Lecture 15

Multi-state models

WILD6900 (Spring 2020)

Readings

> Kéry & Schaub 264-313

What is a "state"

Alive/dead

- CJS model

Occupied/unoccupied

- Occupancy model

Geographic location (location $1, 2, \ldots, S$)

Un-infected/infected/dead

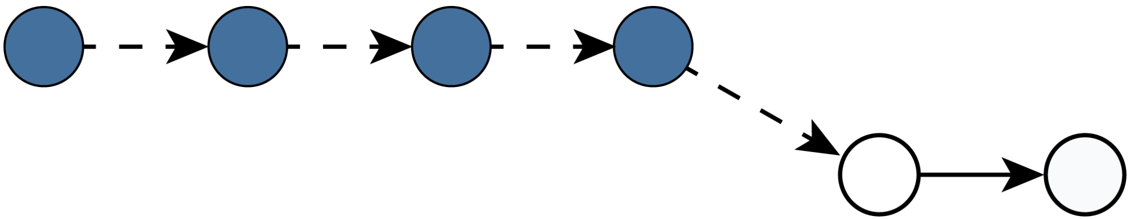
Age class

Any combination of above states*

*At least in theory

Two states

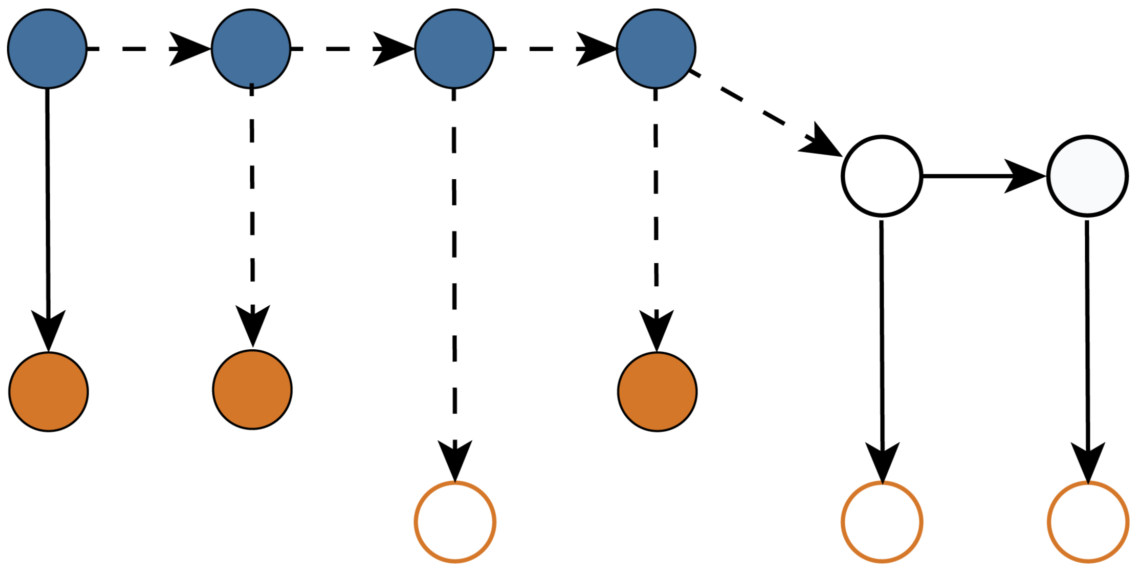
State model



$$z_{i,t} \sim Bernoulli(z_{i,t-1}\phi)$$

Two states

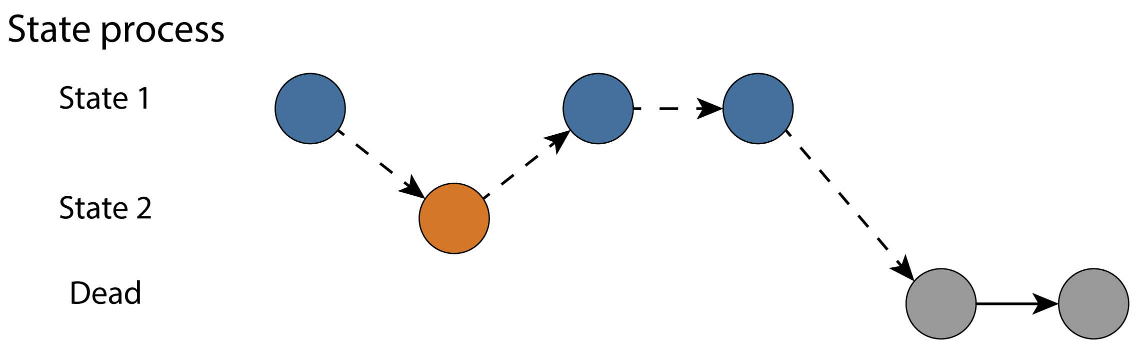
Observation model



$$y_{i,t} \sim Bernoulli(z_{i,t}p)$$

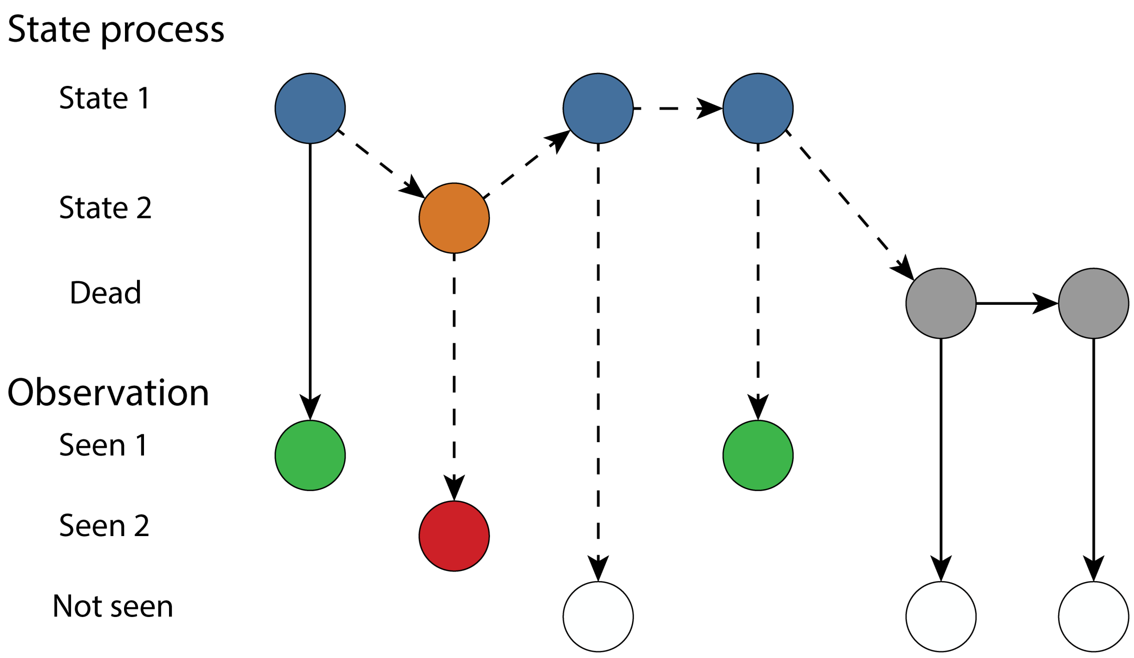
Three states

State model



Three states

State model

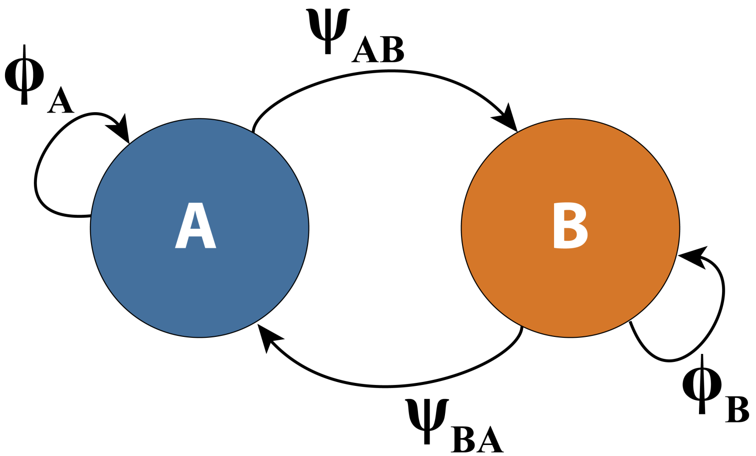


Three states - example

Movement between two sites

Three states - example

Movement between two sites



- ϕ_A: Probability alive at site A
- ϕ_B: Probability alive at site B
- ψ_{AB}: Probability of moving from site A to site B
- ψ_{BA}: Probability of moving from site B to site A

Three states - example

State model

True state at time *t* + *1*

True state at time *t*	Site A	Site B	Dead
Site A	$\phi_A(1-\psi_{AB})$	$\phi_A\psi_{AB}$	$1-\phi_A$
Site B	$\phi_B\psi_{BA}$	$\phi_B(1-\psi_{BA})$	$1-\phi_B$
Dead	0	0	1

Three states - example

Observation model

Observation at time t

True state at time t

	Site A	Site B	Not Seen
Site A	p_A	0	$1 - p_A$
Site B	0	p_B	$1 - p_B$
Dead	0	0	1

Three states - example

The data

- $1 =$ Seen alive at site A
- $2 =$ Seen alive at site B
- $3 =$ Not seen

```
head(rCH)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    1    1    3    1    1    1
## [2,]    1    1    1    1    1    1
## [3,]    1    3    1    2    3    3
## [4,]    1    3    3    3    3    3
## [5,]    1    1    2    3    3    3
## [6,]    1    3    3    2    3    1
```

Three states - example

The data

- $f =$ First capture occasion

```r
tail(rCH)
```

```
##        [,1] [,2] [,3] [,4] [,5] [,6]
## [795,]    3    3    3    3    2    3
## [796,]    3    3    3    3    2    3
## [797,]    3    3    3    3    2    3
## [798,]    3    3    3    3    2    3
## [799,]    3    3    3    3    2    1
## [800,]    3    3    3    3    2    3
```

```r
tail(f)
```

```
## [1] 5 5 5 5 5 5
```

Three states - example

The likelihood

- The Bernoulli distribution describes the probability of one (of two) outcomes
- The categorical distribution describes the probability of one (of N) outcomes
  - Ω and Θ are matrices defining the state/observation probabilities

$$z_{i,t+1}|z_{i,t} \sim categorical(\Omega_{z_{i,t},1:S})$$

$$y_{i,t}|z_{i,t} \sim categorical(\Theta_{z_{i,t},1:S})$$

Analysis in JAGS

```
sink("ms.jags")
cat("
model {
  # Priors
   phiA ~ dunif(0, 1)
   phiB ~ dunif(0, 1)
   psiAB ~ dunif(0, 1)
   psiBA ~ dunif(0, 1)
   pA ~ dunif(0, 1)
   pB ~ dunif(0, 1)
```

Analysis in JAGS

```
# Define state-transition and observation matrices
 # Define probabilities of state S(t+1) given S(t)
    ps[1,1] <- phiA * (1-psiAB)
    ps[1,2] <- phiA * psiAB
    ps[1,3] <- 1-phiA
    ps[2,1] <- phiB * psiBA
    ps[2,2] <- phiB * (1-psiBA)
    ps[2,3] <- 1-phiB
    ps[3,1] <- 0
    ps[3,2] <- 0
    ps[3,3] <- 1
```

Analysis in JAGS

```
# Define probabilities of O(t) given S(t)
po[1,1] <- pA
po[1,2] <- 0
po[1,3] <- 1-pA
po[2,1] <- 0
po[2,2] <- pB
po[2,3] <- 1-pB
po[3,1] <- 0
po[3,2] <- 0
po[3,3] <- 1
```

Analysis in JAGS

```
# Likelihood
for (i in 1:nInd){
   # Define latent state at first capture
   z[i,f[i]] <- y[i,f[i]]
   for (t in (f[i]+1):nOcc){
      # State process: draw S(t) given S(t-1)
      z[i,t] ~ dcat(ps[z[i,t-1],])
      # Observation process: draw O(t) given S(t)
      y[i,t] ~ dcat(po[z[i,t],])
      } #t
   } #i
}
",fill = TRUE)
sink()
```

Multi-state models

Very flexible

- Most capture-recapture models can be formulated as multi-state models
- Can accommodate many states
- Can include covariates and individual/temporal variation

But complexity comes at a cost

- Data hungry
- Often not fully identifiable
- Computationally intensive