Division & Roll No.: ______________ Enrolment No.: ______________

Darshan Institute of Engineering & Technology

B.E. Semester – I ● Pre GTU Examination – February 2021

Subject Code : 3110014 **Date :** 18/02/2021

Subject Name : Mathematics - 1

Time : 11:30 am to 01:30 pm **Total Marks :** 56

Instructions :
1. Attempt any **FOUR** out of **SEVEN** questions.
2. Figure to the right indicate full marks.
3. Don't do any kind of rough work or calculation in Question Paper.

Q. 1 **(A)** Show that $e^x = \sum_{n=1}^{\infty} \frac{2n\pi}{L^2 + n^2\pi^2}\left[1 - (-1)^n e^L\right] \sin\left(\frac{n\pi x}{L}\right)$ using half range series, where $0 < x < L$. **03**

(B) Check the convergence of $\sum_{n=1}^{\infty} \frac{(n+3)!}{3!\, n!\, 3^n}$. **04**

(C) Find the minimum distance from the origin to the plane $3x + 2y + z = 12$. **07**

Q. 2 **(A)** Evaluate: $\lim_{x \to 0} \left[\frac{\pi}{4x} - \frac{\pi}{2x(e^{\pi x} + 1)}\right]$. **03**

(B) If $z = f(x, y), x = e^u + e^{-v}$ & $y = e^{-u} - e^v$, show that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$. **04**

(C) Find Fourier series of $f(x) = x^2$ in $[0, 2\pi]$. **07**

Q. 3 **(A)** Evaluate $\iint e^{y^2} dxdy$ over the region bounded by the triangle with vertices (0,0), (2,1) and (0,1). **03**

(B) Discuss the maxima and minima of the function $3x^2 - y^2 + x^3$. **04**

(C) Find the eigen values and eigen vectors for $A = \begin{bmatrix} 4 & 6 & 6 \\ -8 & -10 & -8 \\ 4 & 4 & 2 \end{bmatrix}$. **07**

Q. 4 **(A)** Find the directional derivative of $f(x, y, z) = xy + yz + zx$ at $(1, 2, 0)$ in the direction of the vector $(1, 2, 2)$. **03**

(B) Solve the following system of linear equations by Gauss − Elimination method: $x + 2y - z = 1, x + y + 2z = 9$ & $2x + y - z = 2$. **04**

(C) State the Cayley − Hamilton theorem and verify it for the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$. **07**

Q. 5 **(A)** Check the convergence of the improper integral $\int_0^{\infty} x^2 e^{-x}\, dx$. **03**

(B) Test the convergence of $\frac{1}{1\cdot 3}+\frac{2}{3\cdot 5}+\frac{3}{5\cdot 7}+\frac{4}{7\cdot 9}+\cdots$. **04**

(C) Expand $f(x)=\begin{cases} 1+\frac{2x}{\pi} & ; -\pi \le x \le 0 \\ 1-\frac{2x}{\pi} & ; \quad 0 \le x \le \pi \end{cases}$ in terms of sine and cosine. **07**

Hence show that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots$ is convergent.

Q. 6 **(A)** Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{(\log n)^n}$. **03**

(B) If $u = e^{xyz}$, show that $\frac{\partial^3 u}{\partial x\, \partial y\, \partial z} = (1+3xyz+x^2y^2z^2)\, e^{xyz}$. **04**

(C) Find the inverse of a matrix $A=\begin{bmatrix} 1 & -1 & 1 \\ 2 & 2 & -3 \\ 1 & -4 & 9 \end{bmatrix}$ by Gauss – Jordan method. **07**

Q. 7 **(A)** Find the rank of the matrix $A=\begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$. **03**

(B) Find the sum of the series $\sum_{n=0}^{\infty} \left(\frac{2^{n+1}}{5^n}\right)$. **04**

(C) Find the radius and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x+2)^n}{2^n \cdot n}$. **07**

Also, for what values of x does the series converges absolutely or conditionally?

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