# CISC 2210 Discrete Structures - Noson S. Yanofsky

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## 1.7

### 1.

Let $S = \{1, 2, 3, 4, 5\}$ and $T = \{a, b, c, d\}$. For each question below: if the answer is Yes, give an example, else explain briefly.

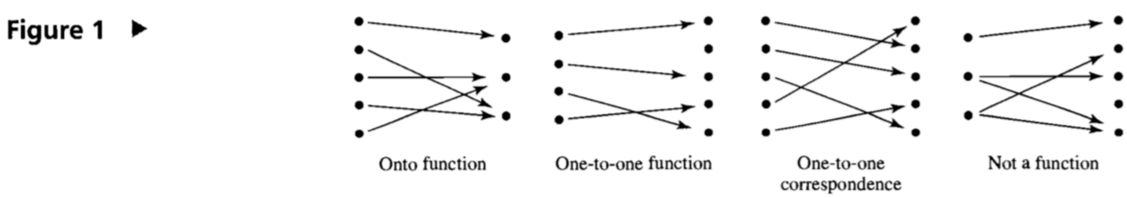



Figure 1: types of functions

*onto: every element in codomain is accounted for*
*one-to-one: every element in domain has a unique spot in codomain*
*one-to-one correspondence: one-to-one between domain-codomain and codomain-domain*

#### (a)

Are there any one-to-one functions from $S$ into $T$?

No, this would be an onto function but doesn't meet the requirements for one-to-one.

#### (b)

Are there any one-to-one functions from $T$ into $S$?

Yes. One element in $S$ will be unused.

#### (c)

Are there any functions mapping $S$ onto $T$?

Yes. Some two elements from $S$ will map onto some single element in $T$.

### (d)

Are there any functions mapping $T$ onto $S$?

No, not enough elements in $T$ to fill up codomain $S$. This could be a one-to-one however.

### (e)

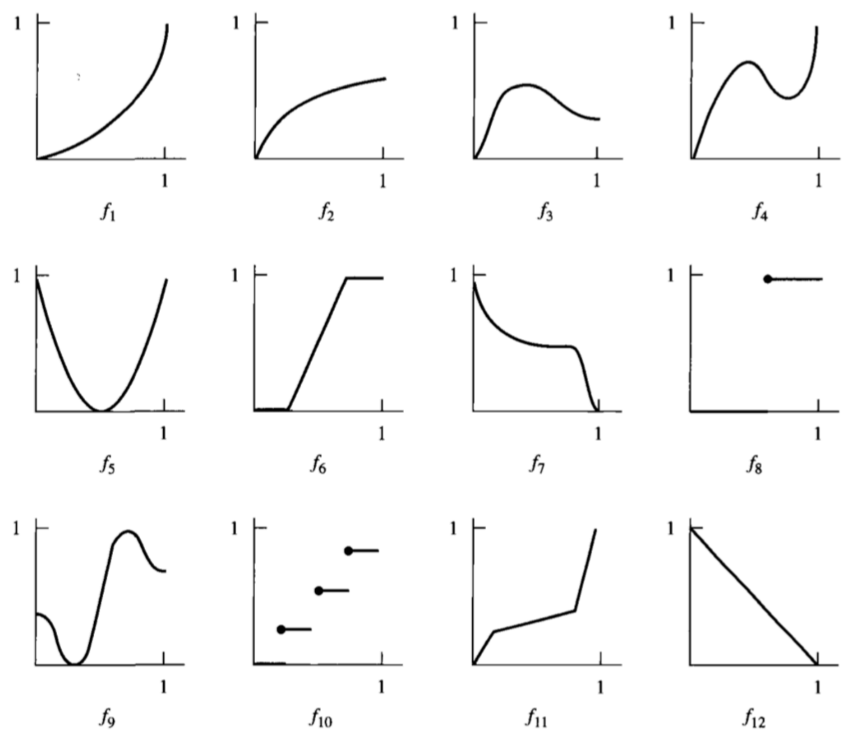
Are there any one-to-one correspondences between $S$ and $T$?

No, $S$ and $T$ have different number of elements.

## 2.

The functions sketched in Figure 3 have domain and codomain both equal to $[0, 1]$

**Figure 3** ▶




*(TODO check with Professor about question 2)*

### (a)

Which of these functions are one-to-one?

$f_1, f_2, f_{11}$ because $x$ and $y$ coordinates are not repeated on $x$ and $y$ axes.

**(b)**

Which of these functions map $[0, 1]$ onto $[0, 1]$?

Might be a bit of a trick question; the original statement tells us that all functions have domain and codomain mapped to $[0, 1]$.

**(c)**

Which of these functions are one-to-one correspondences?

$f_{12}$ because the graph is symmetrical diagonally with no repeated points on $x$ and $y$ axes.

**3.**

The function $f(m, n) = 2^m 3^n$ is a one-to-one function from $\mathbb{N} \times \mathbb{N}$ into $\mathbb{N}$.

**(a)**

Calculate $f(m, n)$ for five different elements $(m, n)$ in $\mathbb{N} \times \mathbb{N}$:

$$f(0, 1) = 2^0 3^1 = 1 \cdot 3 = 3$$
$$f(2, 3) = 2^2 3^3 = 4 \cdot 27 = 108$$
$$f(1, 2) = 2^1 3^2 = 2 \cdot 9 = 18$$
$$f(0, 2) = 2^0 3^2 = 1 \cdot 9 = 9$$
$$f(0, 3) = 2^0 3^3 = 1 \cdot 27 = 27$$

**4.**

Consider the following functions from $\mathbb{N}$ into $\mathbb{N}$:
$1_{\mathbb{N}}(n) = n, f(n) = 3n, g(n) = n + (-1)^n, h(n) = \min[n, 100], k(n) = \max[0, n - 5]$

**(a)**

Which of these functions are one-to-one?

$1_{\mathbb{N}}(n)$
$f(n)$
$g(n)$

**(b)**

Which of these functions map $\mathbb{N}$ into $\mathbb{N}$?

these cover all values in the $\mathbb{N}$ codomain:
$1_{\mathbb{N}}(n)$
$g(n)$

**5.**

Here are two "shift functions" mapping $\mathbb{N}$ into $\mathbb{N}$:
$f(n) = n + 1$ and $g(n) = \max[0, n - 1]$ for $n \in \mathbb{N}$

**(a)**

Calculate $f(n)$ for $n = 0, 1, 2, 3, 4, 73$:

$$f(0) = 1$$
$$f(1) = 2$$
$$f(2) = 3$$
$$f(3) = 4$$
$$f(4) = 5$$
$$f(73) = 74$$

**(b)**

Calculate $g(n)$ for $n = 0, 1, 2, 3, 4, 73$:

$$g(0) = 0$$
$$g(1) = 0$$
$$g(2) = 1$$
$$g(3) = 2$$
$$g(4) = 3$$
$$g(73) = 72$$

**(c)**

Show that $f$ is one-to-one but does not map $\mathbb{N}$ onto $\mathbb{N}$:

$f(n)$ always produces a unique output in codomain so it is one-to-one, but does not plot 0 onto the codomain

**(d)**

Show that $g$ maps $\mathbb{N}$ onto $\mathbb{N}$ but is not one-to-one:

$g(n)$ maps all numbers onto $\mathbb{N}$ codomain but is not one-to-one because $g(0)$ and $g(1)$ both output 0

**(e)**

Show that $g \circ f(n) = n$ for all $n$, but that $f \circ g(n) = n$ does not hold for all $n$:

$g \circ f(n)$ is essentially just $\max[0, n]$, so it will output back $n$
$f \circ g(n)$ does not account for 0

**7.**

**(b)**

**(c)**

**9.**

**(a)**

**10.**

**11.**

**(a)**