 ft. and the length of the base of tower is 1 ft.

- (a) Find the lateral surface area of the pyramid shaped portion.
(b) Find the total surface area of the tower that can be painted.



10. Ram received a spinning top (A toy) from his father as a gift on his sixth birthday. The lower part of the spinning top is conical and the upper part is hemispherical. The total height of the spinning top is 5 cm and diameter of hemisphere is 3.5 cm. Ram is planning to color it. Find the area of the surface that can be colored.

Project Work

Divide the students into various groups of proper size. Construct various types of solid objects using paper, wood, soil, bamboo, wire, sticks or any other locally available materials. Then, find the surface area and volume of such solids. Besides this, construct other new solids combining two or more such solids and also find their area and volume. Test whether the area and volume before and after combination are equal or not. Give reasons in both cases. Finally, prepare a report on the basis of your activities and findings, and also present it in your classroom.

Answers

1. (a) 302 cm^3 (b) 678 cm^2 (c) 48 cm^3 (d) 308 cm^2
2. (a) 92190.47 cm^3 (b) 1296 cm^3 (c) 14373.33 cm^3 (d) 770 cm^3
3. $9.375 \text{ cm}^3, 25.74 \text{ cm}^2$
4. (a) 2262.85 cm^2 (b) 2112 cm^2 (c) 858 cm^2
(d) 3432 cm^2 (e) 4320 cm^2 (f) 3504 cm^2 5. 2890.63 cm^2
6. (a) 1400 cm^3 (b) 760 cm^2 7. 2112 cm^3
8. (a) 12 cm (b) 6 cm
9. (a) $\sqrt{5} \text{ cm}^2$ (b) 26.23 cm^2 10. 39.55 cm^2

5.3.2 Cost Estimation

Activity 1

Form different groups each containing 3 students. Measure the length, breadth and height of your classroom. Also, measure the length, breadth and height of doors and windows. The classroom excluding doors and windows needs to be painted. Now, find the total cost of painting under the following cotation.

- (a) The area of 6 m^2 can be painted with 1 liter color.
- (b) The price of color is Rs. 200 per liter.

Activity 2

The length, breadth and height of a room are 14 ft., 12 ft. and 10 ft. respectively. There is a door of width 3 ft and height 6.6 ft. There is also a window of size $6 \text{ ft} \times 4 \text{ ft}$. The room has to be painted. There are two different conditions regarding the rate of painting and rate of colors. In which cotation can the room be painted in lower cost? Discuss in your group.

House owner provides colors and painting related materials	The painter himself provides colors and all the painting related materials
First cotation	Second cotation
Primer on wall and ceiling = Rs. 7 per sq.ft.	IPrimer on wall and ceiling = Rs.14 per sq.ft.
Painting the walls and ceiling =Rs. 5 per sq.ft	Painting the walls and ceiling =Rs.30 per sq.ft
Enamel painting on doors and windows = Rs.12 per sq.ft	Enamel painting on doors and windows =Rs.24 per square ft
Quantity of colors required = 7.2 liters Cost of colors = Rs 420 per liter	

According to the first cotation,

$$\text{Area of a door } (A_1) = 3 \times 6.6 = 19.8 \text{ ft}^2$$

$$\text{Area of a window } (A_2) = 6 \times 4 = 24 \text{ ft}^2$$

Area of 4 walls and ceiling excluding the door and window

$$\begin{aligned}(A_3) &= 2h(l+b) + l \times b - (A_1 + A_2) \\&= 2 \times 10(14+12) + 14 \times 12 - (19.8 + 24) \\&= (688 - 43.8) \text{ ft}^2 \\&= 644.2 \text{ ft}^2\end{aligned}$$

$$\text{Cost of primer for walls and ceiling } (T_1) = 7 \times 644.2 = \text{Rs. } 4509.4$$

Cost of painting on the door and window (T_2) = $12 \times (19.8 + 24) = \text{Rs. } 525.6$

Cost of panting on the 4 walls and ceiling (T_3) = $5 \times 644.2 = \text{Rs. } 3221$

Cost of colors required for painting (T_4) = $\text{Rs. } 7.2 \times 420 = \text{Rs. } 3024$

$$\begin{aligned}\text{Total cost of painting (T)} &= T_1 + T_2 + T_3 + T_4 \\ &= 4509.4 + 525.6 + 3221 + 3024 \\ &= \text{Rs. } 11280\end{aligned}$$

According to the second cotation,

Cost of primer for walls and ceiling (T_1) = $14 \times 644.2 = \text{Rs. } 9,018.8$

Cost of painting on the door and window (T_2) = $24 (19.8 + 24) = \text{Rs. } 1,051.2$

Cost of panting on the 4 walls and ceiling (T_3) = $30 \times 644.2 = \text{Rs. } 19,326$

Total cost of painting (T) = $T_1 + T_2 + T_3 = 9018.8 + 1051.2 + 19,326 = \text{Rs. } 29,396$

Therefore the cost of painting will be least using first cotation.

Example 1

If the inner length of a square based water tank is 3 m and height is 4 m, find how much water the tank may contain.

Solution

In the square based water tank,

Inner length (l) = 3 m

Inner breadth (b) = 3 m

Area of the base of the tank (A) = $l^2 = (3)^2 = 9 \text{ m}^2$

Volume of the tank (V) = $A \times h = 9 \times 4 = 36 \text{ m}^3$

Volume of tank (V) = volume of water

So, the volume of water (V) = 36 m^3

Again, we know that,

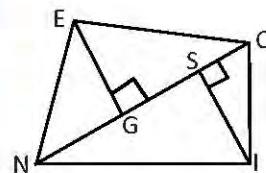
Amount of water in 1 cubic meter = 1000 l

$\therefore 36 \text{ m}^3 = 36 \times 1000 \text{ l} = 36,000 \text{ l}$

Hence, the amount of water that maybe contained in the tank is 36,000 l.

Example 2

The given figure NICE is a quadrilateral shaped land. The length between the corners N and C is 40 m. The length of perpendiculars from the corners E and I to the line NC are $EG = 10 \text{ m}$ and $IS = 15 \text{ m}$ respectively.



- Write the formula to find the area of quadrilateral.
- Find the area of the land.
- A male worker can finish the work of labeling the ground in 2 days and he receives wage at the rate of Rs 1500 per day. After then, a female worker plants the *dubo grass* (Bermuda grass) on it and she receives wages at the rate of Rs. 70 per square meter. Find the total cost of plantation.

Solution

In the quadrilateral NICE,

Length of perpendicular $EG = p_1 = 10 \text{ m}$

Length of perpendicular $= p_2 = 15 \text{ m}$

- The formula to find the area of quadrilateral $(A) = \frac{1}{2} \times d (p_1 + p_2)$
- Area of the ground $(A) = \frac{1}{2} \times 40 (10 + 15) = 20 \times 25 = 500 \text{ m}^2$

Amount to be paid to the female worker for plantation $= 70 \times 500 = \text{Rs. } 35,000$

Amount to be paid to the male worker for labeling the ground $= 1500 \times 2 = \text{Rs. } 3,000$

Therefore, the total cost of *dubo* plantation $= 35,000 + 3,000 = \text{Rs. } 38,000$

Example 3

A rectangular room has length 14 ft, breadth 12 ft. and height 10 ft. There are two square shaped windows of length 3 ft each and two doors of size 6 ft \times 2 ft each

- Write the formula to find the area of 4 walls and ceiling.
- What is the total cost of carpeting the floor at the rate of Rs. 300 per square meter?
- What is the total cost of painting the 4 walls and ceiling excluding the doors and windows, if the rate of painting is Rs. 30 per square feet?
- If the given rate increases by one third, by how much does the total cost of painting increase?

Solution

In the rectangular room

Length (l) = 14 ft

Breadth (b) = 12 ft

Height (h) = 10 ft

Length of the square shaped window = 3 ft There are two doors of size 6 ft. \times 2 ft each.

- (a) The formula to find out the area of 4 walls and ceiling (A) = $2h(l + b) + lb$
- (b) Area of floor of the room $l \times b = 14 \times 12 = 168 \text{ ft}^2$

We know,

$$\text{Area of carpet} = \text{area of floor of the room} = 168 \text{ ft}^2$$

$$\text{Rate of carpet per square meter } (R) = \text{Rs. } 300$$

$$\text{Total cost of carpeting } (T) = 300 \times 168 = \text{Rs. } 50,400$$

- (c) Area of 2 doors (A_1) = $2(6 \times 2) = 24 \text{ ft}^2$

$$\text{Area of 2 windows } (A_2) = 2 \times (3)^2 = 18 \text{ ft}^2$$

Now, area of 4 walls and ceiling excluding windows and doors;

$$\begin{aligned}(A) &= 2h(l + b) + lb - A_1 - A_2 \\&= 2 \times 10(14 + 12) + 14 \times 12 - 24 - 18 \\&= 20 \times 26 + 168 - 42 \\&= 520 + 128 \\&= 646 \text{ ft}^2\end{aligned}$$

Total cost of painting on 4 walls and ceiling at the rate of Rs. 30 per squared meter = $30 \times 646 = \text{Rs. } 19,380$.

- (d) When the rate of painting increases by one third, the new rate = $30 + \frac{1}{3} \times 30$
= Rs. 40

$$\text{Then, the total cost} = 40 \times 646 = \text{Rs. } 25,840$$

$$\text{Increase in total cost} = 25840 - 19380 = \text{Rs. } 6,460$$

Hence, the total cost increases by Rs. 6,460.

Example 4

There are two pillars of height 10 ft each in the gate of a stadium. A pyramid with the same base and height 32 ft is placed on the top of each pillar. If the base of each pillar is 4 ft \times 4 ft then,

- draw two figures base on the given information.
- find the slant height of the pyramid.
- find the total surface area of pillars to be colored. Should we add the area of bases to find the total surface area for the purpose of painting? Give reason.
- how much does it cost to paint the pillars together with pyramids at the rate of Rs. 95 per squared feet?

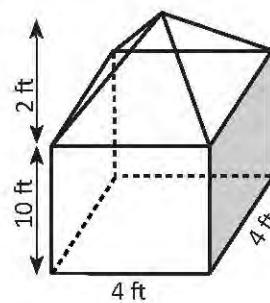
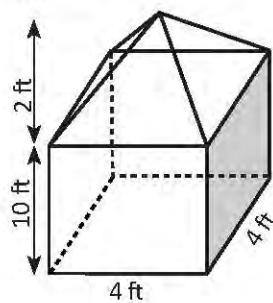
Solution

Here, the height of pillar (h_1) = 10 ft

Height of pyramid (h_2) = 2 ft

Since, the base of pillar is square shaped, its length (a) = 4 ft

(a)



(b) We know,

$$\text{Slant height of pyramid } (l) = \sqrt{(h_2)^2 + \left(\frac{a}{2}\right)^2} = \sqrt{(2)^2 + \left(\frac{4}{2}\right)^2} = \sqrt{4 + 4} = 2.83 \text{ ft}$$

(c) Lateral surface area of prism (A_1) = perimeter of base (P) \times height (h_1)

$$\begin{aligned} &= 4a \times 10 \text{ ft} \\ &= 4 \times 4 \times 10 = 160 \text{ ft}^2 \end{aligned}$$

Lateral surface area of pyramid (A_2) = $2al = 2 \times 4 \times 2.83 = 22.64 \text{ ft}^2$

Therefore, the total surface area of a pillar with a pyramid = $A_1 + A_2$

$$= (160 + 22.64) = 182.64 \text{ ft}^2$$

Total surface area of two pillars containing pyramids = $2 \times 182.64 = 365.28 \text{ ft}^2$

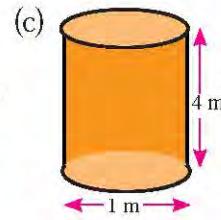
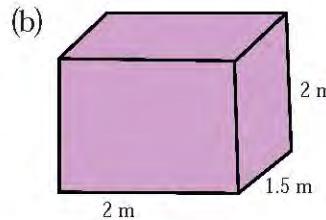
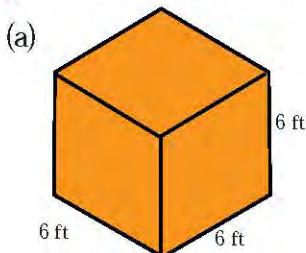
(The base area of pillar is not included to the total surface area as it is lying on the ground.)

- (d) Rate of painting (R) = Rs. 95 per squared ft.

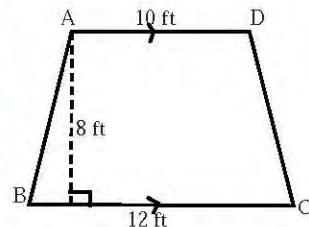
Then, the total cost of painting (T) = $R \times A = 95 \times 365.28 = \text{Rs. } 34,701.6$

Exercise 5.4

1. What is the maximum amount of water that can be contained in the tanks given below. ($1 \text{ ft}^3 = 28.317 \text{ l}$)

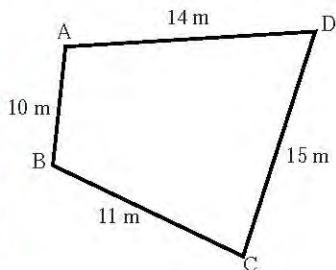


2. The parking area outside the National Insurance Company Limited Nepal is in geometric shape as shown in the figure. It is planning to pave the area with bricks. A brick occupies the area of 0.222 ft^2 and the cost of brick per piece is Rs. 16,



- (a) Find the area of the parking land.
(b) How many bricks are needed to pave the whole parking area?
(c) If 2 workers can complete the work of paving bricks in 3 days and the wage of a worker per day is Rs. 1200, how much does it cost to pave the bricks including the cost of bricks?

3. The figure given aside is a garden which is on the right side of a newly constructed temple. The management committee of temple has decided to fence around it with barbed wire rounding it 5 times. It can be completed by 3 workers in 2 days. The cost of wire per meter is Rs. 80 and the wage per worker per day is Rs. 1500,



- (a) Find the perimeter of the land.
 - (b) How much wire is required to fence the garden rounding it 5 times?
 - (c) Find the total cost of fencing the garden rounding it 5 times including the wages to the worker.
4. **There is a cubical shaped room with length 9 ft which has two square shaped windows of length 2 ft each and two windows of size 5 ft×3 ft each.**
- (a) Find the area of the floor.
 - (b) Find the area of 4 walls excluding the windows and doors.
 - (c) Find the total cost of painting 4 walls including the window and doors at the rate of Rs. 350 per square meter.
 - (d) Another room having length, breadth and height each greater by 1 ft. than that of the first is also to be painted at the rate of Rs. 340 per square ft. Calculate by how much does the total cost increase or decrease while painting the 4 walls including the windows and doors.
5. **It requires 60 concrete rings each of diameter 3.5 ft and height 1 ft. to construct a well. The cost of a ring is Rs. 1200. 2 workers can complete the work of keeping rings in 12 days and the wage of 1 worker is Rs. 1500 per day. (1 cubic foot = 28.317 liters)**
- (a) What is the cost of the rings only?
 - (b) How much does it cost to construct the well?
 - (c) Find the maximum amount of water that can be filled in the well.
 - (d) If the water level is seen at the 35th ring, how much water is there in the well? How many liters of water is needed to fill the well completely?
6. **There are two square based pillars each of height 10 ft placed in the gate of a house. A pyramid of the same base and height 1 ft is placed on the top of each pillar and the length of the base of each pillar is 5 ft.**
- (a) Present the given information in a diagram.
 - (b) Find the slant height of the pyramid.
 - (c) What is the total cost of painting the pillars together with pyramids if the rate of painting is Rs. 94 per square feet?

- 7.** There are two square based pillars each of height 6 ft in the gate of a house. A square based pyramid of same the base and height 1 ft is placed on the top of each pillar. The length of base of each pillar is 1 ft.
- Present the given information in a diagram.
 - Write the formula to find out the lateral surface area of a square based pyramid.
 - What is the slant height of the pyramid shaped part?
 - What is the cost of tiling per square feet if the total cost of tiling the pillars together with pyramids is Rs. 2729?
- 8.** A circular tank with inner diameter 2.80 m and height 3 m is constructed in a nursery to collect rain water. If its upper part is conical shaped with vertical height 0.72 m,
- write the formula to find the volume of cylinder.
 - what is the maximum amount of water that can be contained in the tank?
 - find the total cost to fill up the whole tank if the cost of water is Rs. 1.85 per liter.
- 9.** A pyramid shaped tent is formed using 8 equal triangular shaped pieces of clothes and the lengths of sides of each triangular piece of clothes are 5 m, 6 m and 6 m,
- Find the area of a piece of cloth.
 - Find the total area of clothes required for the tent.
 - If the cost of clothes per square meter is Rs. 600, find the total cost of tent construction.

Project work

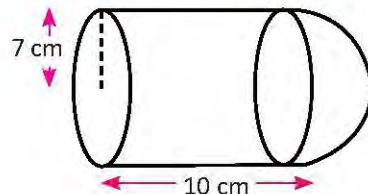
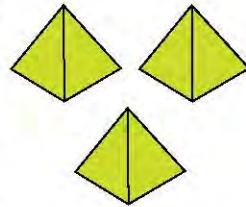
- Form different groups of students with proper size. Each group will visit a shop of construction materials and observe different types of water tanks made from plastic or metal. Identify their geometric shapes. Also, find the capacity of tanks in liters, prices and ratio of their capacity and price. Prepare a report including these details and also present it in your classroom.
- You are planning to construct an underground water tank of capacity 30,000 liter at your home. The tank will be constructed in cuboid shape. What should be the length, breadth and depth of pit so that the tank may contain 30,000 liter water?

Answers

- | | | | |
|----|---|--------------------------------------|--------------------------------------|
| 1. | (a) 6116.47 l | (b) $6,000 \text{ l}$ | (c) 3142.85 l |
| 2. | (a) 88ft^2 | (b) 397 (at least) | (c) Rs. 13,552 |
| 3. | (a) 50 m | (b) 250 m | (c) Rs. 29,000 |
| 4. | (a) 81 ft^2 | (b) 286 ft^2 | (c) Rs. 1,13,400 (d) Rs. 22,600 |
| 5. | (a) ₹. 72,000 | (b) Rs. 1,08,000 | (c) 16,35, 06 l (d) 6813.78 l |
| 6. | (b) $\frac{\sqrt{29}}{2}$ | (c) Rs. 48,109.51 | |
| 7. | (b) $2al$ | (c) $\frac{\sqrt{5}}{2} \text{ ft.}$ | (d) Rs. 52 |
| 8. | (a) $\pi r^2 h$ | (b) 19958.4 l | (c) Rs. 36923.04 |
| 9. | (a) $\frac{5\sqrt{119}}{4} \text{ m}^2$ | (b) $10\sqrt{119} \text{ m}^2$ | (c) Rs. 65,452.27 |

Mixed Exercise

1. A student of grade 10 has submitted three paper made models of square based pyramids to his mathematics teacher as a project work. The length of base and slant height of each pyramid is 12 cm and 10 cm respectively.
 - (a) Write the formula to find out the volume of a pyramid.
 - (b) Find the vertical height of the pyramid.
 - (c) How much space is occupied by the given pyramids?
 - (d) Find the total cost of coloring each pyramid excluding the base if the rate of coloring is Rs. 80 per square cm.
2. If the length of the side of a square based pyramid is 16 cm and volume is 1280 cm^2 ,
 - (a) Write the formula to find out the total surface area of the pyramid.
 - (b) Find the perimeter of the base.
 - (c) Find the slant height of the pyramid.
 - (d) Find the total surface area and the area of triangular surfaces. Which area is greater and by how much? Calculate the area and compare the result.
3. The lateral surface area of a square based pyramid is 540 cm^2 and its slant height is 15 cm.
 - (a) Write the formula to find out the total surface area of a pyramid.
 - (b) Find the length of its base.
 - (c) Find its volume.
4. The figure is a toy formed by combining a cylinder and hemisphere.
 - (a) What is the diameter of its base?
 - (b) Find the circumference of its base.
 - (c) Find the total surface area.
 - (d) Find the volume of the toy.



- 5.** A solid object contains a square based pyramid of slant height 5 cm on the top and a square based prism with length of base 8 cm on its bottom. The volume of object is 1024 cm^3 ,
- Draw the figure as per the information given in the question.
 - Find the vertical height of the pyramid shaped part.
 - Find by how much the vertical height of the pyramid shaped part is greater or less or equal to the height of prism shaped part.
 - Find the total surface area of the object.
- 6.** The lower part of a tent is cylindrical and the upper part is hemispherical. The radius of both parts is the same. If the total height of the tent is 54 m and the height of the cylindrical part is 40 m,
- draw a figure as per the information given in the question.
 - find the radius of the cylindrical part.
 - find the total surface area of the tent.
 - The cost of cloth per meter is Rs. 500, find the total cost to construct the tent.
- 7.** A solid object is formed with a hemisphere and cone combined together. The radius of the base of both parts is 7 cm. The total cost to color the object is Rs. 5148 at the rate of Rs. 6 per square cm.
- Find the total surface area of the combined solid.
 - What is the slant height of the cone?
 - Find the vertical height of the cone.
 - What is the total height of the solid?
- 8.** A toy is formed combining a cylinder and a cone with the same base of diameter 10 cm. The length of the cylindrical part is 14 cm and vertical height of the conical part is 12 cm.
- Find the slant height of the cone.
 - If the length of cylindrical part and the height of conical part are exchanged to form a new toy, which one costs more under the same rate of cost per square meter?
- 9.** A pyramid of vertical height 12 cm is placed on the top of a square based cuboid. If the area of base of cuboid shaped solid is 100 cm^2 and its height is 9 cm,
- What is the length of the base of cuboid shaped object?

- (b) Find the volume of the combined solid.
10. A cylindrical tank with radius 14 cm and height 40 cm is full of cement. A conical shaped pile of height 30 cm is formed while it is poured on the ground.
- Find the radius of the conical shaped pile of cement.
 - Find the surface area of the pile of cement excluding the base.
 - If the weight of 1 cm³ cement is 2.5 gm, find the total weight of the cement.
11. There are two pillars with the base 6 ft × 6 ft and height 8 ft of each in a stadium. A pyramid of height 4 ft is placed on the top of each pillar.
- Draw two figures as per the information given in question.
 - Write the formula to find the lateral surface area of a pyramid.
 - Find the slant height of the pyramid.
 - Find the surface area of the pyramid that need to be calculated for the painting purpose.
 - What will be the total cost to paint the pillars if the rate of cost of painting is Rs. 110 per square feet?
12. There are two cuboid shaped pillars of in the gate of a house, each of which has length 1 ft. breadth 1 ft. and height 6 ft. On the top of each pillar, a pyramid of the same base and height 1 ft is placed.
- Write the formula to calculate the lateral surface area of a prism.
 - What is the perimeter of the base of prism?
 - Find the slant height of the pyramid.
 - Find the area of pillars that need to be calculated for the painting purpose.
 - What will be the total cost to paint the pillars if the rate of cost of painting is Rs. 52 per square feet?
13. The diameter of concrete ring used in a well is 3.5 ft and its height is 1 ft. A ring costs Rs. 1200. If it requires 32 rings to construct the well,
- Find the total cost to construct the well.
 - Find the volume of water.
 - If the water level is seen at the eighteenth ring, find the volume of well above the water level.

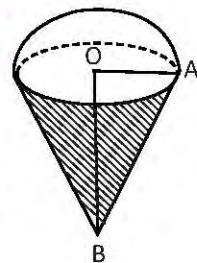
14. A solid object is formed combining two square based pyramids with the same base. The length of its base is 12 cm and the total height is 20 cm.

- (a) Write the formula to calculate the lateral surface area of a pyramid.
- (b) What is the volume of that combined solid object?

15. Nandakishor has established an agricultural farm of cows and goats in his village after he returned from foreign employment. There is a water tank with combined shape of a cylinder and cone. The inner diameter and height of the tank are 1.4 m and 2.1 m respectively. The vertical height of the cone is 0.36 m.

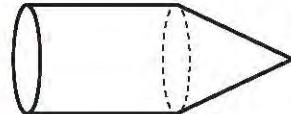
- (a) Write the formula to find the total surface area of a cone.
- (b) Find the perimeter of base of the tank.
- (c) How much water can be contained in the tank? Calculate in liters.

16. A combined solid object with volume $240\pi \text{ cm}^3$ is shown in the figure. The ratio of the slant height of the conical part and radius is 5 : 3.



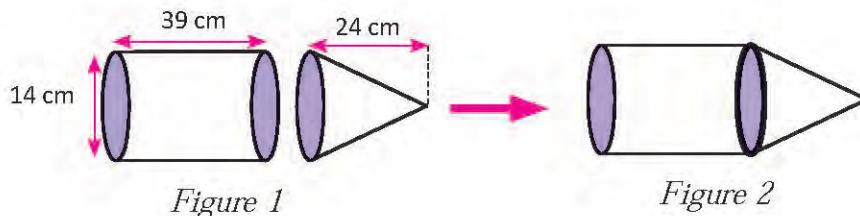
- (a) What are the different shapes contained in the solid?
- (b) Write the formula to calculate the curved surface area of the cone.
- (c) Find the total surface area of the given solid.
- (d) Are the total surface and curved surface area of the solid equal? Justify with reason.

17. The total surface area of a combined solid given in the figure is $840\pi \text{ cm}^2$. The ratio of slant height and vertical height of the conical part is 13 : 12 and the height of cylindrical part is 24 cm.



- (a) What are the different shapes contained in the given solid?
- (b) Write the formula to find the perimeter of base of a cone.
- (c) Find the volume of the solid.
- (d) Is the total surface area of solid equal to its curved surface area? If not, calculate and compare the result.

- 18.** There are a cylinder and a cone with the same base in the first figure. A solid shown in the second figure is formed combining the cylinder and cone of the first figure.



- (a) Write the formula to find the area of a cone.
 - (b) Find the surface area of solid objects of first figure separately and find their sum.
 - (c) Is the sum of area of solids of the first figure equal to the total surface area of combined solid of the second figure?
 - (d) Is the sum of volume of solids of the first figure equal to the volume of combined solid of the second figure? Justify with reasons.
 - (e) Compare the total cost of painting the solids of the first and second figures if the rate of cost of each is Rs. 150 per square meter.
- 19.** A toy contains a cylinder and a cone of the same base with a diameter of 10 cm. The length of the cylindrical part is 14 cm and height of the conical part is 12 cm.
- (a) Write the formula to find the volume of a cone.
 - (b) What is the slant height of the conical part?
 - (c) A new toy is formed by exchanging the measurement of diameter and conical height. Which toy costs more to color its surface at the same rate?
- 20.** A solid object is formed with the combination of a cylinder and a cone with the same radius. The height of the cylinder and slant height of the cone are 28 cm and 17 cm respectively. The total cost of coloring the solid at the rate of Rs 100 per square centimeter is Rs. 2851.20.
- (a) Draw a figure using the information given in the question.
 - (b) What is the shape of the cylinder?

(c) Find the height of the cone.

21. A water tank contains cylindrical and hemispherical parts. The total height of the tank is 20 m and the area of its base is 154 m^2 . If the tank is filled with water at the rate of 45 paisa per liter,

- (a) How much water is contained in the volume of 1 m^3 ?
(b) What is the radius of its base?

Answers

1. (a) $\frac{1}{3} \pi^2 h$ (b) 8 cm (c) 432 cm^2 (d) Rs 57,600
2. (a) $\pi^2 + 2\pi l$ (b) 64 cm (c) 17 cm
(d) $800 \text{ cm}^2, 544 \text{ cm}^2$, greater by 256 cm^2
3. (a) $\pi^2 + 2\pi l$ (b) 18 cm (c) 1296 cm^3
4. (a) 14 cm (b) 44 cm (c) 902 cm^2 (d) 2258.66 cm^3
5. (a) Show to your teacher (b) 3 cm (c) less by 12 cm (d) 624 cm^2
6. (a) Show to your teacher (b) 14 cm (c) 4752 cm^2 (d) Rs. 23,76,000
7. (a) 858 cm^2 (b) 25 cm (c) 24 cm (d) 31 cm
8. (a) 13 cm (b) The new toy costs more as its area is greater.
9. (a) 10 cm (b) 1300 cm^3
10. (a) 28 cm (b) 3611.21 cm^2 (c) 61,600 gm
11. (a) Show to your teacher (b) $2\pi l$ (c) 5 cm (d) 252 cm^2 (e) Rs. 27,720
12. (a) ph (b) 4 ft. (c) $\frac{\sqrt{5}}{2}$ ft (d) $(24 + \sqrt{5}) \text{ cm}^2$ (e) Rs. 1364.27
13. (a) Rs. 38,400 (b) 308 cm^3 (c) 134.75 cm^3
14. (a) $2\pi l$ (b) 960 cm^2
15. (a) $\pi r(r + l)$ (b) 4.4 m (c) 3418.8 l
16. (a) cone and hemisphere (b) πrl (c) 414.86 cm^2
(d) Yes, because the area of both surfaces is same.
17. (a) cylinder and cone (b) $2\pi r$ (c) 10057.14 cm^3
(d) The total surface area is greater by 314.28 cm^2
18. (a) $\frac{1}{3} \pi^2 h$ (b) 2728 cm^2 (c) 308 cm^2
(d) Both have same volume 7238 cm^2 (e) Former costs more by 46200
19. (a) $\pi r^2 h$ (b) 13 cm (c) the second costs more
20. (a) Show to your teacher (b) circle (c) 15 cm
21. (a) $1 \text{ m}^3 = 1000 \text{ l}$ (b) $r = 7 \text{ m}$ (c) 13 m (d) Rs. 12,24,300

6.0 Review

Observe the following sequences and answer the questions.

- | | |
|-------------------------|-------------------------|
| (a) 2, 4, 6, 8, | (b) 1, 3, 5, 7, |
| (c) 1, 4, 16, 64, | (d) 24, 12, 6, 3, |
| (e) 5, 11, 8, 14 | (f) 2, 6, 18, 54, |

Questions

- (i) Identify the above sequences whether they are arithmetic or geometric.
- (ii) Find the common difference or common ratio of the sequences.
- (iii) Find the general term of the sequences.
- (iv) Find the sixth and eight terms of the sequences.
- (v) Express the sequences into series.

6.1 Means of Arithmetic Sequence**Activity 1**

Study the following conversation.

Rita: How can the value of m be found, if 3, m, 7,are in arithmetic sequences?

Raju: The difference of each term to its preceding term of an arithmetic sequence is equal.

$$\text{So, } m - 3 = 7 - m$$

$$m - 3 = 7 - m$$

$$\text{or, } m + m = 7 + 3$$

$$\text{or, } m = \frac{7 + 3}{2} = 5$$

Rita: Yes. The second term is the average of the first and the third terms.

Anil: It can also be solved as follows.

First term (a) = 3

Second term (t_2) = $a + d$, where d is the common difference.

Third term (t_3) = $a + 2d$

or, $7 = 3 + 2d$

or, $2d = 7 - 3$

or, $2d = 4$

$\therefore d = 2$

Putting the value of d in $t_2 = a + d$

$$m = 3 + 2 = 5$$

Rashmi : How can we find more than 1 unknown terms between two given terms?

For example: What are the values of x, y, z in the sequence $3, x, y, z, 19, \dots, \dots$?

Anil : It can be solved In the following way.

First term (a) = 3

Fifth term (t_5) = 19

Common difference (d) = ?

We know that,

$$t_5 = a + (5-1)d$$

$$\text{or, } 19 = 3 + 4d$$

$$\text{or, } 4d = 19 - 3$$

$$\text{or, } 4d = 16$$

$$\therefore d = 4$$

$$\text{Second term } (t_2) = x = a + d = 3 + 4 = 7$$

$$\text{Third term } (t_3) = y = a + 2d = 3 + 2 \times 4 = 11$$

$$\text{Fourth term } (t_4) = z = a + 3d = 3 + 3 \times 4 = 15$$

Rita : Yes, I knew that the three terms between 3 and 19 can be found taking 19 as the fifth term.

Anil : Yes, you are correct.

Total number of terms = number of means + 2

$$n = m + 2$$

(Finally, they reported their discussion to the mathematics teacher.)

Teacher : You are all correct.

The terms lying between the first and the last terms of an arithmetic sequence are called arithmetic means.

- (a) An arithmetic mean between the first and the last terms can be obtained by taking the arithmetic average of the first and the last terms.
- (b) If there are more than one arithmetic means between two terms, they can be obtained by calculating common difference (d).

Example 1

Find an arithmetic mean between the numbers 7 and 17.

Solution

Let m be the arithmetic mean between 7 and 17.

Then, 7, m , 17 are in arithmetic sequence.

First term (a)=7

Third term (t_3) = 17

Arithmetic mean (m) = ?

$$\begin{aligned}\text{We know that } (m) &= \frac{a+b}{2} \\ &= \frac{7+17}{2} \\ &= 12\end{aligned}$$

Alternative method

Third term (t_3) = $a + 2d$

$$\text{or, } 17 = 7 + 2d$$

$$\text{or, } 2d = 17 - 7 = 10$$

$$\text{or, } d = 5$$

$$\text{Now, } (t_2) = a + d = 7 + 5 = 12$$

$$\therefore m = 12$$

Example 2

Find three arithmetic means between the terms 3 and 23.

Solution

The sequence with three arithmetic means is 3, m_1 , m_2 , m_3 , 23

First term (a) = 3

Number of means (m) = 3

Total number of terms (n) = $3 + 2 = 5$

Fifth term (t_5) = 23

We know that

Fifth term (t_5) = $a + (5 - 1)d$

$$\text{or, } 23 = 3 + 4d$$

$$\text{or, } 23 - 3 = 4d$$

$$\text{or, } \frac{20}{4} = d$$

$$\therefore d = 5$$

Now, m_1 = second term $= a + d = 3 + 5 = 8$

m_2 = third term $= a + 2d = 3 + 2 \times 5 = 13$

m_3 = fourth term $= a + 3d = 3 + 3 \times 5 = 18$

Hence, the three arithmetic means between 3 and 23 are 8, 13 and 18.

Example 3

Find 7 arithmetic means between two terms 3 and 43.

Solution

The arithmetic sequence with 7 means is 3, m_1 , m_2 , m_3 , m_4 , m_5 , m_6 , m_7 , 43.

First term (a) = 3

Number of means (m) = 7

Total number of terms (n) = $7 + 2 = 9$

Ninth term (t_9) = 23

We know, ninth term (t_9) = $a + (9 - 1)d$

$$\text{or, } 43 = 3 + 8d$$

$$\text{or, } 8d = 43 - 3 = 40$$

$$\text{or, } d = 5$$

Now, m_1 = second term $= a + d = 3 + 5 = 8$

m_2 = third term $= a + 2d = 3 + 2 \times 5 = 13$

m_3 = fourth term $= a + 3d = 3 + 3 \times 5 = 18$

m_4 = fifth term $= a + 4d = 3 + 4 \times 5 = 23$

m_5 = sixth term $= a + 5d = 3 + 5 \times 5 = 28$

m_6 = seventh term $= a + 6d = 3 + 6 \times 5 = 33$

m_7 = eighth term $= a + 7d = 3 + 7 \times 5 = 38$

6.2 Sum of Arithmetic Series

Activity 2

A shopping centre provides cash incentive to its employees according to their performance. One of the employees receives cash increment in his salary as given in the following table.

Year 2079	Baisakh	Jestha	Ashadh	Shrawan	Bhadra	Ashoj	Kartik
Increment in Salary (Rs.)	800	1000	1200	1400	1600	1800	2000

Study the information given in table and discuss the following questions.

- How much is the incentive amount increasing every month?
- What is the total amount of incentives received from Baisakh to Kartik of 2079 BS.?
- What will be the amount of the incentive in the month of Baisakh, 2080 B.S. if the same rate of growth continues?
- What will be the total amount of incentives that an employee receives upto Baisakh , 2080 BS?

The incentive amount is increased by Rs. 200 every month. The series of incentive amounts of 7 months is $800 + 1000 + 1200 + 1400 + 1600 + 1800 + 2000$.

If the total amount of incentives in seven months is denoted by S_7 ,

$$S_7 = 800 + 1000 + 1200 + 1400 + 1600 + 1800 + 2000 = 9800 \dots\dots (i)$$

First term (a) = 800, Common difference = $1000 - 800 = 200$

Number of term (n) = 7, seventh term (t_7) = 2000

The equation (i) can be expressed as

$$2S_7 = 2000 + 1800 + 1600 + 1400 + 1200 + 1000 + 800 = 9800 \dots\dots (ii)$$

Adding the equations (i) and (ii),

$$S_7 = 2800 + 2800 + 2800 + 2800 + 2800 + 2800 + 2800$$

$$\text{or, } 2S_7 = 7 \times 2800$$

$$\text{or, } S_7 = \frac{7}{2} \times 2800$$

$$= 9800$$

Expressing the value 2800 in terms of first and seventh terms,

$$\text{or, } S_7 = \frac{7}{2} (800 + 2000)$$

$\therefore S_n = \frac{n}{2} (a + t_n)$ where a = incentive amount of first month, t_n = incentive amount of last month.

$$\text{Now, } S_n = \frac{n}{2} (a + t_n)$$

$$= \frac{n}{2} [a + a + (n - 1)d] = \frac{n}{2} [2a + (n - 1)d] \quad [\because t_n = a + (n - 1)d]$$

Total number of months upto Baisakh, 2080 BS is 13

$$\begin{aligned}\text{Incentive amount of 13th month } (t_{13}) &= 800 + (13 - 1) 200 \\ &= 800 + 12 \times 200 \\ &= \text{Rs. 3,200}\end{aligned}$$

Hence, the incentive amount received in Baisakh, 2080 BS is Rs. 3200.

The total amount of incentive received in 13 months (S_{13}) = ?

We know that,

$$\begin{aligned}S_n &= \frac{n}{2} [(2a + (n - 1)d)] \\ &= \frac{13}{2} [2 \times 800 + (13 - 1) 200] \\ &= \frac{13}{2} [1600 + (12 \times 200)] = \frac{13}{2} [1600 + 2400] = \frac{13}{2} \times 4000 \\ &= 26000\end{aligned}$$

Hence, the total amount of incentive received in 13 months = Rs. 26,000.

The sum of the first n terms of an arithmetic series is $S_n = \frac{n}{2} (a + t_n)$,
where first term = a , final term = t_n , common difference = d ,
number of terms = n , and sum of the first n terms = S_n

Again, substituting $t_n = a + (n - 1)d$

$$\begin{aligned}S_n &= \frac{n}{2} (a + t_n) = \frac{n}{2} [a + \{a + (n - 1)d\}] = \frac{n}{2} [2a + (n - 1)d] \\ \therefore S_n &= \frac{n}{2} [2a + (n - 1)d]\end{aligned}$$

Note : The final term can be denoted by either (t_n) or L .

Example 4

Find the sum of the first 20 terms of the series $1 + 2 + 3 + \dots$

Solution

Here, $1 + 2 + 3 + \dots$ is an arithmetic series.

First term (a) = 1

Common difference (d) = $2 - 1 = 1$

Number of terms (n) = 20

Sum of the first 20 terms (S_{20}) = ?

We know that,

$$\begin{aligned} S_n &= \frac{n}{2} [2a + (n-1)d] = \frac{20}{2} [2 \times 1 + (20-1)1] \\ &= 10(2 + 19) \\ &= 10 \times 21 = 210 \end{aligned}$$

Example 5

Find the sum of the first 13 terms of the series $2 + 4 + 6 + \dots$

Solution

Here, $2 + 4 + 6 + \dots$ is an arithmetic series.

First term (a) = 2

Common difference (d) = $4 - 2 = 2$

Number of terms (n) = 13

Sum of the first 13 terms (S_{13}) = ?

We know that,

$$\begin{aligned} S_n &= \frac{n}{2} [2a + (n-1)d] = \frac{13}{2} [2 \times 2 + (13-1)2] \\ &= \frac{13}{2} (4 + 24) \\ &= \frac{13}{2} \times 28 \\ &= 182 \end{aligned}$$

Example 6

Find the sum of the first 28 terms of the series $5 + 17 + 29 + \dots$

Solution

Here, $5 + 17 + 29 + \dots$ is an arithmetic series.

First term (a) = 5

Common difference (d) = $17 - 5 = 12$

Number of terms (n) = 28

Sum of the first 28 terms (S_{28}) = ?

We know that,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{28} = \frac{28}{2} [2 \times 5 + (28-1)12]$$

$$= 14(10 + 324)$$

$$= 14 \times 334 = 4676$$

Example 7

An arithmetic series has the first term 2, the final term 29 and sum of its first n terms is 155. Find the number of terms and the common difference of the series.

Solution

First term (a) = 2,

Final term (t_n) = 29 and,

Sum of first n terms $S_n = 155$

Number of the terms (n) = ?

Common difference (d) = ?

We know that $S_n = \frac{n}{2} (a + t_n)$

$$\text{or, } 155 = \frac{n}{2}(2 + 29)$$

$$\text{or, } 310 = 31n$$

$$\text{or, } n = \frac{310}{31} = 10$$

Again, we have,

$$t_n = a + (n - 1)d$$

$$\text{or, } 29 = 2 + (10 - 1)d$$

$$\text{or, } 27 = 9d$$

$$\text{or, } d = \frac{27}{9} = 3$$

Hence, the common difference is 3.

Example 8

If the sum of certain number of terms in the arithmetic series $24+20+16+\dots\dots\dots\dots$ is 72, find the number of terms.

Solution

Here, first term (a) = 24

Common difference (d) = $20 - 24 = -4$

Sum of first n terms $S_n = 72$

Number of terms (n) = ?

We know that,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$72 = \frac{n}{2}[2 \times 24 + (n - 1)(-4)]$$

$$\text{or, } 144 = n(48 - 4n + 4)$$

$$\text{or, } 144 = 52n - 4n^2$$

$$\text{or, } 4n^2 - 52n + 144 = 0$$

$$\text{or, } n^2 - 13n + 36 = 0$$

$$\text{or, } n^2 - 9n - 4n + 36 = 0$$

$$\text{or, } n(n - 9) - 4(n - 9) = 0$$

$$\text{or, } (n - 9)(n - 4) = 0$$

$$\text{either, } n - 9 = 0 \quad \therefore n = 9$$

$$\text{or, } n - 4 = 0 \quad \therefore n = 4$$

Hence the total number of terms is either 4 or 9.

Example 9

Find the sum of the first 15 terms of an arithmetic series if its third term is 0 and the tenth term is 42.

Solution

Here, third term (t_3) = 0

Tenth term (t_{10}) = 42

Total number of terms (n) = 15

Sum of first 15 terms $S_{15} = ?$

We know that,

$$t_n = a + (n - 1) \cdot d$$

$$\text{or, } t_3 = a + (3 - 1) d$$

$$\text{or, } 0 = a + 2d$$

$$\text{Also, } t_{10} = a + (10 - 1) d$$

$$\text{or, } 42 = a + 9d$$

Subtracting the equation (ii) from (i)

$$a + 2d = 0$$

$$a + 9d = 42$$

$$-7d = -42$$

$$\text{or, } d = \frac{42}{7} = 6$$

$$\therefore d = 6$$

Substituting the value of d in equation (i),

$$a + 2 \times 6 = 0$$

$$a = -12$$

$$\begin{aligned}\text{Now, sum of the first 15 terms } S_{15} &= \frac{15}{2}[2 \times (-12) + (15 - 1)6] \\&= \frac{15}{2}[-24 + 84] \\&= 450\end{aligned}$$

Hence, the sum of the first 15 terms is 450.

Example 10

The salary of a person is Rs. 40,000 at present and he receives a grade of Rs 1,000 on his monthly salary every year. What will be his total income in 6 years if the number of grades goes on increasing as the number of years increases? How much additional amount is required to earn Rs. 35,00,000 in 6 years?

Solution

Here, total income of the first year (a) = $40000 \times 12 = 4,80,000$

Increased amount every year (d) = $1000 \times 12 = 12,000$

Total income in 6 years (S_6) = ?

Number of years (n) = 6

We know that,

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$S_6 = \frac{6}{2} [2 \times 480000 + (6 - 1) \times 12000]$$

$$= 3[960000 + 60000]$$

$$= 3 \times 10,20,000$$

$$= 30,60,000$$

Hence, the total income in 6 years is Rs. 30,60,000.

To earn Rs. 35,00,000 in 6 years, the required amount = $35,00,000 - 30,60,000 = 4,40,000$.

Exercise 6.1

1. (a) What are arithmetic means?
(b) If a, m, b are in arithmetic sequence, express m in terms of a and b .
(c) Find the mid value of the numbers 12 and 18.
2. **Find the arithmetic mean between**
(a) 6 and 10 (b) -2 and 2 (c) -4 and 8 (d) $(a + b)$ and $(a - b)$
3. **Find arithmetic means in the following cases.**
(a) 4 arithmetic means between 5 and 20
(b) 6 arithmetic means between 70 and 14
(c) 6 arithmetic means between 5 and -9

- 4.** The following sequences are arithmetic sequences. Form the sequences, find the value of x .
- $5, x, 9, \dots$
 - $x+1, x+5, 3x+1, \dots$
 - $x+2, 3x, 4x+1, \dots$
- 5.** Find the sum of the following arithmetic series.
- $7 + 11 + 15 + 19 \dots$, to 20 terms
 - $4 - 1 - 6 - 11 - 16 - \dots$, to 7 terms
 - $\frac{1}{2} + \frac{3}{2} + \frac{5}{2} + \dots$, to 16 terms
 - $5 + 10 + 15 + \dots + 65$
 - $-64 - 48 - 32 - \dots + 32$
 - The sum of first 10 odd numbers
 - The sum of first 100 natural numbers
 - The sum of natural numbers from 50 to 100
- 6.** The first term, the last term and the sum of all the terms of an arithmetic series are given below. Find the common difference of the series.
- First term = 1, last term = 50 and sum of all terms = 204
 - First term = 160, last term = 20 and sum of all terms = 440
 - First term = 17, last term = $-\frac{99}{8}$ and sum of all terms = $\frac{407}{16}$
- 7.** The common difference, number of terms and the sum of all the terms of an arithmetic series are given as follows. Find its first term.
- Common difference = -3, number of terms = 10 and sum of all terms = 325
 - Common difference = 9, number of terms = 9 and sum of all terms = 108
 - Common difference = 3, number of terms = 10 and sum of all terms = 155
- 8.**
- How many terms of the series $4 + 10 + 16 + 22 + \dots$ has a sum 374?
 - If the first term of an arithmetic series is 36 and its common difference is 9, how many terms will have a sum 540?
- 9.**
- Find the sum of the first 16 terms of an arithmetic series , if its third term is -15 and eighth term is 10.
 - Find the sum of the first 20 terms of an arithmetic series , if its fifth term is 10 and eleventh term is 22.

- 10.** (a) If the sum of the first 6 terms of an arithmetic series is 75 and the sum of its first 12 terms is 390, find the sum of the first 20 terms.
 (b) If the sum of the first 7 terms of an arithmetic series is 21 and the sum of its first 12 terms is 126, then find the series.
- 11.** (a) A worker in a garment factory produced 1000 Nepali cap (Dhaka Topi) at the first year and increased the production by 100 every year. Can he produce 15,000 caps in 10 years?
 (b) The monthly salary of a person appointed for a new job is Rs. 60,000 initially and it increases by Rs. 24,000 every year. In how many years he will earn the total amount of Rs. 1 crore ? (The interest is not included in his earning)

Project work

Request your parents to deposit some amount on the occasion of your birthday and continue it for 5 years in such a way that the deposit amount for every year increases by the same value. Calculate the total amount deposited in 5 years and compare who will collect more amount in your group of classmates.

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1. (b) $m = \frac{a+b}{2}$ (c) 15 2. (a) 8 (b) 0 (c) 0 (d) a
3. (a) 8, 11, 14, 17 (b) 62, 54, 46, 38, 30, 22 (c) 3, 1, -1, -3, -5, -7
4. (a) 7 (b) 4 (c) 3
5. (a) 900 (b) -77 (c) 128 (d) 455
 (e) -112 (f) 100 (g) 5050 (h) 3825
6. (a) 7 (b) -4 (c) $-\frac{47}{16}$
7. (a) 46 (b) -24 (c) 2
8. (a) 11 (b) 8
9. (a) 200 (b) 420
10. (a) 1050 (b) $-6 -3 + 0 + \dots$
11. (a) No, less by 500 (12) Nearly 12 years

6.3 Means of Geometric Sequence

Activity 1

How can the unknown terms of the following geometric sequences be found? Discuss the solution in each of your pairs and present the conclusion.

- (a) 3, m, 27, ...
or, $m^2 = 3 \times 27$
or, $m = \sqrt{3 \times 27} = 9$
Second term = $\sqrt{\text{first term} \times \text{third term}}$
- (b) 2, m_1 , m_2 , m_3 , 512, ... are in geometric sequence.
First term (a) = 2
Number of means (m) = 3
Total number of terms (n) = $m + 2 = 3 + 2 = 5$
Fifth term (t_5) = 512
We know that, $t_n = ar^{n-1}$
or, $512 = 2r^{5-1}$
or, $256 = r^4$
or, $(4)^4 = r^4$
 $\therefore r = 4$

Then, second term (t_2) = $m_1 = ar = 2 \times 4 = 8$

Third term (t_3) = $m_2 = ar^2 = 2 \times 4^2 = 32$

Fourth term (t_4) = $m_3 = ar^3 = 2 \times 4^3 = 128$

The terms lying between the first and last terms of a geometric sequence are called geometric means.

- (a) The single geometric mean between two terms is the square root of their product.
- (b) Two or more than two geometric means can be calculated using the first term and common ratio. The common ratio of the sequence can be found taking the last term as n^{th} term.

Example 1

Find a geometric mean between two numbers 2 and 32.

Solution

If m is the geometric mean between 2 and 32, then the numbers 2, m , and 32 are in geometric sequence.

First term (a) = 2

Third term (t_3) = 32

Mean (m) = second term =?

We know, mean (m) = $\sqrt{a \times t_3} = \sqrt{2 \times 32} = 8$

Hence, the geometric mean (m) = 8.

Example 2

Find 3 geometric means between two numbers, 5 and 405.

Solution

If m_1, m_2, m_3 are the 3 geometric means between 5 and 405, then 5, $m_1, m_2, m_3, 405$

First term (a) = 5

Number of means (m) = 3

Total number of terms (n) = $3 + 2 = 5$

Fifth term (t_5) = 405

We know that $t_5 = ar^{5-1}$

or, $405 = 5r^4$

or, $405 = 5r^4$

or, $81 = r^4$

or, $3^4 = r^4$

$\therefore r = 3$

Then, $m_1 = \text{second term} = ar = 5 \times 3 = 15$

$m_2 = \text{third term} = ar^2 = 5 \times 3^2 = 45$

$m_3 = \text{fourth term} = ar^3 = 5 \times 3^3 = 135$

Hence, 3 geometric means between 5 and 405 are 15, 45 and 135.

6.4 Sum of Geometric Series

Activity 2

An employee working in a pharmaceutical industry had a salary of Rs. 10,000 initially. Because of the pandemic of Covid 19, the workers were highly demanded and it was decided to increase their salary by 10% every month. How much will he earn in 5 months if the salary increased by 10% every month? Discuss the problem.

Here, the monthly salary at the beginning = 1000

$$\text{Salary of the second month} = 10,000 + 10,000 \times 10\% = 11,000$$

$$\text{Salary of the third month} = 11,000 + 11,000 \times 10\% = 12,100$$

$$\text{Salary of the fourth month} = 12,100 + 12,100 \times 10\% = 13,310$$

$$\text{Salary of the fifth month} = 13,310 + 13,310 \times 10\% = 14,641$$

Now, the salary of 5 months in series is

$$S_5 = 10000 + 11000 + 12100 + 13310 + 14641 \dots \dots \dots \quad (\text{i})$$

$$\text{Common ratio (r)} = \frac{11000}{10000} = \frac{12100}{11000} = \frac{13310}{12100} = \frac{14641}{13310} = 1.1$$

Multiplying the equation (i) by 1.1

$$1.1 \times (S_5) = 11000 + 12100 + 13310 + 14641 + 16105.1 \dots \dots \dots \quad (\text{ii})$$

Subtracting the equation (i) from (ii),

$$\text{or, } (1.1 - 1) S_5 = 16105.1 - 10000$$

$$\text{or, } (1.1 - 1) S_5 = 10000 (1.1)^5 - 10000$$

$$\text{or, } (1.1 - 1) S_5 = 10000 \{(1.1)^5 - 1\}$$

$$\text{Or, } S_5 = \frac{10000[(1.1)^5 - 1]}{(1.1 - 1)}$$

It is in the form of $S_n = \frac{a(r^n - 1)}{(r - 1)}$ where a is the first term, r is the common ratio and n is the total number of terms.

$$\text{or, } S_5 = 61051$$

Hence, the earning of the employee in 5 months is Rs. 61051.

The sum of the first n terms of a geometric sequence is given by $S_n = \frac{a(r^n - 1)}{(r - 1)}$ where $r > 1$ and $S_n = \frac{a(1 - r^n)}{(1 - r)}$ जहाँ $r < 1$.

Here, first term = a, common ratio = r, number of terms = n and the sum of first n terms = S_n .

$$\begin{aligned} \text{Again, } S_n &= \frac{a(r^n - 1)}{(r - 1)} = \frac{a r^n - a}{(r - 1)} = \frac{a r^{n-1} \times r - a}{(r - 1)} \\ &= \frac{t_n r - a}{r - 1} \quad \because t_n = a r^{n-1} \end{aligned}$$

Example 3

Find the sum of the first 5 terms of the geometric series $1 + 3 + 9 + 27 + \dots$

Solution

Here, first term (a) = 1

Common ratio (r) = $\frac{3}{1} = \frac{9}{3} = 3$

Total number of terms (n) = 5

Here, $r > 1$, $S_n = \frac{a(r^n - 1)}{(r - 1)}$

$$\text{So, } S_5 = \frac{1[(3^5 - 1)]}{(3 - 1)}$$

$$= \frac{243 - 1}{2} = 121$$

Hence, the sum of the first 5 terms of the series $1+3+9+27+\dots$ is 121.

Example 4

Find the sum of the first 5 terms of the geometric series $2 + 1 + \frac{1}{2} + \frac{1}{4} + \dots$

Solution

Here, first term (a) = 2

Common ratio (r) = $\frac{1}{2}$

Number of terms (n) = 5

We know that $r < 1$, $S_n = \frac{a(1 - r^n)}{(1 - r)}$

$$S_5 = \frac{2\{1 - (\frac{1}{2})^5\}}{1 - \frac{1}{2}} = \frac{2(1 - \frac{1}{32})}{\frac{1}{2}} = 4(1 - \frac{1}{32}) = \frac{31}{8}$$

Hence, the sum of the first 5 terms of the geometric series $2 + 1 + \frac{1}{2} + \frac{1}{4} + \dots$ is $\frac{31}{8}$.

Example 5

Find the sum of the geometric series $3 + 6 + 12 + 24 + \dots + 768$.

Solution

Here, first term (a) = 3

Common ratio (r) = $\frac{6}{3} = 2$

Last term (t_n) = 768

Since the first term and the last term are given, $S_n = \frac{t_n r - a}{r - 1}$

$$S_n = \frac{768 \times 2 - 3}{(2 - 1)} = 1536 - 3 = 1533$$

Hence, the sum of geometric series $3 + 6 + 12 + 24 + \dots + 768$ is 1533.

Example 6

Find the common ratio of a geometric series, if its first term is 7, last term is 448 and the sum of the terms is 889.

Solution

Here, first term (a) = 7

Last term (l) = 448

Total sum (S_n) = 889

The first and the last terms are given. So, $S_n = \frac{t_n r - a}{r - 1}$.

Therefore, $S_n = \frac{t_n r - a}{r - 1}$

$$\text{or, } 889 = \frac{448 \times r - 7}{r - 1}$$

$$\text{or, } 889r - 889 = 448r - 7$$

$$\text{or, } 889r - 448r = 889 - 7$$

$$\text{or, } 441r = 882$$

$$\text{or, } r = \frac{882}{441} = 2$$

Hence, the common ratio is 2.

Example 7

If the third and sixth terms of a geometric series are 27 and 729 respectively, find the sum of its first 10 terms.

Solution

Here, third term (t_3) = 27, sixth term (t_6) = 729

We know that $t_n = ar^{n-1}$

$$\text{or, } t_3 = ar^{3-1} = ar^2$$

$$\text{or, } 27 = ar^2 \dots \dots \dots \text{(i)}$$

$$\text{or, } t_6 = ar^{6-1} = ar^5$$

$$\text{or, } 729 = ar^5 \dots \dots \dots \text{(ii)}$$

Dividing the equation (ii) by (i)

$$\text{or, } \frac{729}{27} = \frac{ar^5}{ar^2}$$

$$\text{or, } 27 = r^3$$

$$\text{or, } 3^3 = r^3 \quad \text{or, } r = 3$$

$$\therefore r = 3$$

Hence, the common ratio is 3.

Again, putting the value of r in equation (i),

$$27 = a \times 3^2$$

$$\therefore a = 3$$

Since $r > 1$, $S_n = \frac{a(r^n - 1)}{(r - 1)}$

$$\text{or, } S_{10} = \frac{a\{(3)^{10} - 1\}}{(3 - 1)} = \frac{3(59049 - 1)}{2} = 88,572$$

Hence, the sum of first 10 terms is 88,572.

Example 8

How many terms of a geometric series $64 + 32 + 16 + \dots$ will have a sum $\frac{255}{2}$?

Solution

Here, first term (a) = 64

$$\text{Common ratio (r)} = \frac{32}{64} = \frac{1}{2}$$

$$\text{Sum of n terms } S_n = \frac{255}{2}$$

Total number of terms (n) = ?

$$\text{Here, } r < 1, S_n = \frac{a(1 - r^n)}{(1 - r)}$$

$$\text{or, } \frac{255}{2} = \frac{64\{1 - (\frac{1}{2})^n\}}{(1 - \frac{1}{2})} = \frac{64\{1 - (\frac{1}{2})^n\}}{\frac{1}{2}} = 128 \{1 - (\frac{1}{2})^n\}$$

$$\text{or, } \frac{255}{256} = 1 - (\frac{1}{2})^n$$

$$\text{or, } (\frac{1}{2})^n = 1 - \frac{255}{256}$$

$$\text{or, } (\frac{1}{2})^n = \frac{1}{256}$$

$$\text{or, } (\frac{1}{2})^n = (\frac{1}{2})^8$$

$$\therefore n = 8$$

Hence, the number of terms to have a sum $\frac{255}{2}$ is 8.

Example 9

Hari borrowed Rs. 19,682 from Ram Naresh on condition that he would pay back in 9 installments. If the amount of each installment is three times of the former find the difference of first and last installments.

Solution

Here, common ratio (r) = 3 (3 times)

Sum of all the terms (S_n) = 19682

First term (a) = ?

Ninth term (t_9) = ?

$$\text{Here, } r > 1 \text{ So, } S_n = \frac{ar^n - 1}{r - 1}$$

$$\text{or, } 19682 = \frac{a(3^9 - 1)}{(3 - 1)}$$

$$\text{or, } 19682 = \frac{a(19683 - 1)}{2}$$

$$\text{or, } 19682 = \frac{a \times 19682}{2}$$

$$\therefore a = 2$$

The first installment = Rs. 2

Again, the last installment (t_9) = $ar^{n-1} = 2 \times 3^{9-1} = 2 \times 3^8 = 13122$

Then, the difference of the first and last installments is $13122 - 2 = \text{Rs. } 13120$.

Exercise 6.2

1. (a) What is meant by geometric mean?
(b) If the positive numbers a , m and b are in geometric sequence, express m in terms of a and b .
(c) What is the geometric mean between 3 and 27?
2. Find the geometric mean between the given two terms.
(a) -4 and -64 (b) $\frac{1}{5}$ and 125 (c) 7 and 343
3. Find the geometric mean in the following cases.
(a) 4 geometric means between 6 and 192
(b) 3 geometric means between 5 and 405
(c) 3 geometric means between $\frac{9}{4}$ and $\frac{4}{9}$

- 4.** Find the value of x from the following geometric series.
- 4, x and 9
 - x , 4, 8
 - 5, 25 and $x+1$
- 5.** Find the sum of the following geometric series.
- $2 + 4 + 8 + 16 \dots$, to 6 terms
 - $\frac{1}{9} + \frac{1}{3} + 1 + \dots$, to 5 terms
 - $-\frac{1}{4} + \frac{1}{2} - 1 + \dots$, to 6 terms
 - $16 + 8 + 4 + \dots + \frac{1}{16}$
 - $1 + \frac{1}{3} + \frac{1}{9} + \dots + \frac{1}{729}$
- 6.** **The first term, the last term and the sum of all the terms of geometric series are given below. Find their common ratio.**
- First term = 2, last term = 486 and sum of all the terms = 728
 - First term = 5, last term = 1215 and sum of all the terms = 1820
 - First term = 3, last term = 768 and sum of all the terms = 1533
- 7.**
- Find the sum of the first 10 terms of a geometric series , if its second term is 4 and the seventh term is 128.
 - Find the sum of the first 7 terms of a geometric series if its third term is 3 and the fifth term is 81.
- 8.**
- How many terms of a geometric series $32 + 48 + 72 + \dots$ will have a sum 665 ?
 - How many terms of a geometric series $6 - 12 + 24 - 48 + \dots$ will have a sum -2046 ?
- 9.** Sarita borrowed Rs. 43680 from her friend Garima on condition that she would pay back it in 6 installments. If the amount of each installment she paid was three times of the previous one, find the difference of the first and the last installments.

Project Work

Form two groups A and B of your friends in order to solve the mathematical problems assigned by your teacher. Your teacher assigns questions to the different groups for 1 week as given below.

Group A: 3 questions for the first day, after then the number of questions for each day is two times of the previous day.

Group B: 1 question for the first day and the number of questions for each day is three times of the previous day.

Submit the work to your teacher after a week and find how many questions are solved by each group. Discuss the problem and its relation with sequence and series.

Answer

1. (b) $m = \sqrt{a \times b}$ (c) 9
2. (a) -16 (b) 5 (c) 49
3. (a) 12, 24, 48, 96 (b) 15, 45, 135 (c) $\frac{3}{2}, 1, \frac{2}{3}$
4. (a) 6 (b) 2 (c) 124
5. (a) 126 (b) $\frac{121}{9}$ (c) $\frac{21}{4}$
(d) $31\frac{15}{16}$ (e) $1\frac{364}{729}$
6. (a) 3 (b) 3 (c) 2
7. (a) 2046 (b) 1093
8. (a) 6 (b) 10
9. Rs. 29,040