# NM_UNIT_II_Notes

**Roots of Equation** :- The value of x for which an equation f(x)=0 is satisfied, called its root. Geometrically, we can say that a root of an equation f(x)=0 , is that value of x, where the graph of y= f(x) cuts the x- axis. The process of finding the roots of an equation is known as known as the solution of that equation. If f(x) is a Quadratic, cubic or biquadratic equation, Algebraic solutions of these equations are available. But for higher degree or transcendental equations no direct method exist. So those equations can be solve by approximate method such as bisection, secant, Regula-falsi and Newtons-Raphson method.

**Polynomial equation=>** The Polynomial equation has the following forms,

$$a_n x^n + a_{n-1} x^{n-1} + \text{-------------} + a_0 = 0$$

where $a_n \neq 0$ ,this equation has n roots, n roots may be real, equal or imaginary. An equation of the form f(x)=0, where f(x) is the polynomial in x is called as algebraic equation if the degree of the polynomial is more than 1.

Some examples of algebraic equations

$x^3 - x - 1 = 0$

$4x^2 + 2x + 1 = 0$

$5x^4 + 4x^3 - 2x + 1 = 0$

3) **<u>Transcendental equation =</u>** A non-algebraic equation is called transcendental equation. These equations actually have trigonometric, logarithmic or exponential terms.

For ex:-

$\cos x - x + 1 = 0$

$\cos x + xe^{-x} = 0$

$1 + \log x = 0$

This types of equation may have finite or infinite number of roots or may not have any real root.

**Methods for finding roots of an equations**

**Numerical Method** | **Direct Method**

**Or**

**Interactive Method**

**Or**

**Indirect Method**

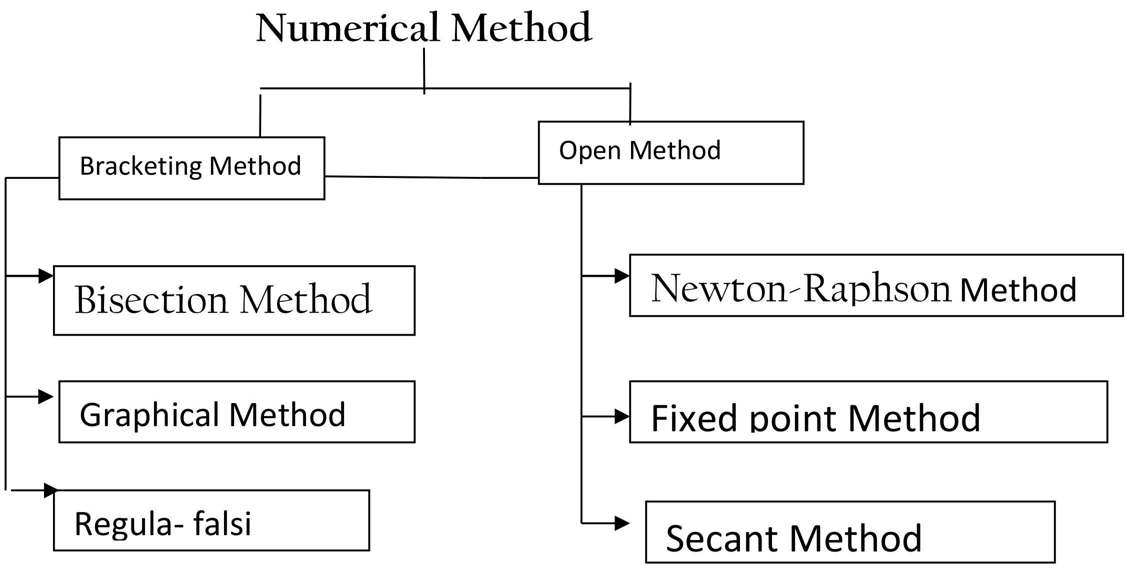
**Direct Method =>** Direct Method give the roots of an equation in a finite number of step, these method are capable in giving all the roots at the same time.

For ex. The roots of the quadratic equation $ax^2 + bx + c = 0$,

Where $a \neq 0$ , where $a \neq 0$ are given by,

$x_1 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ and $x_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

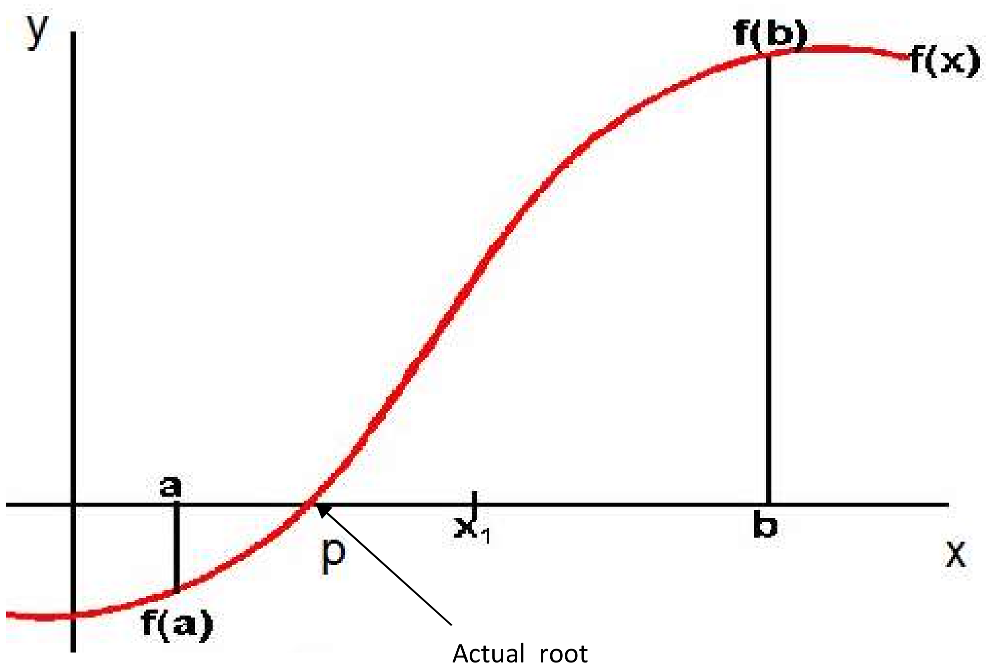
## Numerical Method




**Iterative Method** :- In Iterative or indirect method of obtaining the roots of an equation, we begin with initial guess or an approximation to the true solution. Then step wise this initial guess is improved until it is near to the actual root of equation. That is ,the cycle of repetition will continue till the required accuracy is obtained. This means that the direct method of equation solving the amount of computation is fixed while in an iterative method the amount of computation depends on the accuracy required.

**Bracketing Method** :- All those methods of solving root of equation f(x)=0 in which we take first two roots as a and b for which f(a)f(b)<0 are called bracketing method because initial guess for the root are required. As the name implies these guesses must “bracket”. Or be on either side of the root. The methods cover in this method use the strategy to systematically reduce the width of the bracket and hence give the correct answer.

**Bisection Method :-**



This method is used for finding an approximate solution to an equation f(x)=0 to the desired degree of accuracy. This method is based on concept which states that if a function f(x) is continuous in the closed interval a≤ x ≤b and f(a) , f(b) are of opposite signs, then there must exist at least one real root between x=a and x=b .let f(a) be –ve and f(b) is + value, then at least one root of equation f(x)=0 lies in the open interval and the first approximation to root obtained by bisection method is,

$$x_1 = \frac{a+b}{2}$$

if f(x)=0 then $x_1$ required root of the equation otherwise the root will lie in interval ($x_1$,b), if f($x_1$) is positive value or negative value respectively. Then we bisect the interval as before and continue this process until the root of equation f(x)=0 is found to a desired degree of accuracy

## Bisection Method Algorithm

Follow the below procedure to get the solution for the continuous function:

For any continuous function f(x),

1)Find two points, say a and b such that a < b and f(a)* f(b) < 0

2)Find the midpoint of a and b, say x=(a+b)/2

3)x is the root of the given function if f(x) = 0; else follow the next step

- Divide the interval [a, b]
- If f(x),is negative we replace , a by x.
- Else if f(x) is positive we replace b by x

  Repeat above steps 3 to 5 times until we get accuracy of root or value near to zero.

The bisection method is an approximation method to find the roots of the given equation by repeatedly dividing the interval. This method will divide the interval until the resulting interval is found, which is extremely small.

## Bisection Method Example

**Question:** Determine the root of the given equation $x^2+5x+1 = 0$ between x= -1 and x=0 using bisection.

**Solution:**

Given: $x^2+5x+1 = 0$

Let $f(-1)= (-1)^5+(-5)+1 = -5<0$

$f(0) = (0)^5+(0)+1 = 1>0$

f(a)*f(b)<0

**1st Approximation**

a= -1 , b=0

x= (a+b)/2= (-1+0)/2 = -0.5

$f(-0.5)=(-0.5)^5+5(0.5) +1= -1.5312<0$ -ve

so we replace a by x

**$2^{nd}$ Approximation**

a= -0.5 , b=0

x= (a+b)/2= (-0.5+0)/2 = -0.25

$f(-0.25)=(-0.25)^5+5(-0.25)+1= -0.25<0$ -ve

so we replace a by x

**$3^{rd}$ Approximation**

a= -0.25 , b=0

x= (a+b)/2= (-0.25+0)/2 = -0.125

$f(-0.125)=(-0.125)^5+5(-0.125)+1= 0.375<0$ +ve

so we replace b by x

**$4^{th}$ Approximation**

a= -0.25 , b=-0.125

x= (a+b)/2= (-0.25- 0.125)/2 = -0.1875

$f(-0.1875)=(-0.1875)^5+5(-0.1875)+1= 0.0622<0$ +ve

Hence we stop iteration here , as the value of function x=0.0622 is very close to zero .

There for Approximate root is -0.1875

**Question : Find a root of Equation by using bisection method sinx – x + 2=0 up to two decimal places of accuracy.**

**Solution:** let f(x)= sin x- x +2

Here we take x=2 & x=3

f(2)= sin( 2) -2 +2 = 0.909

f(3) = sin (3 ) – 3 +2 = -0.8588

here a= 3 and b=2 so f(a)*f(b)<0

**$1^{st}$ Approximation**

a= 3 , b=2

x= (a + b)/2= (3+2)/2 = 2.5

f(2.5)=sin(2.5) – 2.5 +2 = 0.0984 + ve

so we replace b by x

**$2^{nd}$ Approximation**

a= 3 , b=2.5

x= (a + b)/2= (3+2.5)/2 = 2.75

f(2.75)=sin(2.75) – 2.75 +2 = -0.3683 - ve

so we replace a by x

**$3^{rd}$ Approximation**

a= 2.75 , b=2.5

x= (a + b)/2= (2.75+2.5)/2 = 2.625

f(2.625)=sin(2.625) – 2.625 +2 = -0.1310 -ve

so we replace a by x

**$4^{th}$ Approximation**

a= 2.625 , b=2.5

x= (a + b)/2= (2.625+2.5)/2 = 2.562

f(2.562)=sin(2.562) – 2.562 +2 = -0.01431 - ve

so we replace a by x

**$5^{th}$ Approximation**

a= 2.562 , b=2.5

x= (a + b)/2= (2.562+2.5)/2 = 2.531

f(2.531)=sin(2.531) – 2.531 +2 = 0.04235 + ve

so we replace b by x

**$6^{th}$ Approximation**

a= 2.562 , b=2.531

x= (a + b)/2= (2.562+2.531)/2 = 2.546

f(2.546)=sin(2.546) – 2.546 +2 = 0.01499 + ve

so we replace b by x

**$7^{th}$ Approximation**

a= 2.562 , b=2.546

x= (a + b)/2= (2.562+2.546)/2 = 2.554

f(2.554)=sin(2.554) – 2.554 +2 = 0.000359 +ve

so we replace b by x

**8th Approximation**

a= 2.562 , b=2.554

x= (a + b)/2= (2.562+2.554)/2 = 2.558

f(2.558)=sin(2.558) – 2.558 +2 = -0.00697 - ve

so we replace a by x

**9th Approximation**

a= 2.558 , b=2.554

x= (a + b)/2= (2.558+2.554)/2 = 2.556

f(2.556)=sin(2.556) – 2.556 +2 = -0.00330 -

so the Approximate value of root up to two decimal places is 2.55

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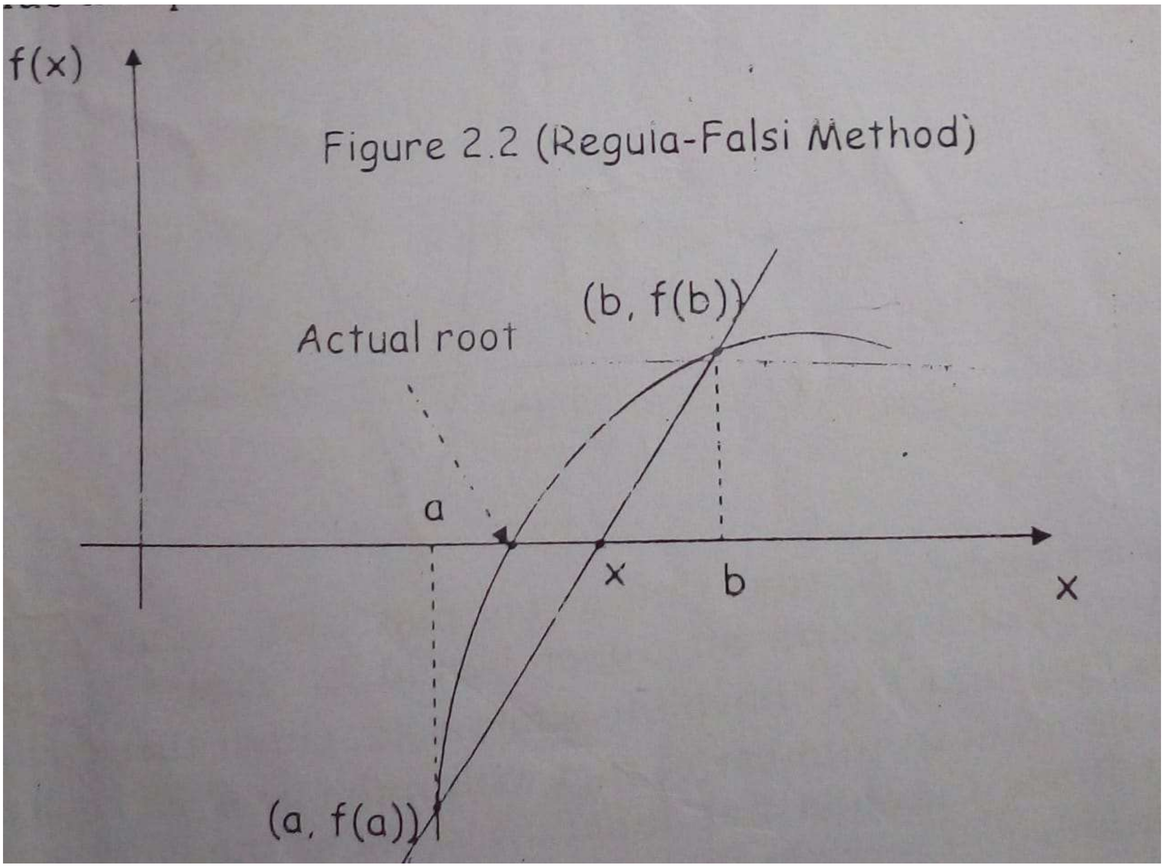
## Regula-falsi Method

This is the oldest method for finding the roots of the equation f(x)=0, this method is similar to the bisection method, in this method we take two points 'a' and 'b' i.e. f(a) and f(b)<0. This means that the graph of y=f(x) will surely cut the x between x=a and x=b. this shows that a root lies between a and b. in this method the part of curve y=f(x) between the points (a,f(a)) and (b,f(b)) is replaced by chord joining these points as shown in the figure.




So that equation of this chord is given by

$$\frac{y-f(a)}{x-a} = \frac{f(b)-f(a)}{b-a} \quad \text{---------------(1)}$$

This chord intersect the x-axis between points where y=0. So by putting y=0 in equation (1), we get

$$x-a = \frac{-f(a)(b-a)}{f(b)-f(a)}$$

$$x = a - \frac{f(a)(b-a)}{f(b)-f(a)}$$

$$x = \frac{a\,f(b)-a\,f(a)-b\,f(a)+a\,f(a)}{fb)-f(a)}$$

$$x=\frac{a*f(b)-b*f(a)}{f(b)-f(a)}\text{-------------}\rightarrow (2)$$

In the figure we can see f(a) is -ve and f(b) is +ve. So now if f(x) is –ve then point a is replace by x and if f(x) is +ve the point b is replace x. now continue this process until we get the root up desired degree of accuracy.

## Algorithm for the Regula–Falsi Method:

1. In the given equation f(x)=0, Find points **a** and **b** such that **a < b** and **f(a) * f(b) < 0**.
2. Take the interval **[a, b]** and determine the next value of **x**.

$$x=\frac{a\,f(b)-b\,f(a)}{f(b)-f(a)}$$

3. Now find f(x) and If **f(x) < 0** then let **a = x**, else if **f(x) > 0** then let **b = x**.

4. Repeat steps 2 & 3 times until the required **accuracy of root** .

# Problems of Regula falsi Method

Prob (1) Find the root of equation $x \tan x = -1$ starting with $a = 2.5$ and $b = 3$, correct up to three decimal places by false Position or Regula falsi Method.

Solution ⇒ $x \tan x = -1$

$$f(x) = x \tan x + 1 = 0$$

when

⇒ $a = 2.5$

$$f(2.5) = 2.5 \tan(2.5) + 1$$
$$= -0.86755$$

⇒ $b = 3$

$$f(3) = 3 \tan(3) + 1$$
$$= 0.57236$$

Here $f(2.5) * f(3) < 0$, so by using false position Method,

$I^{st}$ Approximation,

$$a = 2.5, \quad b = 3$$

$$f(a) = -0.86755, \quad f(b) = 0.57236$$

$$\Rightarrow x = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$= \frac{2.5 \times 0.57236 - 3 \times (-0.86755)}{0.57236 + 0.86755}$$

$$x = 2.80106$$

⇒ **2nd Approximation**

$a = 2.5$ , $b = 2.80106$

$f(a) = -0.86755$, $f(b) = 0.007482$

$$x = \frac{2.5 \times (0.007482) - (2.80106)(-0.86755)}{0.007482 + 0.86755}$$

$\Rightarrow x = 2.79848 \Rightarrow f(x) = 0.0002629$

replace b by x (+ve)

⇒ **3rd Approximation**

$a = 2.5$, $b = 2.79848$

$f(a) = -0.86755$, $f(b) = 0.0002629$

$$x = \frac{2.5 \times 0.0002629 + 2.79848 \times 0.86755}{0.0002629 + 0.86755}$$

$x = 2.7983$

$f(x) = -0.0002407$ (−ve)

Hence after 3rd Approximation (iteration) we have last two values of x same up to three decimal places, so approximate root is 2.7983.

Prob ② Use the false position method find root of an equation $x^2-x-2$

Solⁿ → for $f(1) = 1-1-2 = -2$
$f(2) = 2^2-2-2 = 0$
$f(3) = 3^2-3-2 = 4$

So, 2 is exactly on root of given equation, but we need to solve it by Regula falsi, so by using bracketing method we select two approximate value near to 2 as,

$a = 1.9$ , $b = 2.1$
$f(1.9) = (1.9)^2 - (1.9) - 1 = -0.29$

$f(2.1) = (2.1)^2 - (2.1) - 1 = 0.31$

here $f(a)\,f(b) < 0$, so by Regula falsi or false position Method

$I^{st}$ Approximation ⇒

$$x = \frac{a\,f(b) - b\,f(a)}{f(b) - f(a)}$$

$$x = \frac{(1.9)\times(0.31) - (2.1)(-0.29)}{0.31 - (-0.29)}$$

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$x = 1.99666$

$f(x) = -0.01000$ $\quad -ve$

replace a by x

II^nd Approximation ⇒

$a = 1.99666 \qquad b = 2.1$

$f(a) = -0.01000 \qquad f(b) = 0.31$

$$x = \frac{(0.31)(1.99666) + (2.1)(0.01000)}{0.31 + 0.01000}$$

$x = 1.9998$

$f(x) = -0.00059996$ $\quad -ve$

replace a by x

III^rd Approximation ⇒

$a = 1.9998, \quad b = 2.1$

$f(a) = -0.00059996, \quad f(b) = 0.31$

$x = \cancel{1.99999}\ 1.9999$

$f(x) = -0.00029999$ $\quad -ve$

IV^th Approximation ⇒ $a = 1.9999, \quad b = 2.1$

$f(a) = -0.00029999, \quad f(b) = 0.31$

$x = 1.9999$

⇒ Hence after 3^rd iteration we have values of x same, so, root is 1.9999

# Open Method

for the bracketing method the root is located within an interval prescribed by a lower and an upper bound. Repeated application of these method always results in closer estimates of true value of the root. Such method are said to be convergent because they move closer to the root as the computation progresses

the open methods are bases on formulas that require only a single starting value of x or two starting values that do not necessarily bracket the root. As such they sometimes diverge or move away from the actual root, as the computation progresses. However , when the open methods converge they usually converges more quickly than the bracketing methods.

# Newton Raphson Method/Newton's Iteration formula

Let $x_0$ be an approximate value of the root of the equation f(x)=0 either algebraic or transcendental and let h be a real number sufficiently small.

If $x_1=x_0 + h$ be the exact root of f(x)=0 then

$$f(x_1)=0$$

$$f(x_0+h)=0$$

Now expanding $f(x_0 +h)$ by Taylor's series we get,

$$f(x_0+h)= f(x_0) + h\, f'(x_0) + \frac{h^2}{2!} f''(x_0) + \text{--------} = 0$$

since, h is very small so neglecting $h^2$ and higher term of h,we get

$$f(x_0) + h\, f'(x_0) \cong 0$$

$$h= - f(x_0) / f'(x_0)$$

thus the first Approximation of the root is given by

$$x_1 = x_0 - f(x_0)/ f'(x_0)$$

similarly take $x_1$ as initial approximation so other approximation $x_2$ is obtained as,

$$x_2 = x_1 - f(x_1)/ f'(x_1)$$

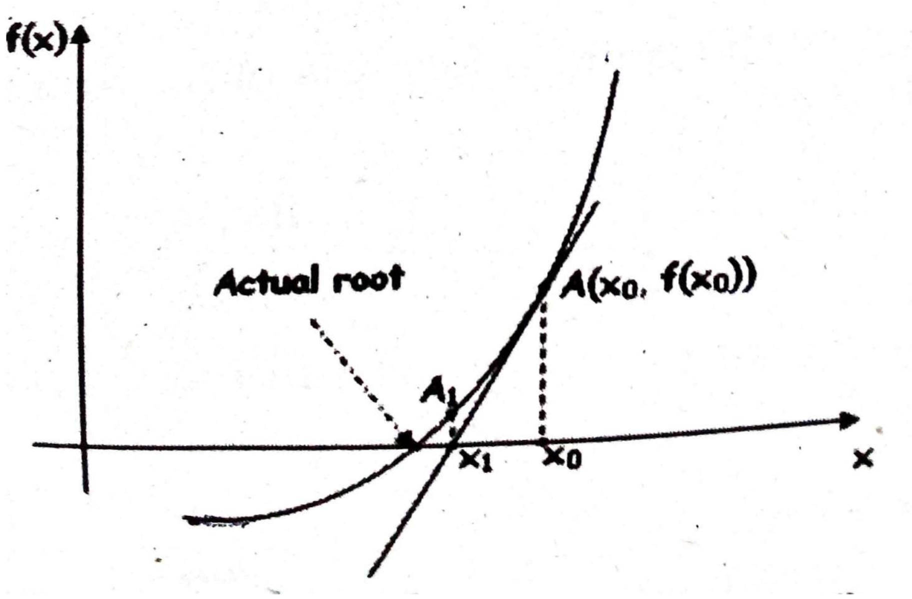
continue this process n-times we get approximation to the root is given by,

$$\mathbf{x_{n+1} = x_n - f(x_n)/ f'(x_n)}$$

for n=0,1,2,----------

this is known as the newton- Raphson formula or Newton's Iteration formula.

## Geometrical Interpretation of Newton's Method




Let A( $x_0$,f($x_0$) ) be the point on curve y=f(x), then the equation of the tangent at A ( $x_0$,f($x_0$) ), is given by,

$$y-f(x_0) = f'(x_0)(x-x_0)$$

this line cuts the x-axis where y=0 hence putting y=0

$$-f(x_0) = f'(x_0)\ (x - x_0)$$

$$(x-x_0) = -\ f(x_0)/\ f'(x_0)$$

$$\mathbf{x = x_0 - f(x_0)/\ f'(x_0)}$$

the point $x_1$ is shown in fig. the point $x_1$ is first approximation to the actual root. Let $A_1(x_1, f(x_1))$ be the point corresponding to $x_1$ then tangent at A1 cuts the x-axis at $x_2$ which is second approximation to the root, by containing this process we get better approximation to the actual root. Hence the general equation is given by

$$\mathbf{x_{n+1} = x_n - f(x_n)/\ f'(x_n)}$$

## Problems of NR Method

1) Compute the positive square root of $x^3 - x - 0.1$ by using Newton – Rapshon method.

Sol$^n$: $f(x) = x^3 - x - 0.1$

Let initial guess of the equation is $x = 1$ because

$f(1) = (1)^3 - 1 - 0.1$

$= -0.1$ (near to zero)

$f'(x) = 3x^2 - 1$

$= 3(1)^2 - 1$

$= 2$

We know the formula of NR method as

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

1$^{st}$ Approximation

$x_1 = 1$, $f(x_1) = -0.1$, $f'(x_1) = 2$

$$x_2 = 1 - \frac{(-0.1)}{2}$$

$$x_2 = 1 + 0.05$$

$$x_2 = \underline{1.05}$$

2$^{nd}$ Approximation

$x_2 = 1.05$ $f(x_2) = (1.05)^3 - 1.05 - 0.1$

$= 0.007625$

$f'(x_2) = 3(1.05)^2 - 1$

$= 2.3075$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 1.05 - \frac{0.007625}{2.3075}$$

$$x_3 = \underline{\underline{1.046696}}$$

<u>3<sup></sup></u>

**$3^{rd}$ Approximation**

$x_3 = 1.046696 \quad f(x_3) = (1.046696)^3 - 1.046696$

$= 0.0003537$

$f'(x_3) = 3(1.046696)^2 - 1$

$= 2.28671$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 1.046696 - \frac{0.00003537}{2.28671}$$

$$x_4 = \underline{1.04668}$$

**$4^{th}$ Approximation**

$x_4 = 1.04668$

$f(x_4) = (1.04668)^3 - 1.04668 - 0.1$

$= -0.00001216$

$f'(x_4) = 3(1.04668)^2 - 1$

$= 2.286617$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)}$$

$$= 1.04668 - \frac{(-0.000001216)}{2.28661}$$

$$x_5 = \underline{\underline{1.04668}}$$

Hence after $3^{rd}$ iteration ,we have last two values of x same up to 5 decimal places ,so approximate root is 1.04668

Prog ⇒ find the real roots of the following equation using Newton Raphson method up to three decimal places.

⇒ $f(x) = \cos x - 3x + 1$

Sol$^n$ ⇒ let initial guess of the equation is $x_0$ is 1 because,

$f(1) = \cos(1) - 3(1) + 1 = -1.4596$ (near to zero)

hence

⇒ $f'(x) = -\sin x - 3$

I$^{st}$ Approximation ⇒

$x_1 = 1$, $f(x_1) = -1.4596$

$f'(x_1) = -3.84147$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1 - \left(\frac{-1.4596}{-3.84147}\right)$$

⇒ $x_2 = 0.62004$

II$^{nd}$ Approximation ⇒ $x_2 = 0.62004$,

$f(x_2) = -0.046269$

$f'(x_2) = -3.5810$

⇒ $$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$x_3 = 0.607121$

$\text{III}^{rd}$ Approximation ⇒

$$x_3 = 0.607121$$
$$f(x_3) = -0.00006909$$
$$f'(x_3) = -3.5705$$

$$\Rightarrow x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$
$$= 0.60710$$

$\text{IV}^{th}$ Approximation ⇒ $x_4 = 0.60710$
$$f(x_4) = -0.00000058845$$
$$f'(x_4) = -3.570489$$

$$\Rightarrow x_5 = x_4 - \frac{f(x_4)}{f'(x_4)}$$
$$= 0.60710$$

Hence after $3^{rd}$ iteration we have ~~latt~~ two values of $x$ are same.
~~He~~ So the approximate root is
0.60710

③ find a Square root of 3 by using N-R Method

Sol$^n$ ⇒ let $x = \sqrt{3}$

$$x^2 = 3$$

$$x^2 - 3 = 0$$

⇒ $f(x) = x^2 - 3$

~~$f'(x) = 2$~~

let initial guess of equation is

$x_0 = 1.5$ (near to zero)

⇒ $f(x) = (1.5)^2 - 3 = -0.75$

⇒ $f'(x) = 2x$

$= 2(1.5) = 3$

I$^{st}$ Approximation -

$$x_1 = 1.5$$

$$f(x_1) = -0.75$$

$$f'(x_1) = 3$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

⇒ $x_2 = 1.75$

2$^{nd}$ Approximation

$$x_2 = 1.75$$

$$f(x_2) = 0.0625$$

$$f'(x_2) = 3.5$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

⇒ $x_3 = 1.73214$

3rd Approximation

$$x_3 = 1.73214$$
$$f(x_3) = 0.0003089$$
$$f'(x_3) = 3.46428$$

$$\Rightarrow x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$x_4 = 1.73205$$

4th Approximation

$$x_4 = 1.73205$$
$$f(x_4) = -0.0000037975$$
$$f'(x_4) = 3.4641$$

$$\Rightarrow x_5 = x_4 - \frac{f(x_4)}{f'(x_4)}$$

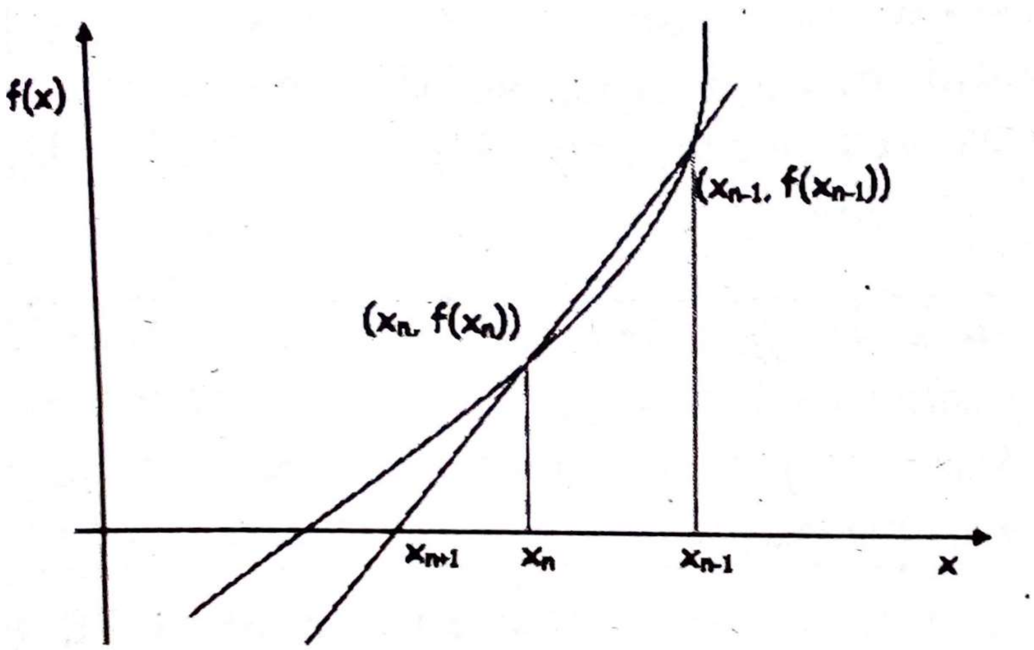
$$= 1.73205$$

hence after 4th iteration we have last two values of $x$ same upto five decimal places, so approximate root is 1.73205

## Secant Method

A serious disadvantage of the Newton-Raphson Method is the need to calculate f'(x) in each iteration. Many situation arrives in which th expression f(x) is long. Hence much compiler times is needed to evaluate it. There are situations where a closed form expression for f(x) is not available. Thus we cannot use Newton's Method , Hence we have secant Method.

This method is improvement over the Regula-falsi Method as it does not require the condition that f(a)*f(b)<0. In this method the graph of function y=f(x) is approximated by secant line at each iteration.



Geometrically in this method we Replace the function f(x) by straight line or chord passing through the points $(x_n, f(x_n))$ and

$(x_{n-1}, f(x_{n-1}))$ and take the point of intersection of this chord with x-axis as the next approximation to the root ,Hence equation of the chord joining two points $(x_n , f(x_n))$ and $(x_{n-1}\ f(x_{n-1}))$ is,

$$\frac{y-f(x_{n-1})}{x-x_{n-1}}=\frac{f(x_n)-f(x_{n-1})}{x_n-x_{n-1}}$$

x is the next point, so $x = x_{n+1}$ at $y = 0$. So,

$$x_{n+1}-x_{n-1}=\frac{-f(x_{n-1})\times(x_n-x_{n-1})}{f(x_n)-f(x_{n-1})}$$

$$x_{n+1}=x_{n-1}-\frac{f(x_{n-1})\times(x_n-x_{n-1})}{f(x_n)-f(x_{n-1})}$$

$$=\frac{x_{n-1}f(x_n)-\cancel{x_{n-1}f(x_{n-1})}-x_nf(x_{n-1})+\cancel{x_{n-1}f(x_{n-1})}}{f(x_n)-f(x_{n-1})}$$

$$x_{n+1}=\frac{x_{n-1}\,f(x_n)-x_nf(x_{n-1})}{f(x_n)-f(x_{n-1})} \qquad \text{---- (1)}$$

**Note :-In the secant Method if $f(x_n)=f(x_{n+1})$ then secant Method fails, so this method not converges necessarily. This is the drawback of this method**

## <u>Problems of secant method</u>

Prob ① Use the Secant Method to find a root of the equation $f(x) = x - e^{-x}$ up to 3 decimal places.

Sol$^n$ ⇒ When $x_0 = 0$ we get $f(x_0) = -1$
and $x_1 = 1$ we get $f(x_1) = 0.63212055$

hence initial guess of roots are,

$$x_0 = 0 \quad \text{and} \quad x_1 = 1$$

I$^{st}$ Approximation ⇒

$$x_0 = 0, \quad x_1 = 1$$

$$f(x_0) = -1, \quad f(x_1) = 0.63212055$$

$$\Rightarrow x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = 0.61304354$$

$$f(x_2) = 0.61304354 - e^{-0.61304354}$$

$$= 0.07134386$$

II$^{nd}$ Approximation ⇒

$$x_1 = 1, \quad x_2 = 0.61304354$$

$$f(x_1) = 0.63212055, \quad f(x_2) = 0.07134386$$

$$\Rightarrow x_3 = x_1 f(x_2) - x_2 f(x_1) / f(x_2) - f(x_1)$$

$$x_3 = 0.56381365$$

$$f(x_3) = -0.005221170$$

3rd Approximation →

$x_2 = 0.61304354$ , $x_3 = 0.56381365$

$f(x_2) = 0.07134867$, $f(x_3) = -0.005221709$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$x_4 = 0.5671707$

$\Rightarrow$ $f(x_4) = 0.0000429545$

4th Approximation →

$x_3 = 0.56381365$ , $x_4 = 0.5671707$

$f(x_3) = -0.005221709$ , $f(x_4) = 0.0000429545$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$x_5 = 0.567143$

Hence after 4th iteration we have last two Values same upto 3 places of decimal

Hence approximate value of root is ~~0.56~~ 0.5671

Prob → Determine the root of equation $f(x) = \cos x - x e^x = 0$ using Secant Method up to four decimal places.

Sol$^n$ → When $x_0 = 0$, we get $f(x_0) = 1$
and $x_1 = 1$, we get $f(x_1) = -2.1779795$

Hence initial guess of the root are $x_0 = 0$ and $x_1 = 1$

1$^{st}$ Approximation ⇒

$x_0 = 0$, $x_1 = 1$
$f(x_0) = 1$, $f(x_1) = -2.1779795$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$\Rightarrow x_2 = 0.31466534$$

$$f(x_2) = \cos(0.31466534) - 0.31466534\, e^{(0.31466534)}$$

$$= 0.51987116$$

2$^{nd}$ Approximation ⇒ $x_1 = 1$, $f(x_1) = -2.1779795$ | $x_2 = 0.31466534$, $f(x_2) = 0.51987116$

$$\Rightarrow x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$x_3 = 0.4467281$$

$$f(x_3) = 0.2035448$$

5th Approximation ⇒

$$x_5 = 0.51690446, \quad x_6 = 0.51772474$$
$$f(x_5) = 0.00259278, \quad f(x_6) = 0.000030310$$

$$x_7 = \frac{x_5 f(x_6) - x_6 f(x_5)}{f(x_6) - f(x_5)}$$

$$x_7 = 0.517757$$

Hence after 5th iteration we have last two values same upto 3 places of decimal, so approximate root is 0.5177

## Comparison between Bisection –Method and N-R Method

1) Bisection method comes under bracketing method and Newton Raphson method comes under the open method.
2) In N-R method we required only one initial guess, while in bisection method we required two initial guesses.
3) N-R method is based on tangent concept, while as concept of bisection method is based on halving the interval.
4) As N-R method comes under the open method so it converge more quickly than bisection method.
5) Two initial guesses must satisfy the condition f(a)*f(b)<0,in bisection method but in N-R method no such condition.

## Comparison between Secant method and Regula Falsi method

1) Formula used to calculate next approximation used by both methods are identical.
2) Both methods need two approximation roots to calculate new estimate of roots.
3) In regula- falsi method both approximation roots has condition f(a)*f(b)<0. But in secant method this condition is not necessary.
4) In secant method next estimate $x_{n+1}$ is used for next iteration and $x_{n-1}$ is left out. But in regula falsi method the sign of f(x) will decide which previous approximation will be left out a or b.
5) Regula-falsi certainly converges in closed interval [a,b] but secant method does not necessarily converge .