CISC 2210 Discrete Structures - Noson S. Yanofsky

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2.2

1.

Give the converse and contrapositive for each of the following propositions:

(a)

$$p \rightarrow (q \wedge r)$$

converse: $(q \wedge r) \rightarrow p$
contrapositive: $\neg(q \wedge r) \rightarrow \neg p$

(b)

if $x + y = 1$, then $x^2 + y^2 \geq 1$

converse: if $x^2 + y^2 \geq 1$, then $x + y = 1$
contrapositive: if $x^2 + y^2 \ngeq 1$, then $x + y \neq 1$

(c)

if $2 + 2 = 4$, then $3 + 3 = 8$

converse: $if 3 + 3 = 8, then 2 + 2 = 4$
contrapositive: $if 3 + 3 \neq 8, then 2 + 2 \neq 4$

3.

Consider the following propositions:

(a)

Which proposition is the converse of $p \rightarrow q$?

$$q \rightarrow p$$

(b)

Which proposition is the contrapositive of $p \to q$?

$$\neg q \to \neg p$$

(c)

Which propositions are logically equivalent to $p \to q$?

$$\neg p \vee q$$
$$\neg q \to \neg p$$
$$\neg(p \wedge \neg q)$$

9.

Construct the truth table for $[(p \vee q) \wedge r] \to (p \wedge \neg q)$

p	q	r	$[(p \vee q)$	$\wedge\, r]$	$\to$	$(p \wedge$	$\neg q)$
0	0	0	0	0	**1**	0	1
0	0	1	0	0	**1**	0	1
0	1	0	1	0	**1**	0	0
0	1	1	1	1	**0**	0	0
1	0	0	1	0	**1**	1	1
1	0	1	1	1	**1**	1	1
1	1	0	1	0	**1**	0	0
1	1	1	1	1	**0**	0	0

11.

Construct truth tables for:

(a)

$$\neg(p \vee q) \to r$$

p	q	r	$\neg$	$(p \vee q)$	$\to r$
0	0	0	1	0	**0**
0	0	1	1	0	**1**
0	1	0	0	1	**1**
0	1	1	0	1	**1**
1	0	0	0	1	**1**
1	0	1	0	1	**1**
1	1	0	0	1	**1**
1	1	1	0	1	**1**

12.

In which of the following statements is the ”or” an ”inclusive or”?

(a)

Choice of soup or salad - Exclusive Or

(b)

To enter the university, a student must have taken a year of chemistry or physics in high school - Inclusive Or

(c)

Publish or Perish - Exclusive Or

(d)

Experience with C++ of Java is desirable - Inclusive Or

(e)

The task will be completed on Thursday or Friday - Exclusive Or

(f)

Discounts are available to persons under 20 or over 60 - Exclusive Or

(g)

No fishing or hunting allowed - Inclusive Or

(h)

The school will not be open in July or August - Inclusive Or

13.

The exclusive or connective $\oplus$, is defined by the truth table:

p	q	$p \oplus q$
0	0	**0**
0	1	**1**
1	0	**1**
1	1	**0**

(a)

Show that $p \oplus q$ has the same truth table as $\neg(p \leftrightarrow q)$:

p	q	$\neg$	$(p \leftrightarrow q)$
0	0	**0**	1
0	1	**1**	0
1	0	**1**	0
1	1	**0**	1

(b)

Construct a truth table for $p \oplus p$:

This is a contradiction

p	p	$p \oplus p$
0	0	**0**
0	0	**0**
1	1	**0**
1	1	**0**

(c)

Construct a truth table for $(p \oplus q) \oplus r$:

p	q	r	$(p \oplus q)$	$\oplus\, r$
0	0	0	0	**0**
0	0	1	0	**1**
0	1	0	1	**1**
0	1	1	1	**0**
1	0	0	1	**1**
1	0	1	1	**0**
1	1	0	0	**0**
1	1	1	0	**1**

(d)

Construct a truth table for $(p \oplus p) \oplus p$:

p	p	p	$(p \oplus p)$	$\oplus\, p$
0	0	0	0	**0**
0	0	0	0	**0**
0	0	0	0	**0**
0	0	0	0	**0**
1	1	1	0	**1**
1	1	1	0	**1**
1	1	1	0	**1**
1	1	1	0	**1**

16.

(a)

Write a compound proposition that is true when exactly one of the three propositions $p, q,$ and r is true:

$$(p \vee q \vee r)$$

18.

Prove or disprove:

(c)

$$[(p \to q) \to r] \Longleftrightarrow [p \to (q \to r)]$$

This is true by way of the transitivity of $\to$ law.

19.

Verify the following logical equivalences using truth tables

(a)

Rule 12a: $[(p \to r) \wedge (q \to r)] \iff [(p \vee q) \to r]$

p	q	r	$[(p \to r)$	$\wedge$	$(q \to r)]$	$\leftrightarrow$	$[(p \vee q)$	$\to r]$
0	0	0	1	1	1	**1**	0	1
0	0	1	1	1	1	**1**	0	1
0	1	0	1	0	0	**1**	1	0
0	1	1	1	1	1	**1**	1	1
1	0	0	0	0	1	**1**	1	0
1	0	1	1	1	1	**1**	1	1
1	1	0	0	0	0	**1**	1	0
1	1	1	1	1	1	**1**	1	1

20.

(b)

22.

(a)

23.

24.

(a)

(b)

(c)

(d)