



# Lecture 3

Introduction to population growth

WILD3810 (Spring 2019)

# Readings:

Mills 79-84

# Abundance

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the number of individual organisms in a population at **a particular time**

- Is the number of individuals of a threatened/endangered species growing or shrinking?
- Is the abundance of a game species stable in the face of hunting pressure?
- Is a non-native species increasing in abundance to the point where it could cause ecosystem harm?

# The BIDE model

Remember from lecture one:

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Abundance can due to:

- births
- deaths
- immigration
- emigration

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Think of these as averages:

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Which can be simplified to:

# Discrete-time population growth model

The terms  $\mu$  and  $r$  is usually expressed as a single parameter :  $\lambda$

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is referred to as the **finite rate of population growth**

# Properties of

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- What is the value of  $\gamma$  when the birth rate equals the death rate ?

# Properties of

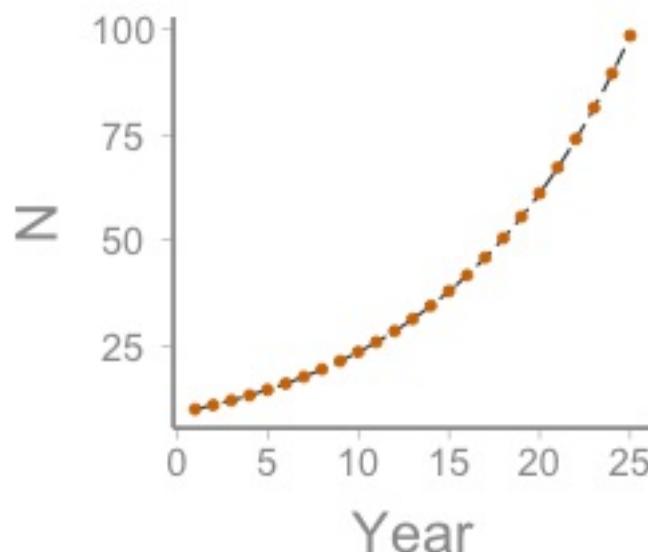
- What is the value of  $\lambda$  when the birth rate equals the death rate ?
- What is the value of  $\lambda$  when the birth rate exceeds the death rate ?

# Properties of

- What is the value of  $r$  when the birth rate equals the death rate ?
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- What is the value of  $r$  when the birth rate is less than the death rate ?

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- What is the value of  $r$  when the birth rate equals the death rate ?
- What is the value of  $r$  when the birth rate exceeds the death rate ?
- What is the value of  $r$  when the birth rate is less than the death rate ?
- What happens to the abundance of the population under each scenario?



# Discrete-time population growth model

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# Discrete-time population growth model

Growth from  $P_0$  to  $P_t$  :

So we get the general form :

where  $t$  is the number of years (or weeks, or months),  $P_t$  is the final population size and  $P_0$  is the initial population size.

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Change in abundance of birth-pulse species happens at discrete point in time

- usually during distinct breeding season

# Discrete vs. continuous time

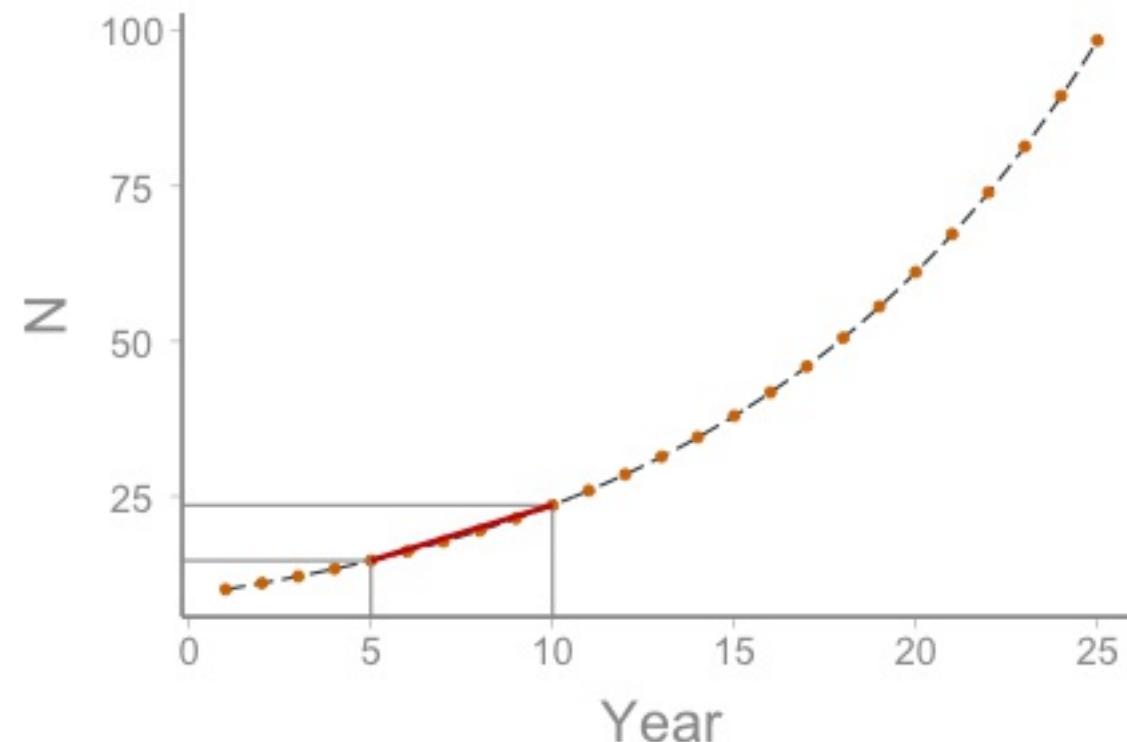
Species that reproduce throughout the year are called **birth-flow** species:

- | births happen continuously throughout the year (i.e, flow)

Abundance of birth-flow species is always changing

# Continuous-time population models

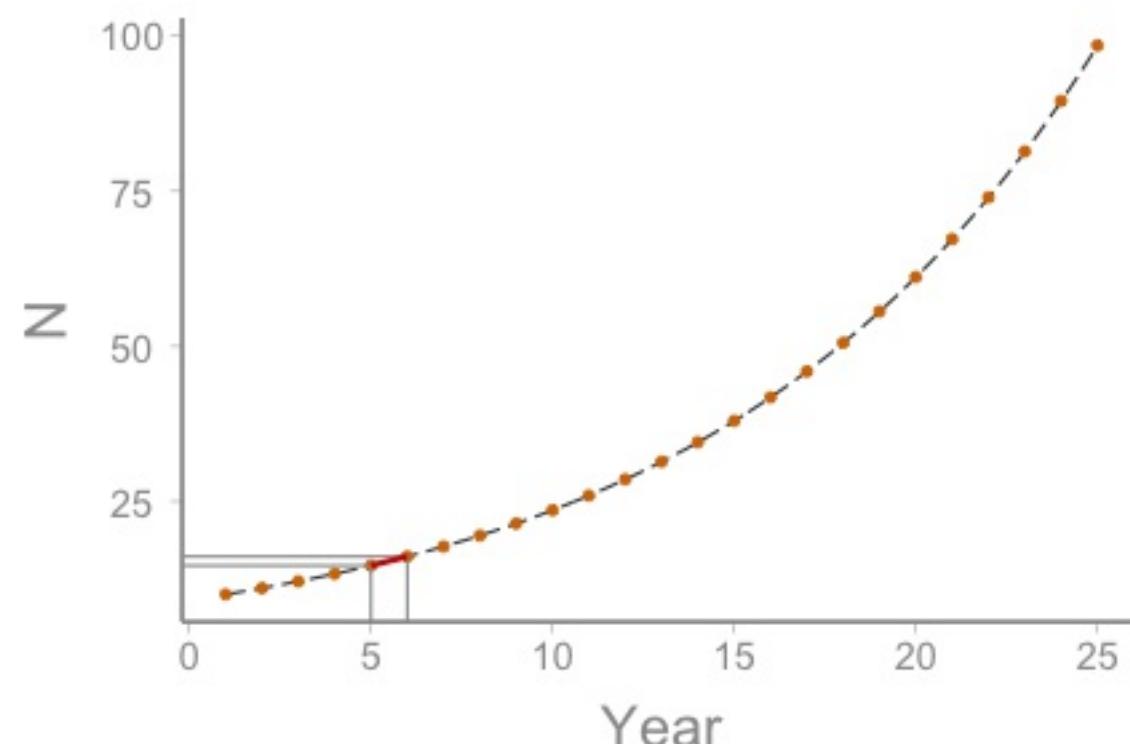
We could model the growth of birth-flow populations using the discrete model and by making  $\Delta t$  very small.



In this figure,  $\Delta t = 1$  and  $\lambda = 0.0894$  individuals  $\Delta t$ . So the population increased by about 61% over a 5 year period. What if we make  $\Delta t$  smaller?

# Continuous-time population models

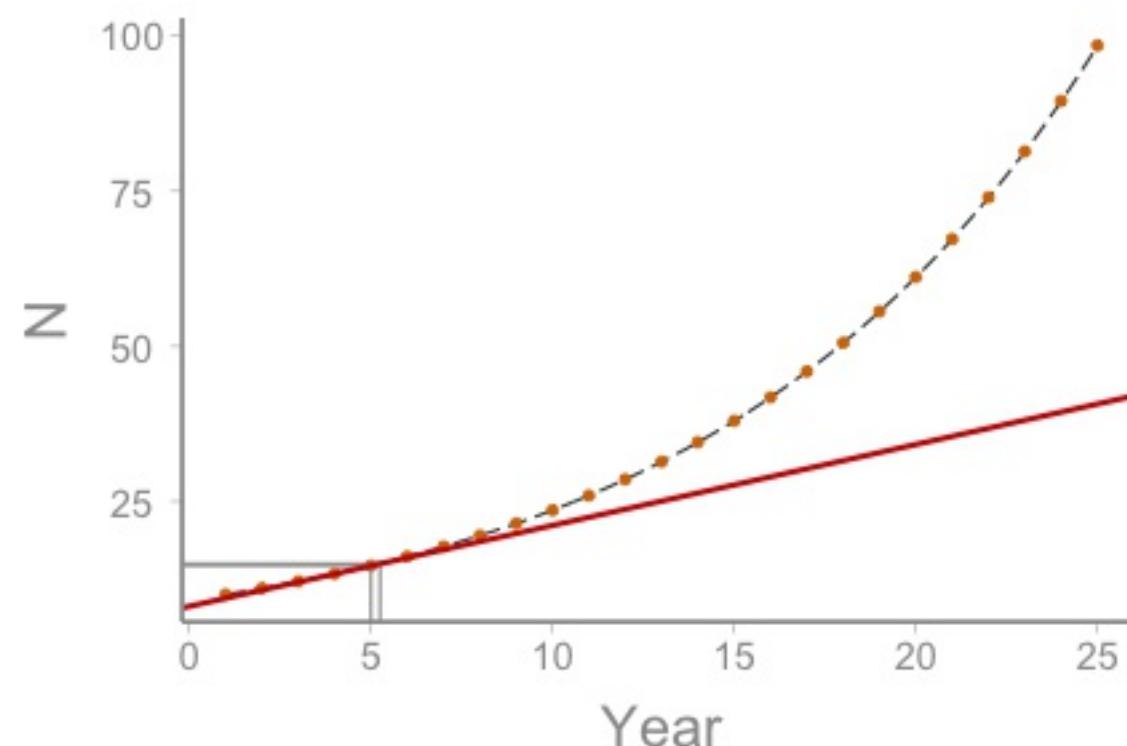
:



$$= 1.46$$

# Continuous-time population models

:



$$= 0.33$$

# Continuous-time population models

Rather than manually making  $\Delta t$  smaller and smaller, we can use calculus to figure out that as  $\Delta t$  becomes really, really small (i.e., approaches zero), we end up with:

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This is the continuous-time equivalent of the discrete model  $P_{t+\Delta t} = P_t e^{r \Delta t}$  and is called the **instantaneous growth rate**

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This is the continuous-time equivalent of the discrete model  $P_{t+\Delta t} = P_t + rP_t\Delta t$  and is called the **instantaneous growth rate**.

Populations with  $r > 0$  remain at the same population size. Populations with  $r < 0$  grow and populations with  $r < 0$  will shrink.

# Continuous-time population model

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So for a continuous-time model, we can project population growth by substituting  $\lambda$  in the equation we used to project discrete population growth:

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In other words, what is  $t$  when  $P = 2P_0$ ?

Start by dividing both sides by  $P_0$ :

# Doubling time

To isolate  $t$ , take the natural log of both sides:

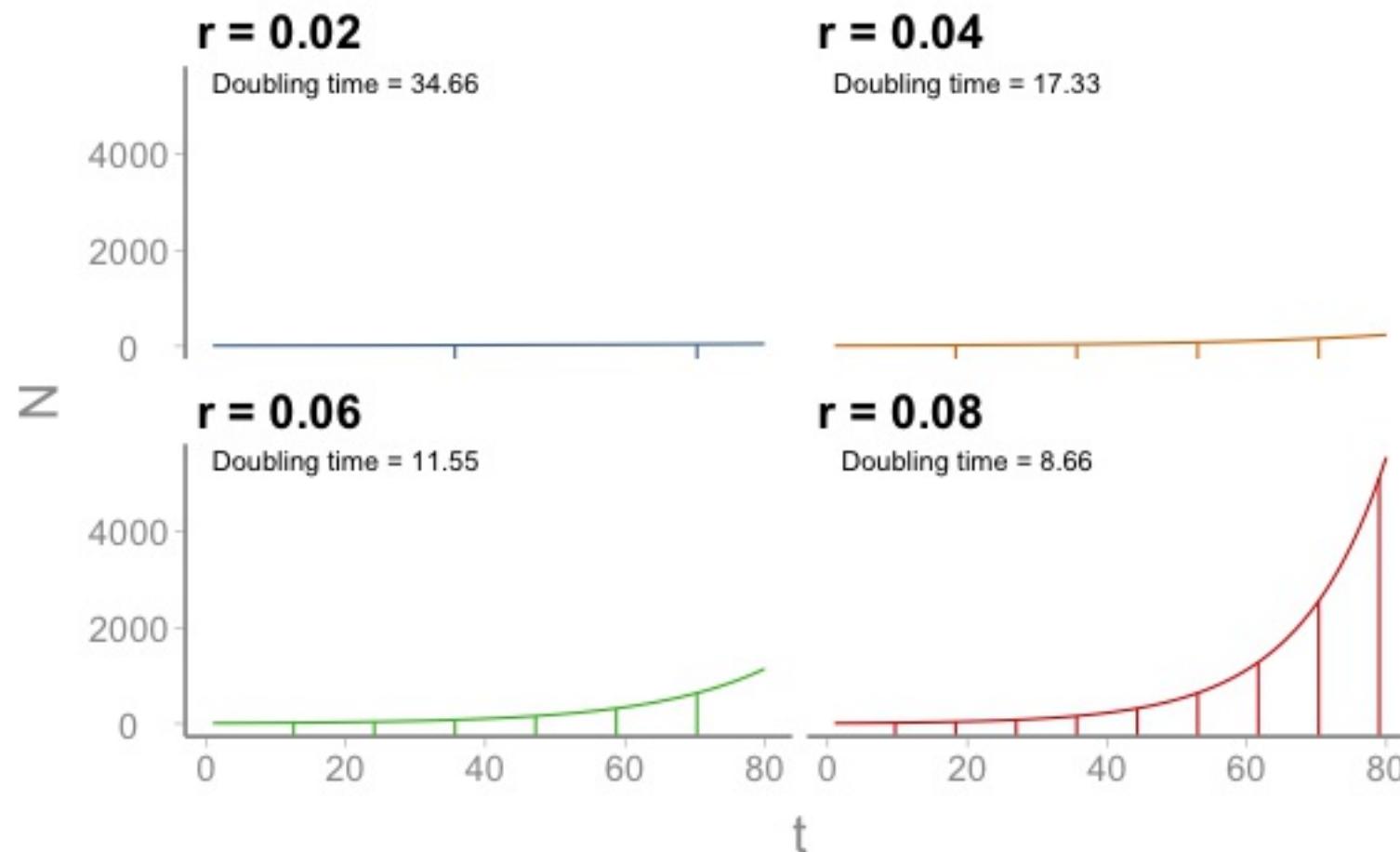
# Doubling time

To isolate  $r$ , take the natural log of both sides:

Now divide both sides by :

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# Doubling time



Relatively small changes in  $r$  can have profound effects on population doubling time :

- The population with  $r = 0.02$  doubled twice in 80 timesteps
- The population with  $r = 0.08$  doubled 9 times!

# Exponential growth

Note that the doubling time does not depend on population size - the population will double every      time steps, no matter what the population size is

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Density-independent population models result in **exponential growth**

- exponential growth occurs because the growth rate is multiplied by the population size at each timestep
- As the population grows, the proportional changes stays the same but the absolute changes get bigger and bigger .

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- After 10 years, the investment will be worth \$1912.69 (~90% return)
- After 20 years, the investment will be worth \$3658.38 (~265% return)
- After 30 years, the investment will be worth \$6997.33 (~600% return)!

"Compound interest is the most powerful force in the universe" - Albert Einstein  
(maybe)

# Exponential growth

## Example

### Tasmanian sheep

- 1820: 200,000 sheep introduced on the Island of Tasmania, Australia
- 1850: 2 million sheep
- 9-fold increase in 30 years



# Exponential growth

## Example

### Ring-necked pheasants

- In 1937, 2 male and 6 female ring-necked pheasants were released on Protection Island, Washington
- 1942: 1,325 adults (Einarson 1942, 1945)
- 220-fold increase in 5 years!



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More often, we have estimates of abundance at different points in time. To estimate population growth for one time step:

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- Pheasants:  $\underline{\quad}$   $\underline{\quad}$  2.94

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- 4) Birth and death rates are constant