Homework 6 . Predicate Logic: PROLOG program. Resolution Proofs.

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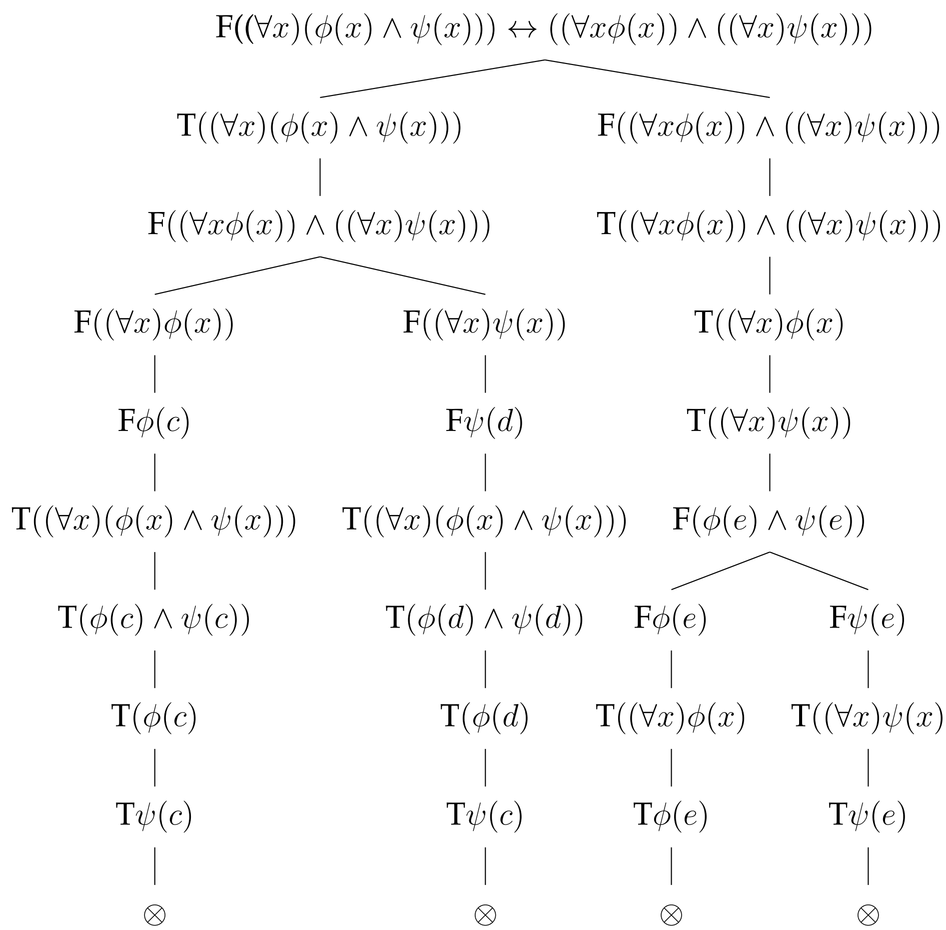
1. **(Tableau Proof). Write a tableau proof for**

$$((\forall x)(\phi(x) \wedge \psi(x))) \leftrightarrow ((\forall x \phi(x)) \wedge ((\forall x)\psi(x)))$$

Solution:



2. **Find the Prenex normal form of the following formula.**

$$\forall x \exists y P(x,y) \vee \neg \exists x \forall y Q(x,y).$$

Solution:

Steps	Steps for Prenex Normal Form
a.	$\forall x \exists y P(x,y) \vee \neg \exists x \forall y Q(x,y)$
b.	$\forall u [\exists y P(u,y) \vee \neg \exists x \forall y Q(x,y)]$
c.	$\forall u \exists v [P(u,v) \vee \neg \exists x \forall y Q(x,y)]$
d.	$\forall u \exists v [P(u,v) \vee \forall x \neg \forall y Q(x,y)]$
e.	$\forall u \exists v [P(u,v) \vee \forall x \exists y \neg Q(x,y)]$
f.	$\forall u \exists v \forall w [P(u,v) \vee \exists y \neg Q(w,y)]$
g.	$\forall u \exists v \forall w \exists z [P(u,v) \vee \neg Q(w,z)]$

3. **Find the Skolemization of the following sentence.**

$$\forall x \forall y \forall z \exists w \varphi(x,w,y,z).$$

Solution:

Skolemization of the given $\forall x \forall y \forall z \exists w \varphi(x,w,y,z)$ sentence is $\forall x \forall y \forall z \varphi(x, d(x,y,z), y, z)$.

4. **Let $\mathcal{L}$ consist of the constant c, function symbol $f/1$ and the unary predicate $R/1$.**

   (a) **What is the Herbrand universe for $\mathcal{L}$?**

   Solution:

   $U(L) : c, f(c), f(f(c)), ...$

   (b) **Give infinitely many Herbrand structures for $\mathcal{L}$. When defining/inventing a relation on the Herbrand universe, you may use the enumeration methods. For example, one binary relation on the Herbrand universe could be $\{\}$ or $\{(c, f(c)\}$ or $\{(c, f(c)), (f(c), f(f(c))), \ldots\}$.**

   Solution:

Sr.No.	Herbrand structures for $\mathcal{L}$
a.	$A = c, f(c), f(f(c)), ...$
b.	`C^` $= c$
c.	`fx^`$(c) = f(c)$
d.	`f^(f^`$(c)) = f(f(c)), ...$

(c) **Give a Herbrand model of $\forall x R(f(x))$.**

Solution:

Sr.No.	**Herbrand model of $\forall x R(f(x))$**
a.	$U(L) : c, f(c), f(f(c)), ...$
b.	$\texttt{C\^{}} = c$
c.	$R = \{c\}$
d.	$\texttt{f\^{}}(x) = c$

5. **Following the algorithm in the slides, list the major steps to find a most general unifier for the following expressions.**

$$\{Q(h(x,y),w), Q(h(g(v),a),f(v)), Q(h(g(v),a),f(b))\}.$$

The unify algorithm accepts term equations only. So, to unify the expressions above, the initial term equations are

$$Q(h(x,y),w) = Q(h(g(v),a),f(v))$$

(making the first two expressions identical) and

$$Q(h(g(v),a),f(v)) = Q(h(g(v),a),f(b))$$

(making the second expression and the third one identical and thus all expressions are identical and thus unified.)

Solution:

(a) $D(S_0) = h(x,y), h(g(v),a), \sigma_0 = x/g(v)$,
Next we obtain $S0 = Q(h(g(v),y),w), Q(h(g(v),a),f(v)), Q(h(g(v),a),f(b))$

(b) $D(S_1) = h(g(v),y), h(g(v),a), \sigma_1 = y/a$,
Next we obtain $S1 = Q(h(g(v),a),w), Q(h(g(v),a),f(v)), Q(h(g(v),a),f(b))$

(c) $D(S_2) = w, f(v), f(b), \sigma_2 = w/f(v)$,
Next we obtain $S2 = Q(h(g(v),a),f(b)), Q(h(g(v),a),f(v)), Q(h(g(v),a),f(b))$

(d) $D(S_3) = f(v), f(v), f(b), \sigma_3 = v/b$,
Next we obtain $S3 = Q(h(g(b),a),f(b)), Q(h(g(v),a),f(b)), Q(h(g(v),a),f(b))$.

(e) $x/g(v), y/a, w/f(b), v/b = x/g(b), y/a, w/f(v), v/b$.

6. **State the Herbrand theorem.**

Solution:

(a) Let $S = \varphi(x_1, ..., x_n)$ be a set of open formulas of a language $\mathcal{L}$.

(b) Either,

i. S has an Herbrand model

ii. S is insatiable, and more specifically, there are a finite number of ground instances of S's constituent parts whose combination is insatiable. There are a limited number of ground instances of the negations of S formulae in which the disjunction is valid, which is equal to the latter case, (ii), (ii'). The disjunction is legitimate if and only if it is a truth-functional tautology (because these ground instances can be thought of as being constructed from propositional letters).

7. **Find a resolvent for**

$$\{P(x, y), P(y, z)\}, \{\neg P(u, f(u))\}$$

where x, y, z, u **are variables.**

Solution:

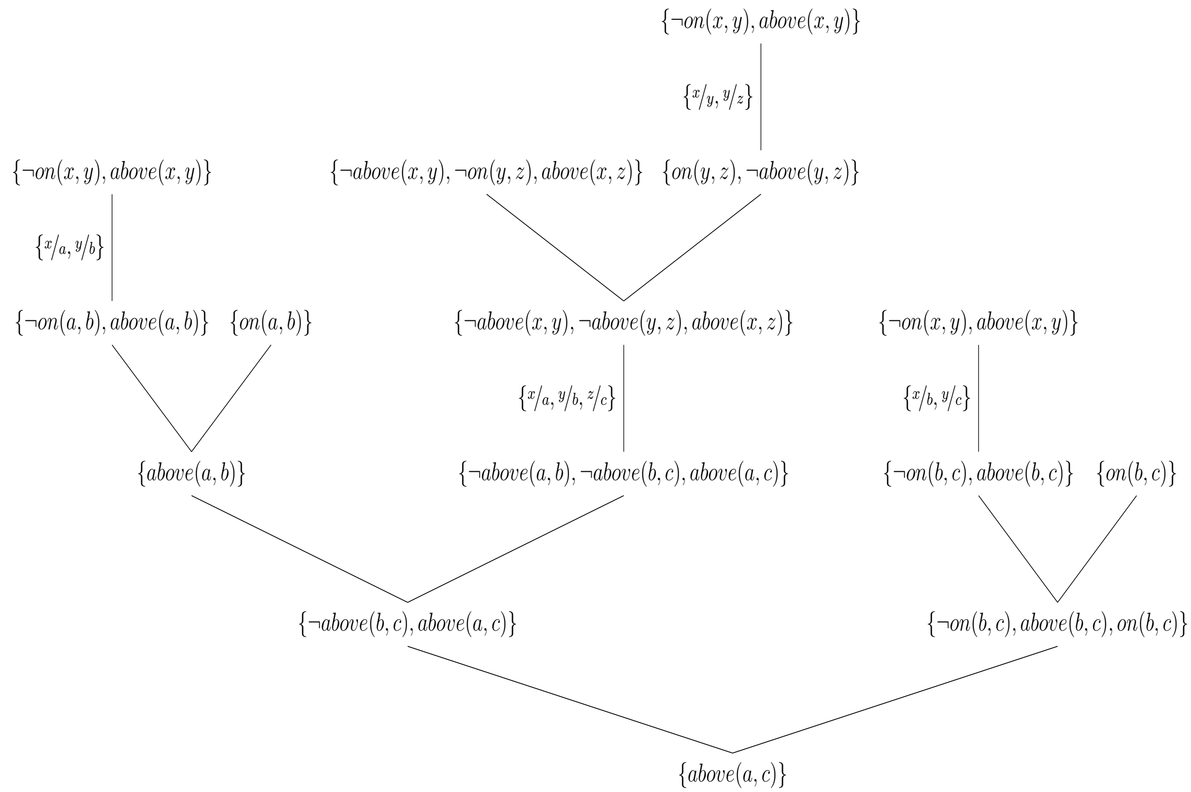
Resolvent for $\{P(x, y), P(y, z)\}, \{\neg P(u, f(u))\}$ = $P(f(u), z)$.

8. **Translate the following formulas into a set** S **of clauses:**

- $\forall x \forall y (above(x, y) \wedge on(y, z) \rightarrow above(x, z))$
- $\forall x \forall y (on(x, y) \rightarrow above(x, y))$
- $on(a, b)$
- $on(b, c)$

Write a resolution tree proof of clause $\{above(a, c)\}$ **from** S**. Indicate the literals being resolved on and the substitutions being made to do the resolution. You may refer to Figure 35 of the textbook.**

Solution:


$\{\neg on(x,y), above(x,y)\}$
$\{x/y, y/z\}$
$\{\neg on(x,y), above(x,y)\}$
$\{\neg above(x,y), \neg on(y,z), above(x,z)\}$
$\{on(y,z), \neg above(y,z)\}$
$\{x/a, y/b\}$
$\{\neg on(a,b), above(a,b)\}$
$\{on(a,b)\}$
$\{\neg above(x,y), \neg above(y,z), above(x,z)\}$
$\{\neg on(x,y), above(x,y)\}$
$\{x/a, y/b, z/c\}$
$\{x/b, y/c\}$
$\{above(a,b)\}$
$\{\neg above(a,b), \neg above(b,c), above(a,c)\}$
$\{\neg on(b,c), above(b,c)\}$
$\{on(b,c)\}$
$\{\neg above(b,c), above(a,c)\}$
$\{\neg on(b,c), above(b,c), on(b,c)\}$
$\{above(a,c)\}$