CISC 2210 Discrete Structures - Noson S. Yanofsky

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2.1

1.

Let p, q and r be the following propositions:
$p =$ "it is raining,"
$q =$ "the sun is shining,"
$r =$ "there are clouds in the sky."

Translate the following into logical notation, using p, q, r, and logical connectives.

(a)

It is raining and the sun is shining.

$$p \wedge q$$

(b)

If It is raining, then there are clouds in the sky.

$$p \rightarrow r$$

(c)

If It is not raining, then the sun is not shining and there are clouds in the sky.

$$\neg p \rightarrow (\neg q \wedge r)$$

(d)

The sun is shining if and only if it is not raining.

$$q \longleftrightarrow \neg p$$

(e)

If there are no clouds in the sky, then the sun is shining.

$$\neg r \rightarrow q$$

2.

Let $p, q,$ and r be as in Exercise 1. Translate the following into English sentences:

(d)

$$\neg(p \longleftrightarrow (q \vee r))$$

It is not raining if and only if the sun is not shining or there are no clouds in the sky.

(e)

$$\neg(p \vee q) \wedge r$$

It is not raining or the sun is not shining, but there are clouds in the sky.

3.

(a)

Give truth values of the propositions in parts (a) to (e) of Example 1:

(a) Julius Caesar was president of the United States: False
(b) $2 + 2 = 4$: True
(c) $2 + 3 = 7$: False
(d) The number 4 is positive and the number 3 is negative: False
(e) If a set has n elements, then it has 2^n subsets: True
(Bonus)
(f) $2^n + n$ is a prime number for infinitely many n: don't know...
(g) Every even integer greater than 2 is the sum of two prime numbers: no one knows... see "Goldbach's conjecture"

(b)

Do the same for parts (a) and (b) of Example 2:

(a) $x + y = y + x$ for all $x, y \in \mathbb{R}$: True commutative property
(b) $2^n = n^2$ for some $n \in \mathbb{N}$: True for $\{2, 4\}$

Note

Converse : flips

Inverse : negates

Contrapositive : flips and negates

6.

Give the converses of the following propositions:

(b)

If I am smart, then I am rich.

If I am rich, then I am smart.

(c)

If $x^2 = x$, then $x = 0$ or $x = 1$.

If $x = 0$ or $x = 1$, then $x^2 = x$.

7.

Give the contrapositives of the propositions in Exercise 6:

(b)

If I am smart, then I am rich.

If I am not rich, then I am not smart.

(c)

If $x^2 = x$, then $x = 0$ or $x = 1$.

If $x \neq 0$ or $x \neq 1$, then $x^2 \neq x$.

9.

(a)

Show that $n = 3$ provides one possible counterexample to the assertion "$n^3 < 3^n \forall n \in \mathbb{N}$":

$$3^3 = 27, 3^3 = 27,$$
$$\text{thus } 27 \not< 27$$

(b)

Can you find any other counterexamples?

I could not find any other counterexamples within the $\mathbb{N}$ domain.

11.

(a)

Show that $x = -1$ is a counterexample to $(x+1)^2 \geq x^2 \forall x \in \mathbb{R}$:

$$(-1+1)^2, -1^2$$
$$0 \not\geq 1$$

(b)

Find another counterexample:

$$x = -2$$
$$(-2+1)^2, -2^2$$
$$1 \not\geq 4$$

(c)

Can a non-negative number serve as an example?

No, because $\forall x \in \mathbb{N}$, $x + 1 > x$

12.

Find counterexamples to the following assertions:

(a)

$2^n - 1$ is prime for every $n \geq 2$

Consider $h(n) = |\{k \in \mathbb{N} : k|n\}|$,
if $h(n) = 2$, then n is prime.
let $n = 4$,
$2^4 - 1 = 15$,
$h(15) = |1, 3, 5, 15| = 4$,
$4 \neq 2$,
thus $n = 4$ is a counterexample.
let $n = 6$,
$2^4 - 1 = 63$,
$h(63) = |1, 3, 7, 9, 21, 63| = 6$,
$6 \neq 2$,
thus $n = 6$ is another counterexample.

14.

Let S be a nonempty set. Determine which of the following assertions are true. For the true ones, give a reason. For the false ones, provide a counterexample.

(a)

$$A \cup B = B \cup A \quad \forall A, B \in \mathcal{P}(S)$$

True - unions are commutative

(b)

$$(A \backslash B) \cup B = A \quad \forall A, B \in \mathcal{P}(S)$$

False - this would equal $A \cup B$

(c)

$$(A \cup B) \backslash A = B \quad \forall A, B \in \mathcal{P}(S)$$

True - you simply remove A from $A \cup B$ leaving B

(d)

$$(A \cap B) \cap C = A \cap (B \cap C) \quad \forall A, B \in \mathcal{P}(S)$$

True - intersections are associative

17.

Each of the following sentences expresses an implication. Rewrite each in the form ”If p, then q.”

(a)

Touch those cookies if you want a spanking.

If you want a spanking, then touch those cookies.

(b)

Touch those cookies and you’ll be sorry.

If you touch those cookies, then you’ll be sorry.

(c)

You leave or I’ll set the dog on you.

If you don’t leave, then I’ll set the dog on you.

(d)

I will if you will.

If you will, then I will.

(e)

I will go unless you stop that.

If you don't stop that, then I will go.