Homework 3. Tableau Proof. Resolution.

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1. **(15) For Definition 6.2, write the following information in the order they occur in the definition**

   - **For each concept defined by this definition, write its name and parameters (if there is any)**
     Solution:

Concept defined by this definition	**Answer**
Name of the concept	Tableau Proof.
Parameter of the concept	Σ and α

   - **For each concept used in this definition, write its name and arguments (if there is any)**

     Solution:

Concept used by this definition	**Answer**
Name of the concept	Tableaux from Premises.
Arguments of the concept	proposition, root entry $F\alpha$, contradictory path, provable

   - **Write meta variables in the definition.**
     Solution:

Meta variables	α and Σ

2. **(15) For Definition 8.4, write the following information in the order they occur in the definition**

   - **For each concept defined by this definition, write its name and parameters (if there is any)**

Solution:

Concept defined by this definition	**Answer**
Name of the concept	Resolution Refutation.
Parameter of the concept	S, C, i, j, k

- **For each concept used in this definition, write its name and arguments (if there is any)**

  Solution:

Concept used by this definition	**Answer**
Name of the concept	Resolution, Deduction
Arguments of the concept	provable, deduction of C, deduction of $\square$, S

- **Write meta variables in the definition.**

  Solution:

Meta variables	$\square$

3. **(15) i) Find the definition of *assignment* from Chapter 8. Write the definition below.**

   Solution:

   An assignment $\mathcal{A}$ is a consistent set of literals, i.e., one not containing both p and $\neg p$ for any propositional letter p. (This, of course, is just the (partial) truth assignment in which those $p \in \mathcal{A}$ are assigned T and those q with $\bar{q} \in \mathcal{A}$ are assigned F.) A complete assignment is one containing p or $\neg p$ for every propositional letter p. It corresponds to what we called a truth assignment in Definition 3.1.

   ii) Write the definition of another concept, whose name contains "assignment", that was defined before (see L04).

   Solution:

   If $\mathcal{A}$ given by p, q, r, s, t, i.e., the (partial) assignment such that $\mathcal{A}(p)$ = T = $\mathcal{A}(q)$ = $\mathcal{A}(r)$ = $\mathcal{A}(s)$ = $\mathcal{A}(t)$, then $\mathcal{A}$ is an assignment not satisfying the formula S in (iii). S is, however, satisfiable.

   iii) Is it precise for us to understand *truth assignment* as the combination of the English meaning of truth and the definition of *assignment* in i)? Why?

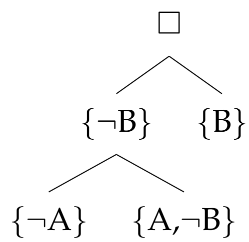
   Solution:

Yes, It is precis for us to understand *truth assignment* as the combination of the English meaning of truth and the definition of *assignment*. The reason to understand the English meaning of truth is that, In normal English statements, your prediction will be either true or false. Every assertion is either True or False in the two-valued logic that mathematicians often apply. A truth assignment demonstrates how the truth or falsity of the basic claims that make up a compound statement affects the truth or falsity of the compound statement. So, every assignment in propositional logic, the propositional letter p contains proposition p and $\neg p$.

4. **(15) i) Write the result of applying the definition of *satisfiable* (see Section 2.3 of L04).**

$$\{\{\neg A\}, \{A, \neg B\}, \{B\}\}.$$

Solution:



In above figure, From first 2 leaf nodes of tree, we can see that set $\neg A$ and set A, $\neg B$, will eliminate clause A and $\neg A$ and keep $\neg B$ as parent node. On the right side, we have another clause B, which results in Empty clause as root value. So, we can say, when A and $\neg A$ are in set of clauses, then set of the clause is unsatisfieble. Because $A \wedge \neg A$ is a contradiction. and, the same case applies for B and $\neg B$.

ii) According to the result above, is the formula satisfiable? If yes, give an assignment satisfying it.

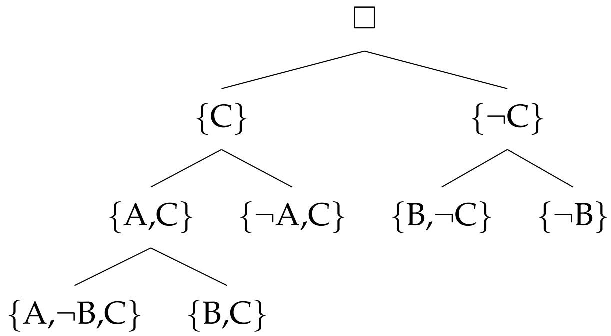
Solution:

From above contradiction in A and $\neg A$ in the formula. This formula is not satisfiable

5. **(20) Find a resolution tree refutation of the following formula:**

$$\{\{A, \neg B, C\}, \{B, C\}, \{\neg A, C\}, \{B, \neg C\}, \{\neg B\}\}.$$

Solution:



6. **(20) Prove that if the formula $S = \{C_1, C_2\}$ is satisfiable and C is the resolvent of C_1 and C_2, then C is satisfiable. Use our proof methodologies and format. Do not copy the proof in the book.**

Solution:

Steps	**Proof**	**Reason**
a.	Assume, $S = \{C_1, C_2\}$ satisfiable.	- Given $\{C_1, C_2\}$ is satisfiable.
b.	Assume, $C = C_1' \cup C_2'$	- Given C is the resolvent of C_1 and C_2.
c.	Assume, α is a proposition, $C_1 = \{\alpha\} \sqcup C_1'$, $C_2 = \{\bar{\alpha}\} \sqcup C_2'$.	- By (b).
d.	$\mathcal{A} \vDash S, \mathcal{A} \vDash C$.	- By statement (a) and (c), $\mathcal{A}$.
e.	$\mathcal{A} \vDash \{C_1, C_2\}$.	- By (d).
f.	$A \vDash C_1, A \vDash C_2$	- By (e).
g.	let $\alpha \notin \mathcal{A}$.	
h.	$\mathcal{A} \vDash C_1'$	-By (g).
i.	$\mathcal{A} \vDash C$	-By (b) and (h).
j.	let $\bar{\alpha} \notin \mathcal{A}$	
k.	$\mathcal{A} \vDash C_1'$	By (j)
l.	$\mathcal{A} \vDash C$	By (b) and (k).
m.	$\mathcal{A} \vDash C$	By (i) and (l).
n.	C is satisfiable.	By (a), (b) and (m).

QED