Lecture 9

State-space models

WILD6900 (Spring 2020)

Readings

> Kéry & Schaub 115-131

Ecological state variables

State variables are the ecological quantities of interest in our model that change over space or time

Abundance

> the number of individual organisms in a population at a particular point in time

Occurrence

> the spatial distribution of organisms with a particular region at a particular point in time

Richness

> the number of co-occurring species at a given location and a particular point in time

Ecological parameters

Parameters determine how the state variables change over space and time

- Survival
- Reproduction
- Movement
- Population growth rate
- Carrying capacity
- Colonization/extinction rate

Process models

$$[z|g(\theta_p, x), \sigma_p^2]$$

- Mathematical description of our hypothesis about how the *state variables* we are interested in change over space and time
- Represent the **true** value of our state variables at any given point in space or time
- Deterministic
- Abstraction

Observation models

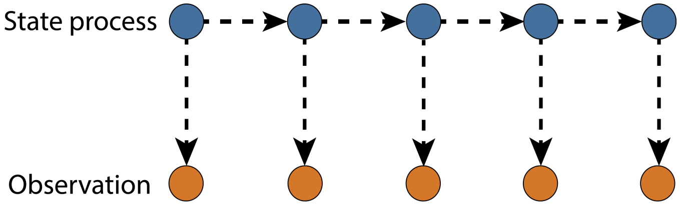
- Rarely observe the true state perfectly
    - Animals are elusive and may hide from observers
    - Even plants may be cryptic and hard to find
    - Measurements may be taken with error
    - May count the same individual > 1
- *Observation uncertainty* (σ_o^2) can lead to biased estimates of model parameters, so generally requires its own model

$$[y_i | d(\Theta_o, z_i), \sigma_o^2]$$

State-space models

State-space models

- Hierarchical models
- Decompose time series into:
  - process variation in state process
  - observation error
- Generally used for *Markovian* state process models
  - Population dynamics
  - Survival
  - Occupancy



Process models

Population dynamics

$$N_{t+1} \sim Poisson(N_t\lambda)$$

$$N_{t+1} \sim Normal(N_t e^{\left[r_0\left(1-\frac{N_t}{K}\right)\right]}, \sigma^2)$$

Survival

$$z_{t+1} \sim Bernoulli\left(z_t\phi_t\right)$$

Occupancy

$$z_{t+1} \sim Bernoulli\left(z_t(1-\epsilon_t) + (1-z_t)\gamma_t\right)$$

Observation models

Population dynamics

$$C_t \sim Normal(N_t, \sigma_o^2)$$

$$C_t \sim Binomial(N_t, p)$$

Survival

$$y_t \sim Bernoulli(z_t p)$$

Occupancy

$$y_t \sim Bernoulli(z_t p_t)$$

Including covariates

For any of these models, we can use GLMs to include covariates effects on parameters:

$$log(\lambda_t) = \alpha + \beta\mathbf{X} + \varepsilon_t$$

$$logit(\phi_t/\gamma_t/\epsilon_t/p_t) = \alpha + \beta\mathbf{X} + \varepsilon_t$$

And random effect structure

$$\varepsilon_t \sim normal\Big(0, \tau_\lambda\Big)$$

$$\tau_\lambda \sim gamma\Big(0.25, 0.25\Big)$$

Simple state-space population growth model

Process model

$$N_{t+1} = N_t \lambda_t$$

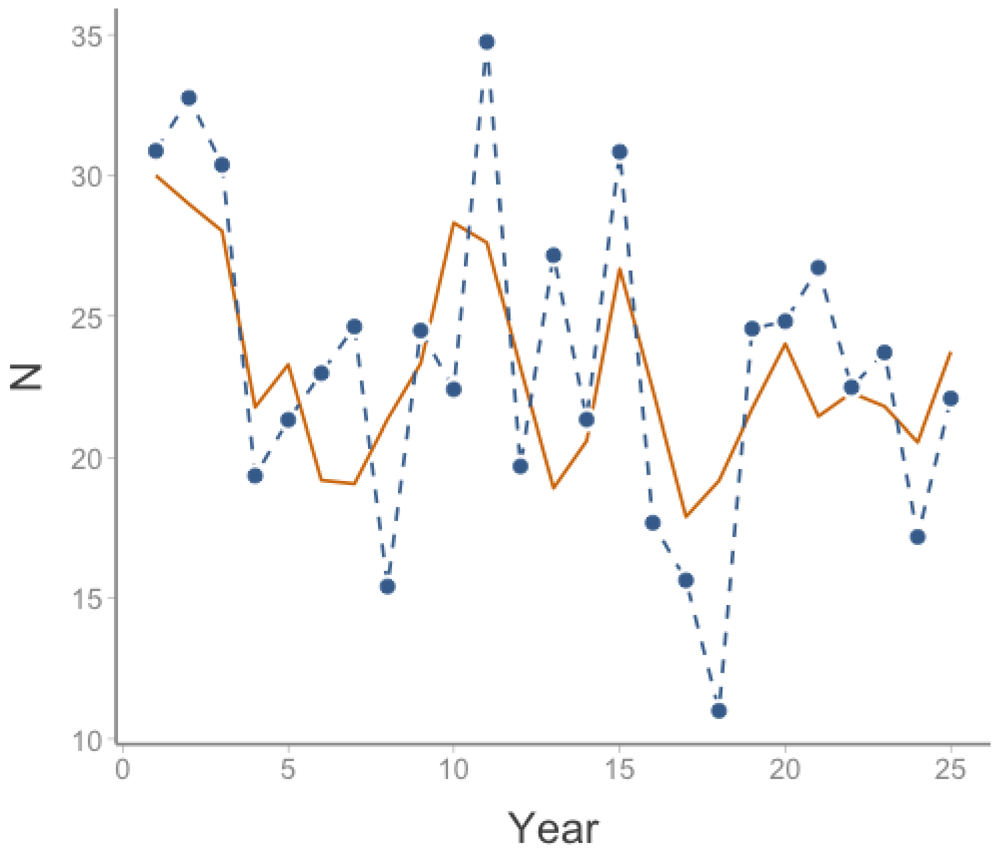
$$\lambda_t \sim normal(\mu_\lambda, \tau_\lambda)$$

Observation model

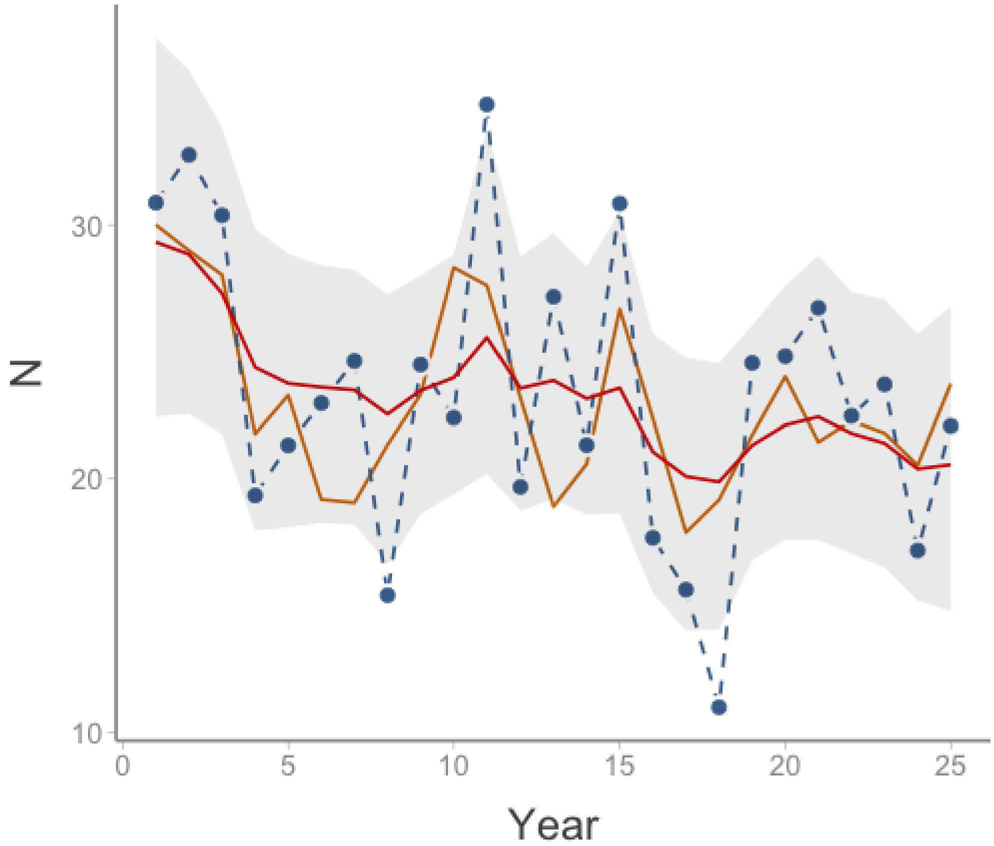
$$C_t = N_t + \epsilon_t$$

$$\epsilon_t \sim Normal(0, \sigma_o^2)$$

Simple state-space population growth model



Simple state-space population growth model



Simple state-space population growth model

What if instead of:

$$C_t = N_t + \epsilon_t$$

$$\epsilon_t \sim Normal(0, \sigma_o^2)$$

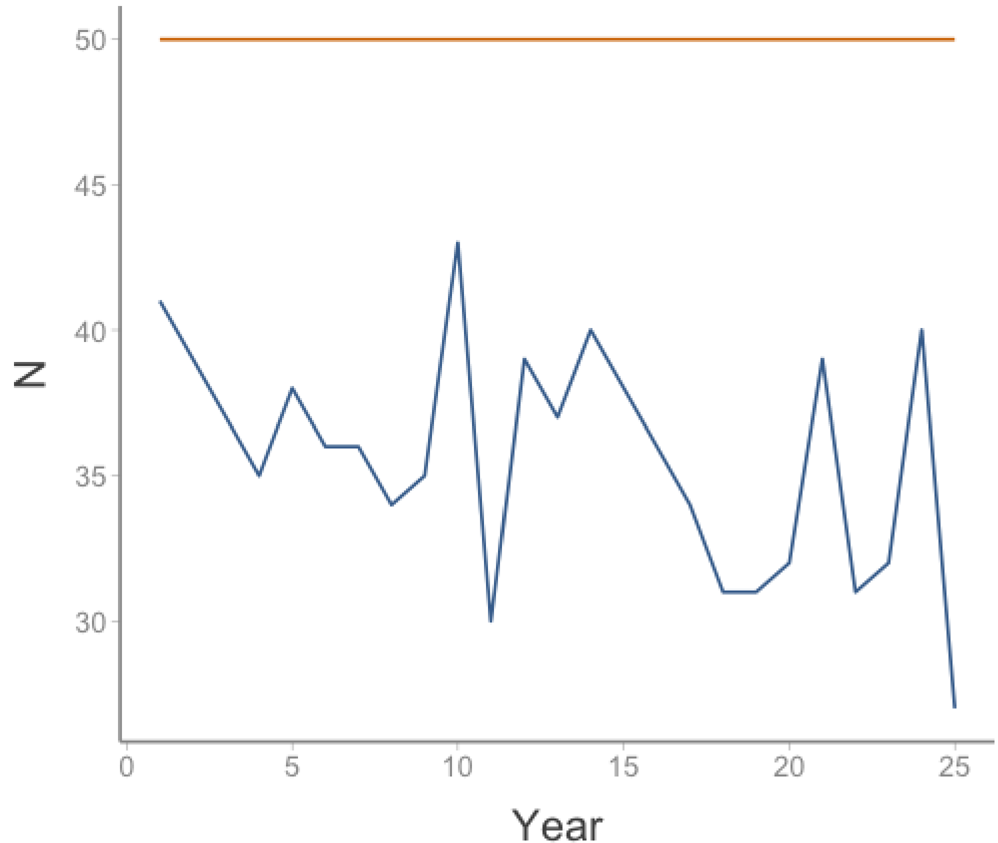
The observation model is:

$$C_t \sim binomial(N_t, p)$$

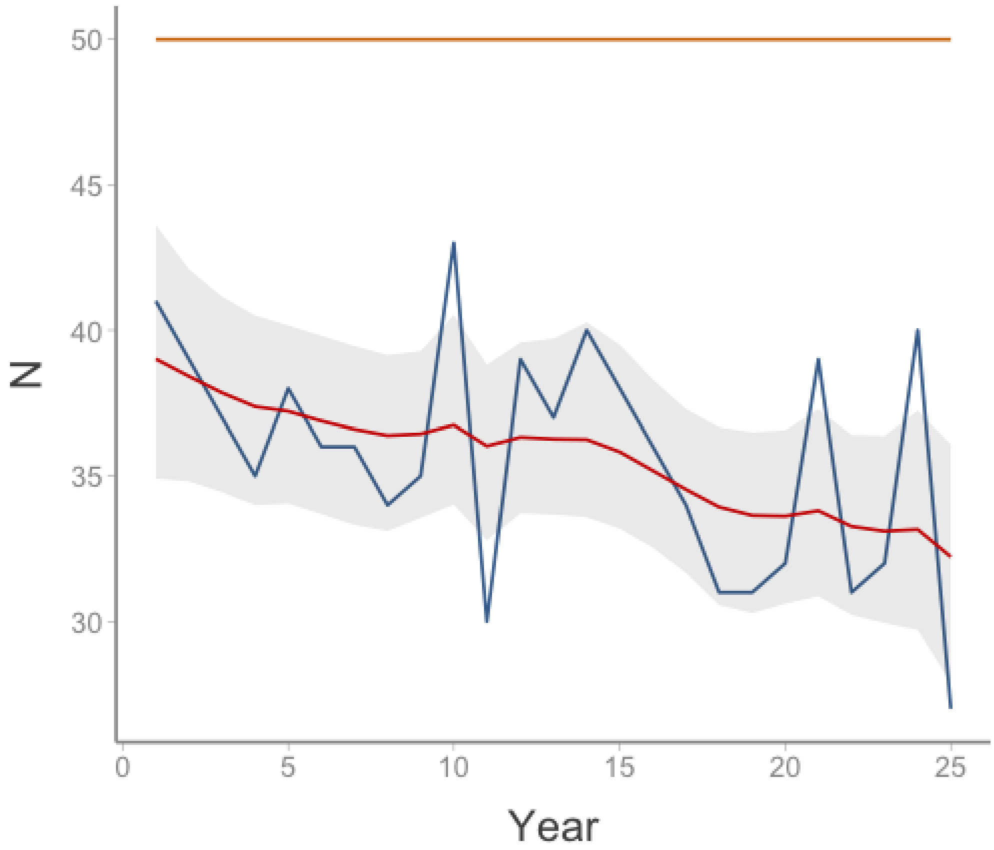
and

$$N_t = 50$$

Systematic bias in state-space models



Systematic bias in state-space models



Systematic bias in state-space models

What if instead of:

$$C_t \sim binomial(N_t, p)$$

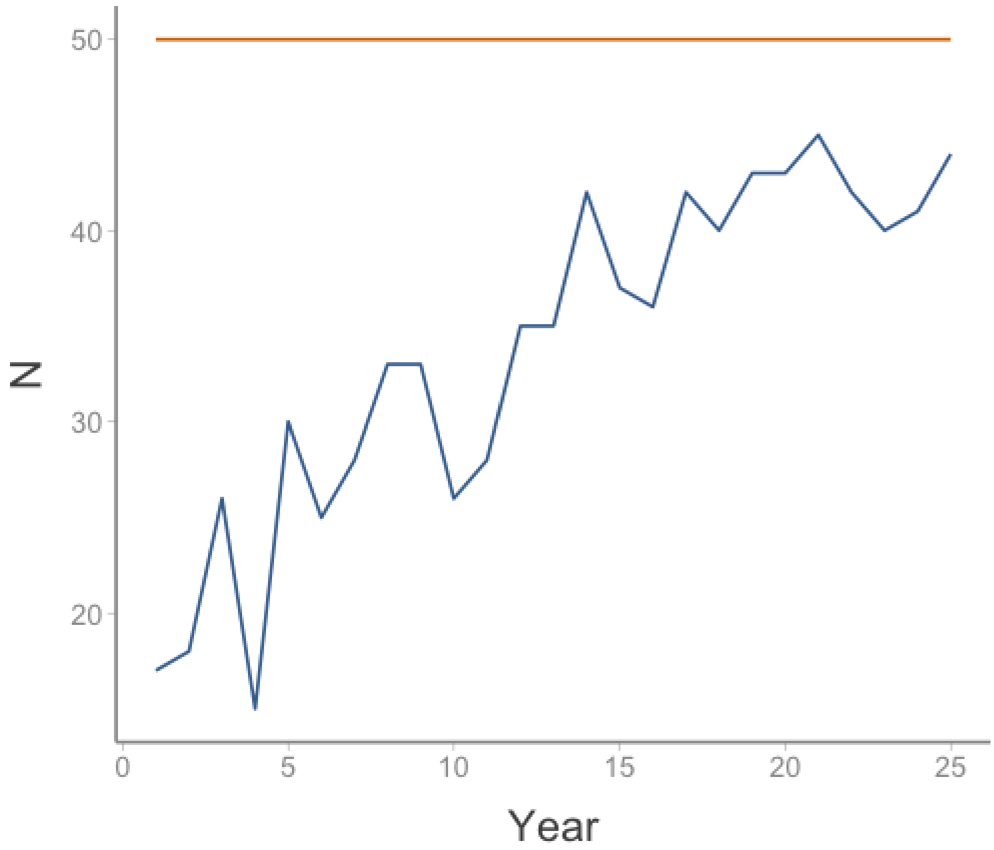
The observation model is:

$$C_t \sim binomial(N_t, p_t)$$

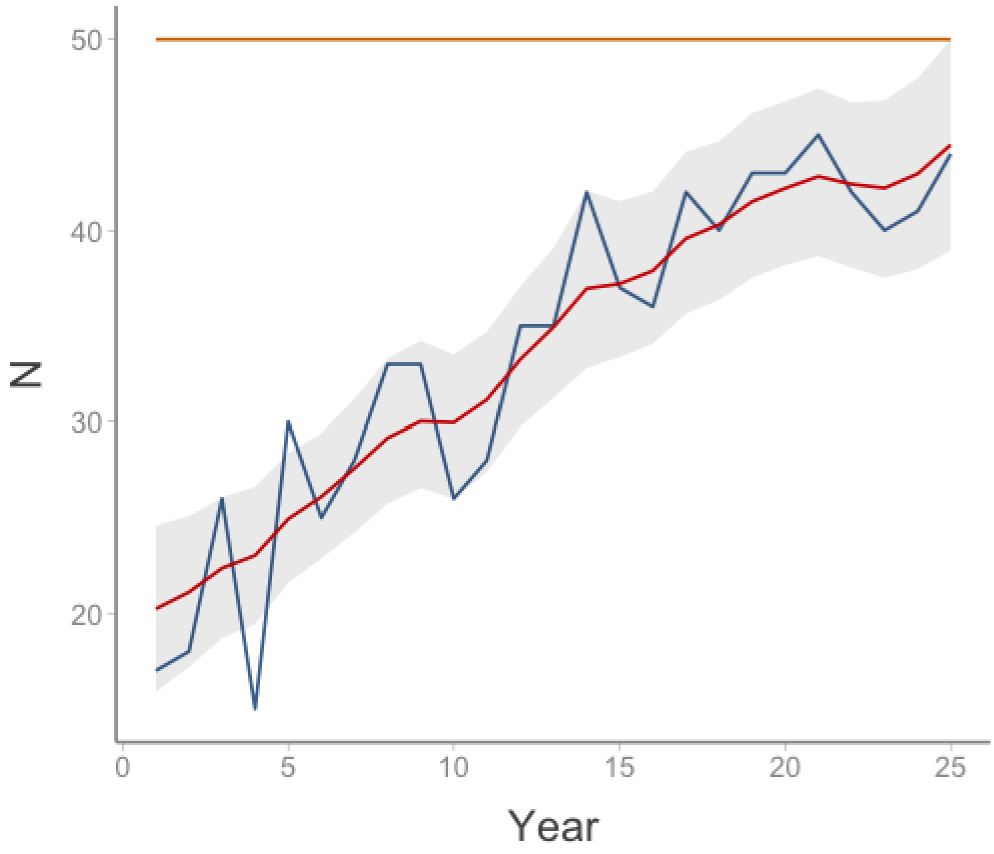
and

$$logit(p_t) = \alpha + \beta \times year_t$$

Systematic bias in state-space models



Systematic bias in state-space models



State-space models

Produce unbiased estimates of N **only** when false-positives and false-negatives cancel each other out on average

Produce unbiased estimates of population **indices** (Np) if detection probability has no pattern over time

Do **not** produce unbiased estimates of N or Np if their are temporal patterns in detection probability or false-positive rates

State-space models

Advantages

- explicit decomposition of process and observation models
- flexible
- mechanistic "smoothing" of process model
- latent state (and uncertainty) can be monitored
- possible to "integrate" data on state/observation parameters

Disadvantages

- computationally intensive
- usually produce biased estimates of N