<u>Marking scheme_Mid Semester Exam of Applied Mathematics (30 Marks)</u>

Question1. A. Find all value of r for whic $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & r \end{bmatrix}$ commutes with $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ **(4 Marks)**

B. Draw a true table for the following Boolean functions **(6 Marks)**

i) $F = x'y'z + (x'yz)' + xy'$

ii) $F = (x + y)(x' + z)(y + z)'$

Answer:

A. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & r \end{bmatrix}$ commutes with $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

iff $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & r \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & r \end{bmatrix}$ /2Marks

$\Leftrightarrow \begin{bmatrix} 2 & 0 & 2 \\ 0 & 1 & 0 \\ r & 0 & r \end{bmatrix} = \begin{bmatrix} 2 & 0 & r \\ 0 & 1 & 0 \\ 2 & 0 & r \end{bmatrix}$ /1 Mark

$\Rightarrow r = 2$ /1 Mark

B.i)

x	y	z	x'	y'	$x'y'z$	$x'yz$	$(x'yz)'$	xy	F
0	0	0	1	1	**0**	0	**1**	**0**	**1**
0	0	1	1	1	**1**	0	**1**	**0**	**1**
0	1	0	1	0	**0**	0	**1**	**0**	**1**
0	1	1	1	0	**0**	1	**0**	**0**	**0**
1	0	0	0	1	**0**	0	**1**	**1**	**1**
1	0	1	0	1	**0**	0	**1**	**1**	**1**
1	1	0	0	0	**0**	0	**1**	**0**	**1**
1	1	1	0	0	**0**	0	**1**	**0**	**1**
					/1 Mark		/1 Mark		/1 Mark

iii)

x	y	z	x'	$x + y$	$x' + z$	$y + z$	$(y + z)'$	F
0	0	0	1	**0**	**1**	0	**1**	**0**
0	0	1	1	**0**	**1**	1	**0**	**0**
0	1	0	1	**1**	**1**	1	**0**	**0**
0	1	1	1	**1**	**1**	1	**0**	**0**
1	0	0	0	**1**	**0**	0	**1**	**0**
1	0	1	0	**1**	**1**	1	**0**	**0**
1	1	0	0	**1**	**0**	1	**0**	**0**
1	1	1	0	**1**	**1**	1	**0**	**0**
				/1 Mark			/1 Mark	/1 Mark

Question2. A. Use Cramer's rule to solve the following systems. **(5 Marks)**

$$\begin{cases} 2\,x_1 + x_2 \ + x_3 \ = 4 \\ x_1 - x_2 + 2x_3 \ \ = 2 \\ 3x_1 - 2x_2 - x_3 = 0 \end{cases}$$

B. Find the domain of definition of the function f(x) given below.

i. $f(x) = \sqrt{-x} + \frac{1}{\sqrt{2+x}}$ **(3 Marks)**

ii. $f(x) = log_2(3-2x)$ **(2 Marks)**

Answer:

A. The system is written in matrix form as $\begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & -2 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$ **/0.5 Mark**

;then the determinant of the system is $\begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & -2 & -1 \end{vmatrix} = 18$. **/0.5 Mark**

The roots are $x_1 = \frac{\begin{vmatrix} 4 & 1 & 1 \\ 2 & -1 & 2 \\ 0 & -2 & -1 \end{vmatrix}}{18}; x_2 = \frac{\begin{vmatrix} 2 & 4 & 1 \\ 1 & 2 & 2 \\ 3 & 0 & -1 \end{vmatrix}}{18}\ ; x_3 = \frac{\begin{vmatrix} 2 & 1 & 4 \\ 1 & -1 & 2 \\ 3 & -2 & 0 \end{vmatrix}}{18}$. **/1.5Marks**

After calculation we find $c = 1; x_2 = \frac{18}{18} = 1\ ; x_3 = \frac{18}{18} = 1;$ **/1.5Marks**

and $S = \{(1, 1,1)\}$ **/1 Mark**

B.i. EC: $\begin{cases} -x \geq 0 \\ 2 + x > 0 \end{cases}$ **/1 Mark**

$\begin{cases} x \leq 0 \\ x > -2 \end{cases}$ **/1 Mark**

$Domf\ =]-2, 0]$ **/1 Mark**

ii. EC: $3 - 2x > 0 \Leftrightarrow x < \frac{3}{2}$ **/1 Mark**

$Domf\ = \left]-\infty, \frac{3}{2}\right[$ **/1 Mark**

Question3. A. Consider the open proposition over the universe $U = \{-5,0,1,2,3,4,5,6,7,8,9,10\}$

$p(x): x^2 < 5, \quad q(x): x \geq 3, \ r(x): is\ a\ multiple\ of\ 2, \quad s(x): x^2 = 25$

Find the truth sets of:

i)$\sim p(x) \vee \{q(x) \wedge \sim r(x)\}$ **(3 Marks)**

$ii)[p(x) \wedge q(x)] \wedge s(x)$ **(3 Marks)**

B. i)How many ways of selecting a president, a vice president, a secretary, and a treasurer in a club consisting of 10 persons? **(2 Marks)**

ii) How many numbers of ordered samples of five cards that can be drawn without replacement from a standard deck of 52 playing cards? **(2 Marks)**

Answer:

A. $P = \{0,1,2\}$; $Q = \{3,4,5,6,7,8,9,10\}$; $R = \{2,4,6,8,10\}$; $S = \{-5,5\}$ **/2 Marks**

i. $\sim p(x) \vee \{q(x) \wedge \sim r(x)\} \equiv P' \cup (Q \cap R')$ **/1 Mark**

$P' \cup (Q \cap R') = \{-5,3,4,5,6,7,8,9,10\}$ **/1 Mark**

ii. $[p(x) \wedge q(x)] \wedge s(x) \equiv (P \cap Q) \cap S$ **/1 Mark**

$(P \cap Q) \cap S = \{\} \cap S = \{\}$ **/1 Mark**

B.i. $10P_4 = \frac{10!}{6!} = 5040$ **/2 Marks**

ii. $\binom{52}{5} = \frac{52!}{5! \times 47!} = 2598960$ **/2 Marks**

END CORRECTIONS!!!!!!!!!