CISC 2210 Discrete Structures - Noson S. Yanofsky

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2.1

1.

Let p, q and r be the following propositions:
$p =$ "it is raining,"
$q =$ "the sun is shining,"
$r =$ "there are clouds in the sky."

Translate the following into logical notation, using p, q, r, and logical connectives.

(a)

It is raining and the sun is shining.

$$p \wedge q$$

(b)

If It is raining, then there are clouds in the sky.

$$p \rightarrow r$$

(c)

If It is not raining, then the sun is not shining and there are clouds in the sky.

$$\neg p \rightarrow (\neg q \wedge r)$$

(d)

The sun is shining if and only if it is not raining.

$$q \longleftrightarrow \neg p$$

(e)

If there are no clouds in the sky, then the sun is shining.

$$\neg r \rightarrow q$$

2.

Let $p, q,$ and r be as in Exercise 1. Translate the following into English sentences:

(d)

$$\neg(p \longleftrightarrow (q \vee r))$$

It is not raining if and only if the sun is not shining or there are no clouds in the sky.

(e)

$$\neg(p \vee q) \wedge r$$

It is not raining or the sun is not shining, but there are clouds in the sky.

3.

(a)

Give truth values of the propositions in parts (a) to (e) of Example 1:

(a) Julius Caesar was president of the United States: False
(b) $2 + 2 = 4$: True
(c) $2 + 3 = 7$: False
(d) The number 4 is positive and the number 3 is negative: False
(e) If a set has n elements, then it has 2^n subsets: True
(Bonus)
(f) $2^n + n$ is a prime number for infinitely many n: don't know...
(g) Every even integer greater than 2 is the sum of two prime numbers: no one knows... see "Goldbach's conjecture"

(b)

Do the same for parts (a) and (b) of Example 2:

(a) $x + y = y + x$ for all $x, y \in \mathbb{R}$: True commutative property
(b) $2^n = n^2$ for some $n \in \mathbb{N}$: True for $\{2, 4\}$

9.

(a)

Show that $n = 3$ provides one possible counterexample to the assertion "$n^3 < 3^n \forall n \in \mathbb{N}$":

(b)

Can you find any other counterexamples?