

Quantum Computing Assignment-1

- RUSHIKESH JAGDISH SONWANE

Problem 1: Quantum Teleportation — Part B

Interpretation and Discussion

Question 1: Histogram Observation After Applying U^\dagger

Is Bob measuring $|0\rangle$ with 100% probability?

Yes. Bob measures $|0\rangle$ with approximately 100% probability. This is because applying U^\dagger after successful teleportation gives:

$$U^\dagger |\psi\rangle = U^\dagger (U |0\rangle) = |0\rangle$$

This result directly confirms that teleportation succeeded — Bob's qubit was correctly reconstructed to match Alice's original unknown state $|\psi\rangle$.

Question 2: Why Does Applying U^\dagger Verify Teleportation?

The unknown state prepared on qubit 0 was:

$$|\psi\rangle = U |0\rangle \quad \text{where} \quad U = R_z(\varphi) \cdot R_y(\theta)$$

After successful teleportation, Bob's qubit (qubit 2) holds exactly $|\psi\rangle$. Applying the inverse operation:

$$U^\dagger = R_y(-\theta) \cdot R_z(-\varphi)$$

reverses the rotation and gives:

$$U^\dagger |\psi\rangle = U^\dagger \cdot U |0\rangle = I |0\rangle = |0\rangle$$

Measuring $|0\rangle$ with 100% certainty confirms the state was faithfully transferred. If teleportation had failed, Bob would hold an incorrect or mixed state, and U^\dagger would **not** return $|0\rangle$ — the histogram would show non-zero probability for $|1\rangle$.

Question 3: Why Were We Able to Avoid Sending Classical Bits?

In the standard teleportation protocol, Alice measures qubits 0 and 1, collapsing the quantum state into one of four outcomes. She then sends 2 classical bits to Bob, who applies conditional X and Z corrections based on those bits.

Instead of measuring and collapsing the state, we keep qubits 0 and 1 in superposition and apply coherent controlled gates:

- $cx(1, 2)$ — Controlled-X performs the X correction coherently
- $cz(0, 2)$ — Controlled-Z performs the Z correction coherently

These two gates simultaneously apply the correct correction for all four possible branches of the superposition at once, without collapsing the state. This eliminates the need for classical communication entirely.

Problem 2: Superdense Coding — Part B

Conceptual Questions

Question 1: What Happens if an Eavesdropper Intercepts Alice's Qubit?

If an eavesdropper (Eve) intercepts the single qubit Alice sends to Bob, she **cannot extract any useful information** from it. There are two key reasons:

1. Maximally Mixed State

Alice's qubit alone, without Bob's paired qubit, is in a maximally mixed state — it looks like pure noise, with equal probability of being $|0\rangle$ or $|1\rangle$ regardless of the encoded message. The information is distributed across both qubits through entanglement; neither qubit alone carries any part of the message. Eve would need both Alice's qubit and Bob's qubit together to decode anything — and Bob never sends his qubit anywhere.

2. Measurement Disturbs the Entanglement

If Eve measures Alice's qubit in an attempt to extract information, she collapses the entangled Bell state. This disturbance corrupts the quantum information, and Bob's decoding will produce wrong results — revealing that the channel was intercepted. This makes superdense coding inherently detectable against eavesdropping.

Question 2: One Application or Implication of Superdense Coding

Superdense coding allows doubling the classical information capacity of a quantum channel — two classical bits are transmitted using only one physical qubit (after pre-sharing entanglement). This has significant implications for quantum networks and the quantum internet, where bandwidth is limited and expensive.

Instead of sending 2 physical qubits to communicate 2 classical bits, you send only 1 qubit, effectively halving the quantum channel usage. This is especially valuable in:

- Satellite-based quantum communication, where transmitting qubits over long distances is costly and lossy.
- Quantum repeater networks, where reducing the number of qubits in transit reduces decoherence.
- Secure financial and government communications that require both high bandwidth and quantum-level security guarantees.

Superdense Coding — Protocol Diagram

The following diagram illustrates the flow of the superdense coding protocol:

