Q1 Probability Part I

8 Points

Below is a table listing the probabilities of three binary random variables.
Fill in the correct values for each marginal or conditional probability below.

X_0	X_1	X_2	$P(X_0, X_1, X_2)$
0	0	0	0.160
1	0	0	0.100
0	1	0	0.120
1	1	0	0.040
0	0	1	0.180
1	0	1	0.200
0	1	1	0.120
1	1	1	0.080

Q1.1

4 Points

$P(X_0 = 1, X_1 = 0, X_2 = 1)$

.2

$P(X_0 = 0, X_1 = 1)$

.24

$P(X_2 = 0)$

.42

Q1.2

4 Points

$P(X_1 = 0 \mid X_0 = 1)$

.714

$P(X_0 = 1, X_1 = 0 \mid X_2 = 1) P(X0 = 1, X1 = 0 \mid X2 = 1)$

.345

$P(X_0 = 1 \mid X_1 = 0, X_2 = 1)$

.526

Q2 Probability, Part II

10 Points

You are given the prior distribution $P(X)$, and two conditional distributions $P(Y \mid X)$ and $P(Z \mid Y)$ as below (you are also given the fact that Z is independent from X given Y).
All variables are binary variables.
Compute the following joint distributions based on the chain rule.

X	$P(X)$
0	0.500
1	0.500

Y	X	$P(Y\|X)$
0	0	0.600
1	0	0.400
0	1	0.900
1	1	0.100

Z	Y	$P(Z\|Y)$
0	0	0.100
1	0	0.900
0	1	0.700
1	1	0.300

Q2.1

Homework 5 GRADED

STUDENT
Rushikesh Machhindra Khamkar

TOTAL POINTS
100 / 100 pts

QUESTION 1
Probability Part I — **8** / 8 pts
1.1 (no title) — **4** / 4 pts
1.2 (no title) — **4** / 4 pts

QUESTION 2
Probability, Part II — **10** / 10 pts
2.1 (no title) — **5** / 5 pts
2.2 (no title) — **5** / 5 pts

QUESTION 3
Probability, Part III — **8** / 8 pts

QUESTION 4
Chain Rule — **8** / 8 pts

QUESTION 5
Bayes' Nets and Probability — 10 / 10 pts

5 Points

$P(X=0, Y=0)$

.3

$P(X=1, Y=0)$

.45

$P(X=0, Y=1)$

.2

$P(X=1, Y=1)$

.05

Q2.2

5 Points

$P(X=0, Y=0, Z=0)$

.03

$P(X=1, Y=1, Z=0)$

.035

$P(X=1, Y=0, Z=1)$

.405

$P(X=1, Y=1, Z=1)$

.015

Q3 Probability, Part III

8 Points

For each of the following four subparts, you are given three joint probability distribution tables. For each distribution, please identify if the given independence / conditional independence assumption is true or false.

For your convenience, we have also provided some marginal and conditional probability distribution tables that could assist you in solving this problem.

X	Y	$P(X,Y)$
0	0	0.240
1	0	0.160
0	1	0.360
1	1	0.240

X	$P(X)$
0	0.600
1	0.400

Y	$P(Y)$
0	0.400
1	0.600

X is independent from Y.

◉ True

○ False

X	Y	$P(X,Y)$

0	0	0.540
1	0	0.360
0	1	0.060
1	1	0.040

X	$P(X)$
0	0.600
1	0.400

X	Y	$P(X\|Y)$
0	0	0.600
1	0	0.400
0	1	0.600
1	1	0.400

X is independent from Y.

◉ True

○ False

X	Y	Z	$P(X,Y,Z)$
0	0	0	0.280
1	0	0	0.070
0	1	0	0.210
1	1	0	0.140
0	0	1	0.060
1	0	1	0.060
0	1	1	0.030
1	1	1	0.150

X	Z	$P(X\|Z)$
0	0	0.700
1	0	0.300
0	1	0.300
1	1	0.700

Y	Z	$P(Y\|Z)$
0	0	0.500
1	0	0.500
0	1	0.400
1	1	0.600

X	Y	Z	$P(X,Y\|Z)$
0	0	0	0.400
1	0	0	0.100
0	1	0	0.300
1	1	0	0.200
0	0	1	0.200
1	0	1	0.200
0	1	1	0.100
1	1	1	0.500

X is independent from Y given Z.

○ True

◉ False

X	Y	Z	$P(X,Y,Z)$
0	0	0	0.140
1	0	0	0.140
0	1	0	0.060
1	1	0	0.060
0	0	1	0.048
1	0	1	0.192
0	1	1	0.072
1	1	1	0.288

X	Z	$P(X\|Z)$
0	0	0.500
1	0	0.500
0	1	0.200
1	1	0.800

Y	Z	$P(Y\|Z)$
0	0	0.700
1	0	0.300
0	1	0.400
1	1	0.600

X	Y	Z	$P(X,Y\|Z)$
0	0	0	0.350
1	0	0	0.350
0	1	0	0.150
1	1	0	0.150
0	0	1	0.080
1	0	1	0.320
0	1	1	0.120
1	1	1	0.480

X is independent from Y given Z.

- (•) True
- ( ) False

Q4 Chain Rule

8 Points

Select all expressions that are equivalent to the specified probability using the given independence assumptions.

Given no independence assumptions, $P(A, B \mid C) =$

- [ ] $\frac{P(C|A)P(A|B)P(B)}{P(C)}$
- [ ] $\frac{P(B,C|A)P(A)}{P(B,C)}$
- [x] $P(A \mid B, C)P(B \mid C)$
- [ ] $\frac{P(A|C)P(B,C)}{P(C)}$

Given that A is independent of B given C, $P(A, B \mid C)$ =

- [ ] $$ $\frac{P(C|A)P(A|B)P(B)}{P(C)}$
- [ ] $\frac{P(B,C|A)P(A)}{P(B,C)}$
- [x] $P(A \mid B, C)P(B \mid C)$
- [x] $\frac{P(A|C)P(B,C)}{P(C)}$

Given no independence assumptions, $P(A \mid B, C)$ =

- [ ] $\frac{P(C|A)P(A|B)P(B)}{P(C)}$
- [x] $\frac{P(B,C|A)P(A)}{P(B,C)}$
- [ ] $\frac{P(A|C)P(C|B)P(B)}{P(B,C)}$
- [x] $\frac{P(C|A,B)P(B|A)P(A)}{P(B|C)P(C)}$

Given that A is independent of B given C, $P(A \mid B, C)$ =

- [ ] $\frac{P(C|A)P(A|B)P(B)}{P(C)}$
- [x] $\frac{P(B,C|A)P(A)}{P(B,C)}$
- [x] $\frac{P(A|C)P(C|B)P(B)}{P(B,C)}$
- [x] $\frac{P(C|A,B)P(B|A)P(A)}{P(B|C)P(C)}$

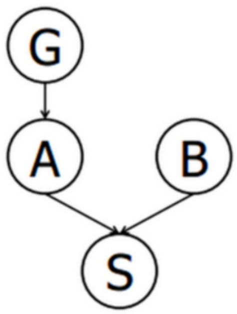
Q5 Bayes' Nets and Probability

10 Points

Suppose that a patient can have a symptom (S) that can be caused by two different diseases (A and B). It is known that the variation of gene G plays a big role in the manifestation of disease A. The Bayes' Net and corresponding probability tables for this situation are shown below.

G	$P(G)$
g	0.10
$\neg g$	0.90

G	A	$P(A \mid G)$
g	a	1.00
g	$\neg a$	0.00
$\neg g$	a	0.10
$\neg g$	$\neg a$	0.90



B	$P(B)$
b	0.40
$\neg b$	0.60

A	B	S	$P(S \mid A, B)$
a	b	s	1.00
a	b	$\neg s$	0.00
a	$\neg b$	s	0.90
a	$\neg b$	$\neg s$	0.10
$\neg a$	b	s	0.80
$\neg a$	b	$\neg s$	0.20
$\neg a$	$\neg b$	s	0.10
$\neg a$	$\neg b$	$\neg s$	0.90

Q5.1

8 Points

Compute $P(g, a, b, s)$.

.04

What is the probability that a patient has disease A?

.19

What is the probability that a patient has disease A given that they have disease B?

.19

What is the probability that a patient has disease A given that they have symptom S and disease B?

.2267

Q5.2

2 Points

What is the probability that a patient has the disease carrying gene variation G given that they have disease A?

.5263

What is the probability that a patient has the disease carrying gene variation G given that they have disease B?

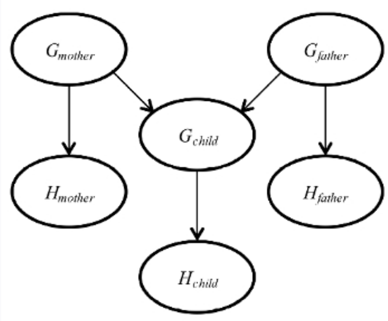
.1

Q6 Bayes' Nets Independence

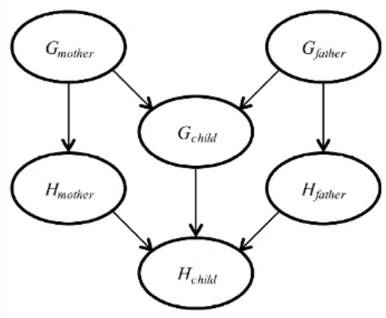
8 Points

Let H_x be a random variable denoting the handedness of an individual x, with possible values l or r. A common hypothesis is that left- or right-handedness is inherited by a simple mechanism; that is, perhaps there is a gene G_x, also with values l or r, and perhaps actual handedness turns out mostly the same (with some probability s) as the gene an individual possesses. Furthermore, perhaps the gene itself is equally likely to be inherited from either of an individual's parents, with a small nonzero probability m of a random mutation flipping the handedness.

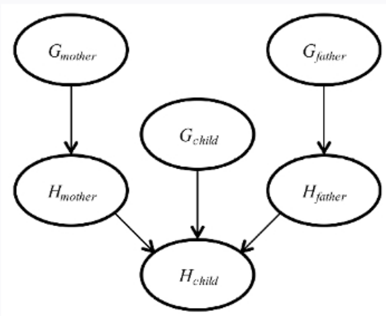
The following three images are possible models involving the genes G and handednesses H.



(a)



(b)



(c)

Which of the three networks above claim that $P(G_{father}, G_{mother}, G_{child}) = P(G_{father})P(G_{mother})P(G_{child})$?

- [ ] (a)
- [ ] (b)
- [x] (c)

Which of the three networks make independence claims that are consistent with the hypothesis about the inheritance of handedness?

- [x] (a)
- [x] (b)
- [ ] (c)

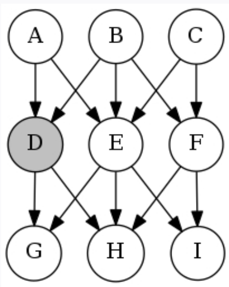
Which of the three networks is the best description of the hypothesis?

- (•) (a)
- ( ) (b)
- ( ) (c)

Q7 D-Separation

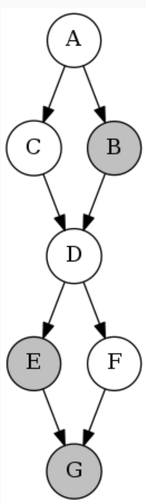
10 Points

You are given several graphical models below, and each graphical model is associated with an independence (or conditional independence) assertion. Please specify if the assertion is true or false.



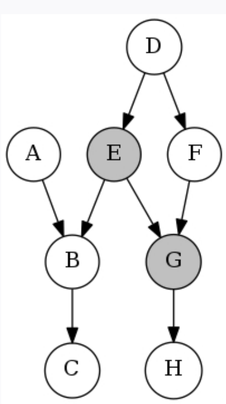
It is guaranteed that G is independent of H given D

- ( ) True
- (•) False



It is guaranteed that A is independent of D given E, B, G

- ( ) True
- (•) False

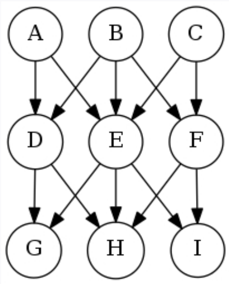


This was mistake that I could not remove. Just mark the following as True

- ◉ True
- ○ False

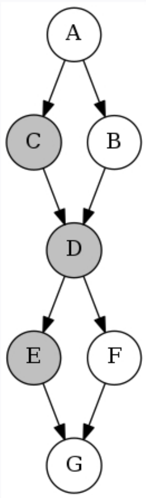
It is guaranteed that H is independent of B given G, E

- ◉ True
- ○ False



It is guaranteed that D is independent of C given F

- ○ True
- ◉ False



It is guaranteed that G is independent of B given C, E, D

- ◉ True
- ○ False

Q8 Combining Factors

10 Points

Given the factors $P(A \mid C)$ and $P(B \mid A, C)$ what is the resulting factor after joining over C ?

- ○ $P(A, B, C)$
- ○ $P(A \mid B, C)$
- ◉ $P(A, B \mid C)$
- ○ None of the above.

Given the factors P(A|C) and P(B|A,C) what is the resulting factor after joining over A and summing over A?

- ○ $P(C)$
- ○ $P(B)$
- ○ $P(B, C)$
- ○ $P(A \mid C)$
- ◉ $P(B \mid C)$

○ None of the above.

Given the factors $P(C|A)$, $P(D|A,B,C)$, $P(B|A,C)$, what is the resulting factor after joining over C and summing over C?

○ $P(D \mid A)$

○ $P(C, D \mid A)$

○ $P(B, C, D \mid A)$

◉ $P(B, D \mid A)$

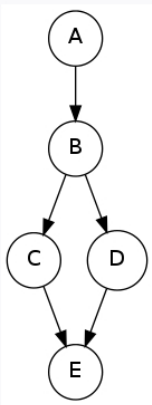
○ $P(C, B \mid A, D) * P(A \mid D)$

○ None of the above.

Q9 Variable Elimination Tables

10 Points

Assume the following Bayes Net and corresponding CPTs. In this exercise, we are given the query $P(C \mid e = 1)$, and we will complete the tables for each factor generated during the elimination process.



A	$P(A)$
0	0.700
1	0.300

B	A	$P(B\|A)$
0	0	0.500
1	0	0.500
0	1	0.900
1	1	0.100

C	B	$P(C\|B)$
0	0	0.500
1	0	0.500
0	1	0.100
1	1	0.900

D	B	$P(D\|B)$
0	0	0.500
1	0	0.500
0	1	0.200
1	1	0.800

C	D	$P(e=1\|C,D)$
0	0	0.900
1	0	0.600
0	1	0.700
1	1	0.500

Three steps are required for elimination, with the resulting factors listed below:

Q9.1

2 Points

Step 1: eliminate A. We get the factor $f_1(B) = \sum_a P(a)P(B|a)$
what is $f_1(B = 0) =$

.62

$f1(B = 1) =$

.38

Q9.2

2 Points

Step 2: eliminate B. We get the factor $f_2(C, D) = \sum_b P(C|b)P(D|b)f_1(b)$
$f_2(C = 0, D = 0) =$

.1626

$f_2(C = 1, D = 0) =$

.2234

$f_2(C = 0, D = 1) = .1854$

$f_2(C=1, D=1) = .4286$

Q9.3

3 Points

Step 3: eliminate D. We get the factor $f_3(C, e=1) = \sum_d P(e=1|C,d) f_2(C,d)$.
$f_3(C=0, e=1) =$

.27612

Q9.4

3 Points

After getting the final factor $f_3(C, e=1)$, a final renormalization step needs to be carried out to obtain the conditional probability $P(C|e=1)$. Fill in the final conditional probabilities below.

$P(C=0 \mid e=1) =$

.442

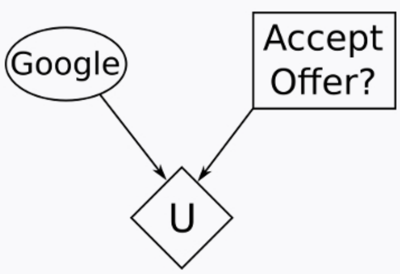
$P(C=1 \mid e=1) =$

.558

Q10 Decision Network

10 Points

You've been job hunting, and you've narrowed your options to two companies:
Acme and Google. You already have an offer from Acme, but it expires today, and you are still waiting for a response from Google. You are faced with the dilemma of whether or not to accept the offer from Acme, which is modeled by the following decision diagram:



Your prior belief about whether Google will hire you and utility over possible outcomes are as follows:

Google outcome	P(Google outcome)
hired	0.25
not hired	0.75

Action	Google outcome	U
accept Acme offer	hired	2000
accept Acme offer	not hired	8000
reject Acme offer	hired	10000
reject Acme offer	not hired	0

Q10.1

2 Points

What is the expected utility of each action? (Note: throughout this problem answers will be evaluated to whole-number precision, so your answer should differ by no more than 1 from the exact answer.)

Action: accept Acme offer

6500

Action: reject Acme offer

2500

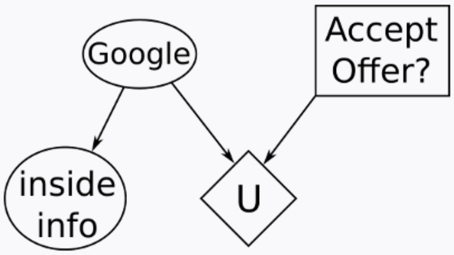
Which action should you take?

- ○ reject
- ◉ accept

Q10.2

8 Points

Suddenly, the phone rings. It's your uncle, who works at Google. Your uncle tells you he has some inside information about the status of your application. Your uncle won't tell you what the information is yet, but he might be willing to divulge it for the right price. You model the new situation by adding a new node to your decision diagram:



You create a CPT to model the relationship between the inside information and Google's future hiring decision:

info	Google outcome	P(info | Google outcome)
good news	hired	0.7
bad news	hired	0.3
good news	not hired	0.1
bad news	not hired	0.9

We'll help grind through the probabilistic inference. The resulting distributions are:

info	P(info)
good news	0.25
bad news	0.75

Google outcome	info	P(Google outcome | info)
hired	good news	0.7
not hired	good news	0.3
hired	bad news	0.1
not hired	bad news	0.9

Fill in the expected utilities for each action, for each possible type of information we could be given:

EU(accept Acme offer | good news)

3800

EU(reject Acme offer | good news)

7000

EU(accept Acme offer | bad news)

7400

EU(reject Acme offer | bad news)

1000

What is the maximum expected utility for each type of information we could be given?

MEU(good news)

7000

MEU(bad news)

7400

If we are given the inside information, what is the expected value of MEU?

7300

What is the value of perfect information of the random variable Inside Info?

800

Q11 Value of Perfect Information

8 Points

Consider the value of perfect information (VPI) of observing some node in an arbitrary decision network. Which of the following are true statements?

- [ ] VPI is guaranteed to be positive (> 0).
- [x] VPI is guaranteed to be nonnegative (≥ 0).
- [ ] VPI is guaranteed to be nonzero.
- [x] The MEU after observing a node could potentially be less than the MEU before observing that node.
- [ ] For any two nodes X and Y, $\mathrm{VPI}(X) + \mathrm{VPI}(Y) \geq \mathrm{VPI}(X, Y)$. That is, the sum of individual VPI's for two nodes is always greater than or equal to the VPI of observing both nodes.
- [x] VPI is guaranteed to be exactly zero for any node that is conditionally independent (given the evidence so far) of all parents of the utility node.