CISC 2210 Discrete Structures - Noson S. Yanofsky

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1.7

1.

Let $S = \{1, 2, 3, 4, 5\}$ and $T = \{a, b, c, d\}$. For each question below: if the answer is Yes, give an example, else explain briefly.

Figure 1 ▶

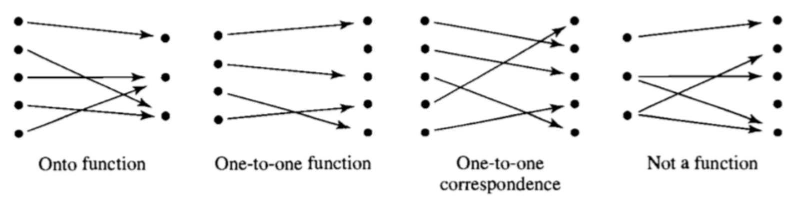


Figure 1: types of functions

onto: every element in codomain is accounted for
one-to-one: every element in domain has a unique spot in codomain
one-to-one correspondence: one-to-one between domain-codomain and codomain-domain

(a)

Are there any one-to-one functions from S into T?

No, this would be an onto function but doesn't meet the requirements for one-to-one.

(b)

Are there any one-to-one functions from T into S?

Yes. One element in S will be unused.

(c)

Are there any functions mapping S onto T?

Yes. Some two elements from S will map onto some single element in T.

(d)

Are there any functions mapping T onto S?

No, not enough elements in T to fill up codomain S. This could be a one-to-one however.

(e)

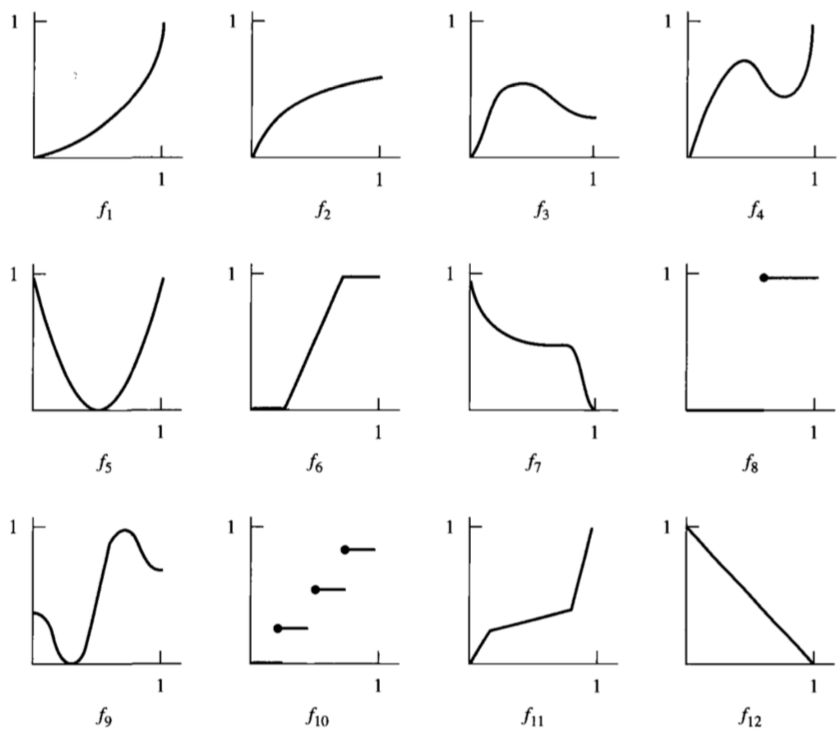
Are there any one-to-one correspondences between S and T?

No, S and T have different number of elements.

2.

The functions sketched in Figure 3 have domain and codomain both equal to $[0, 1]$

Figure 3 ▶



(TODO check with Professor about question 2)

(a)

Which of these functions are one-to-one?

f_1, f_2, f_{11}, f_{12} because they pass the horizontal-line-test; x and y coordinates are not repeated on x and y axes.

(b)

Which of these functions map $[0, 1]$ onto $[0, 1]$?

$f_1, f_4, f_5, f_6, f_9, f_{11}, f_{12}$ because they fill up all the y-axis codomain elements. (f_7 is a bit tricky because there is a tiny gap.

(c)

Which of these functions are one-to-one correspondences?

f_1, f_{11}, f_{12} satisfy bijection requirements of both onto and one-to-one.

3.

The function $f(m, n) = 2^m 3^n$ is a one-to-one function from $\mathbb{N} \times \mathbb{N}$ into $\mathbb{N}$.

(a)

Calculate $f(m, n)$ for five different elements (m, n) in $\mathbb{N} \times \mathbb{N}$:

$$f(0, 1) = 2^0 3^1 = 1 \cdot 3 = 3$$
$$f(2, 3) = 2^2 3^3 = 4 \cdot 27 = 108$$
$$f(1, 2) = 2^1 3^2 = 2 \cdot 9 = 18$$
$$f(0, 2) = 2^0 3^2 = 1 \cdot 9 = 9$$
$$f(0, 3) = 2^0 3^3 = 1 \cdot 27 = 27$$

4.

Consider the following functions from $\mathbb{N}$ into $\mathbb{N}$:
$1_{\mathbb{N}}(n) = n, f(n) = 3n, g(n) = n + (-1)^n, h(n) = \min[n, 100], k(n) = \max[0, n - 5]$

(a)

Which of these functions are one-to-one?

$1_{\mathbb{N}}(n)$
$f(n)$
$g(n)$

(b)

Which of these functions map $\mathbb{N}$ into $\mathbb{N}$?

these cover all values in the $\mathbb{N}$ codomain:
$1_{\mathbb{N}}(n)$
$g(n)$

5.

Here are two "shift functions" mapping $\mathbb{N}$ into $\mathbb{N}$:
$f(n) = n + 1$ and $g(n) = \max[0, n - 1]$ for $n \in \mathbb{N}$

(a)

Calculate $f(n)$ for $n = 0, 1, 2, 3, 4, 73$:

$$f(0) = 1$$
$$f(1) = 2$$
$$f(2) = 3$$
$$f(3) = 4$$
$$f(4) = 5$$
$$f(73) = 74$$

(b)

Calculate $g(n)$ for $n = 0, 1, 2, 3, 4, 73$:

$$g(0) = 0$$
$$g(1) = 0$$
$$g(2) = 1$$
$$g(3) = 2$$
$$g(4) = 3$$
$$g(73) = 72$$

(c)

Show that f is one-to-one but does not map $\mathbb{N}$ onto $\mathbb{N}$:

$f(n)$ always produces a unique output in codomain so it is one-to-one, but does not plot 0 onto the codomain

(d)

Show that g maps $\mathbb{N}$ onto $\mathbb{N}$ but is not one-to-one:

$g(n)$ maps all numbers onto $\mathbb{N}$ codomain but is not one-to-one because $g(0)$ and $g(1)$ both output 0

(e)

Show that $g \circ f(n) = n$ for all n, but that $f \circ g(n) = n$ does not hold for all n:

$g \circ f(n)$ is essentially just $\max[0, n]$, so it will output back n
$f \circ g(n)$ does not account for 0

7.

Find the inverses of the following functions mapping $\mathbb{R}$ into $\mathbb{R}$:

(b)

$$g(x) = x^3 - 2$$

$$g^{-1}(x) = \sqrt[3]{x+2}$$

(c)

$$h(x) = (x-2)^3$$

$$\begin{aligned} y &= (x-2)^3 \\ \sqrt[3]{y} &= x - 2 \\ x &= \sqrt[3]{y} + 2 \\ h^{-1}(x) &= \sqrt[3]{x} + 2 \end{aligned}$$

9.

Show that the following functions are their own inverses:

(a)

The function $f : (0, \infty) \to (0, \infty)$ given by $f(x) = \frac{1}{x}$

suppose $f^{-1}(x) = \frac{1}{x}$
let $x = 5$
$f(5) = \frac{1}{5}$

$$f^{-1}(f(5)) = \frac{1}{(\frac{1}{5})} = 5$$

10.

Let A be a subset of some set S and consider the characteristic function X_A of A.
Find $X_A^{-1}(1)$ and $X_A^{-1}(0)$:

$$X_A^{-1}(1) = A$$

$$X_A^{-1}(0) = A^c$$

11.

Here are some functions from $\mathbb{N} \times \mathbb{N}$ to $\mathbb{N}$:
$\text{SUM}(m, n) = m + n, \text{PROD}(m, n) = m \cdot n, \text{MAX}(m, n) = \max\{m, n\}, \text{MIN}(m, n) = \min\{m, n\}$

(a)

Which of these functions map $\mathbb{N} \times \mathbb{N}$ onto $\mathbb{N}$?

With the right combinations, all of them could cover infinite values for $i \in \mathbb{N}$.