CISC 2210 Discrete Structures - Noson S. Yanofsky

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March 7, 2018

1.6

1.

Calculate

(a)

$$\frac{7!}{5!} = \frac{7 * 6 * 5!}{5!} = 7 * 6 = 42$$

(e)

$$\sum_{k=0}^{5} k! = 5! + 4! + 3! + 2! + 1! + 0! = 120 + 24 + 6 + 2 + 1 + 1 = 154$$

(f)

$$\prod_{j=3}^{6} j = 3 * 4 * 5 * 6 = \frac{6!}{2!} = 360$$

3.

Calculate

(a)

$$\sum_{k=1}^{n} 3^k \text{ for } n = 1, 2, 3, \text{ and } 4$$

for n = 1: 3
for n = 2: 3 + 9 = 12
for n = 3: 3 + 9 + 27 = 39
for n = 4: 3 + 9 + 27 + 81 = 120

(b)

$$\sum_{k=3}^{n} k^3 \text{ for } n = 3, 4 \text{ and } 5$$

for n = 3: 27
for n = 4: 27 + 91 = 118
for n = 5: 27 + 64 + 125 = 216

4.

Calculate

(a)

$$\sum_{i=1}^{10} (-1)^i = -1 + 1 + -1 + 1 + -1 + 1 + -1 + 1 + -1 + 1 = 0$$

(b)

$$\sum_{k=0}^{3} (k^2 + 1) = 1 + 2 + 5 + 10 = 18$$

(c)

$$\left(\sum_{k=0}^{3} k^2\right) + 1 = (0 + 1 + 4 + 9) + 1 = 15$$

7.

Consider the sequence given by $a_n = \frac{n-1}{n+1}$ for $n \in \mathbb{P}$

(a)

List the first six terms of this seq:

$$\tfrac{1-1}{1+1} = 0, \tfrac{2-1}{2+1} = \tfrac{1}{3}, \tfrac{3-1}{3+1} = \tfrac{1}{2}, \tfrac{4-1}{4+1} = \tfrac{3}{5}, \tfrac{5-1}{5+1} = \tfrac{2}{3}, \tfrac{6-1}{6+1} = \tfrac{5}{7}$$

(b)

Claculate $a_{n+1} - a_n$ for $n = 1, 2, 3$

for n = 1: $\frac{1}{3}$
for n = 2: $\frac{3}{6} - \frac{2}{6} = \frac{1}{6}$
for n = 3: $\frac{6}{10} - \frac{5}{10} = \frac{1}{10}$

(c)

Show that $a_{n+1} - a_n = \frac{2}{(n+1)(n+2}$ for $n \in \mathbb{P}$

$$\frac{(n+1)-1}{(n+1)+1} - \frac{n-1}{n+1}$$
$$= \frac{n}{n+2} - \frac{n-1}{n+1}$$
$$= \frac{n(n+1)}{(n+1)(n+2)} - \frac{(n-1)(n+2)}{(n+1)(n+2)}$$
$$= \frac{n^2-n^2+2n-2n+2}{(n+1)(n+2)}$$
$$= \frac{2}{(n+1)(n+2)}$$

8.

Consider the sequence given by $b_n = \frac{1}{2}[1 + (-1)^n]$ for $n \in \mathbb{N}$

(a)

List the first seven terms of this seq:

$$\frac{1}{2}[1 + (-1)^0] = \frac{1}{2}(2) = 1$$
$$\frac{1}{2}[1 + (-1)^1] = \frac{1}{2}(0) = 0$$
$$\frac{1}{2}[1 + (-1)^2] = \frac{1}{2}(2) = 1$$
$$\frac{1}{2}[1 + (-1)^3] = \frac{1}{2}(0) = 0$$
$$\frac{1}{2}[1 + (-1)^4] = \frac{1}{2}(2) = 1$$
$$\frac{1}{2}[1 + (-1)^5] = \frac{1}{2}(0) = 0$$
$$\frac{1}{2}[1 + (-1)^6] = \frac{1}{2}(2) = 1$$

(b)

What is its set of values?

{1, 0}

10.

For n = 1,2,3,..., let SSQ(n) = $\sum_{i=1}^{n} i^2$
(where SSQ = "sum of squares")

(a)

Calculate SSQ(n) for 1,2,3, and 5

for n = 1: 1
for n = 2: 1 + 4 = 5
for n = 3: 1 + 4 + 9 = 14
for n = 5: 1 + 4 + 9 + 16 + 25 = 55

(b)

Observe that SSQ(n + 1) = SSQ(n)+$(n+1)^2$ for $n \geq 1$

SSQ(2) = 5
SSQ(2+1) = 5 + $(2+1)^2$
= 5 + 9 = 14
(Here we're simply adding an n+1 operation outside of $\sum$)

(c)

It turns out that SSQ(73) = 132,349. Use this to calculate SSQ(74) and SSQ(72)

SSQ(74) = SSQ(73) + $(73+1)^2$ = 132,349 + 5,476 = 137,825
SSQ(72) = SSQ(73) - $(72+1)^2$ = 132,349 - 5,329 = 127,020

13. (a)

Using a calculator or other device, complete the table [write E if the calculation is beyond the capability of your computing device]

n	n^4	4^n	n^{20}	20^n	n!
5	$6.25 * 10^2$	$1.02 * 10^3$	$9.54 * 10^{13}$	$3.2 * 10^6$	$1.2 * 10^2$
10	$1 * 10^4$	$1.05 * 10^6$	$1 * 10^{20}$	$1.02 * 10^{13}$	$3.63 * 10^6$
25	$3.91 * 10^5$	$1.13 * 10^{15}$	$9.09 * 10^{27}$	$3.36 * 10^{32}$	$1.55 * 10^{25}$
50	$6.25 * 10^6$	$1.27 * 10^{30}$	$9.54 * 10^{33}$	$1.13 * 10^{65}$	$3.04 * 10^{64}$

14.

Repeat Exercise 13 for the table in Figure 4

n	$\log_{10} n$	$\sqrt{n}$	$20 * \sqrt[4]{n}$	$\sqrt[4]{n} * \log_{10} n$
50	1.70	7.07	53.18	4.52
100	2	10	63.25	6.32
10^4	4	100	200	40
10^6	6	1000	632.46	189.74