CISC 2210 Discrete Structures - Noson S. Yanofsky

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2.1

1.

Let p, q and r be the following propositions:
$p =$ "it is raining,"
$q =$ "the sun is shining,"
$r =$ "there are clouds in the sky."

Translate the following into logical notation, using p, q, r, and logical connectives.

(a)

It is raining and the sun is shining.

$$p \wedge q$$

(b)

If It is raining, then there are clouds in the sky.

$$p \rightarrow r$$

(c)

If It is not raining, then the sun is not shining and there are clouds in the sky.

$$\neg p \rightarrow (\neg q \wedge r)$$

(d)

The sun is shining if and only if it is not raining.

$$q \longleftrightarrow \neg p$$

(e)

If there are no clouds in the sky, then the sun is shining.

$$\neg r \to q$$

2.

Let $p, q,$ and r be as in Exercise 1. Translate the following into English sentences:

(d)

$$\neg(p \longleftrightarrow (q \vee r))$$

It is not raining if and only if the sun is not shining or there are no clouds in the sky.

(e)

$$\neg(p \vee q) \wedge r$$

It is not raining or the sun is not shining, but there are clouds in the sky.

3.

(a)

Give truth values of the propositions in parts (a) to (e) of Example 1:

(a) Julius Caesar was president of the United States: False
(b) $2 + 2 = 4$: True
(c) $2 + 3 = 7$: False
(d) The number 4 is positive and the number 3 is negative: False
(e) If a set has n elements, then it has 2^n subsets: True
(Bonus)
(f) $2^n + n$ is a prime number for infinitely many n: don't know...
(g) Every even integer greater than 2 is the sum of two prime numbers: no one knows... see "Goldbach's conjecture"

(b)

Do the same for parts (a) and (b) of Example 2:

(a) $x + y = y + x$ for all $x, y \in \mathbb{R}$: True commutative property
(b) $2^n = n^2$ for some $n \in \mathbb{N}$: True for $\{2, 4\}$

Note

Converse : flips

Inverse : negates

Contrapositive : flips and negates

6.

Give the converses of the following propositions:

(b)

If I am smart, then I am rich.

If I am rich, then I am smart.

(c)

If $x^2 = x$, then $x = 0$ or $x = 1$.

If $x = 0$ or $x = 1$, then $x^2 = x$.

7.

Give the contrapositives of the propositions in Exercise 6:

(b)

If I am smart, then I am rich.

If I am not rich, then I am not smart.

(c)

If $x^2 = x$, then $x = 0$ or $x = 1$.

If $x \neq 0$ or $x \neq 1$, then $x^2 \neq x$.

9.

(a)

Show that $n = 3$ provides one possible counterexample to the assertion "$n^3 < 3^n \forall n \in \mathbb{N}$":

$$3^3 = 27, 3^3 = 27,$$
$$\text{thus } 27 \not< 27$$

(b)

Can you find any other counterexamples?

I could not find any other counterexamples within the $\mathbb{N}$ domain.

11.

(a)

Show that $x = -1$ is a counterexample to $(x+1)^2 \geq x^2 \forall x \in \mathbb{R}$:

$$(-1+1)^2, -1^2$$
$$0 \ngeq 1$$

(b)

Find another counterexample:

$$x = -2$$
$$(-2+1)^2, -2^2$$
$$1 \ngeq 4$$

(c)

Can a non-negative number serve as an example?

No, because $\forall x \in \mathbb{N}$, $x + 1 > x$

12.

Find counterexamples to the following assertions:

(a)

$2^n - 1$ is prime for every $n \geq 2$

Consider $h(n) = |\{k \in \mathbb{N} : k|n\}|$,
if $h(n) = 2$, then n is prime.
let $n = 4$,
$2^4 - 1 = 15$,
$h(15) = |1, 3, 5, 15| = 4$,
$4 \neq 2$,
thus $n = 4$ is a counterexample.
let $n = 6$,
$2^4 - 1 = 63$,
$h(63) = |1, 3, 7, 9, 21, 63| = 6$,
$6 \neq 2$,
thus $n = 6$ is another counterexample.

14.

(a)

(b)

(c)

(d)

17.

(a)

(b)

(c)

(d)

(e)