Marking scheme-Final Semester Exam of Applied Mathematics (40 Marks)

Question1.

a. Calculate the following limits *(6Marks)*

i) $\lim_{x\to 27}\left(\frac{x-27}{\sqrt[3]{x}-3}\right)$ ii) $\lim_{x\to\infty}\left(\frac{2x+5}{2x+1}\right)^{-x+1}$

b. Calculate the sum of: $3+5+7+9+\cdots+189$ *(4 Marks)*

Answer:

a.i) $\lim_{x\to 27}\left(\frac{x-27}{\sqrt[3]{x}-3}\right)=\frac{0}{0}???$ $\lim_{x\to 27}\frac{(x-27)'}{\left(\sqrt[3]{x}-3\right)'}=\lim_{x\to 27}\left(\frac{1}{\frac{1}{3}x^{\frac{-2}{3}}}\right)$ **/2Marks**

$=3\times 3^2=27$ **/1Mark**

ii) $\lim_{x\to\infty}\left(\frac{2x+5}{2x+1}\right)^{-x+1}=1^{\infty}???$ **/1Mark**

$\lim_{x\to\infty}\left(\frac{2x+5}{2x+1}\right)^{-x+1}=e^{\lim_{x\to\infty}\left(\frac{2x+5}{2x+1}-1\right)(-x+1)}$ **/1Mark**

$=e^{\lim_{x\to\infty}\left(\frac{-4x+4}{2x+1}\right)}=e^{\frac{-4}{2}}=e^{-2}$ **/1Mark**

b. $d=2, u_1=3$ **/1Mark**

$u_n=u_1+d(n-1)\Rightarrow n=\frac{u_n-u_1}{d}+1\Leftrightarrow n=\frac{189-3}{2}+1\Rightarrow n=94$ **/1Mark**

$S_n=\frac{n(u_n+u_1)}{2}$ **/1Mark**

$S_{94}=\frac{94(189+3)}{2}=94\times 96=9024$ **/1Mark**

Question2. Solve the following equation and system of equation: **(10marks)**

a. $\log_3(4.3^x-1)=2x+1$

b. $\begin{cases}\ln(x-2)-3\ln(y-1)=9\\ 2\ln(x-2)-\ln(y-1)=4\end{cases}$

Answer:

a. $\log_3(4.3^x-1)=2x+1\Leftrightarrow 4.3^x-1=3^{2x+1}$ **/1Mark**

$\Leftrightarrow 3t^2-4.t+1=0,\quad 3^x=t\ with$ **/1Mark**

$3^x = \frac{1}{3} \Rightarrow x = -1$ **/1Mark**

$3^x = 1 \Rightarrow x = 0$ **/1Mark**

$S = \{-1, o\}$ **/1Mark**

b. $\begin{cases} \ln(x-2) - 3\ln(y-1) = 9 \\ 2\ln(x-2) - \ln(y-1) = 4 \end{cases} \left| \begin{matrix} -2 \\ 1 \end{matrix} \right. \Rightarrow \ln(y-1) = \frac{-14}{5}$ **/1Mark**

$\Rightarrow y = e^{\frac{-14}{5}} + 1$ **/1Mark**

$\ln(x-2) = \frac{3}{5}$ **/1Mark**

$\Rightarrow x = e^{\frac{3}{5}} + 2$ **/1Mark**

$S = \left\{\left(e^{\frac{3}{5}} + 2, e^{\frac{-14}{5}} + 1\right)\right\}$ **/1Mark**

Question3. Consider the values $A = \int_0^{\frac{\pi}{8}} (e^{-2x} \cos^2 x)\, dx$ and $B = \int_0^{\frac{\pi}{8}} (e^{-2x} \sin^2 x)\, dx$, then evaluate $A + B$, $A - B$,deduce the values of A and B ***(10 Marks)***

Answer:

i) $A + B = \int_0^{\frac{\pi}{8}} (e^{-2x} \cos^2 x)\, dx + \int_0^{\frac{\pi}{8}} (e^{-2x} \sin^2 x)\, dx$

$= \int_0^{\frac{\pi}{8}} e^{-2x} (\cos^2 x + \sin^2 x) dx = \int_0^{\frac{\pi}{8}} e^{-2x} dx$ **/2Marks**

$= \frac{-1}{2} [e^{-2x}]_0^{\frac{\pi}{8}} = \frac{1 - e^{-\frac{\pi}{4}}}{2}$ **/1Mark**

ii) $A - B = \int_0^{\frac{\pi}{8}} (e^{-2x} \cos^2 x)\, dx - \int_0^{\frac{\pi}{8}} (e^{-2x} \sin^2 x)\, dx$

$= \int_0^{\frac{\pi}{8}} e^{-2x} (\cos^2 x - \sin^2 x) dx = \int_0^{\frac{\pi}{8}} e^{-2x} \cos 2x\, dx$ **/1 Mark**

$u = \cos 2x \Rightarrow du = -2 \sin 2x\, dx, \quad dv = e^{-2x} dx \Rightarrow v = \frac{-e^{-2x}}{2}$ **/1 Mark**

$A - B = \frac{-1}{2} [e^{-2x} \cos 2x]_0^{\frac{\pi}{8}} - \int_0^{\frac{\pi}{8}} e^{-2x} \sin 2x\, dx$ **/0.5 Mark**

$A - B = \frac{-1}{2} [e^{-2x} \cos 2x]_0^{\frac{\pi}{8}} - \int_0^{\frac{\pi}{8}} e^{-2x} \sin 2x\, dx$ **/0.5 Mark**

$$= \frac{1-\frac{\sqrt{2}}{2}e^{-\frac{\pi}{4}}}{2} - \int_0^{\frac{\pi}{8}} e^{-2x}\sin 2x\,dx$$ /0.5Mark

$$u = \sin 2x \Longrightarrow du = 2\cos 2x\,dx, \quad dv = e^{-2x}dx \Longrightarrow v = \frac{-e^{-2x}}{2}$$

$$= \frac{1}{2}\left[e^{-2x}\sin 2x\right]_0^{\frac{\pi}{8}} - \int_0^{\frac{\pi}{8}} e^{-2x}\cos 2x\,dx$$ /0.5Mark

$$2(A-B) = \frac{1}{2}\frac{\sqrt{2}}{2}e^{-\frac{\pi}{4}} \Longrightarrow A - B = \frac{\sqrt{2}}{8}e^{-\frac{\pi}{4}}$$ /1Mark

iii) $$\begin{cases} A + B = \frac{1-e^{-\frac{\pi}{4}}}{2} \\ A - B = \frac{\sqrt{2}}{8}e^{-\frac{\pi}{4}} \end{cases} \Longrightarrow A = \frac{4+(\sqrt{2}-4)e^{-\frac{\pi}{4}}}{8}$$ /1Mark $$B = \frac{4-(\sqrt{2}+4)e^{-\frac{\pi}{4}}}{8}$$ /1Mark

Question 4.

a. Find the 12th term of the geometric sequence whose 8th term is 192 and the reason (common ratio) is equal to 2 ***(5 Marks)***

b. Calculate the derivative of the following functions ***(5 Marks)***

i) $f(x) = \sqrt{-2x^3+1}$ ii) $f(x) = \frac{-2x^5+1}{3x+1}$

Answer:

a. $u_8 = r^7 u_1 \Longleftrightarrow 192 = 2^7 u_1$ /2Marks $\Longrightarrow u_1 = \frac{192}{2^7} = \frac{3}{2}$ /1Mark

$u_{12} = r^{11} u_1$ /1Mark $u_{12} = 2^{11} \times \frac{3}{2} = 2^{10} \times 3 = 3072$ /1Mark

b.i) $f'(x) = \left(\sqrt{-2x^3+1}\right)' = \frac{(-2x^3+1)'}{2\sqrt{-2x^3+1}} = \frac{-3x^2}{\sqrt{-2x^3+1}}$ /2Marks

ii) $\left(\frac{-2x^5+1}{3x+1}\right)' = \frac{(-2x^5+1)'(3x+1)-(-2x^5+1)(3x+1)'}{(3x+1)^2}$ /2Marks

$= \frac{-10x^4(3x+1)-(-2x^5+1)3}{(3x+1)^2} = \frac{-24x^5-10x^4-6}{(3x+1)^2}$ /2Marks

Success and God be with you!!!