Lecture 12

Stage-structured populations

WILD3810 (Spring 2019)

Leslie matrix

Leslie matrices are defined as :

- square matrix that summarizes the demography of **age**-specific life cycles
- one column for each **age** class
- matrix elements contain **age-**specific birth and survival rates
- individuals cannot stay in the same **age**-class for more than a single time step [1]

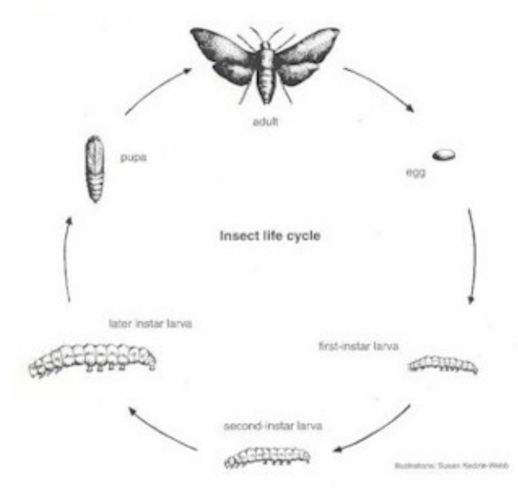
$$\mathbf{A} = \begin{bmatrix} F_1 & F_2 & F_3 \\ P_1 & 0 & 0 \\ 0 & P_2 & 0 \end{bmatrix}$$

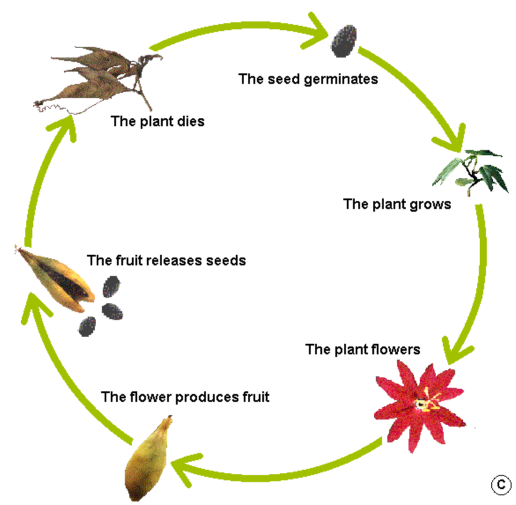
Stage-structured populations

In some cases, age is not a relevant predictor of survival and birth rates

Instead, survival and birth rates vary with **stage**

- life cycle stage



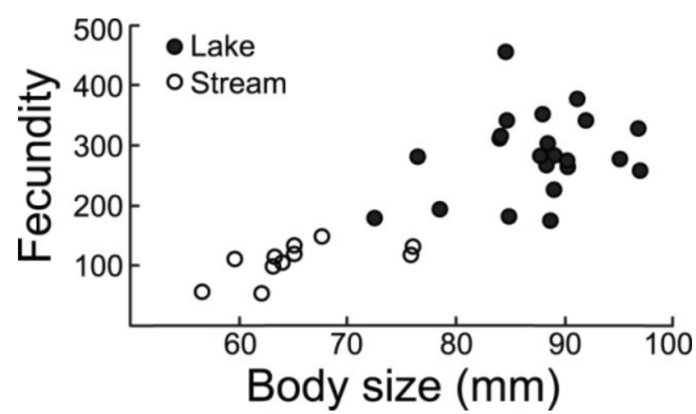


Stage-structured populations

In some cases, age is not a relevant predictor of survival and birth rates

Instead, survival and birth rates vary with **stage**

- life cycle stage
- size

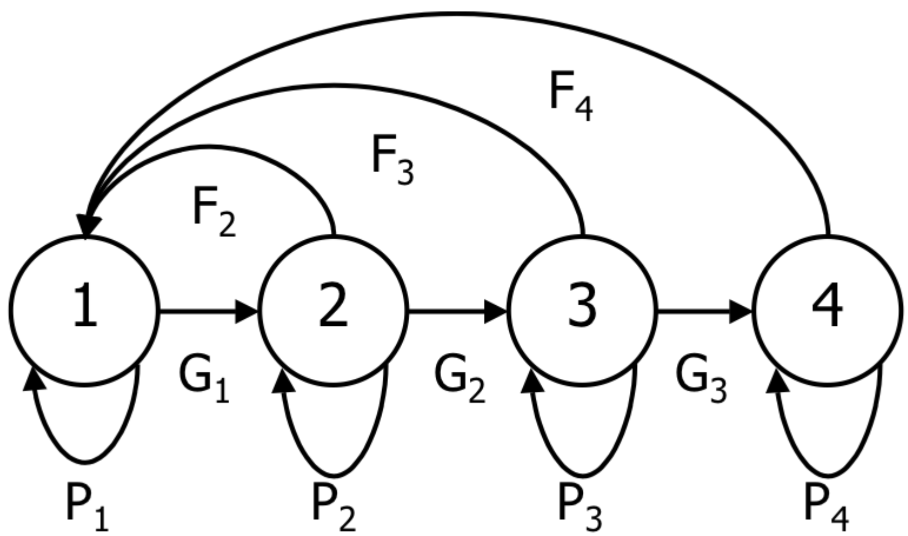


Stage-structured populations

L. Lefkovitch relaxed an assumption of the age-structured matrix model developed by Leslie

Lefkovitch matrices allow individuals to remain in life-stages (or size classes) longer than one time step

Useful for plants and animals with stage-dependent demography



Stage-structured matrix

$$\mathbf{A} = \begin{bmatrix} F_1 & F_2 & F_3 \\ P_1 & 0 & 0 \\ 0 & P_2 & 0 \end{bmatrix} \rightarrow \mathbf{A} = \begin{bmatrix} P_1 & F_2 & F_3 \\ G_1 & P_2 & 0 \\ 0 & G_2 & P_3 \end{bmatrix}$$

- F_x is still **recruitment**, the number of offspring recruited into stage class 1 per adult in stage x
- P_x is the probability of **surviving** from year t until year $t+1$ and **remaining** in stage x
- G_x is the probability of **growing and surviving** to stage $x+1$ during t to $t+1$

Stage-structured matrix model

$$\mathbf{N}_{t+1} = \mathbf{A} \times \mathbf{N}_t$$

$$= \begin{bmatrix} P_1 & F_2 & F_3 \\ G_1 & P_2 & 0 \\ 0 & G_2 & P_3 \end{bmatrix} \times \begin{bmatrix} n_{1,t} \\ n_{2,t} \\ n_{3,t} \end{bmatrix}$$

Matrix multiplication is the same as in the Leslie matrix model!!

Common teasel example

- *Dypsacus sylvestris*
- native to Europe
- invasive species in United States
- stage-structured dynamics studied intensively by Patricia Werner and Hal Caswell

Common teasel example

Complex stage structure

1) Dormant 1st year seeds

2) Dormant 2nd year seeds

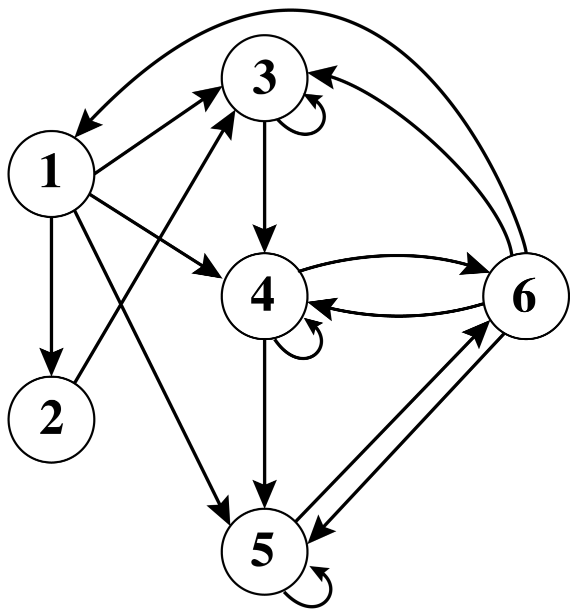
3) Small rosettes $(< 2.5cm)$

4) Medium rosettes $2.5 - 18.9cm$

5) Large rosettes $\geq 19cm$

6) Flowering plants

Common teasel example



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Common teasel example

Seed 1	Seed 2	Small rosette	Medium rosette	Large rosette	Flowering
0.000	0.00	0.000	0.000	0.000	322.280
0.966	0.00	0.000	0.000	0.000	0.000
0.013	0.01	0.125	0.000	0.000	3.448
0.007	0.00	0.125	0.238	0.000	30.170
0.001	0.00	0.000	0.245	0.167	0.862
0.000	0.00	0.000	0.023	0.750	0.000

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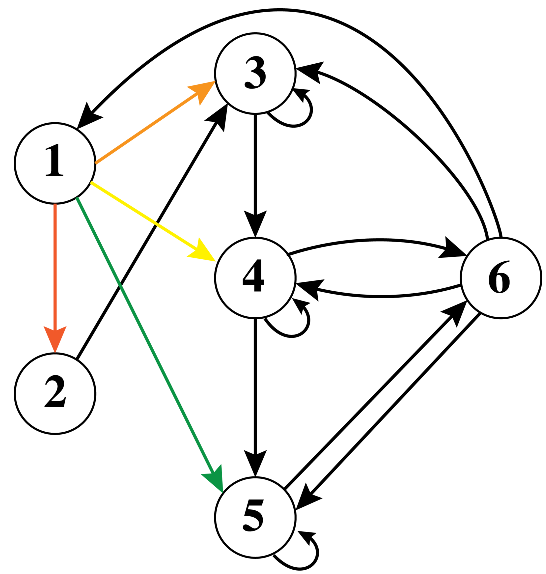

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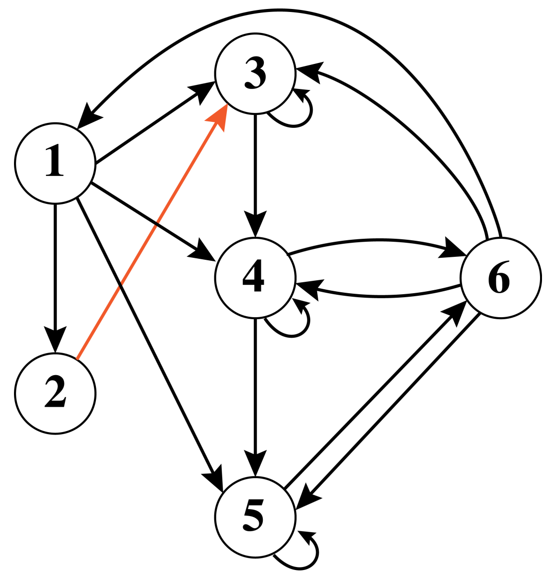

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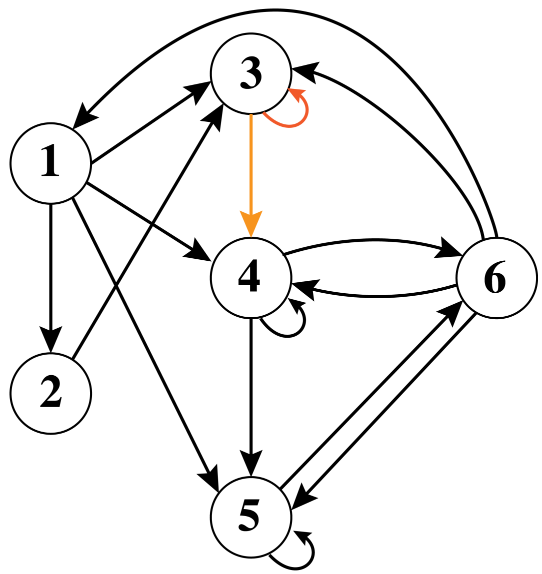

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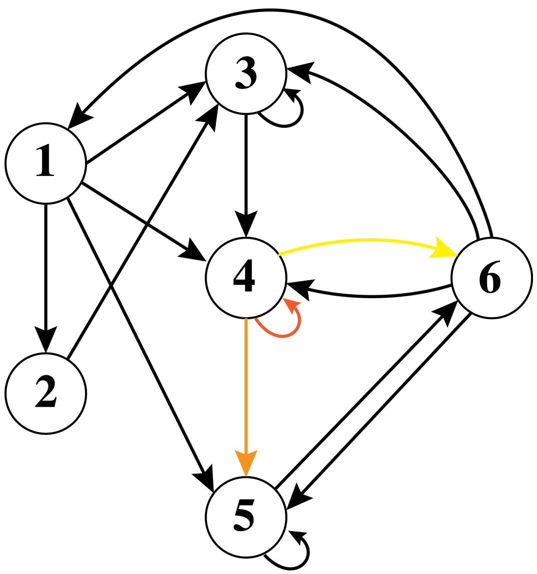

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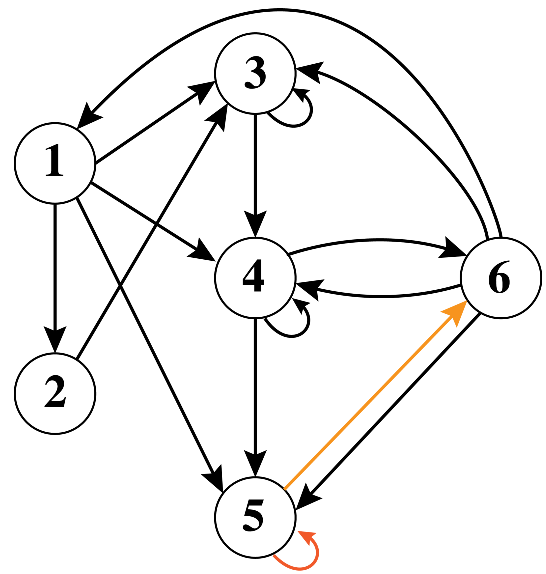
0.000	0.00	0.000	0.023	0.75	0.000



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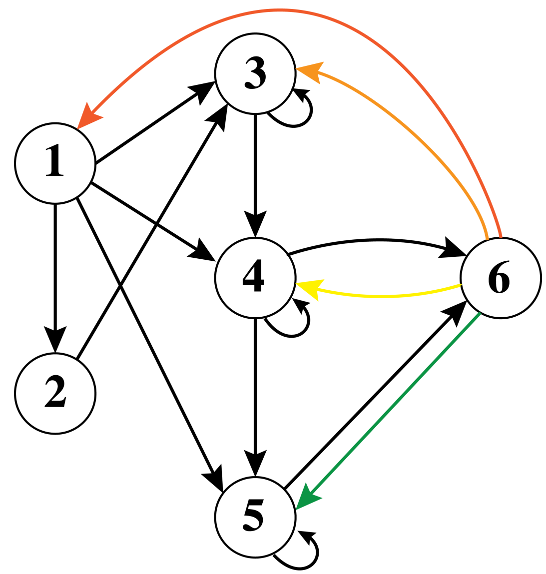


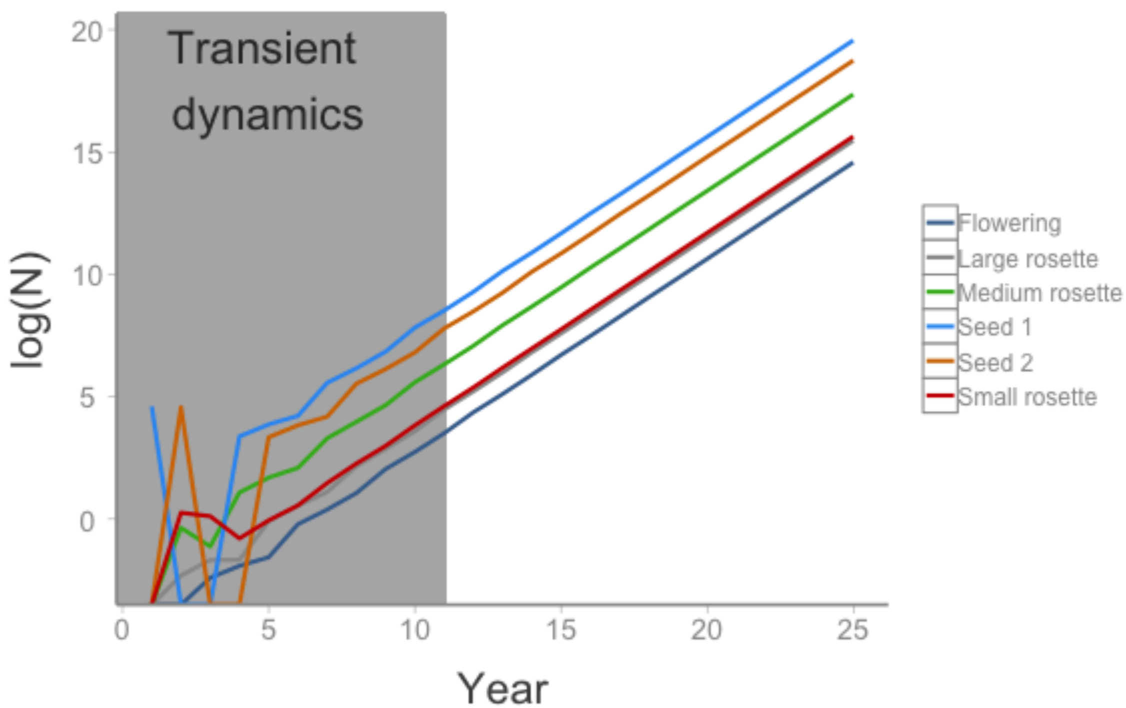
Common teasel example

What happens to a newly established population?

- Assume population starts with 100 1st year seeds

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 322.38 \\ 0.966 & 0 & 0 & 0 & 0 & 0 \\ 0.013 & 0.01 & 0.125 & 0 & 0 & 3.448 \\ 0.007 & 0 & 0.125 & 0.238 & 0 & 30.17 \\ 0.001 & 0 & 0 & 0.245 & 0.167 & 0.862 \\ 0 & 0 & 0 & 0.023 & 0.75 & 0 \end{bmatrix} \times \begin{bmatrix} 100 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Common teasel example



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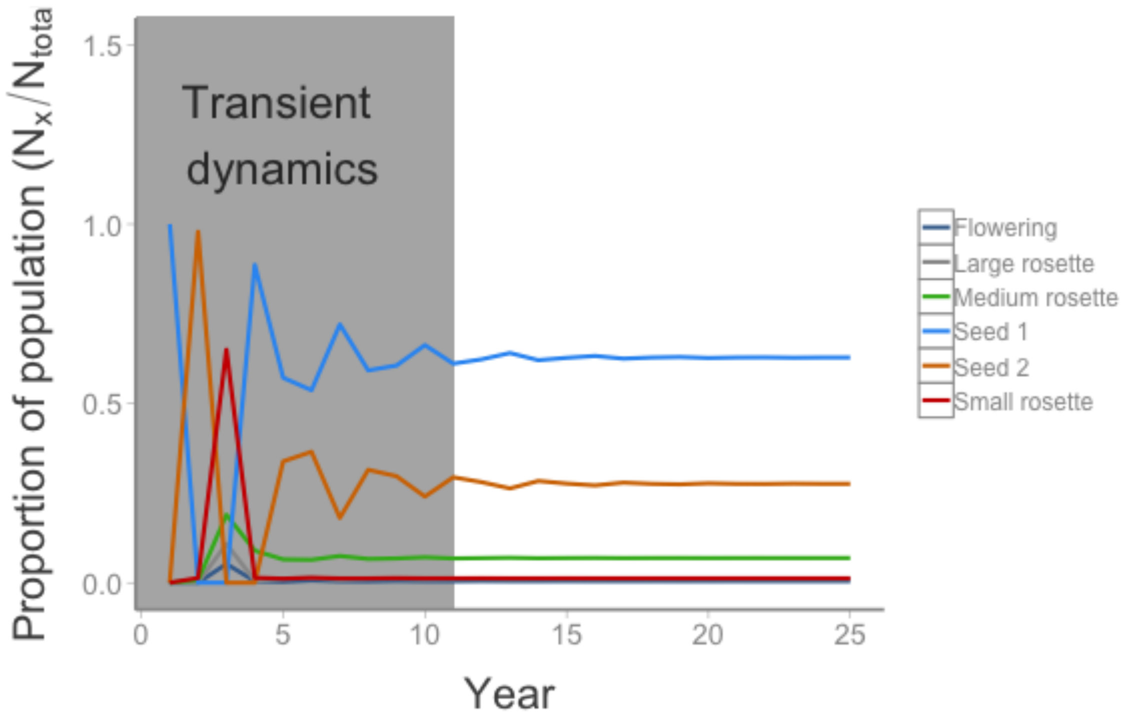

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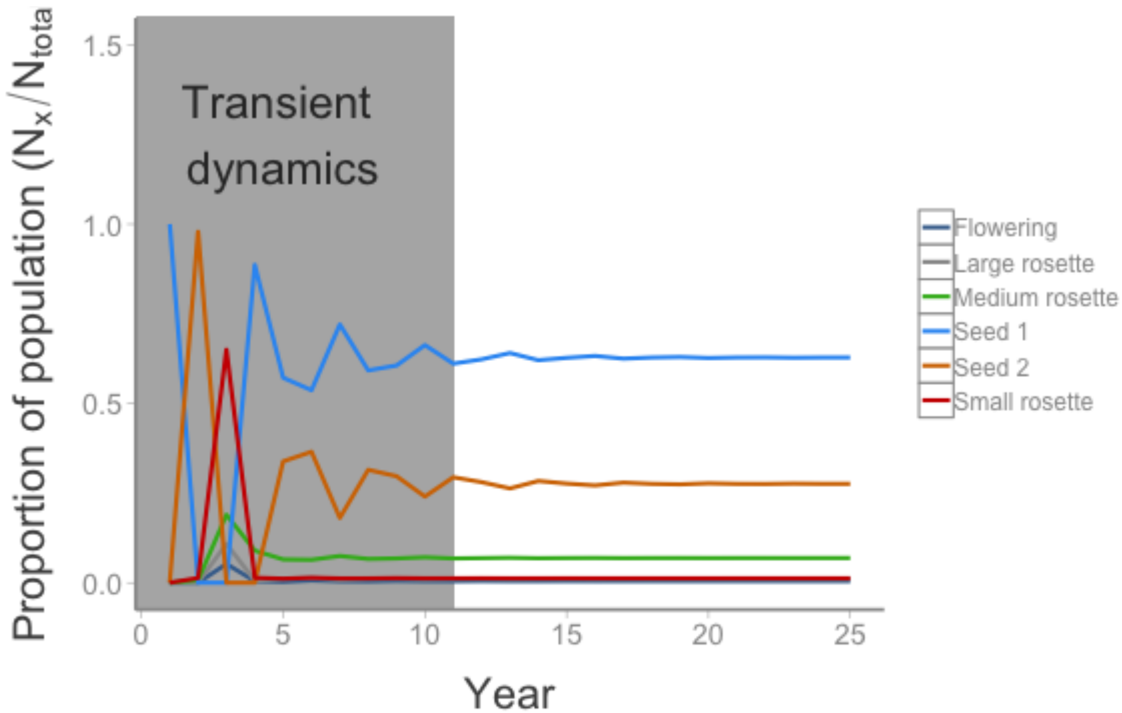
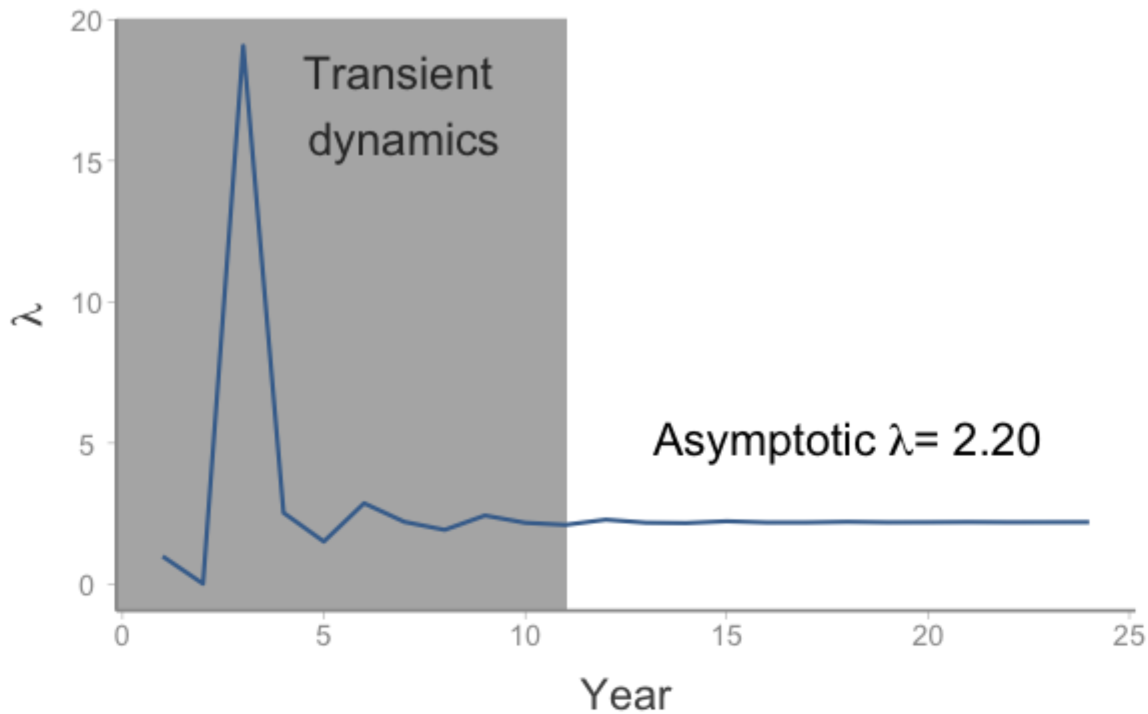


Stable stage distribution

Common teasel example



Asymptotic growth rate

Management questions

What is the short-term growth of this population given the current age/stage structure?

What is the long-term growth of this population given the current vital rates?

Which age/stage contributes most to future population growth?

Which vital rates have the biggest effect on future growth?

How would future population dynamics change if different vital rates were changed?