



Lecture 3

Introduction to population growth

WILD3810 (Spring 2020)

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Readings:

Mills 79-84

Abundance

the number of individual organisms in a population at a particular time

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- Is the abundance of a game species stable in the face of hunting pressure?

Abundance

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- Is the number of individuals of a threatened/endangered species growing or shrinking?
- Is the abundance of a game species stable in the face of hunting pressure?
- Is a non-native species increasing in abundance to the point where it could cause ecosystem harm?

Population growth

Example

Tasmanian sheep

- 1820: 200,000 sheep introduced on the Island of Tasmania, Australia
- 1850: 2 million sheep
- 9-fold increase in 30 years



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Population growth

Example

Ring-necked pheasants

- In 1937, 2 male and 6 female ring-necked pheasants were released on Protection Island, Washington
- 1942: 1,325 adults (Einarson 1942, 1945)
- 220-fold increase in 5 years!



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The BIDE model

Remember from lecture one:

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The BIDE model

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Abundance can due to:

- births
- deaths
- immigration
- emigration

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The BIDE model

The **number** of births or deaths is not usually useful

- is 100 births a lot? Or a little?

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Instead, births or deaths are often expressed as *per capita* (per individual) rates

Think of these as averages:

The BIDE model

Because $\dot{x} = 0$ and $\dot{y} = 0$ (and assuming no movement), the BIDE model can be written as:

The BIDE model

Because $\Delta x = 0$ and $\Delta t = 0$ (and assuming no movement), the BIDE model can be written as:

Which can be simplified to:

Discrete-time population growth model

The terms λ is usually expressed as a single parameter :

Discrete-time population growth model

The terms λ is usually expressed as a single parameter :

is referred to as the **finite rate of population growth**

Properties of

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Properties of

- What is the value of λ when the birth rate equals the death rate?

Properties of

- What is the value of γ when the birth rate equals the death rate ?
- What is the value of γ when the birth rate exceeds the death rate ?

Properties of

- What is the value of π when the birth rate equals the death rate ?
- What is the value of π when the birth rate exceeds the death rate ?
- What is the value of π when the birth rate is less than the death rate ?

Properties of

- What is the value of λ when the birth rate equals the death rate?
- What is the value of λ when the birth rate exceeds the death rate?
- What is the value of λ when the birth rate is less than the death rate?
- What happens to the abundance of the population under each scenario?

Discrete-time population growth model

What if we want to project population growth over longer time periods?

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We know that so:

Discrete-time population growth model

What if we want to project population growth over longer time periods?

First, write the formula for N_t from N_0 :

We know that $N_{t+1} = N_t + \Delta N$ so:

which simplifies to:

Discrete-time population growth model

Growth from N_0 to N_t :

So we get the general form :

where t is the number of years (or weeks, or months), N_t is the final population size and N_0 is the initial population size.

Discrete vs. continuous time

The discrete population growth model is useful for **birth-pulse** species:

- all births happen at a single point in time (i.e, a pulse)

Discrete vs. continuous time

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- all births happen at a single point in time (i.e, a pulse)

Change in abundance of birth-pulse species happens at discrete point in time

- usually during distinct *breeding season*

Discrete vs. continuous time

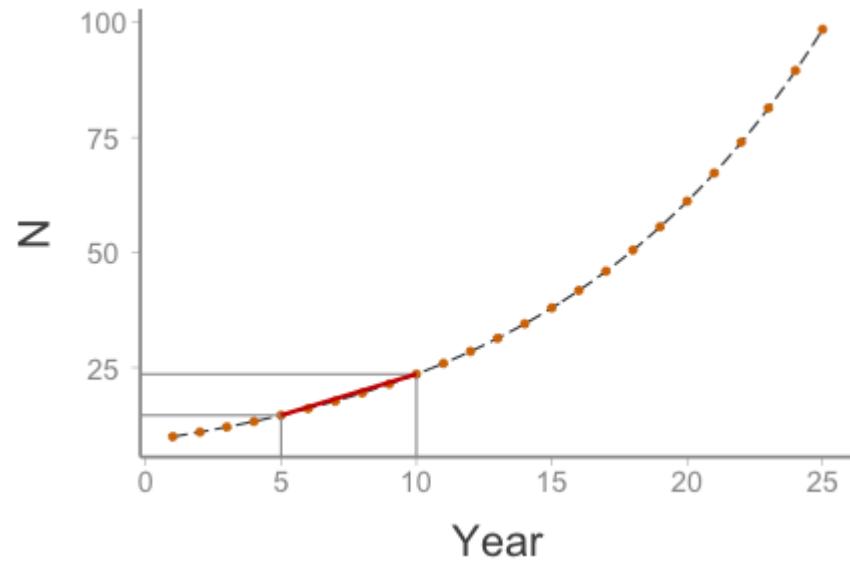
Species that reproduce throughout the year are called **birth-flow species**:

- births happen continuously throughout the year (i.e, flow)

Abundance of birth-flow species is *always* changing

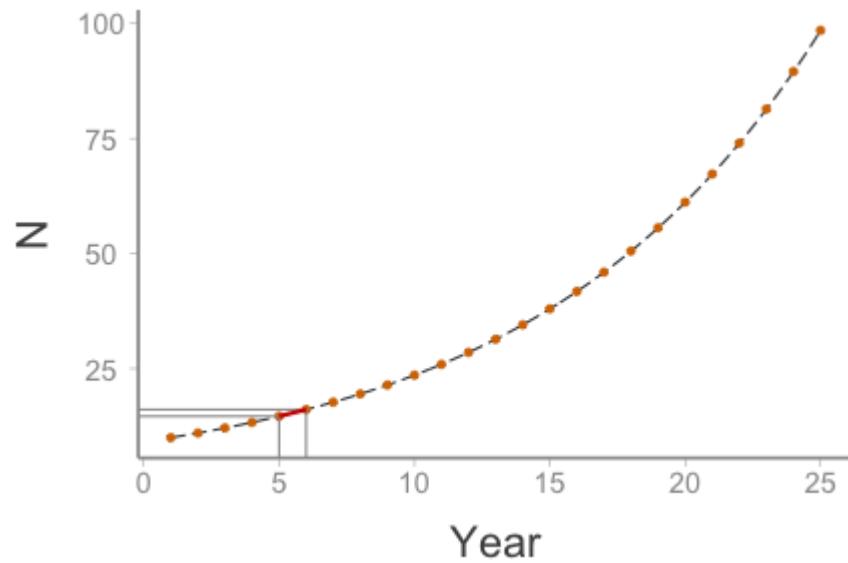
Continuous-time population models

We *could* model the growth of birth-flow populations using the discrete model and by making Δt very small.



Continuous-time population models

:

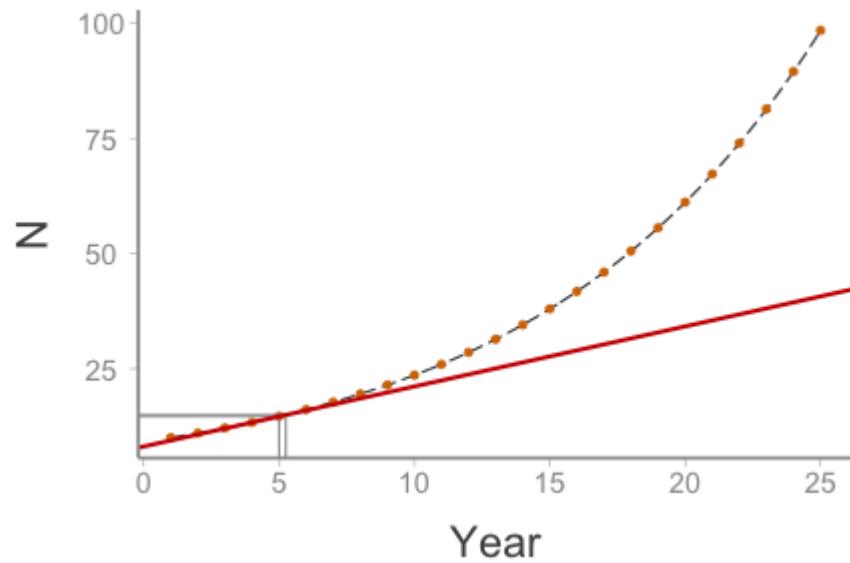


= 1.46

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Continuous-time population models

:



$$= 0.33$$

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Continuous-time population models

Rather than manually making Δt smaller and smaller, we can use calculus to figure out that as Δt becomes really, really small (i.e., *approaches zero*), we end up with:



This is the continuous-time equivalent of the discrete model ΔP and is called the **instantaneous growth rate**

Continuous-time population models

Rather than manually making Δt smaller and smaller, we can use calculus to figure out that as Δt becomes really, really small (i.e., *approaches zero*), we end up with:



This is the continuous-time equivalent of the discrete model ΔP and is called the **instantaneous growth rate**.

Populations with $r > 0$ remain at the same population size. Populations with $r = 0$ grow and populations with $r < 0$ will shrink.

Continuous-time population model

How does λ compare to μ ?

Continuous-time population model

How does λ compare to r ?

So for a continuous-time model, we can project population growth by substituting λ in the equation we used to project discrete population growth:

Doubling time

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Start by dividing both sides by :

Doubling time

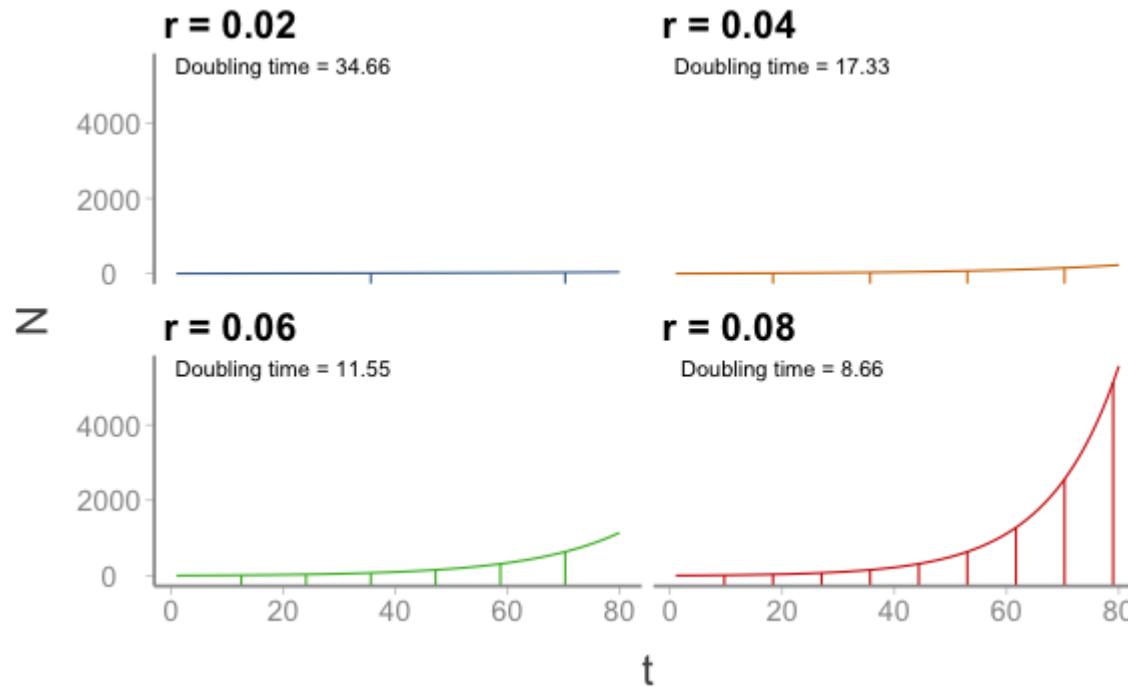
To isolate t , take the natural log of both sides:

Doubling time

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Now divide both sides by :

Doubling time



Relatively small changes in r can have profound effects on population doubling time :

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Exponential growth

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Density-independent population models result in **exponential** growth

- exponential growth occurs because the growth rate is *multiplied* by the population size at each timestep
- As the population grows, the proportional changes stays the same but the absolute changes get bigger and bigger .

Exponential growth

Exponential growth has profound implications

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- After 20 years, the investment will be worth \$3658.38 (~265% return)

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- After 10 years, the investment will be worth \$1912.69 (~90% return)
- After 20 years, the investment will be worth \$3658.38 (~265% return)
- After 30 years, the investment will be worth \$6997.33 (~600% return)!

"Compound interest is the most powerful force in the universe"
- Albert Einstein (maybe)

Estimating and

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More often, we have estimates of abundance at different points in time. To estimate population growth for one time step:

Estimating and

If we have a longer series of abundance estimates $\underline{\underline{}}$, we use:

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- Sheep: $\underline{\quad}$ 1.08
- Pheasants: $\underline{\quad}$ 2.94

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- 1) Population closed to immigration and emigration
- 2) Model pertains to only the limiting sex, usually females
- 3) Birth and death rates are independent of an individual's age or biological stage
- 4) Birth and death rates are constant