Homework 4. Resolution. Resolution Refutation.

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1. **State the soundness and completeness results about resolution, T-resolution, $\mathcal{A}$-resolution, SLD-resolution. (Make sure what "formula" your result is talking about.)**

   Solution:

Name	**Soundness Results**	**Completeness Results**
Resolution $\Rightarrow S$: set of clauses	If there is a *resolution refutation* of S, then S is unsatisfiable.	If S is unsatisfiable, then there is a resolution refutation of S.
T-resolution $\Rightarrow$ T-resolutions are resolutions in which neither parent clause is a tautology. RT(S) = closure of S under T-resolutions.	If $\Box \in$ RT(S), then S is unsatisfiable.	If S is unsatisfiable, then $\Box \in$ RT(S).
$\mathcal{A}$-resolution $\Rightarrow \mathcal{A}$: Assignment and S : Formula RA(S) = closure of S under $\mathcal{A}$-resolution.	For any $\mathcal{A}$ and S, if $\Box \in$ RA(S), then $S \in$ UNSAT.	For any $\mathcal{A}$ and S, if $S \in$ UNSAT, then $\Box \in$ RA(S).
SLD-resolution $\Rightarrow$ P is PROLOG Program and $\{G\}$ is Goal clause and $S = P \cup \{G\}$ is Set of Horn clauses which contain at most one positive literal.	If there is a SLD-resolution refutation of $P \cup \{G\}$	If $P \cup \{G\} \in$ UNSAT and R is any selection rule, then there is a SLD-resolution refutation of $P \cup \{G\}$ via R.

2. **Let** $S = \{\{A, \neg B, C\}, \{B, C\}, \{\neg A, C\}, \{B, \neg C\}, \{\neg B\}\}$. **What is** S^B**? What is** $S^{\neg B}$**?**

   Solution:

$$S : \{\{A, \neg B, C\}, \{B, C\}, \{\neg A, C\}, \{B, \neg C\}, \{\neg B\}\}$$

S^B: {{A, C}, $\{\neg A, C\}, \{\square\}\}$ $\quad$ $S^{\neg B}$: {{C}, $\{\neg A, C\}, \{\neg C\}\}$

3. **Follow our proof methods, prove the following.**

   If P is a PROLOG program and $G = \{\neg q_1, \ldots, \neg q_n\}$ a goal clause, then every q_i ($i \in 1..n$) is a consequence of P if and only if $P \cup \{G\}$ is unsatisfiable.

   You may use the following definition of consequence in your proof: a literal l is a *consequence of a formula* S if for every assignment $\mathcal{A}$ that satisfies S, $\mathcal{A}$ satisfies l, i.e., $l \in \mathcal{A}$.

   Solution:

Steps	Proof	Reason
a.	Assume, Assignment $\mathcal{A}$ for which $\mathcal{A}(P)$ is true	
b.	The goal clause G consists of negative literals	-Given $G = \{\neg q_1, \ldots, \neg q_n\}$ a goal clause.
c.	$\neg q_i$ and q_i will be true for all $i > 0$, $i \leq n$.	-By (b).
d.	All q_i are facts from the *prolog* program P. Thus it is a consequence of $\mathcal{A}(P)$ is true.	-By (a) and (b).
e.	$P \cup \{G\}$ is unsatisfiable.	-Given every q_i ($i \in 1..n$) is a consequence of P if and only if $P \cup \{G\}$ is unsatisfiable.
f.	$\mathcal{A}$ satisfies P ($\mathcal{A} \vdash P$) and makes G=$False$ as q_i=$True$ which and its negation $\neg q_i$=$False$, which implies G=$False$.	-By (e).
g.	P is a $PROLOG$ program.	-Given
h.	Every q_i ($i \in 1..n$) is a consequence of P if and only if $P \cup \{G\}$ is unsatisfiable.	-By (f) and (g)

   QED

4. **Consider a PROLOG program P and a conjunction of propositional letters $p_1, ..., p_n$. To prove that $p_1, ..., p_n$ is a consequence of P, what is the set of clauses that the resolution refutations (discussed in class) are based on? Explain why these methods works.**

Solution:

Method: A task that yields G $false$ and satisfies P is taken into consideration. The approach taken by $PROLOG$ to demonstrate that $p_1...p_n$ is a result of P is to formulate a goal clause $G = \neg p1...$ and add the goal clause to the program P to confirm the set $P \cup \{G\}$ of the horn clause is unsatisfiable.

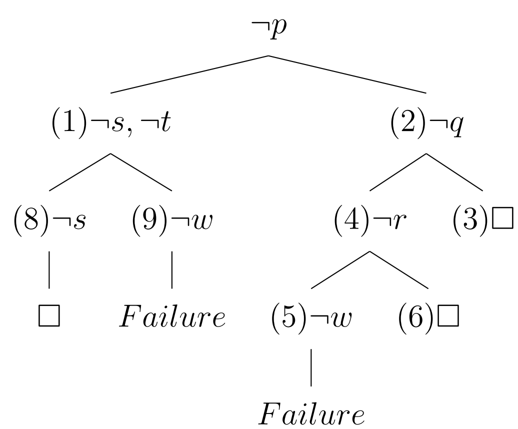
Why these methods works: Let us consider that the resulting set $P \cup \{G\}$ of horn clauses is unsatisfiable. All $\neg p_i$ must be $false$ for G to be $false$, which implies that p_i is *true*. Taking this into account, we can state that the propositional letter is *true* in PROLOG P, demonstrating the consequence relationship between the propositional letters and P. As a result of our assumption that P will be *true* in the event that $P \cup \{G\}$, we know that the assignment $\mathcal{A}$ on P p_i is *true*. If the assignment satisfies the program clause, then every propositional letter is also *true*.

5. **Consider a PROLOG program P:**

$\{\mathtt{p :- s, t.} \quad \mathtt{p :- q.} \quad \mathtt{q.} \quad \mathtt{q :- r.} \quad \mathtt{r.} \quad \mathtt{r :- w.} \quad \mathtt{r.} \quad \mathtt{s.} \quad \mathtt{t :- w.}\}$

(1) Draw an SLD-tree for goal ¬p in terms of P.

Solution:



(2) What is an SLD resolution refutation of $P \cup \{\{\neg \mathtt{p}\}\}$?

Solution:
A SLD resolution refutation of $P \cup \{\{\neg \mathtt{p}\}\}$ via the selection rule R is an LD resolution proof of
$< G_0(\neg p),\ C_0(p : \neg s, \neg t) >$,
$< G_1(\neg p),\ C_1(p : q) >$,
$< G_2(\neg s, \neg t),\ C_2(s) >$,
$< G_3(\neg q),\ C_3(q : r) >$,
$< G_4(\neg q),\ C_4(q) >$,
$< G_5(\neg r),\ C_5(r : \neg w) >$,

$< G_6(\neg r), C_6(r) >$
with $G_0 = \neg p$ and $G_n + 1$ in which $R(G_i)$ is the literal resolved at the $(i+1)$ step in the proof.