Homework 1. Basics on definitions, proofs and propositional logic

Rushikesh Khamkar

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1. **(10 points) Everyone is required to sign this form: https://forms.gle/EZRNcWq2TWXaJkZDA**

   Solution: I have signed the form that is available at the link above.

2. **(10) 1) Which of the following strings are official propositions (according to our definition of propositions).**

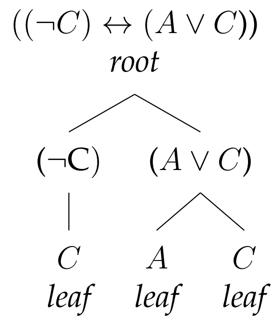
   (a) $((\neg(A \vee B)) \wedge C)$
   (b) $(A \wedge B) \vee C$
   (c) $(A \wedge (B \wedge C)))$

   Solution:

Is it Proposition?	**Answer**
$((\neg(A \vee B)) \wedge C)$	The answer is yes, as it adheres to the Propositional Logic Language (connective symbols, parenthesis and propositional letters).
$(A \wedge (B \wedge C)))$	That's not a proposition, either.
$(A \wedge (B \wedge C)))$	That's not a proposition, either.

   2) Draw the formation tree of $((\neg C) \leftrightarrow (A \vee C))$**.**

   Solution:



3. **(20) Following our proof methodology prove**

$((A \to B) \leftrightarrow C)$ **is a proposition.**

Goals for this questions:

- **Understand well the working backward proof method. Write steps for working backward on a scratch paper. During working backward, also practice application of definitions and decomposition of the current statement to prove into the "main concept" or logical connective and the rest (similar to the formation tree of a proposition).**
- **From your working backward steps, write the proof steps in a "forward" way. See the format of the "Final proof" in L3-ProofExamples.pdf available on blackboard.**

Solution:

Steps	**Proof**	**Reason**
a.	A is a proposition letter.	-By definition of proposition letter.
b.	A is a proposition.	-By (a) and definition of proposition.
c.	B is a proposition letter.	-By definition of proposition letter.
d.	B is a proposition.	-By (c) and definition of proposition.
e.	A and B are propositions.	-By (b),(d) and logical connective $\to$ **(implies)**.
f.	If A and B are propositions, $(A \to B)$ is a proposition.	-By definition of proposition with $\alpha = A$ and $\beta = B$. -By (d) and (e)
g.	$(A \to B)$ is a proposition.	-By (f)
h.	C is a proposition letter.	-By definition of proposition letter.
i.	C is a proposition.	-By (h) and definition of proposition.
j.	$(A \to B)$ and C are propositions.	-By (g),(i) and logical connective $\leftrightarrow$ **(if and only if)**.
k.	If $(A \to B)$ and C are propositions, $((A \to B) \leftrightarrow C)$ is a proposition.	-By Definition of proposition with $\alpha = (A \to B)$ and $\beta = C$.
l.	$((A \to B) \leftrightarrow C)$ is a proposition.	-By (g),(i), (k)

4. **(10) Although we follow the content in the book, but our class covers much more (e.g., identifying concepts and their parameters, precise definitions and etc.) than what is printed on the textbook. Also the materials in this class are so special that one unlikely can answer the questions in homework or tests without understanding what is discussed during class and studying the notes and textbook. After you complete this homework, do you strongly agree, agree, keep neutral, disagree or strongly disagree with the statements above? Explain why you answer so.**

   Solution:
   Yes, I wholeheartedly concur that the things taught in the class are so valuable and in-depth that we don't need a textbook or notes. I believe this since the definition of propositional logic and the explanation of Question (2) from this homework are both excellent. The explanation of propositional logic from the accompanying notes made it simple to answer question 2 in this homework assignment. Additionally, drawing a tree's shape was simple because you had given examples in class. I found it simple to demonstrate the given example for Question 3 of this homework because of the concluding evidence provided in your Lecture 03 notes. I fervently concur that our course covers a lot more material than what is covered in the textbook because of this.

5. **(50) (Read and write definitions)**

   - **The definition of "Definition 3.2" is not as explicit and complete as we would like. Write a precise and complete definition. (Recall the discussion of how to define valuation during class. Also recall the definition of propositions to master the definition methods there.)**

     Solution:
     A truth valuation V function gives each proposition a distinct truth value V(a) in order to determine the value of a proposition's value on a compound proposition (i.e., one with a connective). The pertinent truth tables are then used to calculate this value.

     Consider the propositional letters A and B. as defined by propositional logic. Letters with a proposal are always propositions. If the assertions A and B are true, then the propositions $(\neg A)$, $($A $\vee B)$, $(A \wedge B)$, $(A \rightarrow B)$, and $(A \leftrightarrow B)$ are also true. The Truth values of a compound statement of A and B are always True or False according to propositional logic. Additionally, based on their compound statements and connectives, their truth values will be determined.

     Consider the statement, "If and only if A, then B," in the proposition $(A \leftrightarrow B)$. This statement will be understood as (($A \rightarrow B$) wedge ($B \rightarrow A$)) or ((neg A $\vee B) \wedge (A \vee \neg B)$) if it is rewritten.

According to the truth table below, B is true if and only if A is true. Alternatively, B is false if and only if A is false.

A	B	$(A \leftrightarrow B)$
T	T	T
T	F	F
F	T	F
F	F	T

- **Write the precise definition of the notation a_i (two lines below Figure 6 in Page 19 of the text book) in the proof for Theorem 2.8 (also see appendix). For any notations in your definition, find its meaning in the proof and write its precise definition here. Repeat this process until all notations are defined in this proof or outside the proof (e.g., you don't need to define $\wedge$).**

  Solution:

Definition of the notation a_i
$(A1, ..., Ak)$ are a series of propositional letters, with $A1$ serving as the first and Ak as the last.
$(A_1^{a_{i1}} \wedge ... \wedge A_k^{a_{ik}})$ is a_i.
The Truth Table's a_{ij} signifies the True or False value for the i^{th} row and j^{th} column (T,F).
I stands for row, and J for column.
For α, $\alpha^T = \alpha$ and $\alpha^F = (\neg\alpha)$

- **Write the following information for each of the concepts: the a_i above, and support in "Definition 2.5 (ii)" in the text book.**
  - **The name of the concept:**
  - **The parameter(s) of the concept:**
  - **Meta variables in the definition of the concept:**
  - **The concepts used in the definition (and defined before) (including names and their parameters):**

    Solution:
  - **For a_i:**

Concept	**Information**
Name of the concept	Adequacy connectives.
Parameter of the concept	Propositional Letter $(A_1, A_2, A_3, ...A_k)$, i row and j column.
Meta Variables	a(i,j) = (α^T, α^F)
Concepts to be defined	Propositional Letters, Propositional Functions, Conjunction Statements, and Connectives $(\neg, \vee, \wedge, \rightarrow, \leftrightarrow)$ are all types of propositions.

- **For Definition 2.5 (ii) :**

Concept	**Information**
Name of the concept	Proposition Tree Formation.
Parameter of the concept	Compound Proposition (S, C, T, D, Propositional Letter (s1, s2,s3, ...sn), Connectives $(\neg, \vee, \wedge, \rightarrow, \leftrightarrow)$
Meta Variables	Compound Proposition (String of Symbols) (Example - $s1 \wedge s2 \wedge s3 \wedge$ ..., or $(s1 \rightarrow s2 \leftrightarrow (s3 \wedge s4))$
Concepts to be defined	Propositional Letters, Propositional Functions, Conjunction Statements, and Connectives $(\neg, \vee, \wedge, \rightarrow, \leftrightarrow)$ are all types of propositions.