Assignment 1: Entanglement-Based Quantum Communication

Total Marks: 30
Teleportation: 18 (14 Qiskit + 4 Concepts)
Superdense Coding: 12 (10 Qiskit + 5 Concepts)
Submission for Part-A (Qikit codes): GitHub repository link
Submission for Part B (Conceptual Questions) as a separate pdf file

Problem 1: Quantum Teleportation (Bell State)

Marks: 15 = 12 (Qiskit) + 3 (Concepts)

Objective

In this assignment, you will implement quantum teleportation in Qiskit and verify its correctness by undoing the unitary operation used to generate the unknown state $|\psi\rangle$.
Unlike what was taught in class, this problem avoids classical feed-forward correction (Alice does not call up Bob to tell him the two classical bits she measured). Instead, you will implement an equivalent fully quantum (unitary) correction method.
You are expected to write the full circuit yourself (except for the rotation gate syntax provided below).

Part A — Qiskit Implementation (14 Marks)

We introduce a new single-qubit gate: the rotation gate.

Rotation About the Y-Axis:
$R_y(\theta)$ rotates the qubit around the Y-axis of the Bloch sphere by an angle θ.

$$R_y(\theta) = \begin{bmatrix} \cos\theta/2 & -\sin\theta/2 \\ \sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

Similarly, $R_y(\varphi)$ rotates the qubit around the Y-axis of the Bloch sphere by an angle φ

$$R_z(\varphi) = \begin{bmatrix} e^{-i\varphi/2} & 0 \\ 0 & e^{+i\varphi/2} \end{bmatrix}$$

<u>Your Task:</u>

Step 1: Arbitrary Qubit Preparation (3 Marks)

1. Start with qubit 0 initialized in $|0\rangle$
2. Apply $R_y(\theta)$ followed by a $R_z(\varphi)$ rotation of your choice.
3. Choose θ in (0.5, 2.5) and φ in (0.5, 2.5).
4. Do NOT choose special angles such as 0, pi/2, pi, or pi/4.

For the θ and φ rotations, you can use the Qiskit commands

qc.ry(theta, 0)
qc.rz(phi, 0)

Clearly state your chosen values in your notebook.
This state is your unknown state $|\psi\rangle$.

Step 2: Create a Bell Pair (2 Marks)

Using qubits 1 and 2, create the Bell state:

$$|\phi^{+}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Label clearly:

Qubit 0 → Alice's unknown state

Qubit 1 → Alice's entangled qubit

Qubit 2 → Bob's entangled qubit

Step 3: Perform the Teleportation Circuit (2 Marks)

Apply the Bell-measurement circuit between qubits 0 and 1 as taught in class – First CNOT followed by a Hadamard

Important Difference From Class:

In class, Alice measured qubits 0 and 1 and sent classical bits to Bob.

In this assignment:

- Do NOT measure qubits 0 and 1.
- Do NOT use classical bits or conditional gates.

Instead, implement the equivalent correction using quantum controlled gates only (Step4)

Step 4: Quantum Correction Step (3 Marks)

In this assignment, we do something different. You must:

- **NOT measure qubits 0 and 1**
- **NOT use classical bits**
- **NOT use conditional gates**
-

Instead, apply the following controlled gates:

qc.cx(1, 2)
qc.cz(0, 2)

These gates perform the teleportation correction coherently in superposition.
This replaces the classical communication step and keeps the entire circuit unitary.

Step 5: Undo the Unitary U (2 Marks)

Let U be the rotations you applied to qubit 0.
You must now apply the inverse rotation operation to Bob's qubit (qubit 2) that is $U^{\dagger}$

Important principles:
- Reverse the order of gates.
- Use negative angles for rotation gates on qubit 2.

For example, if you used qc.ry(theta, 0) to rotate the qubit 0 in Step 1, you need to implement qc.ry(− theta, 2) to rotate back (that is undo the operation) on Bob's qubit. Similarly, do the qc.ry(− phi, 2) step.

You must determine and apply the remaining inverse operations correctly.

Step 5: Measurement and Histogram (2 Mark)

1. Measure Bob's qubit only.
2. Run the circuit with at least 5000 shots.
3. Plot the histogram.
4. Draw the full circuit diagram.

Part B — Interpretation and Discussion (4 Marks)

1. What do you observe in the histogram after applying U†?
   Is Bob measuring 0 with 100% probability?

2. Why does applying U† verify teleportation?
   Use: $U^{\dagger}U = |0\rangle$ in your explanation.

3. Why were we able to avoid sending classical bits in this assignment?
   Explain how the fully quantum correction differs from the real protocol.

Expected Observation

If your implementation is correct, Bob should measure $|0\rangle$ with probability approximately 100%. This confirms that teleportation worked and Bob successfully reconstructed $|\psi\rangle$.

Problem 2: Superdense Coding (Bell State)

Marks: 10 = 8 (Qiskit) + 2 (Concepts)

Part A: Qiskit Implementation (8 marks)

Objective:
Demonstrate how two classical bits can be sent using one qubit and a shared Bell state.

Qubit roles:
- Qubit 0: Alice's qubit
- Qubit 1: Bob's qubit

Tasks:
1. Create a Bell pair shared between Alice and Bob.
2. Choose any particular 2-bit classical message (either 00, or 01, or 10, or 11)
3. Encode the message using a single-qubit gate on Alice's qubit, based on the encoding rule given below
4. Perform Bell-basis decoding.
5. Measure both qubits and plot a histogram.

Encoding rule:
00 → Identity
01 → Z
10 → X
11 → X followed by Z

Part B: Conceptual Questions (2 marks)

Answer briefly:
1. Explain what happens if an eavesdropper intercepts the qubit sent from Alice to Bob?
2. Give one application or implication of superdense coding.