Kernel: SageMath 10.3

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EXPERIMENT NO : 05

AIM : To find range space and null space of a linear transformation .

QUESTION . 01 (a)

Question 1 :-Define a linear treanformation T(x1,x2,x3) = (x1 - x2 - x3 , x1 + 3x2 + x3 , -3x1 + x2 - x3) from Q^3 to Q^3.Hence find the image and kernal of T .Check if T is one-one ,onto ,or byjection.

In [1]:
```
V = W = QQ^3
var('x1, x2, x3')
```

Out[1]:
```
(x1, x2, x3)
```

In [2]:
```
f(x1, x2, x3) = [x1-x2-x3, x1+3*x2+x3, -3*x1+x2-x3]
T = linear_transformation(V, W, f)
T
```

Expand

Image of any vector u can be obtained byT(u)

In [3]:
```
u = vector(QQ, [1, 2, -1])
T(u)
```

Out[3]:
```
(0, 6, 0)
```

In [4]:
```
T.image()
```

Out[4]:
```
Vector space of degree 3 and dimension 3 over Rational Field
Basis matrix:
[1 0 0]
[0 1 0]
[0 0 1]
```

In [5]:
```
T.kernel()
```

Out[5]:
```
Vector space of degree 3 and dimension 0 over Rational Field
Basis matrix:
[]
```

QUESTION . 01 (b)

Question 2 :-Define a linear treanformation T(x1,x2,x3) = (3x1 + 5x2 + x3 , x1 - 3x2 + 7x3 , -3x1 - 20x2 - x3) from Q^3 to Q^3.Hence find the image and kernal of T .Check if T is one-one ,onto ,or byjection.

In [6]:
```
V = W = QQ^3
var('x1, x2, x3')
```

Out[6]:
```
(x1, x2, x3)
```

In [11]:
```
f(x1, x2, x3) = [3*x1+5*x2+x3, x1-3*x2+7*x3, -3*x1-20*x2-x3]
T = linear_transformation(V, W, f)
T
```

Out[11]:
```
Vector space morphism represented by the matrix:
[  3   1  -3]
[  5  -3 -20]
[  1   7  -1]
Domain: Vector space of dimension 3 over Rational Field
Codomain: Vector space of dimension 3 over Rational Field
```

Image of any vector u can be obtained byT(u)

In [12]:
```
u = vector(QQ, [1, 2, -1])
T(u)
```

Out[12]:
```
(12, -12, -42)
```

In [13]:
```
T.image()
```

Out[13]:
```
Vector space of degree 3 and dimension 3 over Rational Field
Basis matrix:
[1 0 0]
[0 1 0]
[0 0 1]
```

In [14]:
```
T.kernel()
```

Out[14]:
```
Vector space of degree 3 and dimension 0 over Rational Field
Basis matrix:
[]
```

Conclusion: Problems on linear transformation, range space and null space are successfully executed.