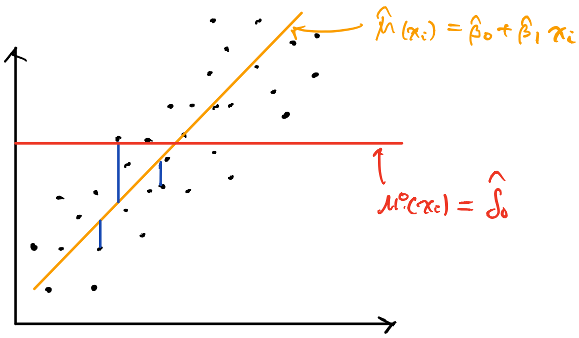
Lecture 12: Oct. 18

$N = 50$
$\beta_0 = 1, \ \beta_1 = 1, \ \sigma = 4$
$x = 1:N$
$\mu_x = \beta_0 + \beta_1 x$
$y = \mu_x + \text{rnorm}(N, 0, \text{sigma})$

Generate Random Data



$\mathcal{M}_1$: $Y_i = \mu(x_i) + \epsilon_i$
$\mu(x_i) = \beta_0 + \beta_1 x_i$
$\epsilon_i \sim N(0, \sigma^2)$

$H_0: \beta_1 = 0$
$H_1: \beta_1 \neq 0$

$\mathcal{M}_0$: $Y_i = \mu^0(x_i) + \epsilon_i$
$\mu^0(x_i) = \gamma_0$ (No slope)

Note: Models are "nested" ← $\mathcal{M}_0 \subset \mathcal{M}_1$

Parameter Space of $\mathcal{M}_0$ ⊂ Parameter Space of $\mathcal{M}_1$

Recall:

$$F \overset{D}{=} \frac{\chi^2_{(n_1)}}{\chi^2_{(n_2)}}, \quad \chi^2_{(n_1)} \perp \chi^2_{(n_2)}$$

$\mathcal{M}_1$:
Let $X = \text{cbind}(\mathbb{1}, x)$.

$$\hat{\beta} = (X^T X)^{-1} X^T y = \begin{pmatrix} 1.978 \\ 0.996 \end{pmatrix}$$

$$\hat{y} = X\hat{\beta}$$

$$R = y - \hat{y}$$

$$SSE^1 = \text{sum}(R^2), \quad dF1 = N-2$$
$$= 726.42$$

$\mathcal{M}_0$:
Let $X^0 = \text{rep}(1, N)$

$$\hat{\beta}^0 = (X^T X)^{-1} X^T y = \bar{y} = 27.374$$

$$\hat{y}^0 = X^0 \hat{\beta}^0$$

$$R^0 = y - \hat{y}^0$$

$$SSE^0 = \text{sum}(R^{0^2}), \quad dF0 = N-1$$
$$= 11053.84$$

$$F_{obs} = \frac{\frac{SSE0 - SSE1}{dF0 - dF1}}{\frac{SSE1}{dF1}} = 682.4$$

Note:
2 dF
$c = F(0.95, 1, 48)$

p-value:

$$p\text{-value} = \Pr\{F > F_{obs} \mid H_0 \text{ is True}\}$$
$$\cong 0$$

Connection to t-statistic

$$T = \frac{\hat{\beta}_1}{\sqrt{\hat{Var}(\hat{\beta}_1)}} \Rightarrow T = \frac{Z}{\sqrt{\frac{Q}{q}}} \Rightarrow \boxed{T^2 = \frac{Z^2/1}{Q/q} \sim F_{(1,q)}}$$

don't know σ^2, which we need to get $Var(\hat{\beta})$.