



Lecture 3

Introduction to population growth

WILD3810 (Spring 2020)

Readings:

Mills 79-84

Abundance

the number of individual organisms in a population at **a particular time**

- Is the number of individuals of a threatened/endangered species growing or shrinking?
- Is the abundance of a game species stable in the face of hunting pressure?
- Is a non-native species increasing in abundance to the point where it could cause ecosystem harm?

Population growth

Example

Tasmanian sheep

- 1820: 200,000 sheep introduced on the Island of Tasmania, Australia
- 1850: 2 million sheep
- 9-fold increase in 30 years



Population growth

Example

Ring-necked pheasants

- In 1937, 2 male and 6 female ring-necked pheasants were released on Protection Island, Washington
- 1942: 1,325 adults (Einarson 1942, 1945)
- 220-fold increase in 5 years!



The BIDE model

Remember from lecture one:

Abundance can due to:

- births
- deaths
- immigration
- emigration

The BIDE model

The **number** of births or deaths is not usually useful

- is 100 births a lot? Or a little?

Instead, births or deaths are often expressed as per capita (per individual) rates

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Think of these as averages:

The BIDE model

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Which can be simplified to:

Discrete-time population growth model

The terms μ and r is usually expressed as a single parameter : λ

Discrete-time population growth model

The terms λ is usually expressed as a single parameter :

is referred to as the **finite rate of population growth**

Properties of

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- What is the value of γ when the birth rate equals the death rate ?

Properties of

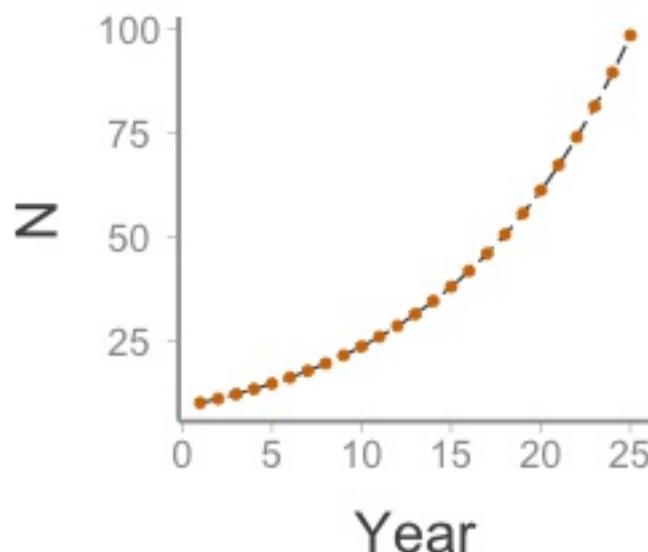
- What is the value of λ when the birth rate equals the death rate ?
- What is the value of λ when the birth rate exceeds the death rate ?

Properties of

- What is the value of r when the birth rate equals the death rate ?
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- What is the value of r when the birth rate is less than the death rate ?

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- What is the value of r when the birth rate equals the death rate ?
- What is the value of r when the birth rate exceeds the death rate ?
- What is the value of r when the birth rate is less than the death rate ?
- What happens to the abundance of the population under each scenario?



Discrete-time population growth model

What if we want to project population growth over longer time periods?

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which simplifies to:

Discrete-time population growth model

Growth from P_0 to P_t :

So we get the general form :

where t is the number of years (or weeks, or months), P_t is the final population size and P_0 is the initial population size.

Discrete vs. continuous time

The discrete population growth model is useful for **birth-pulse** species:

- | all births happen at a single point in time (i.e, a pulse)

Discrete vs. continuous time

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Change in abundance of birth-pulse species happens at discrete point in time

- usually during distinct breeding season

Discrete vs. continuous time

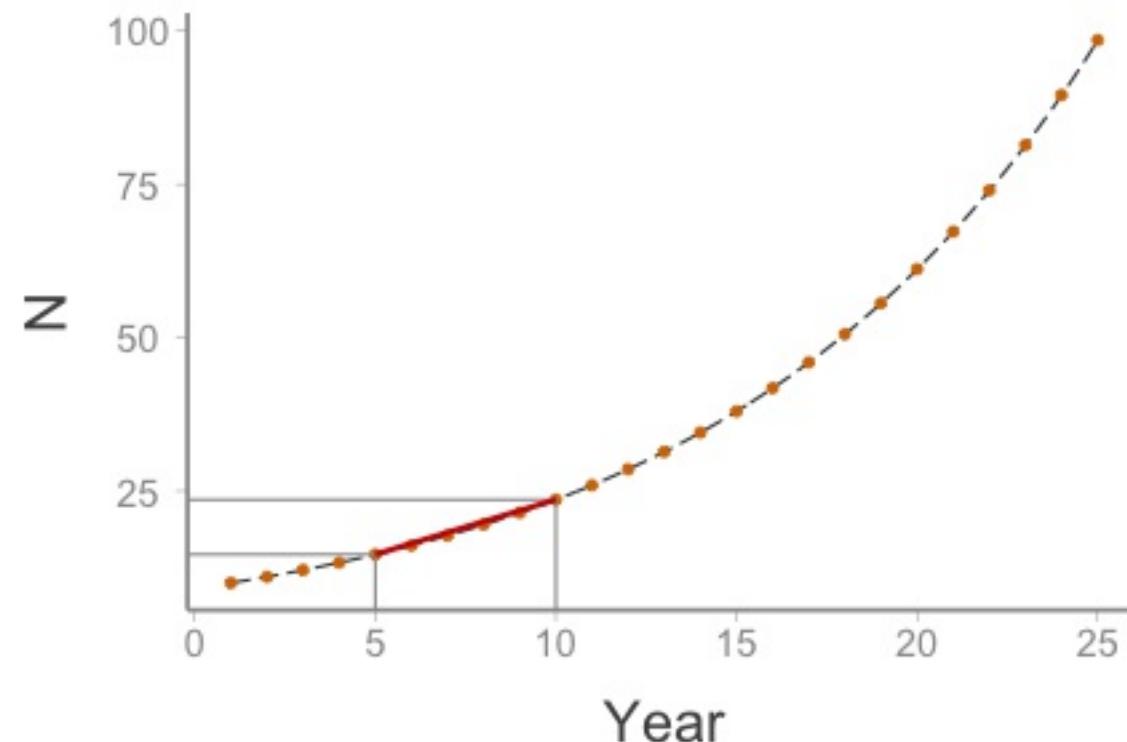
Species that reproduce throughout the year are called **birth-flow** species:

- | births happen continuously throughout the year (i.e, flow)

Abundance of birth-flow species is always changing

Continuous-time population models

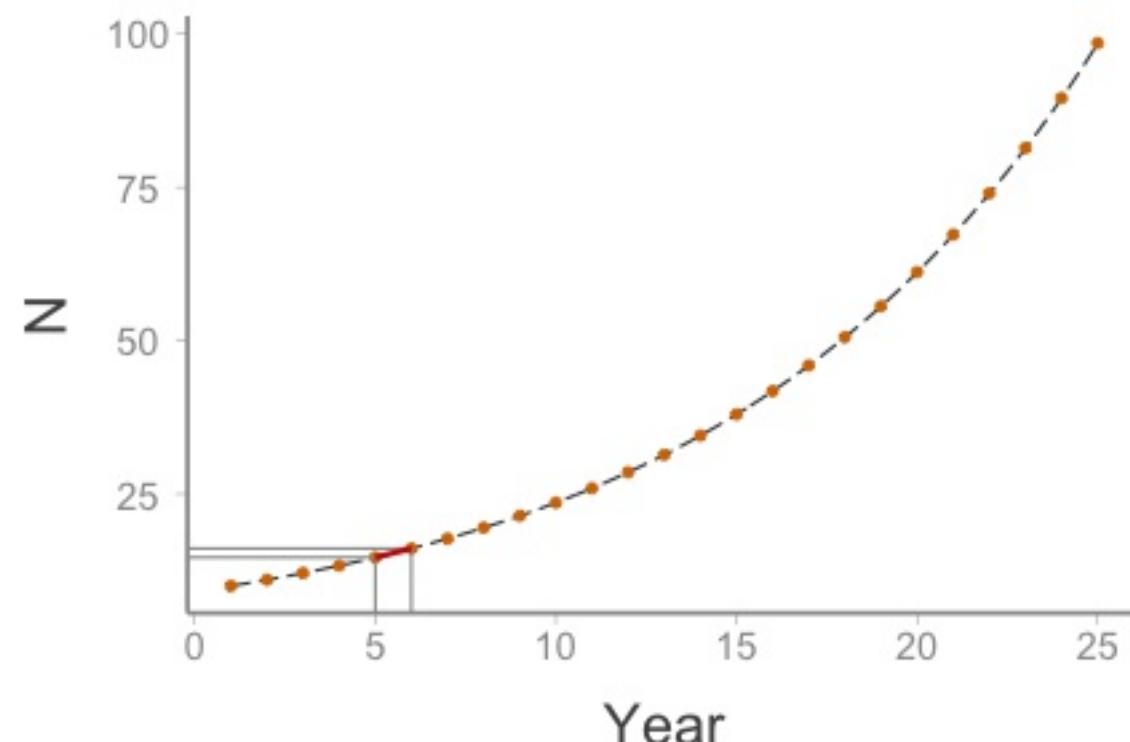
We could model the growth of birth-flow populations using the discrete model and by making Δt very small.



In this figure, $\Delta t = 1$ and $\lambda = 0.0894$ individuals Δt . So the population increased by about 61% over a 5 year period. What if we make Δt smaller?

Continuous-time population models

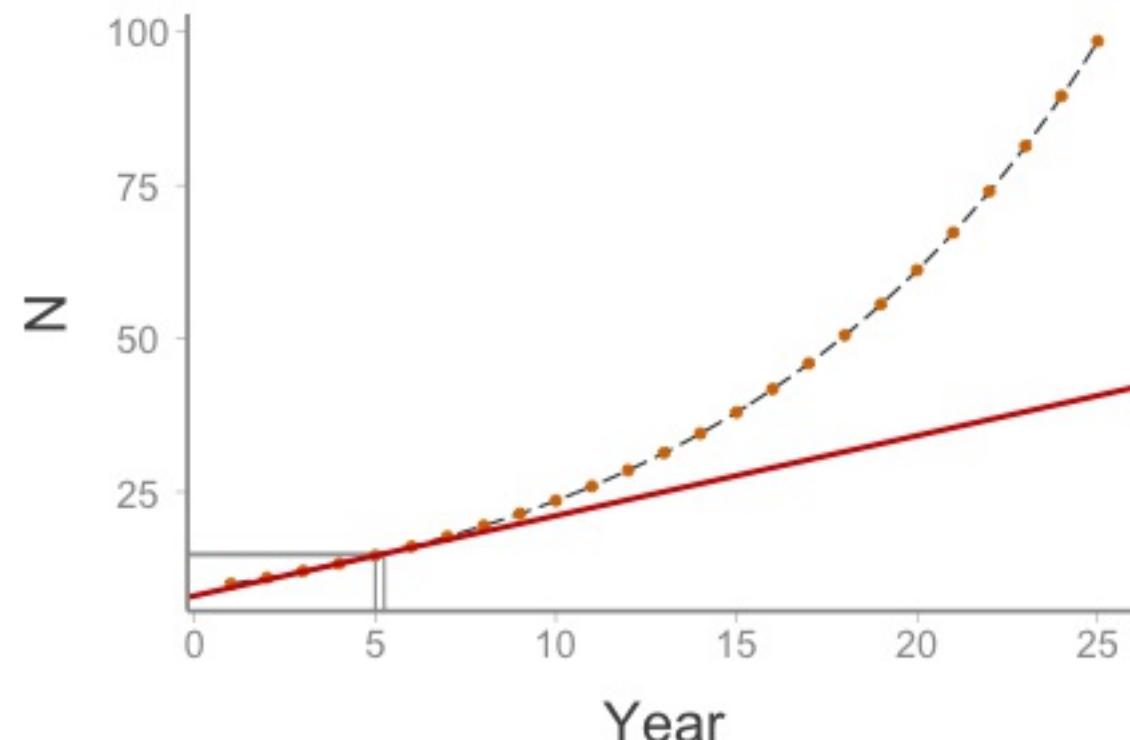
:



$$= 1.46$$

Continuous-time population models

:



$$= 0.33$$

Continuous-time population models

Rather than manually making Δt smaller and smaller, we can use calculus to figure out that as Δt becomes really, really small (i.e., approaches zero), we end up with:

This is the continuous-time equivalent of the discrete model $P_{t+\Delta t} = P_t e^{r \Delta t}$ and is called the **instantaneous growth rate**.

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This is the continuous-time equivalent of the discrete model $P_{t+\Delta t} = P_t + rP_t\Delta t$ and is called the **instantaneous growth rate**.

Populations with $r > 0$ remain at the same population size. Populations with $r < 0$ grow and populations with $r < 0$ will shrink.

Continuous-time population model

How does λ compare to μ ?

Continuous-time population model

How does λ compare to r ?

So for a continuous-time model, we can project population growth by substituting λ in the equation we used to project discrete population growth:

Doubling time

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In other words, what is t when $P = 2P_0$?

Start by dividing both sides by P_0 :

Doubling time

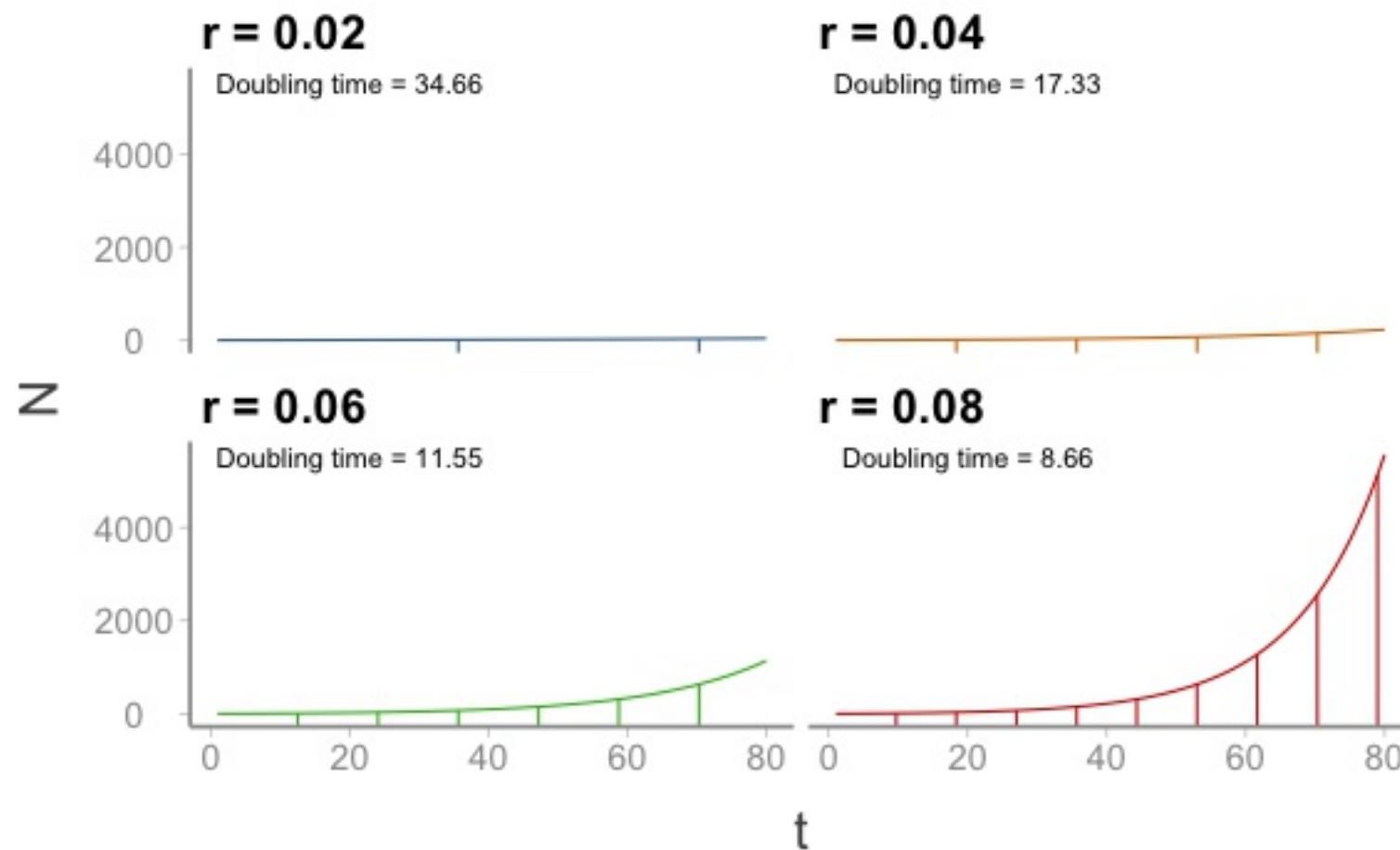
To isolate t , take the natural log of both sides:

Doubling time

To isolate r , take the natural log of both sides:

Now divide both sides by :

Doubling time



Relatively small changes in r can have profound effects on population doubling time :

- The population with $r = 0.02$ doubled twice in 80 timesteps
- The population with $r = 0.08$ doubled 9 times!

Exponential growth

Note that the doubling time does not depend on population size - the population will double every time steps, no matter what the population size is

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Density-independent population models result in **exponential growth**

- exponential growth occurs because the growth rate is multiplied by the population size at each timestep
- As the population grows, the proportional changes stays the same but the absolute changes get bigger and bigger .

Exponential growth

Exponential growth has profound implications

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- After 20 years, the investment will be worth \$3658.38 (~265% return)
- After 30 years, the investment will be worth \$6997.33 (~600% return)!

"Compound interest is the most powerful force in the universe" - Albert Einstein
(maybe)

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More often, we have estimates of abundance at different points in time. To estimate population growth for one time step:

Estimating and

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What was the growth rate of the sheep and pheasant populations?

- Sheep: $\underline{\quad}$ — 1.08
- Pheasants: $\underline{\quad}$ — 2.94

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- 2) Model pertains to only the limiting sex, usually females
- 3) Birth and death rates are independent of an individual's age or biological stage
- 4) Birth and death rates are constant