What are random effects?

- Fixed effects are constant across observational units, random effects vary across units
- Fixed effects are used when you are interested in the specific factor levels, random effects are used when you are interested in the underlying population
- When factors levels are exhaustive, they are fixed. When they are a sample of the possible levels, they are random
- Random effects are the realized values of a random variable
- Fixed effects are estimated by maximum likelihood, random effects are estimated with shrinkage

A simple linear model

$$y_{ij} = \beta_{[j]} + \epsilon_i$$

$$\epsilon_i \sim normal(0, \tau)$$

- If $\beta_{[1]} = \beta_{[2]} = \beta_{[3]} = \ldots = \beta_{[J]}$

```
model {
# Priors
beta0 ~ dnorm(0, 0.33)
tau ~ dgamma(0.25, 0.25)

# Likelihood
for (i in 1:N){
y[i] ~ dnorm(mu[i], tau)
mu[i] <- beta0
} #i
}
```

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$$y_{ij} = \beta_{[j]} + \epsilon_i$$

$$\epsilon_i \sim normal(0, \tau)$$

- If $\beta_{[1]} \perp \beta_{[2]} \perp \beta_{[3]} \perp \ldots \perp \beta_{[J]}$

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model {
# Priors
for(j in 1:J){
beta0[j] ~ dnorm(0, 0.33)
}
tau ~ dgamma(0.25, 0.25)

# Likelihood
for (i in 1:N){
y[i] ~ dnorm(mu[i], tau)
mu[i] <- beta0[group[j]]
} #i
}
```

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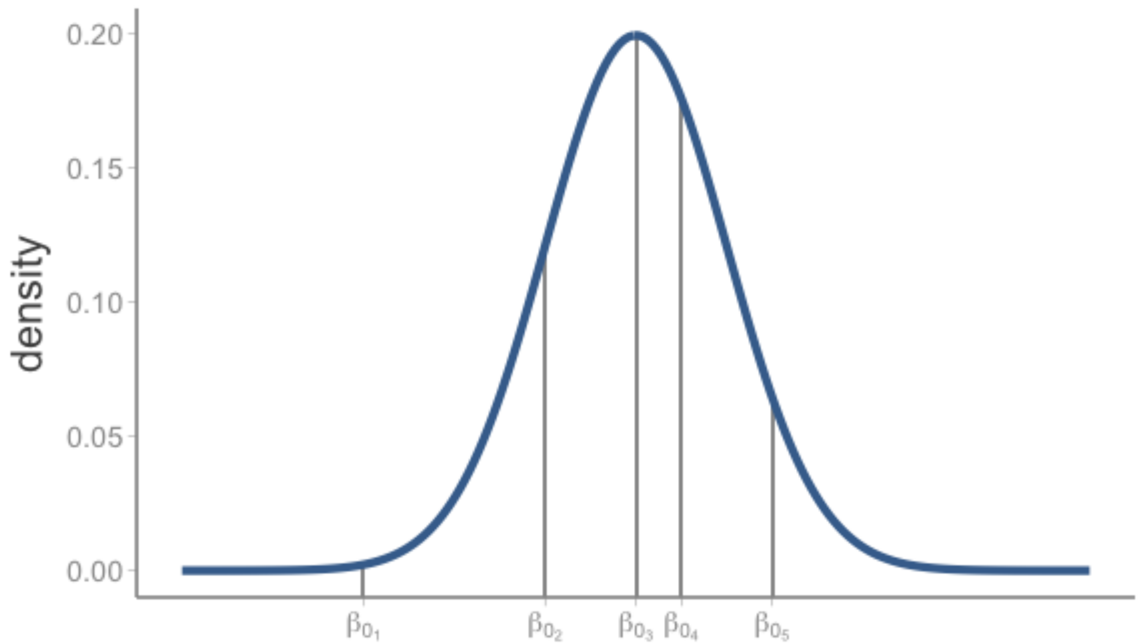
$$\epsilon_i \sim normal(0, \tau)$$

- If $\beta_{[j]} \sim normal(\mu_{\beta 0}, \tau_{\beta 0})$



A simple linear model

$$y_{ij} = \beta_{[j]} + \epsilon_i$$

$$\epsilon_i \sim normal(0, \tau)$$

- If $\beta_{[j]} \sim normal(\mu_{\beta 0}, \tau_{\beta 0})$

```
model {
# Priors
for(j in 1:J){
beta0[j] ~ dnorm(mu.beta, tau.beta)
}
mu.beta ~ dnorm(0, 0.33)
tau.beta ~ dgamma(0.25, 0.25)
tau ~ dgamma(0.25, 0.25)

# Likelihood
for (i in 1:N){
y[i] ~ dnorm(mu[i], tau)
mu[i] <- beta0[group[j]]
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```

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- Only apply to factors
- Imply grouped effects

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- Imply grouped effects
- Can include intercept, slope, and variance parameters
- Assume exchangeability
- Represent a compromise between total pooling ($\beta_{0[1]} = \beta_{0[2]} = \ldots = \beta_{0[J]}$) and no pooling ($\beta_{[1]} \perp \beta_{[2]} \perp \ldots \perp \beta_{[J]}$)
- Typically require $> 5 - 10$ factor levels

Random effects = hierarchical model

$$[\beta_{0[j]}, \mu_{\beta 0}, \tau_{\beta 0}, \tau | y_{ij}] = [y_{ij}|\beta_{0[j]}, \tau][\beta_{0[j]}|\mu_{\beta 0}, \tau_{\beta 0}][\tau][\mu_{\beta 0}][\tau_{\beta 0}]$$

- Can include multiple random effects
- Can include fixed and random effects (mixed-models)
- Can include multiple levels of hierarchy

Why use random effects?

1) Scope of inference

- learn about $\beta_{0[j]}$ **and** μ_{beta_0}, τ_{beta_0}
- prediction to unsampled groups (in space or time)

Why use random effects?

1) Scope of inference

2) Partitioning of variance

- Account for variability among groups vs. among observational units

Why use random effects?

1) Scope of inference

2) Partitioning of variance

3) Accounting for uncertainty

- modeling τ_{beta_0} recognizes uncertainty from sampling groups

Why use random effects?

1) Scope of inference

2) Partitioning of variance

3) Accounting for uncertainty

4) Avoiding psuedo-replication

- Account for non-independence within groups

Why use random effects?

1) Scope of inference

2) Partitioning of variance

3) Accounting for uncertainty

4) Avoiding psuedo-replication

5) Borrowing strength

- $\beta_{0[j]}$ estimating from group j observations + all other groups
- "shrinkage"" towards mean
    - degree of shrinkage inversely proportional to precision

Why not use random effects?

Always use random effects (Gelman & Hill 2007)

but...

- Assumption of exchangeability
- Requires 5-10 levels
- Computationally intensive
- Harder to interpret