

INDIAN INSTITUTE OF SPACE SCIENCE AND TECHNOLOGY
THIRUVANANTHAPURAM 695 547

Quiz II - June 2022

B.Tech - II Semester

MA121 - Vector Calculus and Ordinary Differential Equations

Date: 01/06/2022

Time: 09.00 am - 10.00 am

Max. Marks: 20

Answer all questions

1. (a) Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{otherwise} \end{cases}$$

Show that for each directions \bar{v} , $D_{\bar{v}(f)}(0,0)$ exists. Is f differentiable at $(0,0)$? Find the maximum and minimum values of $|D_{\bar{v}(f)}(0,0)|$, and also find the corresponding vectors \bar{v} for which the maximum and minimum values of $|D_{\bar{v}(f)}(0,0)|$ are attained. [0.5 + 1 + 1]

- (b) Let C be a parametric curve defined by the set of equations $x = y^2, 9z^2 = 16y^3, y \geq 0$ with initial point $(1, 1, 4/3)$ and the final point $(16, 4, 32/3)$. Parametrize the curve C . Is the parametrization you have got is the arc-length parametrization? Find the corresponding arc length function, and therefore, find the length of the curve. Find the curvature of the curve, if exists. [0.5 + 0.5 + 1 + 0.5]

- (c) Let $f(x, y) = \sqrt{x^2 + y^2}$ for all $(x, y) \in \mathbb{R}^2$ and C is a semicircle $x^2 + y^2 = 1; y \geq 0$ with initial point $(1, 0)$. With explanation find the value of the line integral $\int_{\dot{C} \circ C^{-1}} f \, ds$, if it exists. Can you give an example of a curve C_1 such that $\int_{C_1} f \, ds = 0$? [1.5 + 1]

- (d) Let $F(x, y, z) = \left(\frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2}, z^2 + 1 \right)$. Calculate $\int_C F \cdot d\bar{s}$, if exists where C is given by $x^2 + y^2 = 4, z = 4$, oriented anti-clockwise. Is F a conservative vector field? [1.5 + 1]

2. (a) Let Q_1 and Q_2 be two real valued functions such that they are one time differentiable on the interval $[a, b]$. Prove that if the Wronskian $W(x) \neq 0$, for some $x \in [a, b]$, then Q_1 and Q_2 are linearly independent on $[a, b]$. [2]
- (b) Use method of undetermined coefficients to find particular solutions of the differential equation

$$y'' + y = e^x \cos x$$

- (c) Find the general solution of the differential equation

$$\frac{d^2 y}{dx^2} + 6 \frac{dy}{dx} + 9y = \frac{e^{-3x}}{x}, \quad x > 0$$

END

[6]