

# Assumptions of the B-D models

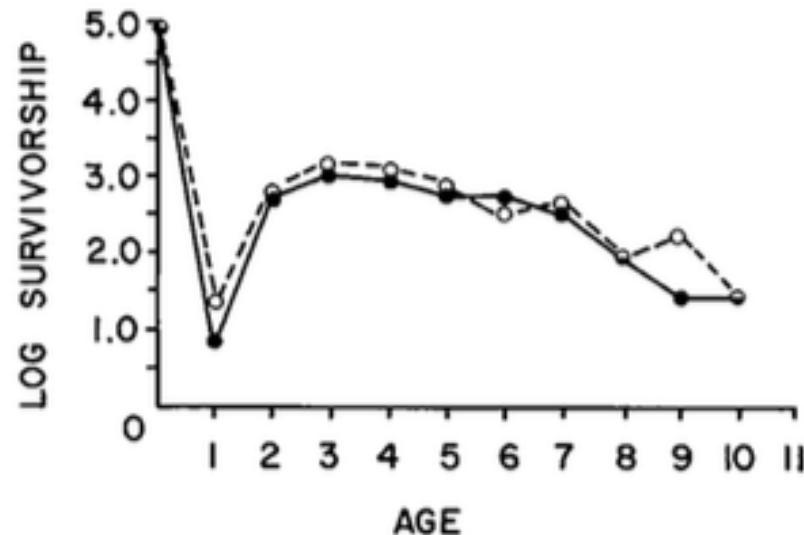
Over the coming weeks, we will learn about why and how to relax assumption 3:

- 1) Population closed to immigration and emigration
- 2) Model pertains to only the limiting sex, usually females
- 3) Birth and death rates are independent of an individual's age or biological stage**
- 4) Birth and death rates are constant

# Structured populations

# Age-structured populations

Survival and birth rates often vary with age

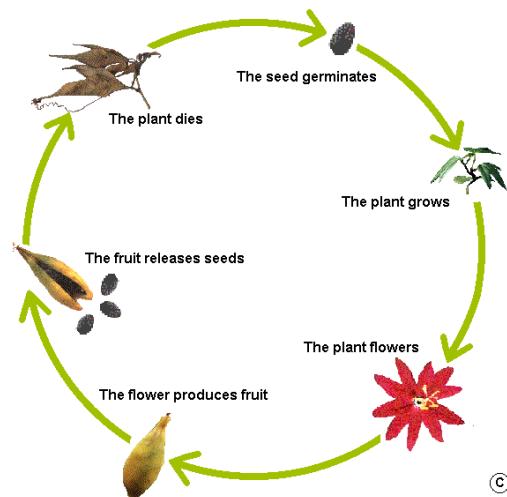
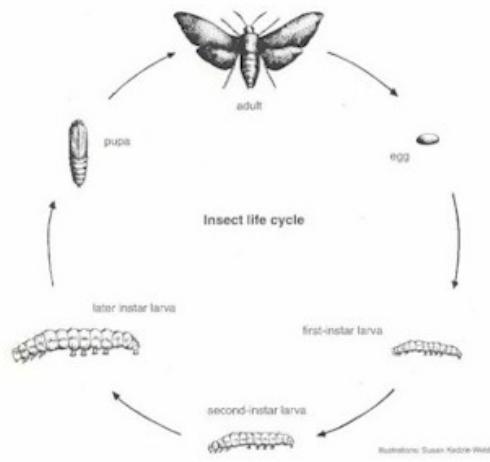


# Stage-structured populations

In some cases, age is not a relevant predictor of survival and birth rates

Instead, survival and birth rates vary with **stage**

- life cycle stage

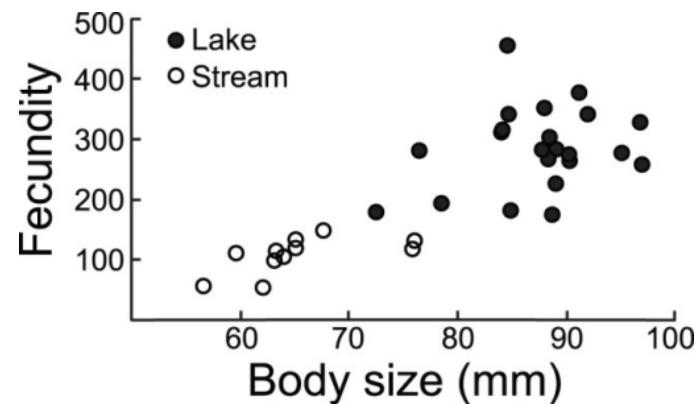


# Stage-structured populations

In some cases, age is not a relevant predictor of survival and birth rates

Instead, survival and birth rates vary with **stage**

- life cycle stage
- size



# Age-structured populations

# Age-structured populations

## Life tables

- earliest accounting tool for calculating age-specific survival and mortality

## Applications to:

- Human demography
- Insurance industry (actuarial sciences)
- Health professions

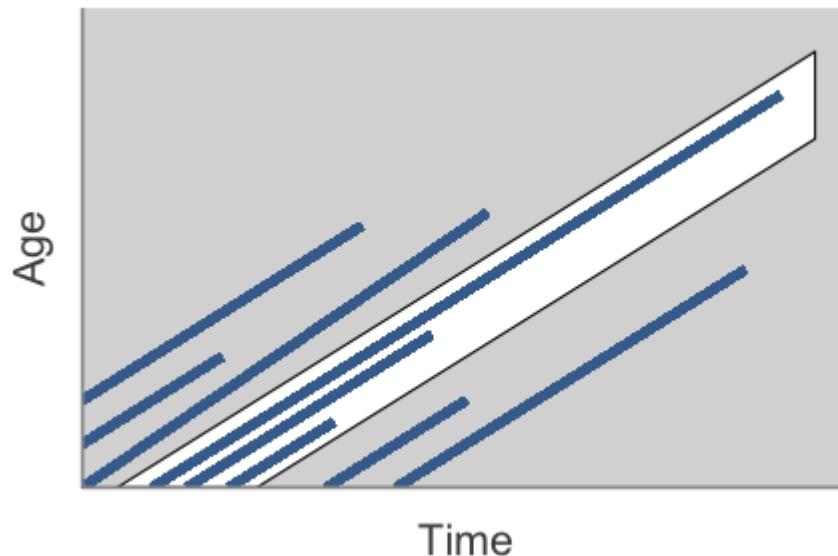
Pearl and Parker (1921) were the first to calculate a non-human life table

- *Drosophila melanogaster* (a fruit fly)
- Since used in ecology, evolution, and natural resource management

# Life tables

## Cohort life table

- follow group of individuals born within short period (a **cohort**) until each individual's death
- develop life table for groups of individuals with similar traits (e.g., one for males, one for females)
- **longitudinal study design**



# Life tables

- $\mathcal{X}$ : age of individuals
- $N_x$ : number of individuals observed alive at age  $\mathcal{X}$  (sometimes called  $S_x$ )

$x$	$N_x$					
0	100					
1	75					
2	50					
3	40					
4	0					

# Life tables

- $l_x$ : Probability of surviving to age  $x$ 
  - Can also be interpreted as the proportion of individuals still alive at age  $x$
- $l_x = N_x/N_0$

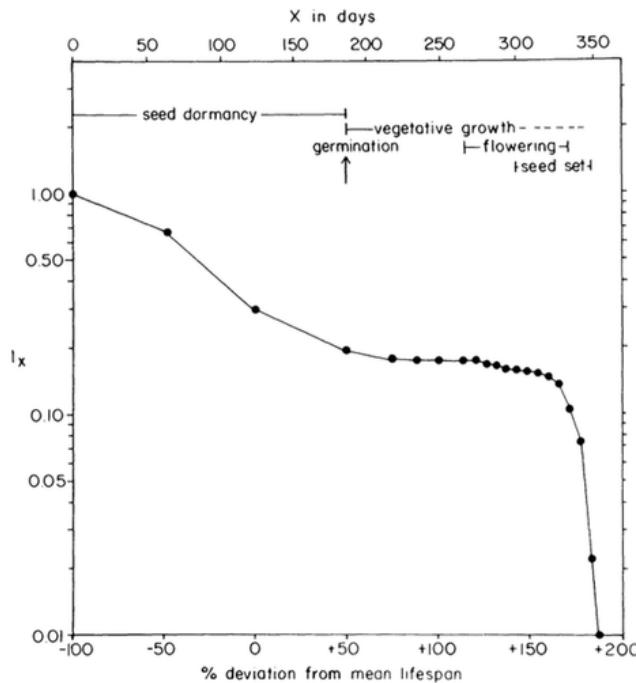
$x$	$N_x$	$l_x$			
0	100				
1	75				
2	50				
3	40				
4	0				

# Life tables

- $l_x$ : Probability of surviving to age  $x$
- $l_x = N_x / N_0$

$x$	$N_x$	$l_x$			
0	100	1.00			
1	75	0.75			
2	50	0.50			
3	40	0.40			
4	0	0.00			

# Life tables



# Life tables

- $P_x$ : Probability of surviving **from** age  $x$  **to** the next age (of those alive at age  $x$ )
- $P_x = l_{x+1}/l_x$

$x$	$N_x$	$l_x$	$P_x$		
0	100	1.00	0.75		
1	75	0.75	0.667		
2	50	0.50	0.8		
3	40	0.40			
4	0	0.00			

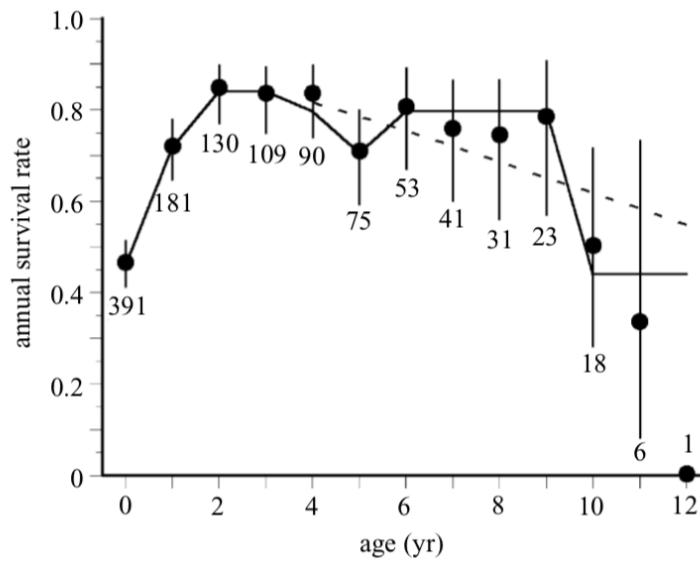
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1	75	0.75	0.667		
2	50	0.50	0.8		
3	40	0.40	0		
4	0	0.00	-		

# Life tables

Age-specific survival of fallow deer (*Dama dama*)



# Life tables

- $q_x$ : Probability of dying **between** age  $x$  the next age (of those alive at age  $x$ )
- $q_x = 1 - P_x$

$x$	$N_x$	$l_x$	$P_x$	$q_x$	
0	100	1.00	0.75	0.25	
1	75	0.75	0.667	0.333	
2	50	0.50	0.8	0.2	
3	40	0.40	0		
4	0	0.00	-		

# Life tables

- $q_x$ : Probability of dying **between** age  $x$  the next age (of those alive at age  $x$ )
- $q_x = 1 - P_x$

$x$	$N_x$	$l_x$	$P_x$	$q_x$	
0	100	1.00	0.75	0.25	
1	75	0.75	0.667	0.333	
2	50	0.50	0.8	0.2	
3	40	0.40	0	1	
4	0	0.00	-	-	

# Life tables

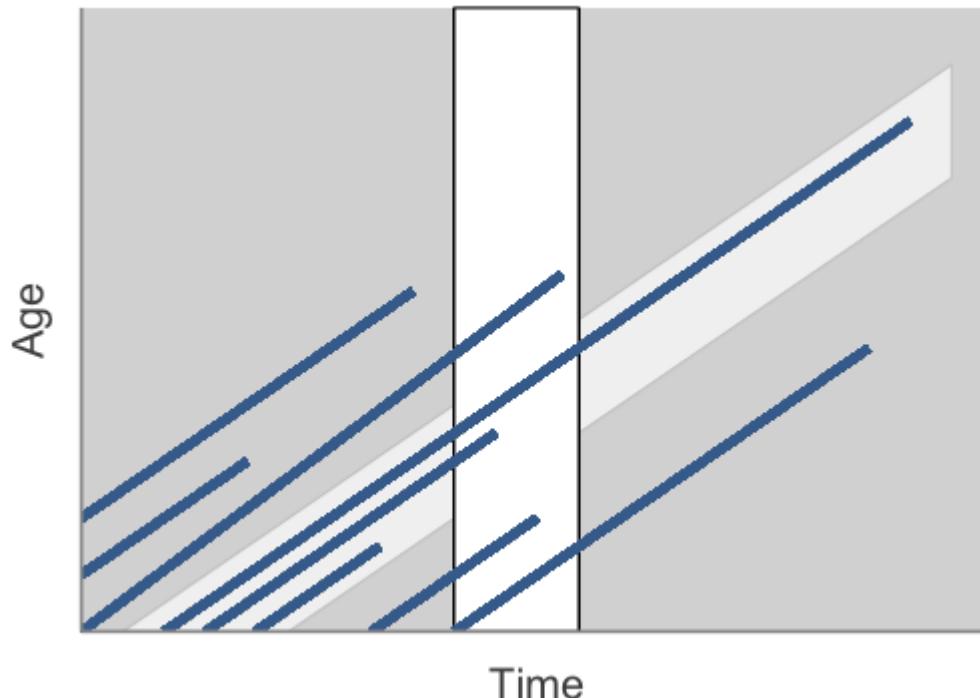
## Cohort tables

- Assume that all live individuals are available for observation at every age until death
- Most readily applied to:
  - Human populations (e.g., CDC NCHS)
  - Plants
  - Sessile animals
  - Mobile animals on small islands with high observer detection
  - Animals in captivity (zoos)

# Life tables

## Static tables

- assess number of individuals of known age at one point in time or the age of individuals dying at any point in time



# Static life tables

- $a_x$ : Deaths between age  $x - 1$  and age  $x$

$x$	$a_x$					
0						
1	25					
2	15					
3	10					
4	7					

$$\max_{x=0}^{\infty} a_x = 57$$

# Static life tables

- $a_x$ : Deaths between age  $x - 1$  and age  $x$

$x$	$a_x$	$N_x$			
0		57			
1	25	57-25=32			
2	15	32-15=17			
3	10	17-10=7			
4	7	7-7=0			

$$\max_{x=0}^{\infty} a_x = 57$$

# Static life tables

- $a_x$ : Deaths between age  $x - 1$  and age  $x$

$x$	$a_x$	$N_x$	$l_x$	$P_x$	etc.
0		57	1.00	0.56	
1	25	57-25=32	0.56	0.54	
2	15	32-15=17	0.30	0.4	
3	10	17-10=7	0.12	0	
4	7	7-7=0	0.00	-	

# Static life tables

Static life tables most readily applied to:

- Plants and animals where age can be accurately determined for all individuals at a given moment in time
- e.g., using the rings/layering in trees, teeth, otoliths, horns, and maybe even telomere length on genes

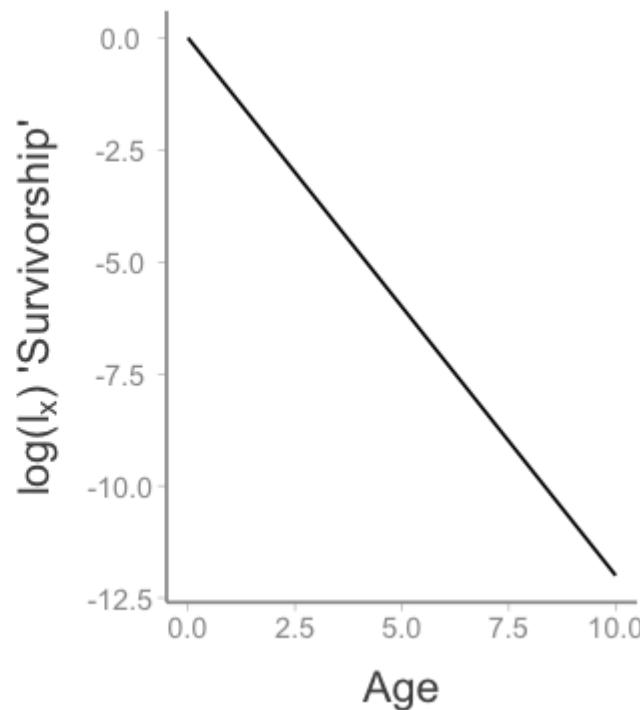
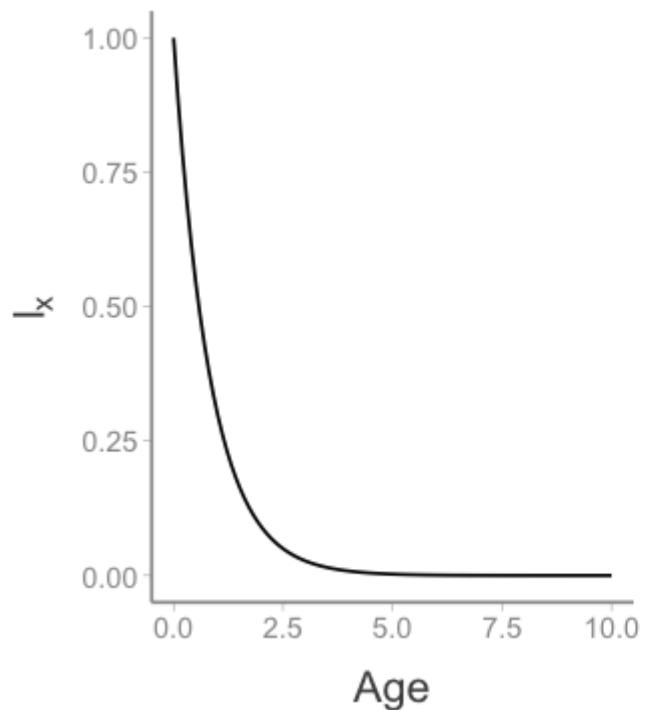
# Static life tables

Static life table assumptions:

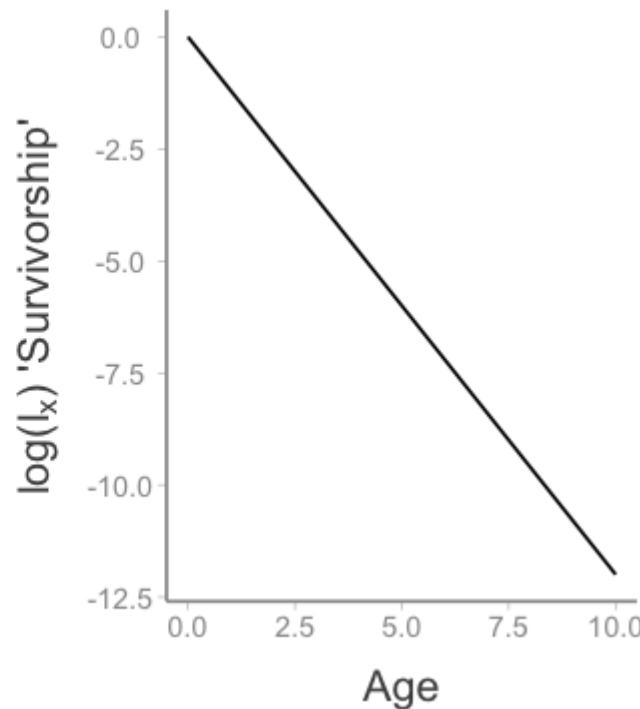
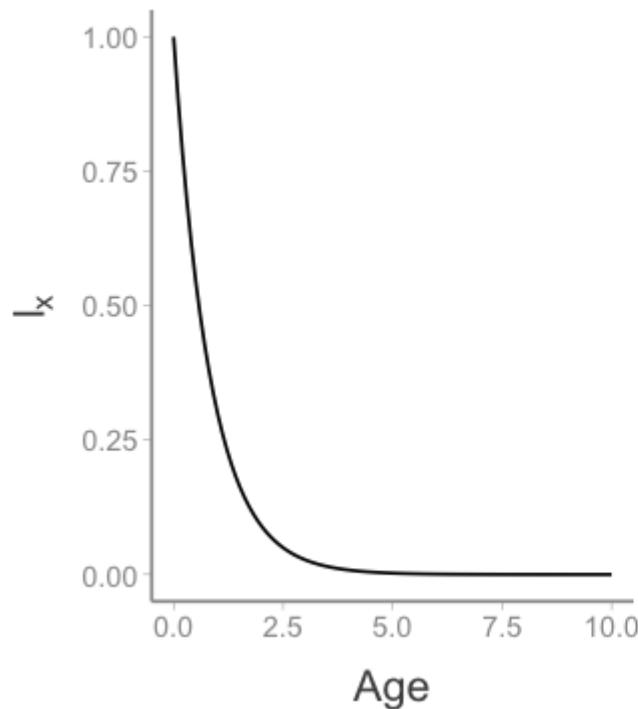
- All individuals have equivalent availability for observation, regardless of age
- Stable age distribution
- Age-specific survival ( $P_x$ ) does not change over time
  - If it does, time effects may appear as ‘age effects’ and lead to biased results

# Survivorship curves

# Survivorship curves

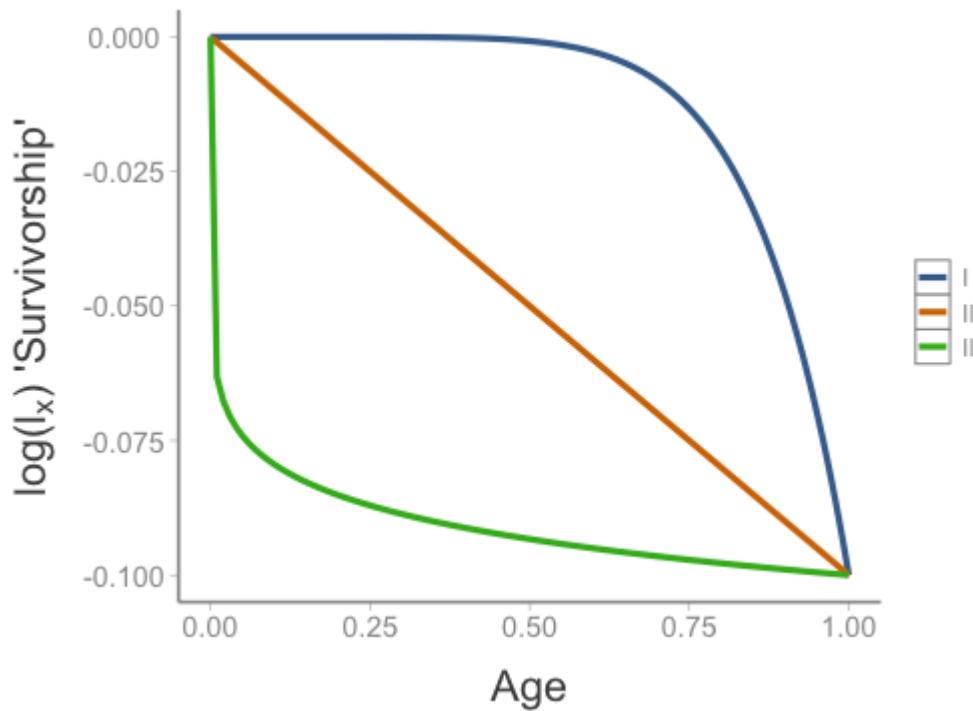


# Survivorship curves



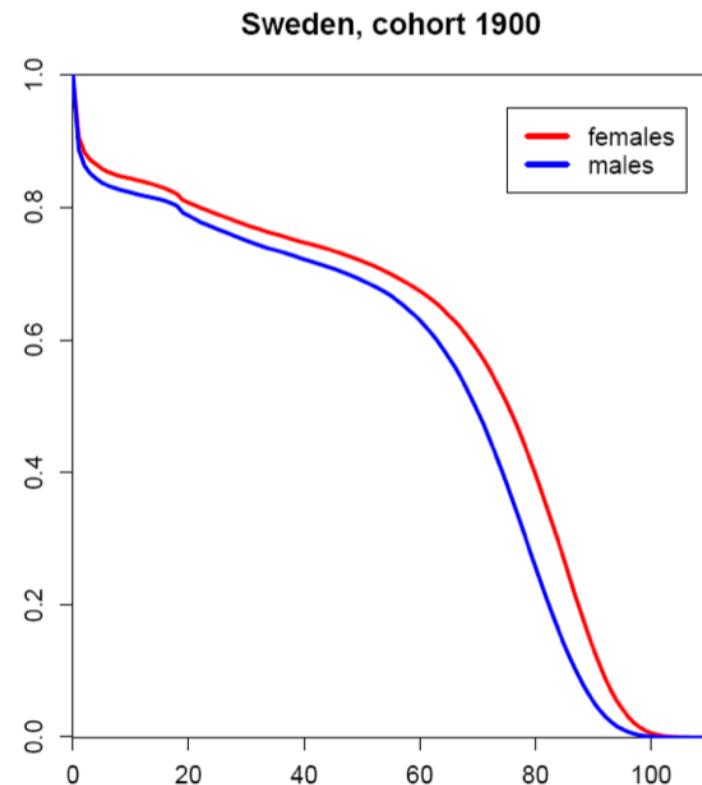
Type II Survivorship curve (Pearl 1928)

# Survivorship curves



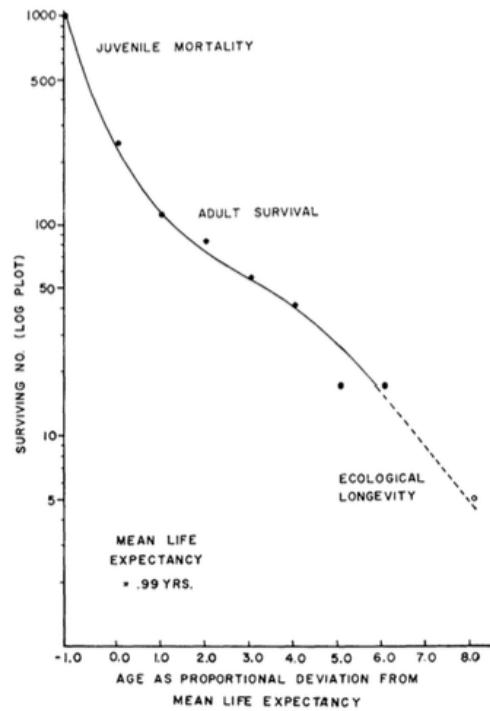
# Survivorship curves

What types of species do you expect have type 1 survivorship curves?



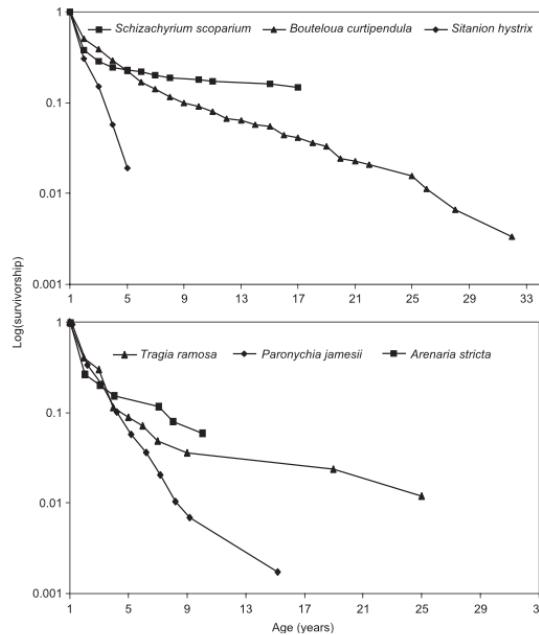
# Survivorship curves

What types of species do you expect have type 2 survivorship curves?



# Survivorship curves

What types of species do you expect have type 3 survivorship curves?



# Life expectancy

# Life expectancy

## Life Expectancy

The average length of time an individual is expected to live, given they are currently  $x$  years old

- Directly related to the probability of surviving to a given age

Utah has the lowest per capita crude death rate in the U.S. (averaged across all ages)

Can Utahns expect to live longer than in all other states?

# Life expectancy

To estimate life expectancy, start with  $l_x$

- $l_x$  = Proportion of individuals still alive at age  $x$

$x$	$N_x$	$l_x$			
0	100	1.00			
1	75	0.75			
2	50	0.50			
3	40	0.40			
4	0	0.00			

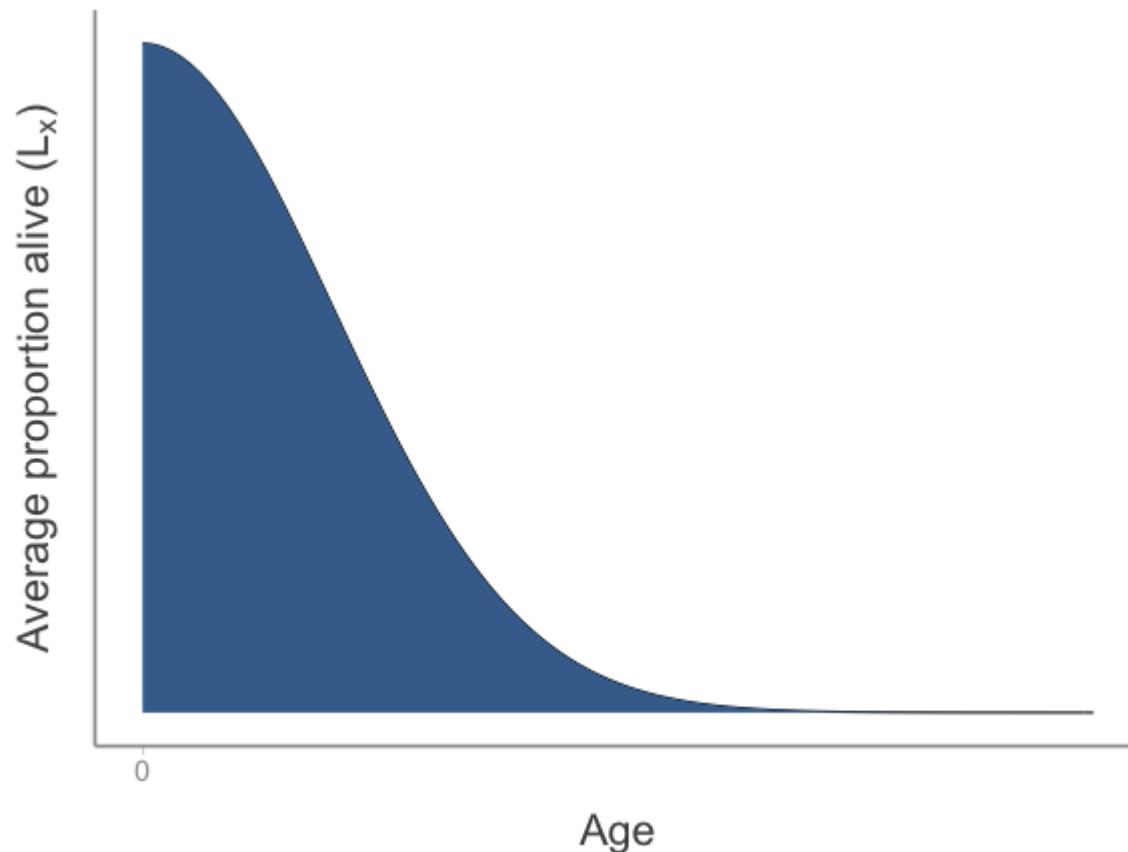
# Life expectancy

- $L_x$ : Average proportion of living individuals across successive age classes
- $L_x = \frac{l_x + l_{x+1}}{2}$

$x$	$N_x$	$l_x$	$L_x$		
0	100	1.00	0.875		
1	75	0.75	0.65		
2	50	0.50	0.5		
3	40	0.40	0.2		
4	0	0.00	-		

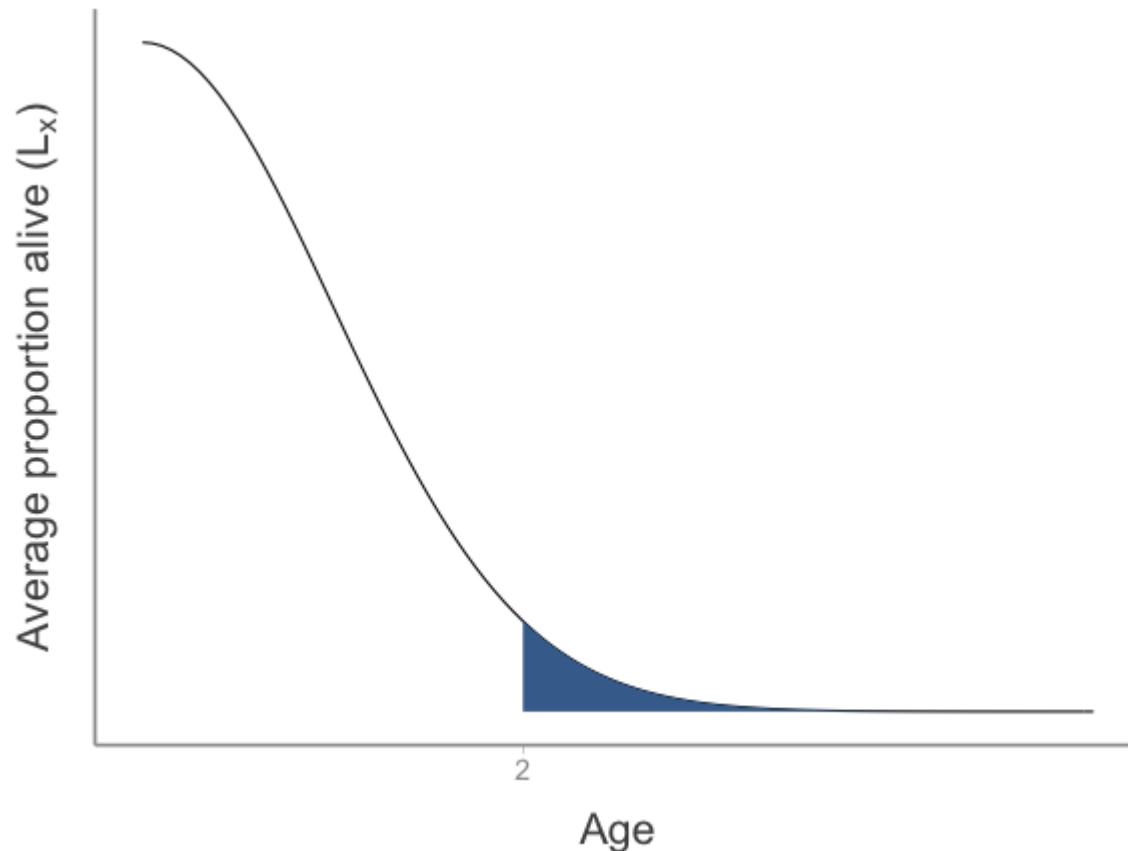
# Life expectancy

- $T_x$ : total proportion of living individuals from age  $x$  to  $\max(x)$



# Life expectancy

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# Life expectancy

- $T_x$ : total proportion of living individuals from age  $x$  to  $\max(x)$
- $T_x = \frac{\max(x)}{x} L_x$

$x$	$N_x$	$l_x$	$L_x$	$T_x$	
0	100	1.00	0.875	2.23	
1	75	0.75	0.65	1.35	
2	50	0.50	0.5	0.7	
3	40	0.40	0.2	0.2	
4	0	0.00	-	-	

# Life expectancy

- $E_x$ : Number of additional years an individual of age  $x$  is expected to survive
- $E_x = T_x / l_x$

$x$	$N_x$	$l_x$	$L_x$	$T_x$	$E_x$
0	100	1.00	0.875	2.23	2.23
1	75	0.75	0.65	1.35	1.93
2	50	0.50	0.5	0.7	1.17
3	40	0.40	0.2	0.2	0.5
4	0	0.00	-	-	-

# Utah life expectancy

Utah has the lowest per capita crude death rate d in the U.S

Can Utahns expect to live longer than all other states?

Not exactly

- Life expectancy of a newborn Utahn is 80.2
- Ranked 10th in the U.S., not first

# Utah life expectancy

Why is Utah's crude per capita death rate  $d$  so low?

Because Utah has the highest birth rate and consequently the youngest age structure

- % of Utahns < 18 yrs old is 32%
- % of U.S. < 18 yrs old is 25%
- % of Utahns  $\geq 65$  is 11%
- % of U.S.  $\geq 65$  = 15%

Young individuals have highest chances of survival

Having many young individuals in a population will skew the crude average  $d$