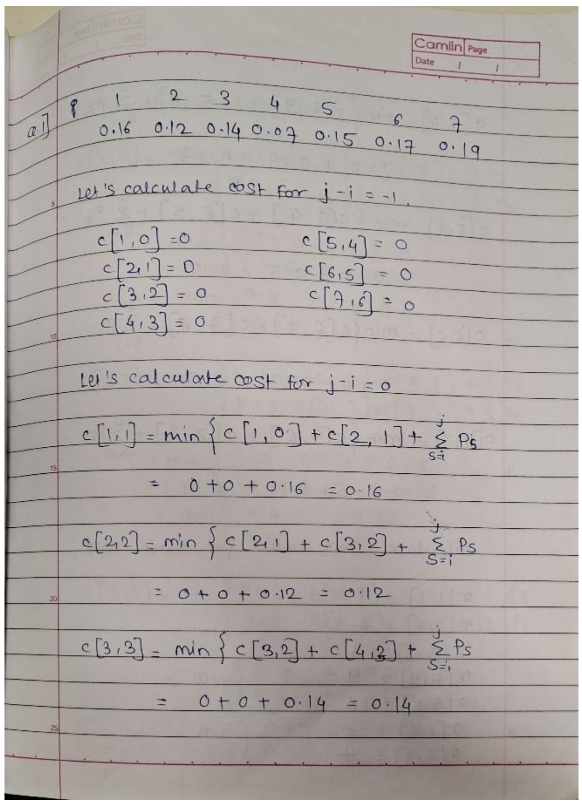
Assignment-3

Due: 11:59 pm on Thursday (04/14/2022)

1. **Determine the cost of an optimal binary search tree for a set of n = 7 keys with the following probabilities: (20p)**

i	1	2	3	4	5	6	7
p_i	0.16	0.12	0.14	0.07	0.15	0.17	0.19



a.1] i 1 2 3 4 5 6 7
0.16 0.12 0.14 0.07 0.15 0.17 0.19

Let's calculate cost for j - i = -1.

$c[1,0] = 0$ $c[5,4] = 0$
$c[2,1] = 0$ $c[6,5] = 0$
$c[3,2] = 0$ $c[7,6] = 0$
$c[4,3] = 0$

Let's calculate cost for j - i = 0

$c[1,1] = \min\{c[1,0] + c[2,1] + \sum_{s=i}^{j} p_s$

$= 0 + 0 + 0.16 = 0.16$

$c[2,2] = \min\{c[2,1] + c[3,2] + \sum_{s=i}^{j} p_s$

$= 0 + 0 + 0.12 = 0.12$

$c[3,3] = \min\{c[3,2] + c[4,3] + \sum_{s=i}^{j} p_s$

$= 0 + 0 + 0.14 = 0.14$

$c[4,4] = \min\{c[4,3] + c[5,4] + \sum_{s=i}^{j} P_s$

$= 0 + 0 + 0.07 = 0.07$

$c[5,5] = \min\{c[5,4] + c[6,5] + \sum_{s=i}^{j} P_s$

$= 0 + 0 + 0.15 = 0.15$

$c[6,6] = \min\{c[6,5] + c[7,6] + \sum_{s=i}^{j} P_s$

$= 0 + 0 + 0.17 = 0.17$

$c[7,7] = \min\{c[7,6] + c[8,7] + \sum_{s=i}^{j} P_s$

$= 0 + 0 + 0.19 = 0.19$

calculate root value for above cost

$r[1,1] = 1$
$r[2,2] = 2$
$r[3,3] = 3$
$r[4,4] = 4$
$r[5,5] = 5$
$r[6,6] = 6$
$r[7,7] = 7$

let's calculate cost for $j-i=1$

$$c[1,2] = \min\begin{cases} k=1, & c[1,0] + c[2,2] + \sum_{s=i}^{j} P_s \\ k=2, & c[1,1] + c[3,2] + \sum_{s=i}^{j} P_s \end{cases}$$

$$= \min\begin{cases} (0 + 0.12 + 0.28) \\ (0.16 + 0 + 0.28) \end{cases}$$

$$= \min\begin{cases} 0.4 \\ 0.44 \end{cases} = 0.4$$

$$c[2,3] = \min\begin{cases} k=2, & c[2,1] + c[3,3] + \sum_{s=i}^{j} P_s \\ k=3, & c[2,2] + c[4,3] + \sum_{s=i}^{j} P_s \end{cases}$$

$$= \min\begin{cases} (0 + 0.14 + 0.26) \\ (0.12 + 0 + 0.26) \end{cases}$$

$$= \min\begin{cases} 0.4 \\ 0.38 \end{cases} = 0.38$$

$$c[3,4] = \min\begin{cases} k=3, & c[3,2] + c[4,4] + P_s \\ k=4, & c[3,3] + c[5,4] + P_i \end{cases}$$

$$= \min\begin{cases} (0 + 0.07 + 0.21) \\ (0.14 + 0 + 0.21) \end{cases}$$

$$= \min\begin{cases} 0.28 \\ 0.35 \end{cases} = 0.28$$

$$c[4,5] = \min \begin{cases} k=4, & c[4,3] + c[5,5] + P_i \\ k=5, & c[4,4] + c[6,5] + P_i \end{cases}$$

$$= \min \begin{cases} (0 + 0.15 + 0.22) \\ (0.07 + 0 + 0.22) \end{cases}$$

$$= \min \begin{cases} (0.37) \\ (0.29) \end{cases} = 0.29$$

$$c[5,6] = \min \begin{cases} k=5, & c[5,4] + c[6,6] + P_i \\ k=6, & c[5,5] + c[7,6] + P_i \end{cases}$$

$$= \min \begin{cases} (0 + 0.17 + 0.32) \\ (0.15 + 0 + 0.32) \end{cases}$$

$$= \min \begin{cases} 0.49 \\ 0.47 \end{cases} = 0.47$$

$$c[6,7] = \min \begin{cases} k=6, & c[6,5] + c[7,7] + P_i \\ k=7, & c[6,6] + c[8,7] + P_i \end{cases}$$

$$= \min \begin{cases} (0 + 0.19 + 0.36) \\ (0.17 + 0 + 0.36) \end{cases}$$

$$= \min \begin{cases} 0.55 \\ 0.53 \end{cases} = 0.53$$

let's calculate root for above calculated

$r[1,2] = 1$ $\quad$ $r[4,5] = 5$

$r[2,3] = 3$ $\quad$ $r[5,6] = 6$

$r[3,4] = 3$ $\quad$ $r[6,7] = 7$

let's calculate cost for $j - i = 2$

$$c[1,3] = \min \begin{cases} k=1, & c[1,0] + c[2,3] + P_i \\ k=2, & c[1,1] + c[3,3] + P_i \\ k=3, & c[1,2] + c[4,3] + P_i \end{cases}$$

$$= \min \begin{cases} (0 + 0.38 + 0.42) \\ (0.16 + 0.14 + 0.42) \\ (0.4 + 0 + 0.42) \end{cases}$$

$$= \min \begin{cases} 0.8 \\ 0.72 \\ 0.82 \end{cases} = 0.72$$

$$c[2,4] = \min \begin{cases} k=2, & c[2,1] + c[3,4] + P_i \\ k=3, & c[2,2] + c[4,4] + P_i \\ k=4, & c[2,3] + c[5,4] + P_i \end{cases}$$

$$= \min \begin{cases} (0 + 0.28 + 0.33) \\ (0.12 + 0.07 + 0.33) \\ (0.38 + 0 + 0.33) \end{cases}$$

$$= \min \begin{cases} 0.61 \\ 0.52 \\ 0.71 \end{cases} = 0.52$$

$$c[3,5] = \min \begin{cases} k=3, & c[3,2] + c[4,5] + P_i \\ k=4, & c[3,3] + c[5,5] + P_i \\ k=5, & c[3,4] + c[6,5] + P_i \end{cases}$$

$$= \min \begin{cases} (0 + 0.29 + 0.36) \\ (0.14 + 0.15 + 0.36) \\ (0.28 + 0 + 0.36) \end{cases}$$

$$= \min \begin{cases} 0.65 \\ 0.65 \\ 0.64 \end{cases} = 0.64$$

$$c[4,6] = \min \begin{cases} k=4, & c[4,3] + c[5,6] + P_i \\ k=5, & c[4,4] + c[6,6] + P_i \\ k=6, & c[4,5] + c[7,6] + P_i \end{cases}$$

$$= \min \begin{cases} (0 + 0.47 + 0.39) \\ (0.07 + 0.17 + 0.39) \\ (0.29 + 0 + 0.39) \end{cases}$$

$$= \min \begin{cases} 0.86 \\ 0.63 \\ 0.68 \end{cases} = 0.63$$

$$c[5,7] = \min \begin{cases} k=5, & c[5,4] + c[6,7] + P_i \\ k=6, & c[5,5] + c[7,7] + P_i \\ k=7, & c[5,6] + c[8,7] + P_i \end{cases}$$

$$= \min \begin{cases} (0 + 0.53 + 0.51) \\ (0.15 + 0.19 + 0.51) \\ (0.47 + 0 + 0.51) \end{cases}$$

$$= \min \begin{cases} 1.04 \\ 0.85 \\ 0.98 \end{cases} = 0.85$$

Let's calculate root for above cast

$r[1,3] = 2$ $\qquad$ $r[4,6] = 5$

$r[2,4] = 3$ $\qquad$ $r[5,7] = 6$

$r[3,5] = 5$

let's calculate cost for $j-i=3$.

$$c[1,4] = \min \begin{cases} k=1,\ c[1,0] + c[2,4] + P_i \\ k=2,\ c[1,1] + c[3,4] + P_i \\ k=3,\ c[1,2] + c[4,4] + P_i \\ k=4,\ c[1,3] + c[5,4] + P_i \end{cases}$$

$$= \min \begin{cases} (0 + 0.52 + 0.49) \\ (0.16 + 0.28 + 0.49) \\ (0.4 + 0.07 + 0.49) \\ (0.72 + 0 + 0.49) \end{cases}$$

$$= \min \begin{cases} 1.01 \\ 0.93 \\ 0.96 \\ 1.21 \end{cases} = 0.93$$

$$c[2,5] = \min \begin{cases} c[2,1] + c[3,5] + P_i \\ c[2,2] + c[4,5] + P_i \\ c[2,3] + c[5,5] + P_i \\ c[2,4] + c[6,5] + P_i \end{cases}$$

$$= \min \begin{cases} (0 + 0.64 + 0.48) \\ (0.12 + 0.29 + 0.48) \\ (0.38 + 0.15 + 0.48) \\ (0.52 + 0 + 0.48) \end{cases}$$

$$= \min \begin{cases} 1.12 \\ 0.89 \\ 1.01 \\ 1 \end{cases} = 0.89$$

$$c[3,6] = \min \begin{cases} c[3,2] + c[4,6] + P_i \\ c[3,3] + c[5,6] + P_i \\ c[3,4] + c[6,6] + P_i \\ c[3,5] + c[7,6] + P_i \end{cases}$$

$$= \min \begin{cases} (0 + 0.63 + 0.53) \\ (0.14 + 0.47 + 0.53) \\ (0.28 + 0.17 + 0.53) \\ (0.64 + 0 + 0.53) \end{cases}$$

$$= \min \begin{cases} 1.16 \\ 1.14 \\ 0.98 \\ 1.17 \end{cases} = 0.98$$

$$c[4,7] = \min \begin{cases} c[4,3] + c[5,7] + P_i \\ c[4,4] + c[6,7] + P_i \\ c[4,5] + c[7,7] + P_i \\ c[4,6] + c[8,7] + P_i \end{cases}$$

$$= \min \begin{cases} (0 + 0.85 + 0.58) \\ (0.07 + 0.53 + 0.58) \\ (0.29 + 0.19 + 0.58) \\ (0.63 + 0 + 0.58) \end{cases}$$

$$= \min \begin{cases} 1.43 \\ 1.18 \\ 1.06 \\ 1.21 \end{cases} = 1.06$$

let's calculate root for above cost.

$r[1,4] = 2$
$r[2,5] = 3$
$r[3,6] = 5$
$r[4,7] = 6$

Let's calculate cost for $j-i = 4$

$$c[1,5] = \min \begin{cases} k=1, & c[1,0] + c[2,5] + P_i \\ k=2, & c[1,1] + c[3,5] + P_i \\ k=3, & c[1,2] + c[4,5] + P_i \\ k=4, & c[1,3] + c[5,5] + P_i \\ k=5, & c[1,4] + c[6,5] + P_i \end{cases}$$

$$= \min \begin{cases} (0 + 0.89 + 0.64) \\ (0.16 + 0.64 + 0.64) \\ (0.4 + 0.29 + 0.64) \\ (0.72 + 0.15 + 0.64) \\ (0.93 + 0 + 0.64) \end{cases}$$

$$= \min \begin{cases} 1.53 \\ 1.44 \\ 1.33 \\ 1.51 \\ 1.57 \end{cases} = 1.33$$

$$c[2,6] = \min \begin{cases} k=2, & c[2,1] + c[3,6] + P_i \\ k=3, & c[2,2] + c[4,6] + P_i \\ k=4, & c[2,3] + c[5,6] + P_i \\ k=5, & c[2,4] + c[6,6] + P_i \\ k=6, & c[2,5] + c[7,6] + P_i \end{cases}$$

$$= \min \begin{cases} (0 + 0.98 + 0.65) \\ (0.12 + 0.63 + 0.65) \\ (0.38 + 0.47 + 0.65) \\ (0.52 + 0.17 + 0.65) \\ (0.89 + 0 + 0.65) \end{cases}$$

$$= \min \begin{cases} 1.63 \\ 1.4 \\ 1.5 \\ 1.34 \\ 1.54 \end{cases} = 1.34$$

$$c[3,7] = \min \begin{cases} k=3,\ c[3,2] + c[4,7] + P_i \\ k=4,\ c[3,3] + c[5,7] + P_i \\ k=5,\ c[3,4] + c[6,7] + P_i \\ k=6,\ c[3,5] + c[7,7] + P_i \\ k=7,\ c[3,6] + c[8,7] + P_i \end{cases}$$

$$= \min \begin{cases} (0 + 1.06 + 0.72) \\ (0.14 + 0.85 + 0.72) \\ (0.28 + 0.53 + 0.72) \\ (0.64 + 0.19 + 0.72) \\ (0.98 + 0 + 0.72) \end{cases}$$

$$= \min \begin{cases} 1.78 \\ 1.71 \\ 1.53 \\ 1.55 \\ 1.7 \end{cases} = 1.53$$

let's calculate root for above cost.

$r[1,5] = 3$
$r[2,6] = 5$
$r[3,7] = 5$

let's calculate cost for $j-i=5$

$$c[1,6] = \min \begin{cases} k=1,\ c[1,0] + c[2,6] + P_i \\ k=2,\ c[1,1] + c[3,6] + P_i \\ k=3,\ c[1,2] + c[4,6] + P_i \\ k=4,\ c[1,3] + c[5,6] + P_i \\ k=5,\ c[1,4] + c[6,6] + P_i \\ k=6,\ c[1,5] + c[7,6] + P_i \end{cases}$$

$$= \min \begin{cases} (0 + 1.34 + 0.81) \\ (0.16 + 0.98 + 0.81) \\ (0.4 + 0.63 + 0.81) \\ (0.72 + 0.47 + 0.81) \\ (0.93 + 0.17 + 0.81) \\ (1.33 + 0 + 0.81) \end{cases}$$

$$= \min \begin{cases} 2.15 \\ 1.95 \\ 1.84 \\ 2 \\ 1.91 \\ 2.14 \end{cases} = 1.84$$

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$$c[2,7] = \min \begin{cases} k=2, c[2,1] + c[3,7] + p_i \\ k=3, c[2,2] + c[4,7] + p_i \\ k=4, c[2,3] + c[5,7] + p_i \\ k=5, c[2,4] + c[6,7] + p_i \\ k=6, c[2,5] + c[7,7] + p_i \\ k=7, c[2,6] + c[8,7] + p_i \end{cases}$$

$$= \min \begin{cases} (0 + 1.53 + 0.84) \\ (0.12 + 1.06 + 0.84) \\ (0.38 + 0.86 + 0.84) \\ (0.52 + 0.53 + 0.84) \\ (0.89 + 0.19 + 0.84) \\ (1.34 + 0 + 0.84) \end{cases}$$

$$= \min \begin{cases} 2.37 \\ 2.02 \\ 2.08 \\ 1.89 \\ 1.92 \\ 2.18 \end{cases} = 1.89$$

Let's calculate root for above cost

$r[1,6] = 3$ $r[2,7] = 5$

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let's calculate cost for $j-i = 6$

$$c[1,7] = \min \begin{cases} k=1, c[1,0] + c[2,7] + p_i \\ k=2, c[1,1] + c[3,7] + p_i \\ k=3, c[1,2] + c[4,7] + p_i \\ k=4, c[1,3] + c[5,7] + p_i \\ k=5, c[1,4] + c[6,7] + p_i \\ k=6, c[1,5] + c[7,7] + p_i \\ k=7, c[1,6] + c[8,7] + p_i \end{cases}$$

$$= \min \begin{cases} (0 + 1.89 + 1) \\ (0.16 + 1.53 + 1) \\ (0.4 + 1.06 + 1) \\ (0.72 + 0.85 + 1) \\ (0.93 + 0.53 + 1) \\ (1.33 + 0.19 + 1) \\ (1.84 + 0 + 1) \end{cases}$$

$$= \min \begin{cases} 2.89 \\ 2.69 \\ 2.46 \\ 2.57 \\ 2.46 \\ 2.52 \\ 2.84 \end{cases} = 2.46$$

and root value for above cost is

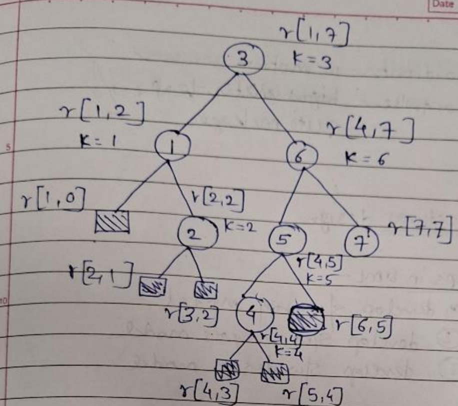
$r[1,7] = 3$

cost table -

	0	1	2	3	4	5	6	7
1	0	0.16	0.4	0.72	0.93	1.33	1.84	2.46
2		0	0.12	0.38	0.52	0.89	1.34	1.89
3			0	0.14	0.28	0.64	0.98	1.53
4				0	0.07	0.29	0.63	1.06
5					0	0.15	0.47	0.85
6						0	0.17	0.53
7							0	0.19
8								0

root

	1	2	3	4	5	6	7
1	1	1	2	2	3	3	3
2		2	3	3	3	5	5
3			3	3	5	5	5
4				4	5	5	6
5					5	6	6
6					.	6	7
7							7



2. **Determine an LCS of {1; 0; 0; 1; 0; 1; 0; 1} and {0; 1; 0; 1; 1; 0; 1; 1; 0}. (20p)**

Q.2] →

x = {1, 0, 0, 1, 0, 1, 0, 1}

y = {0, 1, 0, 1, 1, 0, 1, 1, 0}

Step 1 -

		0	1	0	1	1	0	1	1	0
	0	0	0	0	0	0	0	0	0	0
1	0	→0	↘1	→1	↘1	↘1	→1	↘1	↘1	→1
0	0									
0	0									
1	0									
0	0									
1	0									
0	0									
1	0									

Step 2 -

		0	1	0	1	1	0	1	1	0
	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	1	1	1	1
0	0	↘1	→1	↘2	→2	→2	↘2	→2	→2	↘2
0	0									
1	0									
0	0									
1	0									
0	0									
1	0									

Step 3 -

		0	1	0	1	1	0	1	1	0
	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	1	1	1	1
0	0	1	1	2	2	2	2	2	2	2
0	0	↘1	→1	↘2	→2	→2	↘3	→3	→3	↘3
1	0									
0	0									
1	0									
0	0									
1	0									

Step 4 -

		0	1	0	1	1	0	1	1	0
	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	1	1	1	1
0	0	1	1	2	2	2	2	2	2	2
0	0	1	1	2	2	2	3	3	3	3
1	0	↓1	2	→2	↘3	↘3	→3	↘4	↘4	→4
0	0									
1	0									
0	0									
1	0									

Step 5 -

		0	1	0	1	1	0	1	1	0
	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	1	1	1	1
0	0	1	1	2	2	2	2	2	2	2
0	0	1	1	2	2	2	3	3	3	3
1	0	1	2	2	3	3	3	4	4	4
0	0	1	2	3	3	3	4	4	4	5
1	0									
0	0									
1	0									

Step 6 -

		0	1	0	1	1	0	1	1	0
	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	1	1	1	1
0	0	1	1	2	2	2	2	2	2	2
0	0	1	1	2	2	2	3	3	3	3
1	0	1	2	2	3	3	3	4	4	4
0	0	1	2	3	3	3	4	4	4	5
1	0	1	2	3	4	4	4	5	5	5
0	0									
1	0									

Step 7 -

		0	1	0	1	1	0	1	1	0
	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	1	1	1	1
0	0	1	1	2	2	2	2	2	2	2
0	0	1	1	2	2	2	3	3	3	3
1	0	1	2	2	3	3	3	4	4	4
0	0	1	2	3	3	3	4	4	4	5
1	0	1	2	3	4	4	4	5	5	5
0	0	1	2	3	4	4	5	5	5	6
1	0									

Step 8 -

		0	1	0	1	1	0	1	1	0
	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	1	1	1	1
0	0	1	1	2	2	2	2	2	2	2
0	0	1	1	2	2	2	3	3	3	3
1	0	1	2	2	3	3	3	4	4	4
0	0	1	2	3	3	3	4	4	4	5
1	0	1	2	3	4	4	4	5	5	5
0	0	1	2	3	4	4	5	5	5	6
1	0	1	2	3	4	5	5	6	6	6

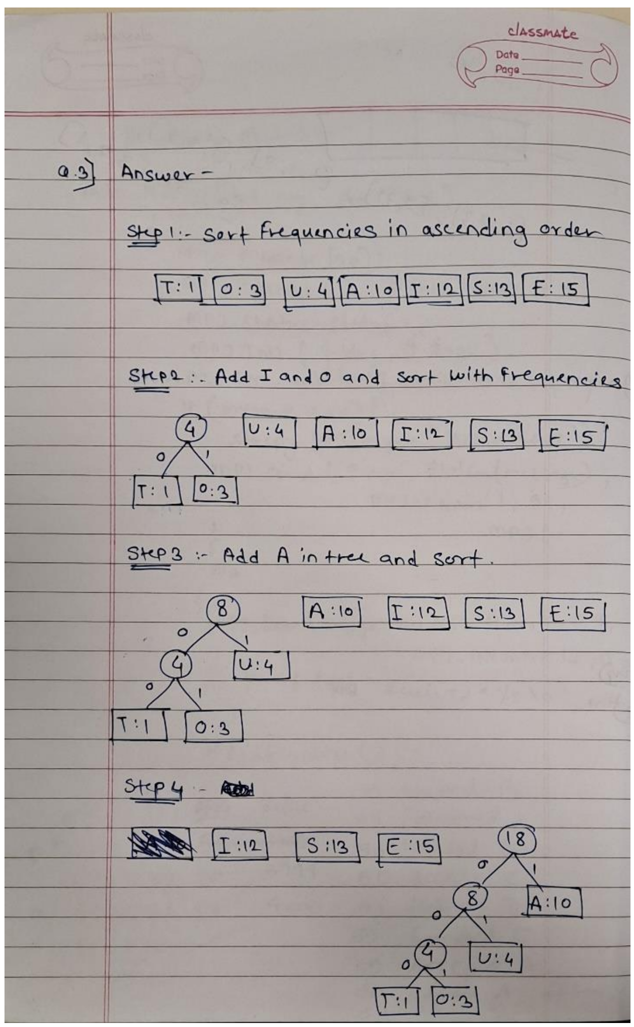
Final- Get the sequence using Backtrack.

		$\boxed{0}$	$\boxed{1}$	$\boxed{0}$	1	$\boxed{1}$	$\boxed{0}$	1	$\boxed{1}$	0
	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	1	1	1	1
0	0	1	1	2	2	2	2	2	2	2
$\boxed{0}$	0	1	1	2	2	2	3	3	3	3
$\boxed{1}$	0	1	2	2	3	3	3	4	4	4
$\boxed{0}$	0	1	2	3	3	3	4	4	4	5
$\boxed{1}$	0	1	2	3	4	4	4	5	5	5
$\boxed{0}$	0	1	2	3	4	4	5	5	5	6
$\boxed{1}$	0	1	2	3	4	5	5	6	6	6

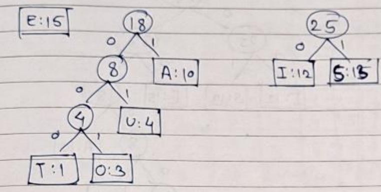
so, common subsequence is <u>010101</u>

3. Construct a Huffman tree for the following table (20p)

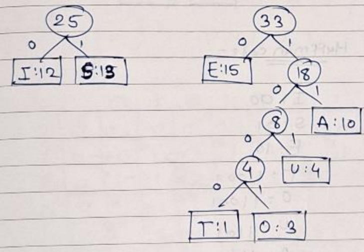
Characters	Frequencies
a	10
e	15
i	12
o	3
u	4
s	13
t	1



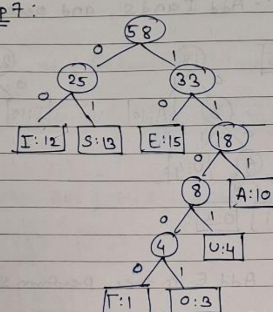
Step 5 :- Add I and S, and perform sort



Step 6 :- Add E in tree, perform sort.



Step 7 :



Huffman code -

I : 00
S : 01
E : 10
T : 11000
O : 11001
U : 1101
A : 111

4. **From the following algorithm design techniques which one is used to find all the pairs of shortest distances in a graph? (10p)**

   a) Backtracking
   b) Greedy
   c) Dynamic programming
   d) Divide and Conquer

5. **In Huffman coding, data in a tree always occur? (10p)**
   a) roots
   b) leaves
   c) left sub trees
   d) right sub trees

6. **Which of the following is/are element/elements of a dynamic programming problem?**
   a) Optimal substructure (10p)
   b) Overlapping subproblems
   c) Greedy approach
   d) Both optimal substructure and overlapping subproblems

7. **If a problem can be broken into subproblems which are reused several times, the problem possesses _____________ property.(10p)**
   a) Overlapping subproblems
   b) Optimal substructure
   c) Memoization
   d) Greedy