22 Sept 2016 JERRY KANG

Compare reaction times for the White Atty vs Asian Atty

Two Set-ups:

(1) Two-independent sample

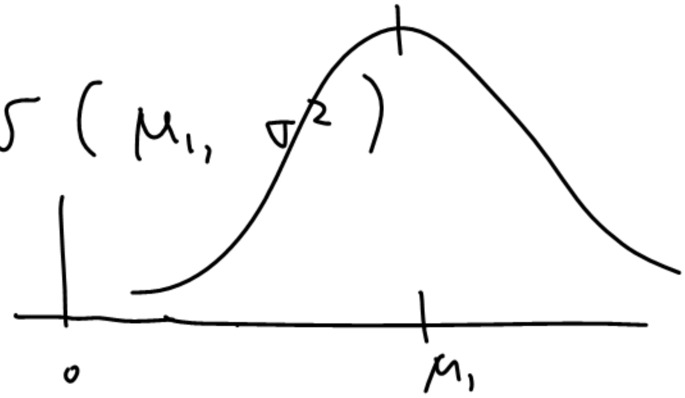
(2) Paired design

Two-indep sample.

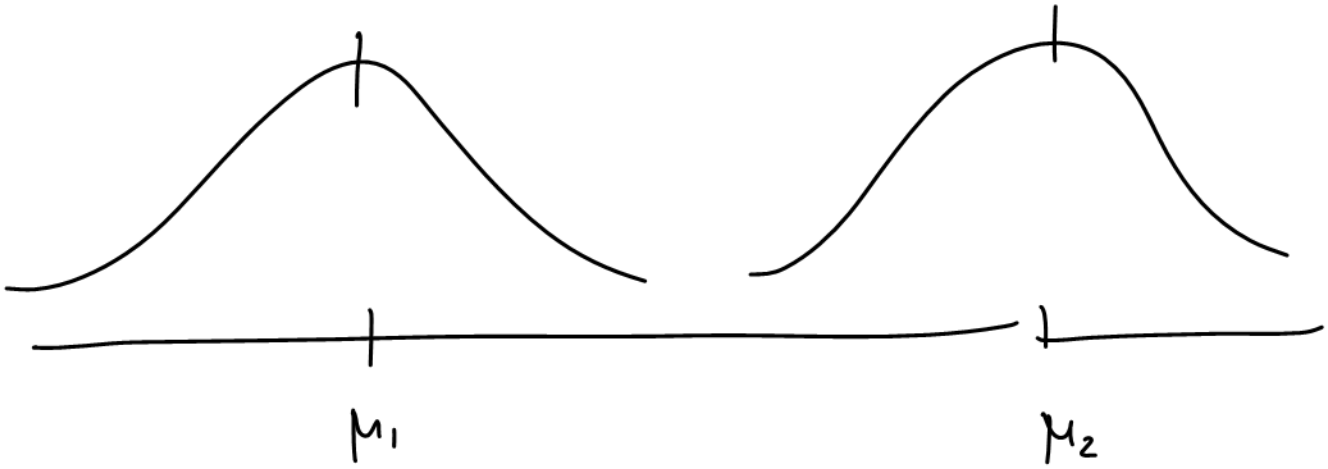
$Y^1_1, \ldots, Y^1_{n_1}$ reaction time of subjects $1, \ldots, n_1$ in the White Atty Group

$Y^2_1, \ldots, Y^2_{n_2}$ ——— $1, \ldots, n_2$ in the Asian Atty Group

$Y^1_1, \ldots, Y^1_{n_1}$ iid $N(\mu_1, \sigma^2)$



$Y^2_1, \ldots, Y^2_{n_2}$ iid $N(\mu_2, \sigma^2)$




$\mu_1 = E(Y^1_i) \quad \forall\, i = 1 \ldots, n_1$

= mean of the distn of rx times of the White Atty group

$\mu_2 = E(Y^2_i) \quad \forall i = 1 \ldots, n_2$

= ______________ Asian Atty Group

$\Delta = \mu_2 - \mu_1$ difference between the popn rx times

$$\hat{\Delta} = \hat{\mu}_2 - \hat{\mu}_1$$

difference between the sample estimates

$$\hat{\mu}_1 = \bar{Y}^1 = \frac{1}{n_1} \sum_{i=1}^{n_1} Y^1_i$$

$$\hat{\mu}_2 = \bar{Y}^2 = \frac{1}{n_2} \sum_{i=1}^{n_2} Y^2_i$$

Sample means estimators for $\mu_1$ & $\mu_2$.

Inference

Hypothesis Testing

Confidence Interval Estimation

$H_0$ : $\Delta = 0 \iff \mu_1 = \mu_2$

null hypothesis

$H_1$ : $\Delta \neq 0 \iff \mu_1 > \mu_2$ OR $\mu_2 > \mu_1$

alternative

Observe data: $y_1^1 \dots y_{n_1}^1 \longrightarrow \bar{y}^1$ 600 ms

$y_1^2 \dots y_{n_2}^2 \longrightarrow \bar{y}^2$ 900 ms

observed difference is $\hat{\Delta} = 900\text{ ms} - 600\text{ ms}$

$= 300\text{ ms}$

Compare 300 ms with a reference distribution.

Assume that $\sigma^2 = (100)^2 (\text{ms})^2$ known.

$n_1 = n_2 = \underline{100}$

Test Statistic:

$$\hat{\Delta} = \bar{Y}^2 - \bar{Y}^1$$

$$\bar{Y}^2 \sim N(\mu_2, \sigma^2/n_2)$$

$$\bar{Y}^1 \sim N(\mu_1, \sigma^2/n_1)$$

$$\hat{\Delta} = \underbrace{\bar{Y}^2 - \bar{Y}^1}$$

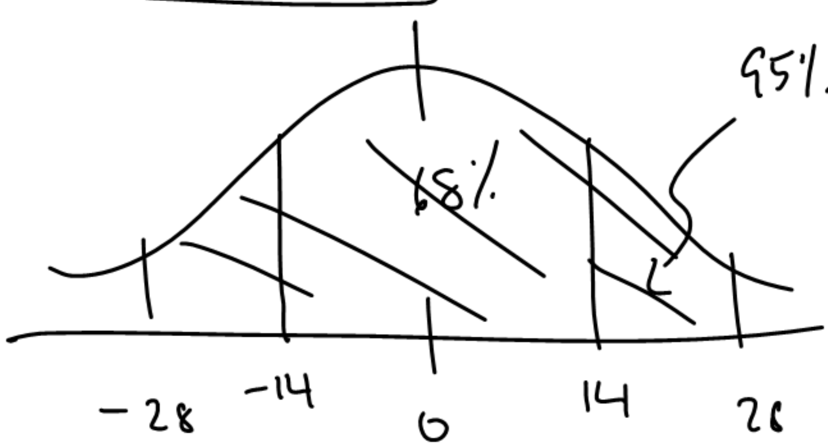
$$\sim N\left(\Delta = \mu_2 - \mu_1,\ Var(\hat{\Delta}) = \sigma^2/n_1 + \sigma^2/n_2\right)$$

$$\overset{H_0}{\sim} N\left(0,\ (100)^2\left(\frac{1}{100} + \frac{1}{100}\right)\right)$$

$$\overset{H_0}{\sim} N(0, \underset{\sim}{200})$$ Ref distn of $\hat{\Delta}$

$$\text{Std dev}(\hat{\Delta}) = \sqrt{200} \cong 14$$

$\boxed{\text{Under } H_0}$



Here, the observed difference $\hat{\Delta} = 300$ lies at the tail of the reference distn under $H_0$.

Two possibilities:

(1) the $H_0$ is true but we just observed an unusual events

(2) The $H_0$ is not true.

<u>Decision</u>: We reject $H_0$

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<u>Recall</u> we reject $H_0$ if

$$|\hat{\Delta}| = |\bar{Y}^2 - \bar{Y}^1| \text{ is "large"}$$

$$\Leftrightarrow \quad \frac{|\hat{\Delta}|}{\sqrt{\operatorname{Var}\hat{\Delta}}} = \frac{|\bar{Y}^2 - \bar{Y}^1|}{\sqrt{\operatorname{Var}\hat{\Delta}}} \text{ is "large"}$$

$$\hat{\Delta} \overset{H_0}{\sim} N\left(\delta = 0,\ \mathrm{Var}\,\hat{\Delta} = \sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)\right)$$

$$\Rightarrow \frac{\hat{\Delta}}{\sqrt{\sigma^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \checkmark \overset{H_0}{\sim} \underbrace{N(0, 1)}$$

Suppose that $\sigma^2$ is not known.

From the White Afty group:

$$\sigma^2 = E\left(Y_i^1 - \mu_1\right)^2$$

$$\bar{Y}^1$$

$$S_1^2 = \frac{1}{n_1 - 1} \sum_{i=1}^{n_1} \left(Y_i^1 - \bar{Y}^1\right)^2$$

Results: $Y_1^1 \ldots Y_{n_1}^1$ iid $N(\mu_1, \sigma^2)$

$\Rightarrow$ (1) $\bar{Y}^1 \sim N(\mu_1, \sigma^2/n_1)$

(2) $\underline{\frac{(n_1 - 1) S_1^2}{\sigma^2}} = \underline{\sum_{i=1}^{n_1} \frac{(Y_i^1 - \bar{Y}^1)^2}{\sigma^2}} \sim \underline{\chi^2(n_1 - 1)}$

(3) $\bar{Y}^1$ and $S_1^2$ independent

From (2):

$$E\left(\frac{(n_1-1)S_1^2}{\sigma^2}\right) = E\left[\chi^2(n_1-1)\right]$$

$$\frac{\cancel{(n_1-1)}}{\sigma^2} E S_1^2 = \cancel{(n_1-1)}$$

$$\Rightarrow \quad E S_1^2 = \sigma^2$$

Moreover, from $Y_1^2 \ldots Y_{n_2}^2$ iid $N(\mu_2, \sigma^2)$

$$\Rightarrow \quad E S_2^2 = \sigma^2$$