

## Assignment 1

$$\textcircled{1} \quad A = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}$$

$$\det[A - \lambda I] = 0$$

$$\det \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\det \begin{bmatrix} 2-\lambda & -12 \\ 1 & -5-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)(-5-\lambda) - (1)(-12) = 0$$

$$-10 - 2\lambda + 5\lambda + \lambda^2 + 12 = 0$$

$$\lambda^2 + 3\lambda + 2 = 0$$

$$(\lambda+2)(\lambda+1) = 0$$

$$\boxed{\lambda = -2} \quad \boxed{\lambda = -1} \rightarrow \text{eigen values}$$

$$\underline{\lambda = -2} \text{ in } \det(A - \lambda I)$$

$$\begin{pmatrix} 2-(-2) & -12 \\ 1 & -5-(-2) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$\begin{pmatrix} 4 & -12 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0 \Rightarrow \begin{cases} 4x - 12y = 0 \\ x - 3y = 0 \end{cases} \quad x = 3y$$

$$\text{let } \boxed{y = 1}$$

then

$$\boxed{x = 3}$$

eigen vector when  $\lambda = -2$  is  $\underline{\underline{\begin{pmatrix} 3 \\ 1 \end{pmatrix}}}$ .

$$\underline{\lambda = -1}$$

$$\begin{pmatrix} 2 - (-1) & -12 \\ 1 & -5 - (-1) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$\begin{pmatrix} 3 & -12 \\ 1 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0 \Rightarrow \begin{cases} 3x - 12y = 0 \\ x - 4y = 0 \end{cases} \left. \begin{array}{l} x = 4y \\ x = 4, y = 1 \end{array} \right\}$$

eigen vector when  $\lambda = -1$  is  $\underline{\underline{\begin{pmatrix} 4 \\ 1 \end{pmatrix}}}$ .

$\therefore$  The eigen values are  $\boxed{\lambda = -2}$  and  $\boxed{\lambda = -1}$

The eigen vectors are  $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$  and  $\underline{\underline{\begin{bmatrix} 4 \\ 1 \end{bmatrix}}}$ .