

CISC 2210 Discrete Structures - Noson S. Yanofsky

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1.7

1.

Let $S = \{1, 2, 3, 4, 5\}$ and $T = \{a, b, c, d\}$. For each question below: if the answer is Yes, give an example, else explain briefly.

Figure 1 ►

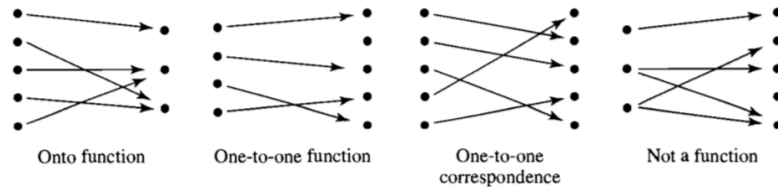


Figure 1: types of functions

onto: every element in codomain is accounted for

one-to-one: every element in domain has a unique spot in codomain

one-to-one correspondence: one-to-one between domain-codomain and codomain-domain

(a)

Are there any one-to-one functions from S into T ?

No, this would be an onto function but doesn't meet the requirements for one-to-one.

(b)

Are there any one-to-one functions from T into S ?

Yes. One element in S will be unused.

(c)

Are there any functions mapping S onto T ?

Yes. Some two elements from S will map onto some single element in T .

(d)

Are there any functions mapping T onto S ?

No, not enough elements in T to fill up codomain S . This could be a one-to-one however.

(e)

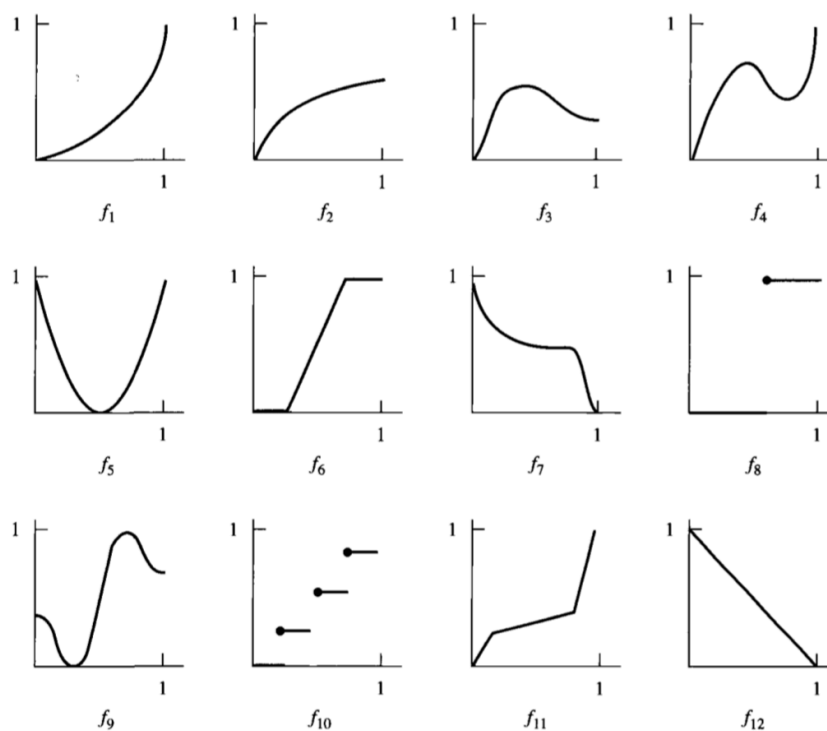
Are there any one-to-one correspondences between S and T ?

No, S and T have different number of elements.

2.

The functions sketched in Figure 3 have domain and codomain both equal to $[0, 1]$

Figure 3 ►



(TODO check with Professor about question 2)

(a)

Which of these functions are one-to-one?

f_1, f_2, f_{11}, f_{12} because they pass the horizontal-line-test; x and y coordinates are not repeated on x and y axes.

(b)

Which of these functions map $[0, 1]$ onto $[0, 1]$?

$f_1, f_4, f_5, f_6, f_9, f_{11}, f_{12}$ because they fill up all the y-axis codomain elements. (f_7 is a bit tricky because there is a tiny gap.

(c)

Which of these functions are one-to-one correspondences?

f_1, f_{11}, f_{12} satisfy bijection requirements of both onto and one-to-one.

3.

The function $f(m, n) = 2^m 3^n$ is a one-to-one function from $\mathbb{N} \times \mathbb{N}$ into \mathbb{N} .

(a)

Calculate $f(m, n)$ for five different elements (m, n) in $\mathbb{N} \times \mathbb{N}$:

$$\begin{aligned}f(0, 1) &= 2^0 3^1 = 1 \cdot 3 = 3 \\f(2, 3) &= 2^2 3^3 = 4 \cdot 27 = 108 \\f(1, 2) &= 2^1 3^2 = 2 \cdot 9 = 18 \\f(0, 2) &= 2^0 3^2 = 1 \cdot 9 = 9 \\f(0, 3) &= 2^0 3^3 = 1 \cdot 27 = 27\end{aligned}$$

4.

Consider the following functions from \mathbb{N} into \mathbb{N} :

$$1_{\mathbb{N}}(n) = n, f(n) = 3n, g(n) = n + (-1)^n, h(n) = \min[n, 100], k(n) = \max[0, n - 5]$$

(a)

Which of these functions are one-to-one?

$$\begin{aligned}1_{\mathbb{N}}(n) \\f(n) \\g(n)\end{aligned}$$

(b)

Which of these functions map \mathbb{N} into \mathbb{N} ?

these cover all values in the \mathbb{N} codomain:

$$\begin{aligned}1_{\mathbb{N}}(n) \\g(n)\end{aligned}$$

5.

Here are two "shift functions" mapping \mathbb{N} into \mathbb{N} :

$f(n) = n + 1$ and $g(n) = \max[0, n - 1]$ for $n \in \mathbb{N}$

(a)

Calculate $f(n)$ for $n = 0, 1, 2, 3, 4, 73$:

$$\begin{aligned}f(0) &= 1 \\f(1) &= 2 \\f(2) &= 3 \\f(3) &= 4 \\f(4) &= 5 \\f(73) &= 74\end{aligned}$$

(b)

Calculate $g(n)$ for $n = 0, 1, 2, 3, 4, 73$:

$$\begin{aligned}g(0) &= 0 \\g(1) &= 0 \\g(2) &= 1 \\g(3) &= 2 \\g(4) &= 3 \\g(73) &= 72\end{aligned}$$

(c)

Show that f is one-to-one but does not map \mathbb{N} onto \mathbb{N} :

$f(n)$ always produces a unique output in codomain so it is one-to-one, but does not plot 0 onto the codomain

(d)

Show that g maps \mathbb{N} onto \mathbb{N} but is not one-to-one:

$g(n)$ maps all numbers onto \mathbb{N} codomain but is not one-to-one because $g(0)$ and $g(1)$ both output 0

(e)

Show that $g \circ f(n) = n$ for all n , but that $f \circ g(n) = n$ does not hold for all n :

$g \circ f(n)$ is essentially just $\max[0, n]$, so it will output back n
 $f \circ g(n)$ does not account for 0

7.

Find the inverses of the following functions mapping \mathbb{R} into \mathbb{R} :

(b)

$$g(x) = x^3 - 2$$

$$g^{-1}(x) = \sqrt[3]{x+2}$$

(c)

$$h(x) = (x-2)^3$$

$$y = (x-2)^3$$

$$\sqrt[3]{y} = x - 2$$

$$x = \sqrt[3]{y} + 2$$

$$h^{-1}(x) = \sqrt[3]{x} + 2$$

9.

Show that the following functions are their own inverses:

(a)

The function $f : (0, \infty) \rightarrow (0, \infty)$ given by $f(x) = \frac{1}{x}$

$$\text{suppose } f^{-1}(x) = \frac{1}{x}$$

$$\text{let } x = 5$$

$$f(5) = \frac{1}{5}$$

$$f^{-1}(f(5)) = \frac{1}{(\frac{1}{5})} = 5$$

10.

Let A be a subset of some set S and consider the characteristic function X_A of A .

Find $X_A^{-1}(1)$ and $X_A^{-1}(0)$:

11.

(a)