Lecture 3

Introduction to population growth

WILD3810 (Spring 2020)

Readings:

> Mills 79-84

Abundance

> the number of individual organisms in a population at **a particular time**

- Is the number of individuals of a threatened/endangered species growing or shrinking?
- Is the abundance of a game species stable in the face of hunting pressure?
- Is a non-native species increasing in abundance to the point where it could cause ecosystem harm?

Population growth

Example

Tasmanian sheep

- 1820: 200,000 sheep introduced on the Island of Tasmania, Australia
- 1850: 2 million sheep
- 9-fold increase in 30 years

Population growth

Example

Ring-necked pheasants

- In 1937, 2 male and 6 female ring-necked pheasants were released on Protection Island, Washington
- 1942: 1,325 adults (Einarson 1942, 1945)
- 220-fold increase in 5 years!

The BIDE model

Remember from lecture one:

$$N_{t+1} = N_t + B + I - D - E$$

Abundance can due to:

- births (B)
- deaths (D)
- immigration (I)
- emigration (E)

The BIDE model

The **number** of births or deaths is not usually useful

- is 100 births a lot? Or a little?

Instead, births (B) or deaths (D) are often expressed as *per capita* (per individual) rates

Think of these as averages:

$$b = \frac{B}{N}$$

$$d = \frac{D}{N}$$

The BIDE model

Because $b \times N_t = B$ and $d \times N_t = D$ (and assuming no movement), the BIDE model can be written as:

$$N_{t+1} = N_t + \underbrace{(b \times N_t)}_{\text{births}} - \underbrace{(d \times N_t)}_{\text{deaths}}$$

Which can be simplied to:

$$N_{t+1} = N_t \times (1 + b - d)$$

Discrete-time population growth model

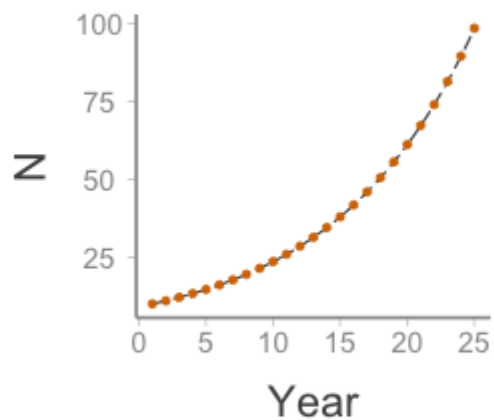
The terms $1 + b - d$ is usually expressed as a single parameter λ:

$$N_{t+1} = N_t \times \underbrace{(1 + b - d)}_{\lambda}$$

λ is referred to as the **finite rate of population growth**

Properties of λ

- What is the value of λ when the birth rate equals the death rate $(b - d = 0)$?
- What is the value of λ when the birth rate exceeds the death rate $(b - d > 0)$?
- What is the value of λ when the birth rate is less than the death rate $(b - d < 0)$?
- What happens to the abundance of the population under each scenario? [1]



Discrete-time population growth model

$$N_{t+1} = N_t \times \lambda$$

What if we want to project population growth over longer time periods?
First, write the forumula for N_{t+2} from N_{t+1}:

$$N_{t+2} = N_{t+1} \times \lambda$$

We know that $N_{t+1} = N_t \times \lambda$ so:

$$N_{t+2} = (N_t \times \lambda) \times \lambda$$

which simplifies to:

$$N_{t+2} = N_t \times \lambda^2$$

Discrete-time population growth model

Growth from t to $t + 3$:

$$N_{t+3} = N_t \times \lambda^3$$

So we get the general form [2]:

$$N_T = N_0 \times \lambda^T$$

where T is the number of years (or weeks, or months), N_T is the final population size and N_0 is the initial population size.

Discrete vs. continuous time

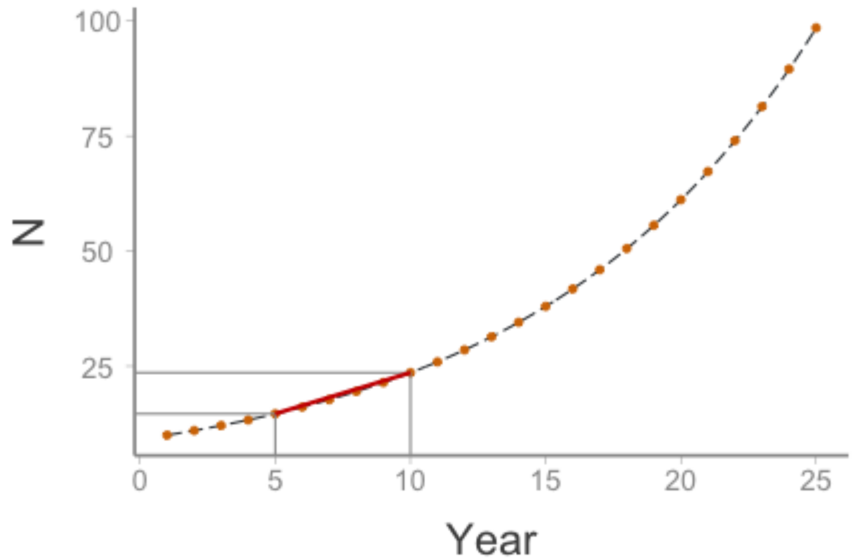
Species that reproduce throughout the year are called **birth-flow** species:

> births happen continuously throughout the year (i.e, flow)

Abundance of birth-flow species is *always* changing

Continuous-time population models

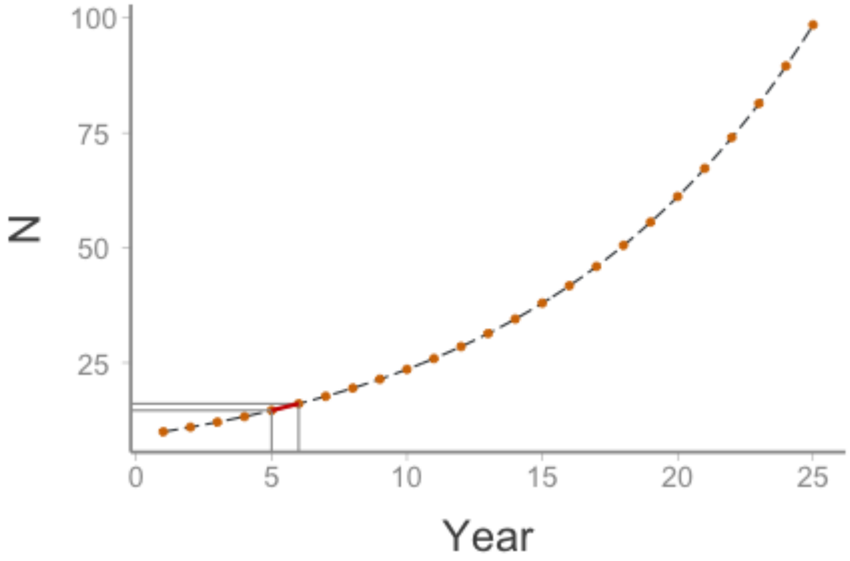
We *could* model the growth of birth-flow populations using the discrete model and by making Δ_t very small.



In this figure, $\Delta_t = 5$ and Δ_N = 8.94 individuals [3]. So the population increased by about 61% over a 5 year period. What if we make Δ_t smaller?

Continuous-time population models

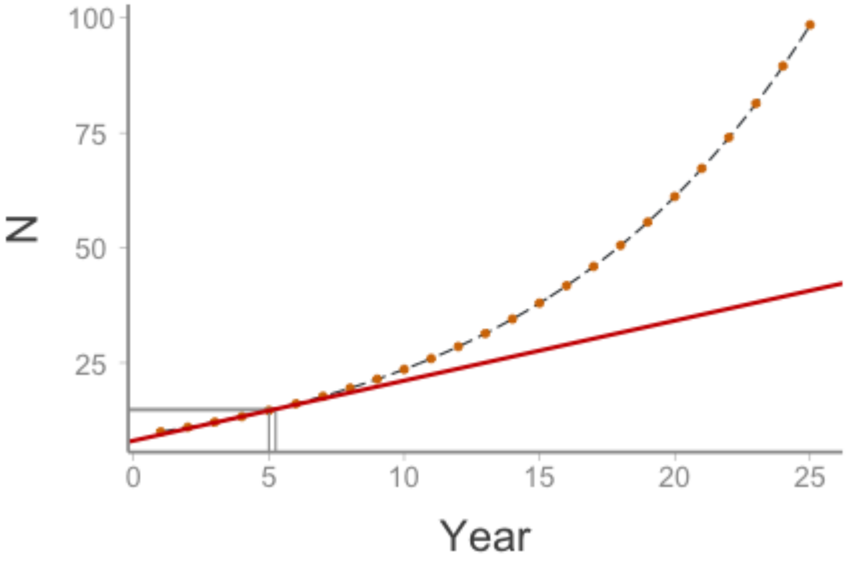
$\Delta_t = 1$:



$\Delta_N = 1.46$ [4]

Continuous-time population models

$\Delta_t = 0.25$:



$\Delta_N = 0.33$

Continuous-time population models

Rather than manually making Δ_t smaller and smaller, we can use calculus to figure out that as Δ_t becomes really, really small (i.e., *approaches zero*), we end up with:

$$\frac{dN}{dt} = N \times (b - d) = N \times r$$

This the continuous-time equivalent of the discrete model [5] and r is called the **instantaneous growth rate**

Populations with $r = 0$ remain at the same population size. Populations with $r > 0$ grow and populations with $r < 0$ will shrink.

Continuous-time population model

How does r compare to λ?

$$r = \log(\lambda)$$

$$\lambda = e^{r}$$

So for a continuous-time model, we can project population growth by substituting e^{r} in the equation we used to project discrete population growth:

$$N_T = N_0 \times e^{rT}$$

Doubling time

Doubling time

How long will it take a population to double?

In other words, what is T when $N_T = 2 \times N_0$?

$$2N_0 = N_0 \times e^{rT}$$

Start by dividing both sides by N_0:

$$2 = e^{rT}$$

Doubling time

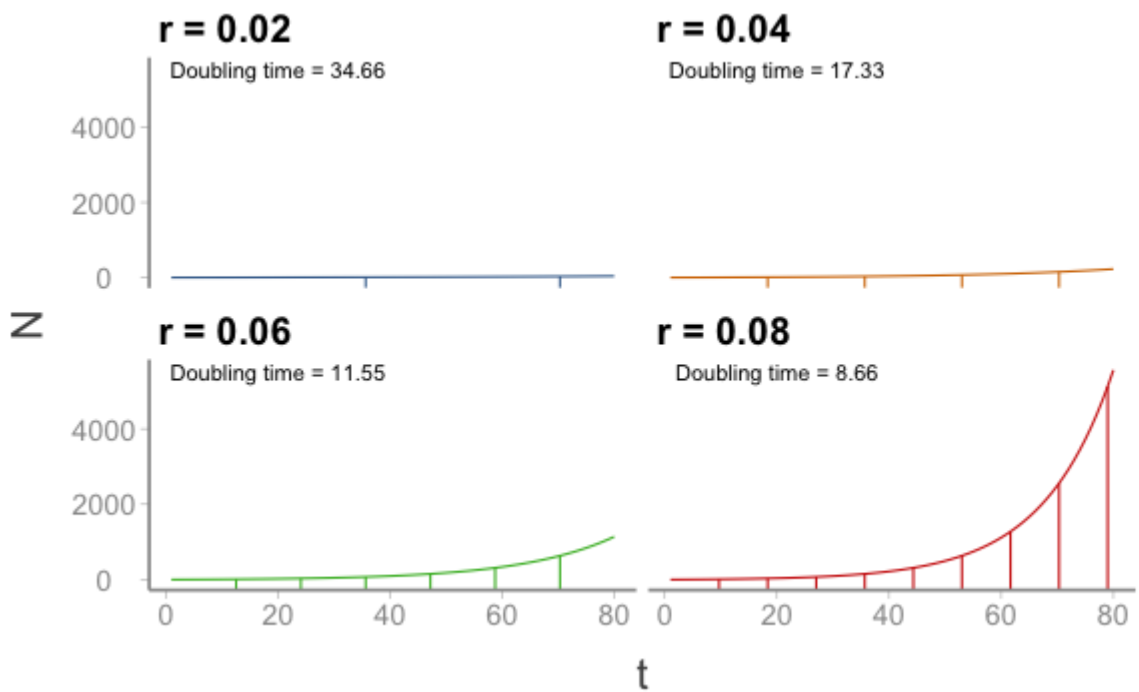
To isolate rT, take the natural log [6] of both sides:

$$\ln(2) = \ln(e^{rT}) = rT$$

Now divide both sides by r:

$$\frac{0.693}{r} = T_{\mathrm{double}}$$

Doubling time



Relatively small changes in r can have profound effects on population doubling time [7]:

- The population with $r = 0.02$ doubled twice in 80 timesteps
- The population with $r = 0.08$ doubled 9 times!

Exponential growth

Note that the doubling time does not depend on population size - the population will double every t_{double} time steps, no matter what the population size is

Because the growth rate (r or λ) does not depend on abundance, these models are **density-independent**

Density-independent population models result in **exponential** growth [8]

- exponential growth occurs because the growth rate is *multiplied* by the population size at each timestep
- As the population grows, the proportional changes stays the same but the absolute changes get bigger and bigger [9].

Exponential growth

Exponential growth has profound implications

Imagine putting $1000 in a investment that grows by 6.5% per year:

- After 10 years, the investment will be worth $1912.69 (~90% return)
- After 20 years, the investment will be worth $3658.38 (~265% return)
- After 30 years, the investment will be worth $6997.33 (~600% return)!

> "Compound interest is the most powerful force in the universe" - Albert Einstein (maybe)

Estimating λ and r

Estimating λ and r

We can estimate λ two ways depending on our interests and what type of data we have available

As we learned above, we can estimate λ directly from the birth and death rates using:

$$\lambda = 1 + b - d$$

More often, we have estimates of abundance at different points in time. To estimate population growth for one time step:

$$\lambda = \frac{N_{t+1}}{N_t}$$

Estimating λ and r

If we have a longer series of abundance estimates $t = 1, 2, 3 ... T$), we use:

$$\lambda = (\frac{N_T}{N_0})^{\frac{1}{T}}$$

What was the growth rate of the sheep and pheasant populations?

- Sheep: $(\frac{2000000}{200000})^{\frac{1}{30}}$ = 1.08
- Pheasants: $(\frac{1325}{6})^{\frac{1}{6}}$ = 2.94

Assumptions of the B-D models

1) Population closed to immigration and emigration

2) Model pertains to only the limiting sex, usually females

3) Birth and death rates are independent of an individual's age or biological stage

4) Birth and death rates are constant