Lecture 12

Reproductive value, sensitivity, and elasticity

WILD3810 (Spring 2020)

Readings

> Mills 103-109

Management questions

What is the short-term growth of this population given the current age/stage structure?

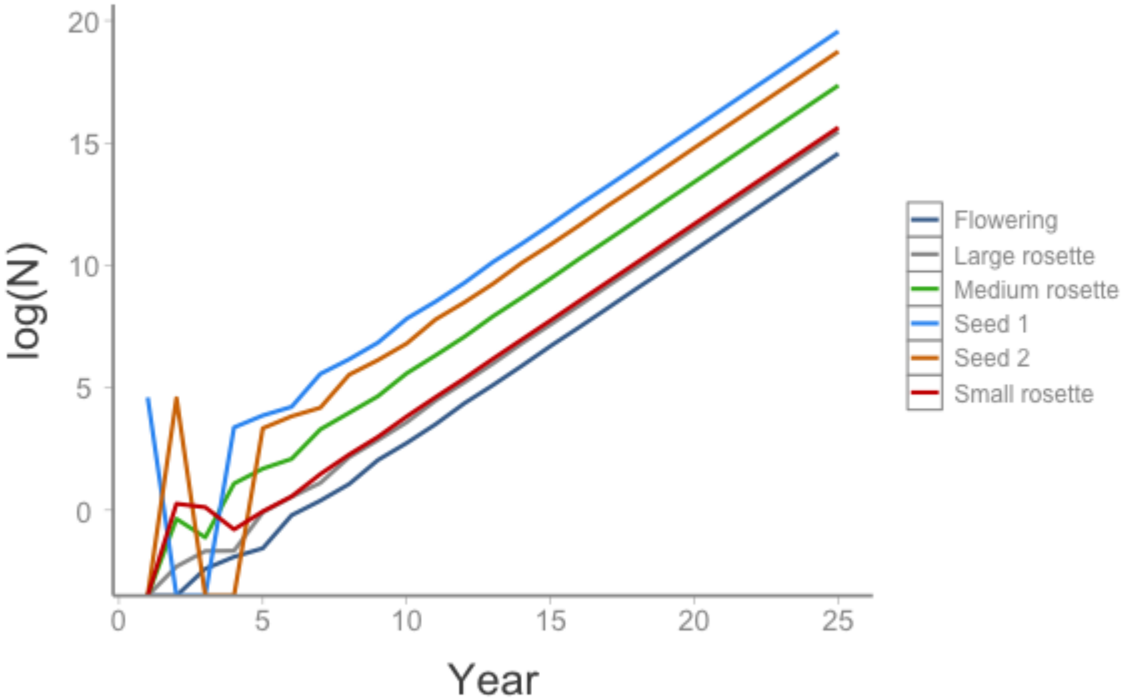
What is the long-term growth of this population given the current vital rates?

Which age/stage contributes most to future population growth?

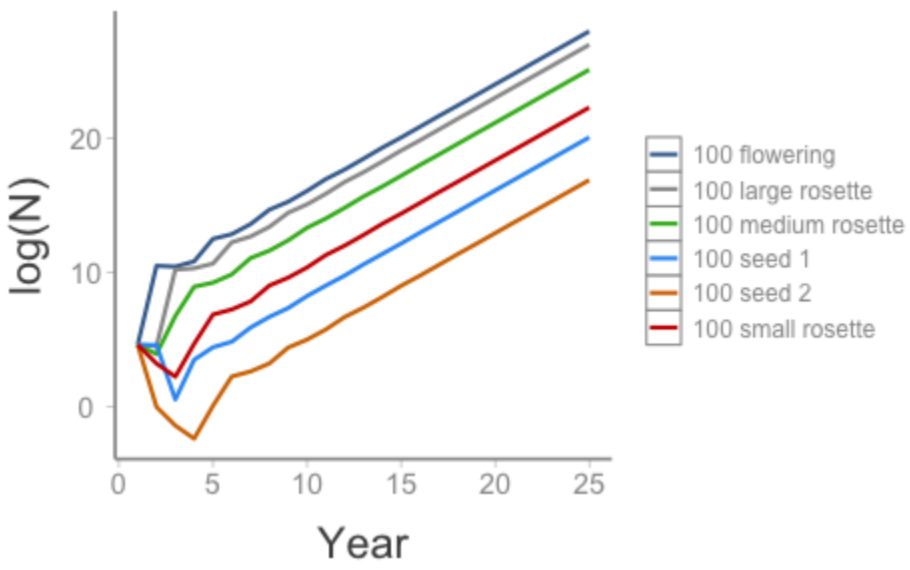
Which vital rates have the biggest effect on future growth?

How would future population dynamics change if different vital rates were changed?

Common teasel example



How does initial stage-distribution effect growth?



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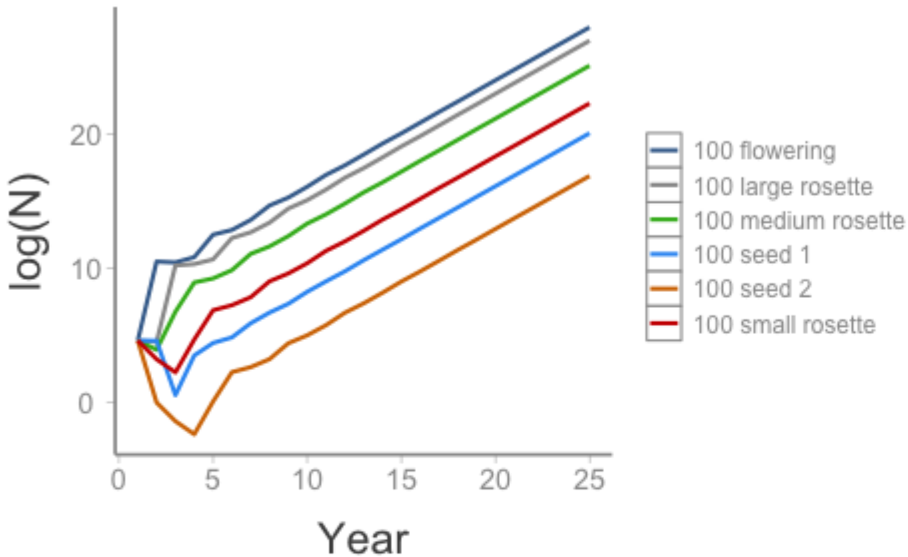
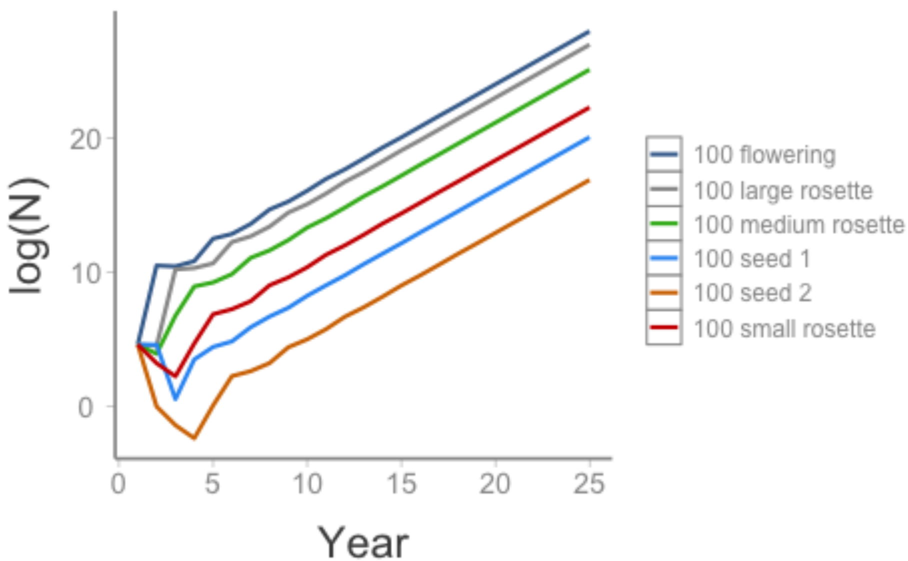


All populations reach the same stable stage distribution and have the same λ_{SSD}

How does initial stage-distribution effect growth?



All populations reach the same stable stage distribution and have the same λ_{SSD}

- But they do not have the same N_T

Reproductive value

Reproductive value

> the number of offspring that an individual is expected to contribute to a population over its remaining life span (after adjusting for the growth rate of the population)

Intuitive, but complex to compute

Factors that influence reproductive value:

- Expected future reproductive output
- Survival probability
- Age at maturity

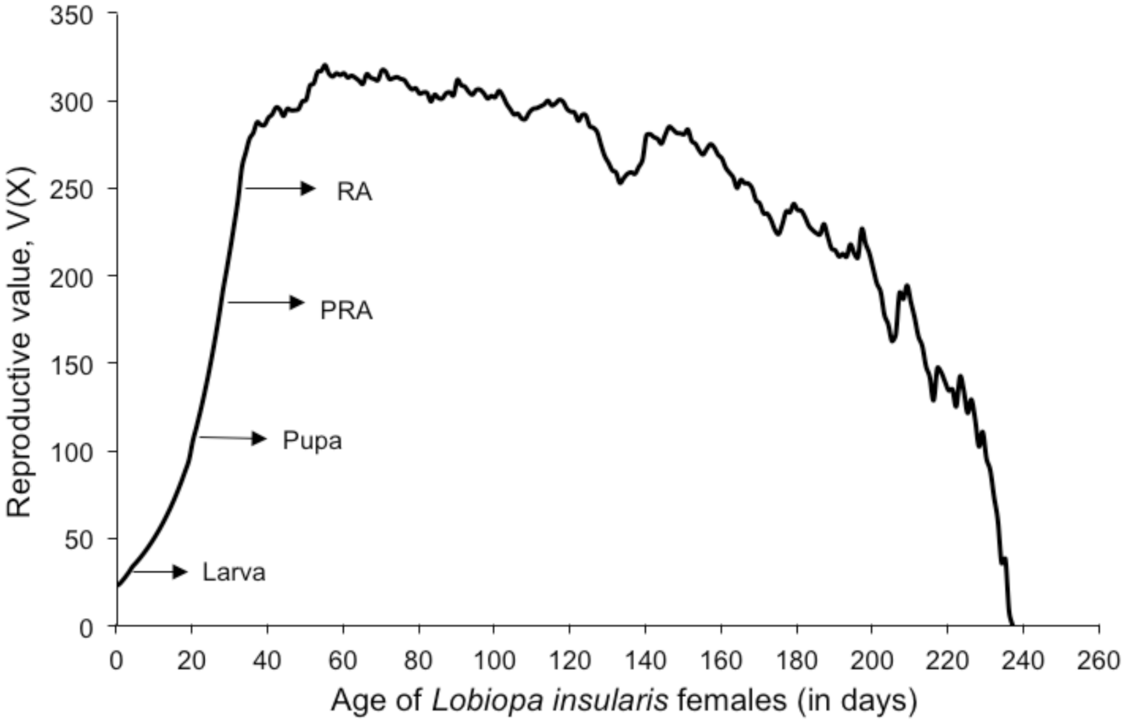
- Population growth rate
    - if a population is growing, future offspring will be smaller contribution to N
    - if a population is shrinking, future offspringwill be larger contribution to N

Reproductive value

Teasel example

- Dormant seeds, year 1: 1.00
- Dormant seeds, year 2: 0.04
- Small rosette: 9.19
- Medium rosette: 152.4
- Large rosette: 972.1
- Flowering plant: $2,633$

Reproductive value

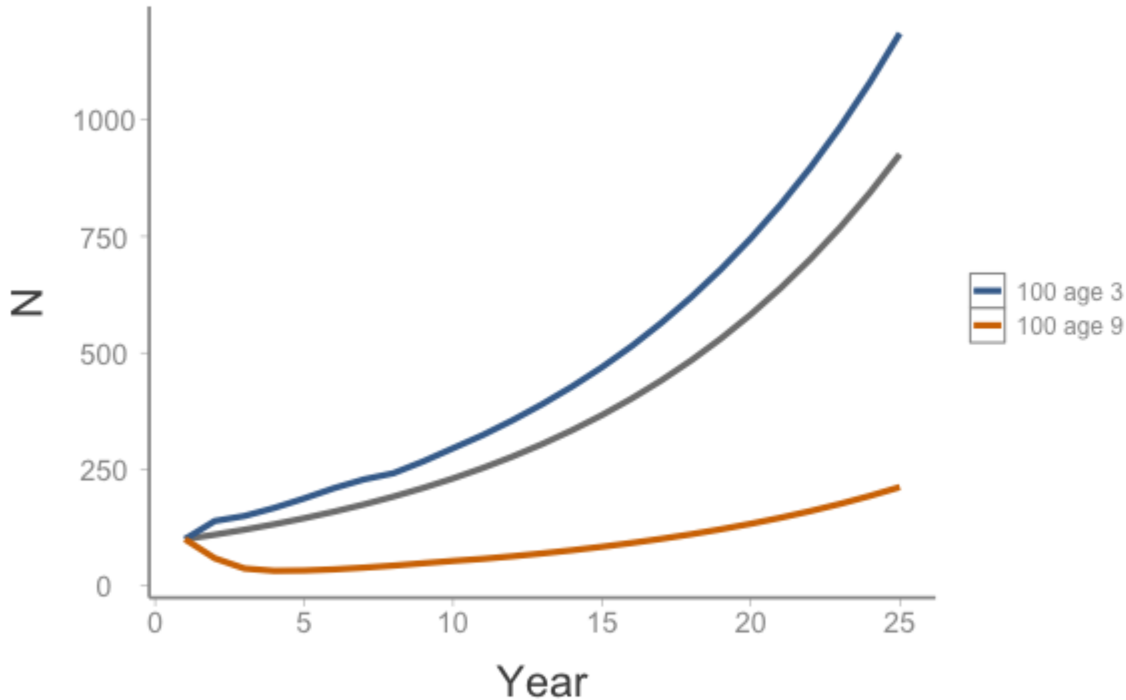


Reproductive value

Population inertia

Population inertia

> difference between the long-term population size of a population that experiences transient dynamics and the long-term population size of a population that grows at the SSD



Population inertia

Grey wolf population matrix

$$\mathbf{A} = \begin{bmatrix} 0 & 0.44 & 0.62 & 0.62 & 0.62 & 0.62 & 0.62 & 0.62 & 0.22 \\ 0.69 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.77 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.77 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.77 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.77 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.77 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.77 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.55 & 0.37 \end{bmatrix}$$

$$\lambda_{SAD} = 1.097$$

Reproductive values:

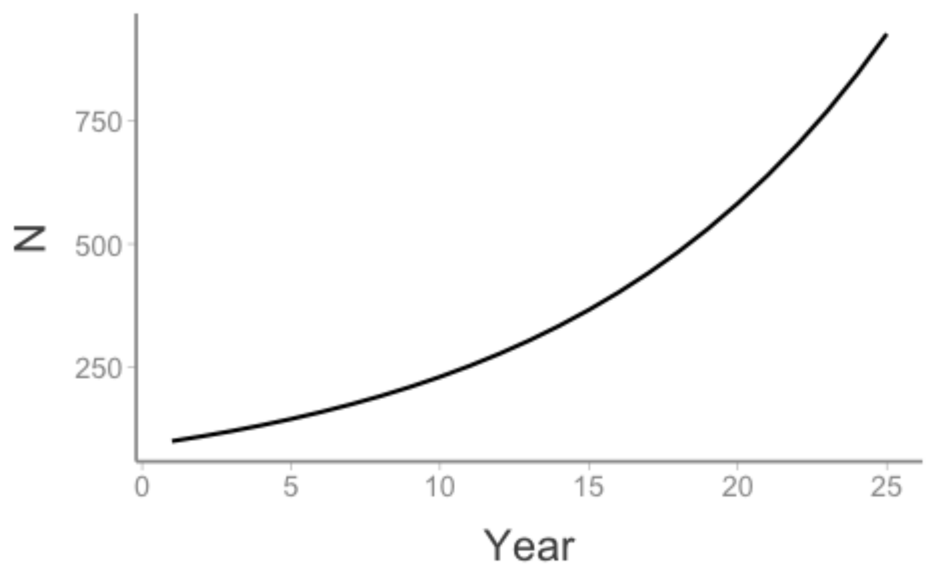
$$[1.00, 1.59, 1.69, 1.61, 1.49, 1.31, 1.07, 0.72, 0.30]$$

Population inertia

What is the projected growth if reintroduced population starts at SAD?

SAD:

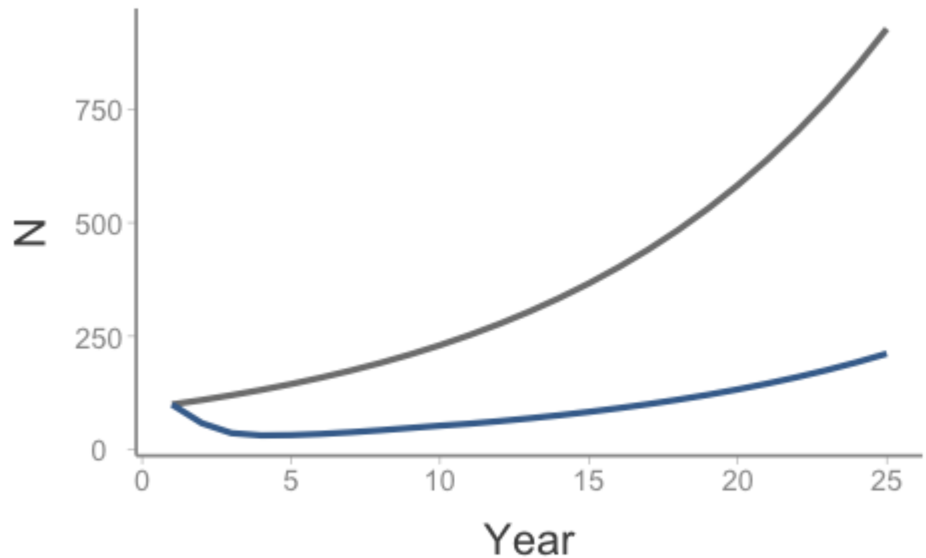
$$\begin{bmatrix} 0.33 \\ 0.21 \\ 0.15 \\ 0.10 \\ 0.07 \\ 0.05 \\ 0.04 \\ 0.03 \\ 0.02 \end{bmatrix}$$



Population inertia

What is the projected growth if reintroduced population starts with all 9 year olds?

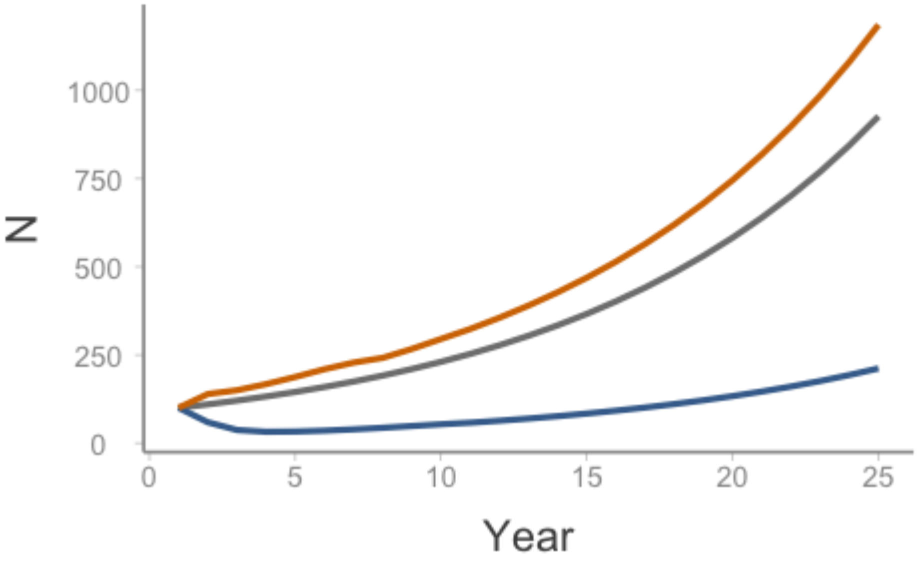
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$



Population inertia

What is the projected growth if reintroduced population starts with all 3 year olds?

$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



Population inertia and reproductive value in practice

What age/stage distribution will lead to largest N for introduced population?

What age/stage should be the target for reducing invasive species?

What age/stage distribution can be harvested to maintain stable population of game species?

Sensitivity and elasticity

Sensitivity

Remember that λ is determined by the birth and death rates of a population

$$\lambda = 1 + b - d$$

As managers, we might want to increase or decrease λ of certain species by manipulating age-specific b and d rates

But λ does not respond equally to all vital rates

- in some cases, a small change in adult survival may result in a large change in λ
- in other cases, a small change in fecundity or juvenile survival may result in a large change in λ

Which vital rates should we focus our management efforts on?

Sensitivity

Sensitivity

> the change in λ caused by a small change in a vital rate

$$s_{i,j} = \frac{\delta\lambda}{\delta a_{i,j}} = \frac{v_i w_j}{\sum_{k=1} v_k w_k}$$

where v_i and w_j are the reproductive value and stable stage distribution of stage i

- large reproductive values or large stable stable distribution lead to large sensitivity

Sensitivity

Common frog (*Rana temporaria*)

3 stages:

- pre-juvenile (egg - tadpole)
- juvenile (tadpole - 2 years)
- adult (> 2 years)

$$\mathbf{A} = \begin{bmatrix} 0 & 52 & 279.5 \\ 0.019 & 0.25 & 0 \\ 0 & 0.08 & 0.43 \end{bmatrix}$$

$$\lambda = 1.338$$

Estimating sensitivity

By hand

$$a_{2,1} = 0.019, \lambda = 1.338$$

$$a'_{2,1} = 0.029, \lambda = 1.571$$

$$s_{2,1} = \frac{1.571 - 1.338}{0.01} = 23.3$$

Analytically

`popbio::sensitivity(A)[2,1] = 26.05`

Estimating sensitivity

By hand

$$a_{1,2} = 52, \lambda = 1.338$$

$$a'_{1,2} = 52.01, \lambda = 1.338$$

$$s_{1,2} = \frac{1.338 - 1.338}{0.01} = 0.00$$

Analytically

`popbio::sensitivity(A)[1,2] = 0.006`

Elasticity

Does it make sense to compare a change of 0.01 in a survival value to a change of 0.01 in fecundity?

- 0.01 is about 52% of 0.019
- 0.01 is about 0.02% of 52

Elasticity

> the change in λ caused by a small *proportional* change in a vital rate

$$e_{i,j} = s_{i,j}\left[\frac{a_{i,j}}{\lambda}\right]$$

- Elasticity scales sensitivity to account for the magnitude of the vital rate

Elasticity

Common frog example

By hand

$$e_{2,1} = 26.05 \left[\frac{0.019}{1.338} \right] = 0.3699$$

$$e_{2,1} = 0.006 \left[\frac{52}{1.338} \right] = 0.2332$$

Analytically

`popbio::elasticity(A)[2,1] = 0.3699`

`popbio::elasticity(A)[1,2] = 0.251`

Life history variation

Organisms have limited resources to investment between growth, reproduction, and survivorship

Evolution selects for different combinations of *life history traits*

> Demographic traits that influence fitness (i.e., λ)

- size at birth
- growth pattern
- age at maturity
- fecundity schedule
- mortality schedule
- length of life

Trade-offs between traits directly related to sensitivity/elasticity

We'll explore variation in life history traits during the next lecture