INDIAN INSTITUTE OF SPACE SCIENCE AND TECHNOLOGY
THIRUVANANTHAPURAM 695 547

Quiz I - May 2022

B.Tech - II Semester

MA121 - Vector Calculus and Ordinary Differential Equations

Date: 29/04/2022 **Time: 09.00 am - 10.00 am** **Max. Marks: 20**

Answer all questions

1. (a) Let $f_n \xrightarrow{u} f$ and f_n is continuous at $x_0 \; \forall \; n$. Show that f is continuous at x_0. [2.5]

   (b) Examine whether the series $\sum u_n(x), u_n(x) = x^n - x^{n-1} \; |x| \leq \alpha < 1$ converges uniformly. Justify your answer. [2.5]

2. (a) Prove or disprove by giving a counter example that $f_n(x) \xrightarrow{p} f(x)$ in $[a,b] \Longrightarrow \int_a^b f_n(x)\,dx \longrightarrow \int_a^b f(x)\,dx$. [2.5]

   (b) Prove or disprove $f_n \xrightarrow{u} f$ in $(a,b) \Rightarrow f_n'(x) \xrightarrow{p} f'(x)$ in (a,b). [2.5]

3. (a) Let $L = \{(x,0) \in \mathbb{R}^2 \mid x \in (0,1)\}$. Is L open in $\mathbb{R}^2$? [1.5]

   (b) Show that the singleton set $\{(0,0)\}$ in $\mathbb{R}^2$ is closed. [1.5]

   (c) Show that the interval $(0,1]$ is neither closed nor open in $\mathbb{R}$. [2]

4. (a) Let $f : \mathbb{R}^2 \longrightarrow R$ be given by

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^4 + y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{otherwise} \end{cases}$$

   Show that the function f is not continuous at $(0,0)$ and the directional derivatives of f exist along all the directions at $(0,0)$. [2]

   (b) Let $g(x,y) = \sqrt{x^2 + y^4}$ for all $(x,y) \in \mathbb{R}^2$. Show that g is continuous on $\mathbb{R}^2$. Find all directions $\vec{v}$ for which $D_{\vec{v}}(f)|_{(0,0)}$ exists. [2]

   (c) Let $h(x,y,z) = xe^z + \cos y$ for all $(x,y,z) \in \mathbb{R}^3$. Show $D_{\vec{v}}(h)|_{(0,0,0)}$ exists for any direction $\vec{v}$. [1]

END