CISC 2210 Discrete Structures - Noson S. Yanofsky

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1.7

1.

Let $S = \{1, 2, 3, 4, 5\}$ and $T = \{a, b, c, d\}$. For each question below: if the answer is Yes, give an example, else explain briefly.

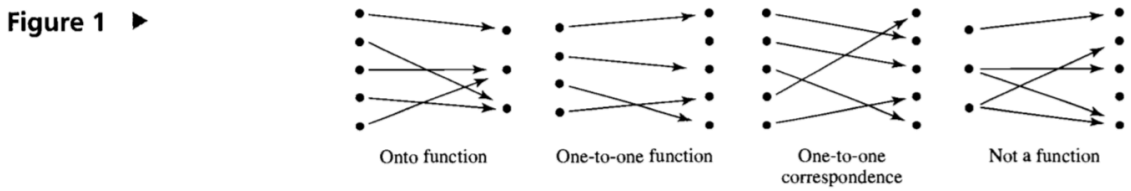


Figure 1: types of functions

onto: every element in codomain is accounted for
one-to-one: every element in domain has a unique spot in codomain
one-to-one correspondence: one-to-one between domain-codomain and codomain-domain

(a)

Are there any one-to-one functions from S into T?

No, this would be an onto function but doesn't meet the requirements for one-to-one.

(b)

Are there any one-to-one functions from T into S?

Yes. One element in S will be unused.

(c)

Are there any functions mapping S onto T?

Yes. Some two elements from S will map onto some single element in T.

(d)

Are there any functions mapping T onto S?

No, not enough elements in T to fill up codomain S. This could be a one-to-one however.

(e)

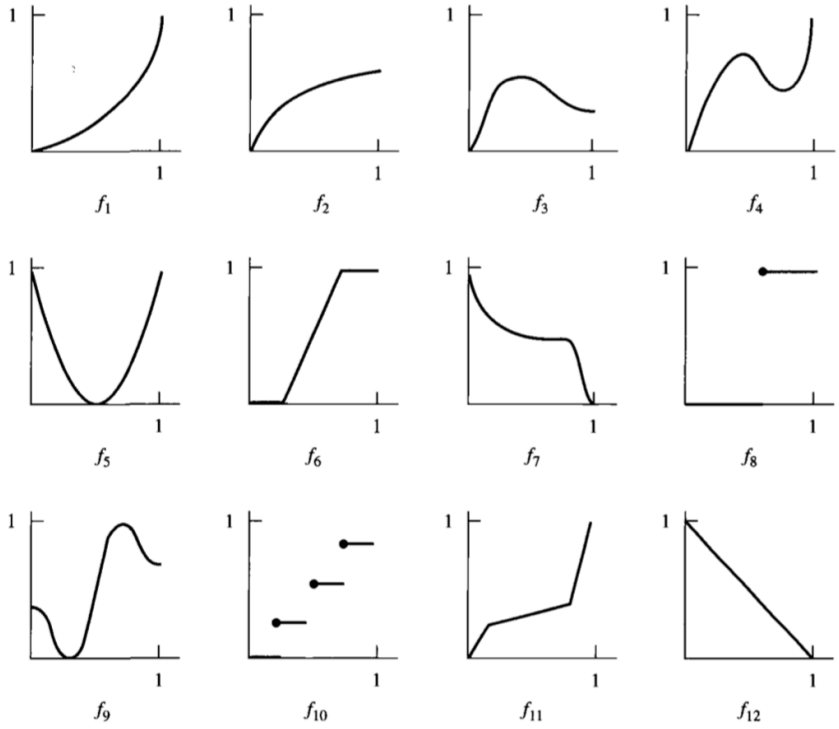
Are there any one-to-one correspondences between S and T?

No, S and T have different number of elements.

2.

The functions sketched in Figure 3 have domain and codomain both equal to $[0, 1]$

Figure 3 ▶



(TODO check with Professor about question 2)

(a)

Which of these functions are one-to-one?

f_1, f_2, f_{11} because x and y coordinates are not repeated on x and y axes.

(b)

Which of these functions map $[0, 1]$ onto $[0, 1]$?

Might be a bit of a trick question; the original statement tells us that all functions have domain and codomain mapped to $[0, 1]$.

(c)

Which of these functions are one-to-one correspondences?

f_{12} because the graph is symmetrical diagonally with no repeated points on x and y axes.

3.

The function $f(m, n) = 2^m 3^n$ is a one-to-one function from $\mathbb{N} \times \mathbb{N}$ into $\mathbb{N}$.

(a)

Calculate $f(m, n)$ for five different elements (m, n) in $\mathbb{N} \times \mathbb{N}$:

$$f(0,1) = 2^0 3^1 = 1 \cdot 3 = 3$$
$$f(2,3) = 2^2 3^3 = 4 \cdot 27 = 108$$
$$f(1,2) = 2^1 3^2 = 2 \cdot 9 = 18$$
$$f(0,2) = 2^0 3^2 = 1 \cdot 9 = 9$$
$$f(0,3) = 2^0 3^3 = 1 \cdot 27 = 27$$

4.

Consider the following functions from $\mathbb{N}$ into $\mathbb{N}$:
$1_{\mathbb{N}}(n) = n, f(n) = 3n, g(n) = n + (-1)^n, h(n) = \min[n, 100], k(n) = \max[0, n - 5]$

(a)

Which of these functions are one-to-one?

$1_{\mathbb{N}}(n)$
$f(n)$
$g(n)$

(a)

Which of these functions map $\mathbb{N}$ into $\mathbb{N}$?

these cover all values in the $\mathbb{N}$ codomain:
$1_{\mathbb{N}}(n)$
$g(n)$