Discrete maths project

(SC-205)

Project-2023

CURRENCY ARBITRAGE

Group Members:

202201117 - Rushi Makadiya
202201526 - Elvis Kotadiya
202201430 - Ridham Patel
202201325 - Keyur Padia
202201236 - Fenil Amin
202201484 - Jaikrit Sanandiya

Instructor:
Prof. Manish K. Gupta
Prof.Manoj Raut

Introduction :

The currency exchange rate is one of the most important determinants of a country's relative level of economic health. Exchange rates play a vital role in a country's level of trade, which is critical to most every free market economy in the world. For this reason, exchange rates are among the most watched, analyzed and Governmentally manipulated economic measures. But exchange rates matter on a smaller scale as well: they impact the real return of an investor's portfolio.

Traders usually make money, or look forward to making money, by buying stocks at low prices and selling them at high. The same principle is followed in currency trading as well. There is, however, one strategy – currency arbitrage – where traders buy and sell at the same time at different prices to make money from the variation in prices at which two transactions are carried out.

Example :

we can make it more clear by having an example so let's say at one particular place **A**. we have price of one apple 15, and at some other place **B**, price for same apple is 25, so we will purchase from site A and sell it at B,so we can make considerable profit,now what is going to happen is supply of apples at site A will reduce by a certain amount against demand of apples at that place so automatically price will escalate. On the other hand,supply of apples has increased against demand of apples at B so eventually prise of apple at B will reduce, after some time situation is gonna come where price of apple at both the sites would be nearly equal,so we will able to make a good profit in that time period.The amount calculated using this method is considered neglecting the transportation and other miscellaneous cost.

Definition of Arbitrage :

Arbitrage[1] is a financial strategy that involves taking advantage of price differences for the same asset or financial instrument in different markets or at different times.

By exploiting these price disparities, arbitrageurs aim to eliminate any potential risk and secure risk-free profits. Arbitrage opportunities can arise due to various factors, such as variations in supply and demand, market inefficiencies, or delays in information dissemination. However, as markets become more efficient, arbitrage opportunities tend to diminish.

What is an Arbitrage ?

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[6] Let's say that we are provided exchange rate estimates for USA, EUROPE and INDIA.

The spot exchange rates offered at three distinct currency markets are listed below:

1 ₹= 0.012 $

1 ₹= 0.0113 €

1 €= 88.68 $

Suppose we have 1Cr ₹Assuming that there are no transaction costs.

We can determine €/ $ by cross rate formula as below:

$ / €= ($ / ₹) / (€/ ₹)
. = (0.012)/(0.0113)
. = 1.07

Arbitrage Process :

- Sell 1,000,000 ₹at the rate of 0.0113 per €and receive €1,13,000 through arbitrage.
- From €1,13,000 purchase 1,20,910 $ in USA at the rate of 1 €= 0.93 $.
- Exchange £1,20,910 for $ 1,00,07,720.7 IND at the rate of 1 $ = 82.77 ₹

$$\text{Arbitrage Gain} = ₹1{,}00{,}07{,}720.7 - ₹1{,}00{,}00{,}000 = ₹7{,}720.7$$

this type of triangular arbitrage opportunities rarely exist in the real world.

Observation :

In order to find a cross-relation between any two currencies and then spot potential arbitrage opportunities, we can therefore employ an intermediate currency. The fact that we can still turn a tiny profit despite the rate difference being so small is another interesting to notice.

But in the real world, we have actually more than 3 different currencies. so here comes the concept of graph theory that will help to resolve this problem.

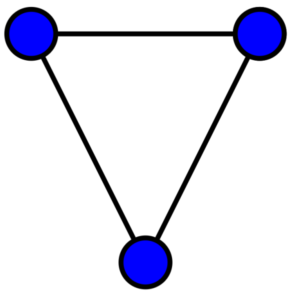
Previously we can arbitrage between 3 currency which is very rare to make profit but further we can use mathematical approach for more than three currency arbitrage to generate more profit from that.

Mathematics Behind Problem Statement:

Undiretced graph :

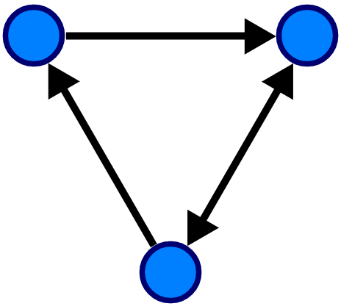
A graph[3] (sometimes called an undirected graph to distinguish it from a directed graph, or a simple graph to distinguish it from a multigraph) is a pair G = (V, E), where V is a set whose elements are called vertices (singular: vertex), and E is a set of paired vertices, whose elements are called edges (sometimes links or lines).

The vertices x and y of an edge x, y are called the endpoints of the edge. The edge is said to join x and y and to be incident on x and y. A vertex may belong to no edge, in which case it is not joined to any other vertex.



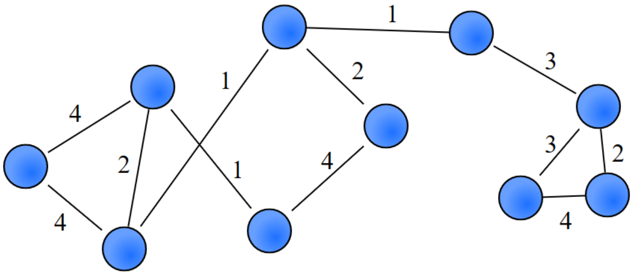
Directed graph :

A directed graph or digraph is a graph in which edges have orientations.



Weighted graph :

A weighted graph or a network is a graph in which a number (the weight) is assigned to each edge. Such weights might represent for example costs, lengths or capacities, depending on the problem at hand. Such graphs arise in many contexts, for example in shortest path problems such as the traveling salesman problem.



Way to Find Arbitrage :

[4]Let's write edge weights as W1 for edge 1, W2 for edge 2, W3 for edge 3 and Wn for edge n.

When a cycle2 occurs, arbitrage possibilities appear. and the graph both show that Edge weights meet the criteria in the following equation:

$$X1 * X2 * X3 * ... * Xn > 1 \quad (1)$$

However, in order to use Graph Algorithms and reduce complexity, we will alter the aforementioned equation by taking natural logarithms on both sides.

$$\log(X1) + \log(X2) + \log(X3) + ... + \log(Xn) > 0 \quad (2)$$

Now, by multiplying the aforementioned equation by -1, we obtain the following result:

$$(-\log(X1)) + (-\log(X2)) + (-\log(X3)) + ... + (-\log(Xn)) < 0 \quad (3)$$

So that an arbitrage scenario is achievable if we can identify a cycle of vertices where the total of their weights is negative.

Algorithmic Methodology :

It is also possible to find the shortest path between a specific source vertex and all other vertices in the graph using methods like Bellman-Ford and Dijkstra's.

The Bellman–Ford algorithm is an algorithm that computes shortest paths from a single source vertex to all of the other vertices in a weighted digraph.It is slower than Dijkstra's algorithm for the same problem, but more versatile, as it is capable of handling graphs in which some of the edge weights are negative numbers

Negative edge weights are found in various applications of graphs, hence the usefulness of this algorithm.if a graph contains a ”negative cycle” (i.e. a cycle whose edges sum to a negative value) that is reachable from the source, then there is no cheapest path: any path that has a point on the negative cycle can be made cheaper by one more walk around the negative cycle. In such a case, the Bellman–Ford algorithm can detect and report the negative cycle.

Bellman–Ford algorithm :

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[2]We can use the Bellman-Ford technique to find the shortest path through a weighted graph.

Despite being asymptotically quicker than the Bellman-Ford algorithm, Dijkstra's approach is unable to handle negative edge weights. Therefore, in order to discover the negative weight cycle, we apply the Bellman-Ford algorithm rather than the former over the former. The importance of the negative weight cycle in our issue will be discussed in the following sections.

Now we move forward to concept of algorithm :

[9]Bellman–Ford is denoted in terms of (G,E,V,S) where graph is denoted as G,v is denoted as vertex, E is denoted as edge and S is denoted as source element

Step 1

1. for each vertex $v \in a$ do
   dist[v] = infinite
   item dist[source] = 0

Step 2

1. for i=1 to $|v|$-1,
   for each edge $(u, v) \in G$ (where u,v are vertex)
   relax(u,v,w)

Step 2

1. for each edge $(u, v) \in G$
   if(dist[u] + W(u,v) ¡dist[v])
   return ”graph contain negative weigh cycle”
   return distance

Now we are trying to understand how we approach this theorem,Let We denote the weight of vertex u and W ; where u,v denote the weight of edge from source vertex u to destination vertex v.

1. Calculate Number Of Iterations which is equal to V́ - 1 ', where V is the number of vertices.

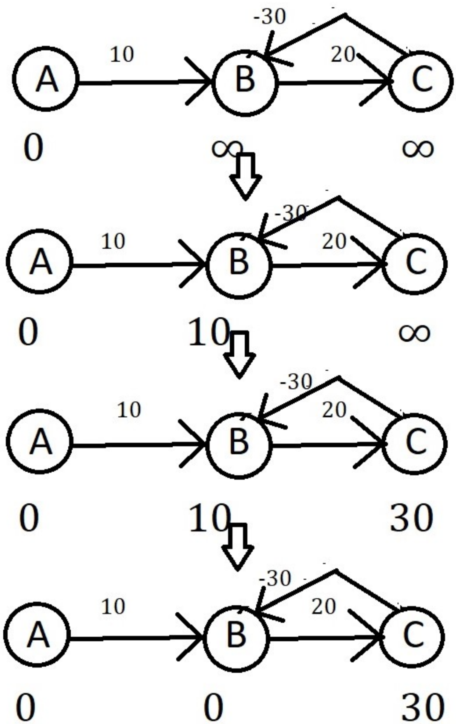
2. Choose any Source vertex and assign distance value of source to 0& 3 and other vertices to infinity.

3. In each iteration,if Wu $+$W $<$ Wv, then Wv $=$ Wu $+$Wu,v.Therefore every edge is relaxed and the weight of each vertex is updated accordingly.

4. By the end of the last iteration, we will end up with some shortest path from Source to every vertex.

Illustration :

Let's use an example to better comprehend the aforementioned method.



A weighted directed graph can also include negative cycles, which can be found using the Bellman-Ford approach.The Bellman-Ford approach can also be used to locate negative cycles in a directed weighted graph.By checking to see if the V edge path solution and the V -1 edge solution are same, Bellman-Ford can be utilised to detect negative weight cycles. If it is smaller, there is a negative weight cycle!

Complexity Of Bellman-Ford Algorithm

Time Complexity

- Best Case : O($|E|$)
- Average Case: O($|V| * |E|$)
- Worst Case: O($|V| * |E|$)

Space Complexity

[7]The Bellman-Ford algorithm has O($|V|$) space complexity. $|V|$,or the quantity of currencies, is typically a negligible figure.

Adaption of it in our problem :

Our goal is to develop a technique that can identify arbitrage scenarios using graph theory. Arbitrage opportunities arise in the face of a negative weight cycle. here we use previously discussed Way to find Arbitrage.

The Bellman-Ford approach can be used to identify negative cycles. There are some We just apply the Bellman-Ford algorithm on our obtained Graph.

The route weight is calculated by Bellman-Ford by multiplying each edge weight by itself. To make this work for multiplicative exchange rates, we must utilise the natural logarithms of each edge weight. Exchange rates are multiplied by edge weights.

When they are added together along a path. The sum's exponentiation can be utilised to find the multiplied amount.

$\log(X1)+\log(X2)+\log(X3)+ \ldots +\log(Xn)>0$.

In our arbitrage scenario, the goal is to maximise the amount of cash received, whereas Bellman-Ford seeks to uncover least weight pathways and negative edge cycles.

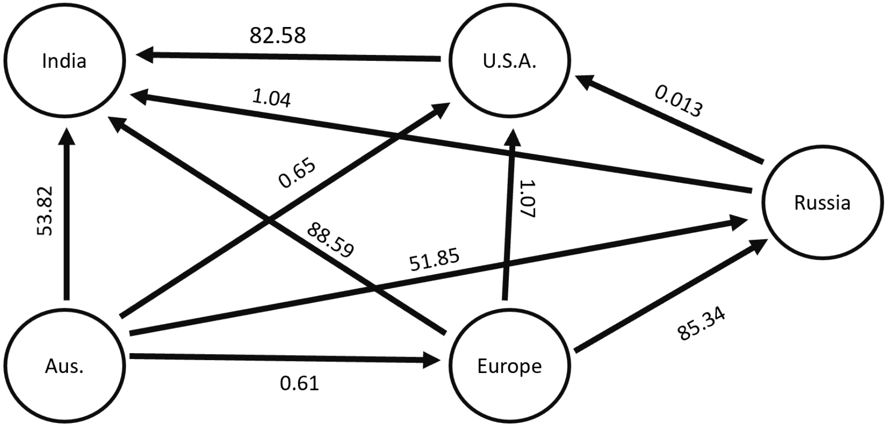
$(-\log(X1))+(-\log(X2))+(-\log(X3))+\ldots+(-\log(Xn))<0$.

Coming back in our problem :

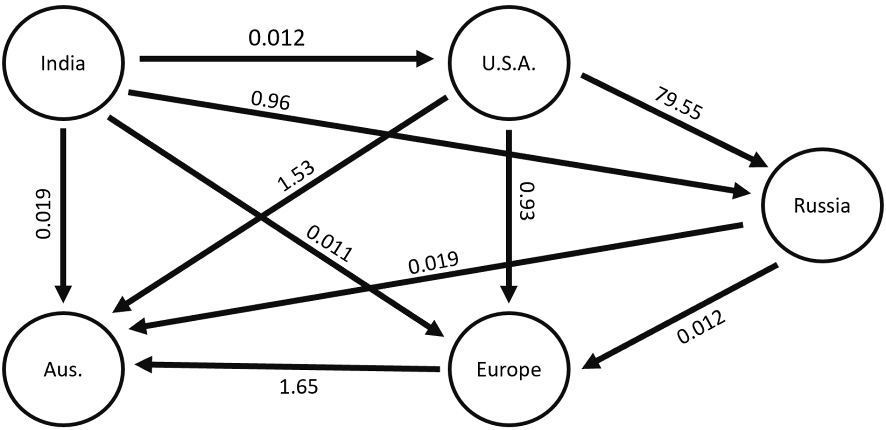
Assume we are given a table of currency exchange rates and must decide whether there is any potential for arbitrage, that is, starting with some money in one currency and trading it for more money in other currencies to finish up with more money in the same currency.

Currency Exchange Rates					
	IND	USA	RUS	EURO	AUS
IND	1	0.012	0.96	0.011	0.019
USA	82.58	1	79.55	0.93	1.53
RUS	1.04	0.013	1	0.012	0.019
EURO	88.59	1.07	85.34	1	1.65
AUS	53.82	0.65	51.85	0.61	1

The vertices of the graph serve as a representation of the Currencies. The graph's edge weights serve as a representation of the exchange rates.



India
U.S.A.
Russia
Aus.
Europe
82.58
1.04
0.013
0.65
53.82
1.07
88.59
51.85
85.34
0.61

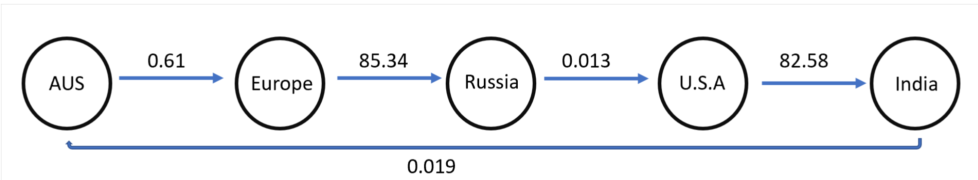


India
U.S.A.
Russia
Aus.
Europe
0.012
0.96
79.55
1.53
0.019
0.93
0.011
0.019
0.012
1.65

If we combined these two diagram we can make all possible way to reach each and every vertex.because every edge here becomes bidirectional so we can choose any of that as source.

The Bellman-Ford technique can be used to quickly find any negative weighted cycles that correspond to arbitrage possibilities now that we have our Graph.

One arbitrage path can be seen on our graph using Hand Analysis, as shown on the following graph.



So from this graph we can use equation for calculation of arbitrage,

$$X1 * X2 * X3 * ...Xn>1$$
$$0.61 * 85.34 * 0.013 * 82.58 * 0.019 = 1.06 > 1$$

if we take 1 Aus Doller and convert it threw AUS.→EUROPE→RUSSIA →U.S.A→INDIA→AUS. we can make 1.06 Aus Doller we can make 6 persent profit.

This is only one way in which we make profit there are such more ways for making profit.

Conclusion :

We may use graph theory to apply different graph algorithms for currencies, which also allows us to generalise the concept.

Triangular arbitrage is a concept. Graph Theory allows us to generalise the concept of triangular arbitrage and to employ various graph algorithms for currency exchange. We may discover negative cycles in the network by representing different currencies as vertices of the graph, edge weights as the exchange rate between two currencies, and then taking the negative logarithm of edge weights.

A Commercialization Perspective:

From a commercialization perspective, currency arbitrage refers to the practice of taking advantage of price differences between different currency pairs in order to generate profits. It involves buying a currency at a lower price in one market and simultaneously selling it at a higher price in another market, capitalizing on the exchange rate discrepancies.

Here are some key aspects of currency arbitrage from a commercialization standpoint:

Market Efficiency: Currency arbitrage helps improve market efficiency by aligning prices across different markets. When traders engage in arbitrage, they effectively equalize prices between markets, as the demand for the undervalued currency increases and its price rises, while the supply of the overvalued currency increases, leading to its price decline.

Risk Management: Commercial entities, such as banks, hedge funds, and financial institutions, often engage in currency arbitrage to manage their foreign exchange risk. By taking positions in multiple currency pairs, they can offset potential losses in one currency with gains in another, thereby mitigating their overall risk exposure.

Technology and Speed: Currency arbitrage has become highly technology-dependent, with automated trading systems and algorithmic strategies being widely used. High-frequency trading (HFT) platforms allow traders to execute trades at lightning speed, exploiting price discrepancies within fractions of a second. The ability to react swiftly to market changes is crucial in the competitive world of currency arbitrage.

Regulatory Considerations: Commercialization of currency arbitrage is subject to various regulatory considerations. Each country has its own regulations governing foreign exchange trading, and traders must comply with legal requirements and licensing obligations. Additionally, regulatory bodies monitor trading activities to prevent market manipulation and ensure fair practices.

Liquidity and Market Depth: Successful commercialization of currency arbitrage relies on the availability of liquidity and market depth. Liquid markets with sufficient trading volume provide opportunities for traders to enter and exit positions quickly without significantly affecting prices. Thinly traded or illiquid markets may pose challenges for executing trades profitably.

Transaction Costs: Currency arbitrage involves transaction costs, including spreads, commissions, and fees associated with trading currencies. These costs can impact the profitability of arbitrage strategies. Traders need to carefully evaluate these costs and consider their impact on potential profits.

Economic Factors: Currency arbitrage is influenced by various economic factors, such as interest rates, inflation rates, geopolitical events, and macroeconomic indicators. Traders must closely monitor these factors to identify opportunities and assess the risks associated with their arbitrage positions.

Overall, from a commercialization perspective, currency arbitrage offers profit opportunities, risk management benefits, and liquidity advantages for market participants. It requires sophisticated technology, market knowledge, and compliance with regulatory requirements to successfully capitalize on price discrepancies in the foreign exchange markets.

Our youtube video on this project :

[8] ← click here

Our wesite link :

[5] ← click here

References

[1] Arbitrage.

[2] Bellman Ford algorithm.

[3] Graph(discrete mathematics).

[4] Arbitrage as a shortest path problem.

[5] Our website.

[6] Algorithm of shortest path.

[7] Time complexity.

[8] Our youtube video on this project.

[9] Dijkstra's Vs Bellman Ford.