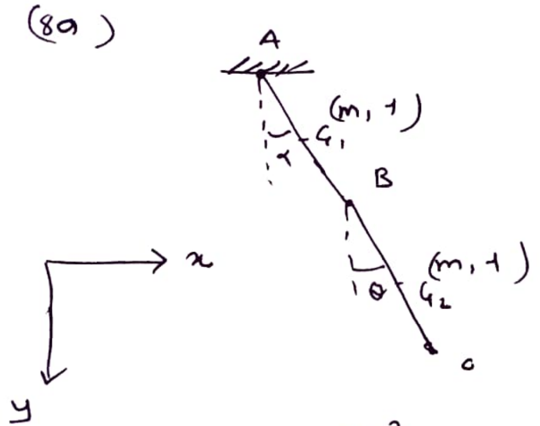
(8a)



$$G_1 = \left(\frac{l \sin\alpha}{2}, \frac{l \cos\alpha}{2} \right)$$

$$G_2 = \left[l\left(\sin\alpha + \frac{\sin\theta}{2}\right), l\left(\cos\alpha + \frac{\cos\theta}{2}\right) \right]$$

$$\cancel{T = \frac{1}{2 \times 4} m l^2 \dot{\alpha}^2 + \frac{1}{2} \times m l^2 \left(1 + \frac{1}{4} + \cos\alpha \cdot \cos\theta - \sin\alpha \sin\theta\right)}$$

$$\cancel{+ \frac{1}{2} \times \frac{m l^2}{12} \left(\dot{\alpha}^2 + \dot{\theta}^2\right)}$$

$$\vec{v_1} = \frac{l \dot{\alpha}}{2} (\cos\alpha, -\sin\alpha)$$

$$\vec{v_2} = l \left(\cos\alpha \cdot \dot{\alpha} + \frac{\cos\theta}{2} \dot{\theta}, -\sin\alpha \cdot \dot{\alpha} - \frac{\sin\theta}{2} \dot{\theta} \right)$$

$$T = \cancel{\frac{1}{2} m} \frac{m l^2}{2} \left\{ \frac{\dot{\alpha}^2}{4} + \dot{\alpha}^2 + \frac{\dot{\theta}^2}{4} + \cos(\theta \mp \alpha) \dot{\alpha} \dot{\theta} \right\}$$

$$+ \frac{1}{2} \times \frac{m l^2}{12} \left(\dot{\alpha}^2 + \dot{\theta}^2\right)$$

$$= \frac{m l^2}{2} \left\{ \frac{4}{3} \cancel{\frac{26}{12}} \dot{\alpha}^2 + \frac{\dot{\theta}^2}{3} + \cos(\theta \mp \alpha) \dot{\alpha} \dot{\theta} \right\}$$

$$V = -mgl \left(\frac{\cos\alpha}{2} + \cos\alpha + \frac{\cos\theta}{2} \right) = -\frac{mgl}{2} (3\cos\alpha + \cos\theta)$$

$$L \cancel{\phi} = T - V = \frac{m l^2}{2} \left(\frac{4}{3}\cancel{\frac{26}{12}} \dot{\alpha}^2 + \frac{\dot{\theta}^2}{3} + \cos(\theta \mp \alpha) \dot{\alpha} \dot{\theta} \right) + \frac{mgl}{2} (3\cos\alpha + \cos\theta)$$

$\theta - \alpha \approx 0$ ∵ small oscillations

$$\Rightarrow L = \frac{m l^2}{2} \left(\frac{4}{3} \dot{\alpha}^2 + \frac{\dot{\theta}^2}{3} + \dot{\alpha} \dot{\theta} \right) + \frac{mgl}{2} (3\cos\alpha + \cos\theta)$$

$$\frac{\partial L}{\partial \dot{\alpha}} = \frac{4 m l^2}{3} \dot{\alpha} + \frac{m l^2 \dot{\theta}}{2} \qquad \frac{\partial L}{\partial \alpha} = \frac{-3mgl \sin\alpha}{2}$$

$$\frac{4}{3} m l^2 \ddot{\alpha} + \frac{m l^2 \ddot{\theta}}{2} + \frac{3mgl \sin\alpha}{2} = 0$$

$$\frac{\partial L}{\partial \dot\theta} = \frac{ml^2\dot\theta}{3} + \frac{ml^2\dot\alpha}{2} \qquad \frac{\partial L}{\partial \theta} = -\frac{mgl\sin\theta}{2}$$

$$\therefore \quad ml^2\left(\frac{\ddot\theta}{3} + \frac{\ddot\alpha}{2}\right) + \frac{mgl\sin\theta}{2} = 0 \qquad -(2)$$

$$ml^2\left(\frac{\ddot\theta}{2} + \frac{4\ddot\alpha}{3}\right) + \frac{mgl}{2}\left(3\sin\alpha \cancel{+\sin\theta}\right) = 0 \qquad (1)$$

$$D^2\theta + \frac{8}{3}D^2\alpha = -3(\sin\alpha)\frac{g}{l}$$

$$\frac{2}{3}D^2\theta + D^2\alpha = -\frac{g}{l}(\sin\theta)$$

$$\cancel{3D^2\theta + 2D^2\alpha =}$$

$$3D^2\theta + \left(8D^2 + 9\frac{g}{l}\right)\alpha = 0$$

$$\cancel{3D^2 + 9\frac{g}{l}}$$

$$\left(2D^2 + 3\frac{g}{l}\right)\theta + 3D^2\alpha = 0$$

$$\left[\left(8D^2 + 9\frac{g}{l}\right)\left(2D^2 + \frac{3g}{l}\right) - 9D^4\right]\alpha = 0$$

$$\left(7D^4 + 42\frac{g}{l}D^2 + 27\frac{g^2}{l^2}\right)\alpha = 0$$

$$\left[9D^4 - \left(8D^2 + 9\frac{g}{l}\right)\left(2D^2 + \frac{3g}{l}\right)\right]\theta = 0$$

$$\left(7D^4 + 42\frac{g}{l}D^2 + 27\frac{g^2}{l^2}\right)\theta = 0$$

$$\left[D^2 - \left(-3 + \frac{\sqrt{441-189}}{7}\right)\frac{g}{l}\right]\left[D^2 - \left(-3 - \frac{\sqrt{441-189}}{7}\right)\frac{g}{l}\right]\theta = 0$$

$$\left(D^2 + \left(3 - \frac{6}{\sqrt7}\right)\frac{g}{l}\right)\left(D^2 - \left(3 + \frac{6}{\sqrt7}\right)\frac{g}{l}\right)\theta = 0 \Rightarrow T = \frac{2\pi}{n^2} \quad n = \left(3 \pm \frac{6}{\sqrt7}\right)\frac{g}{l}$$