

# CISC 2210 Discrete Structures - Noson S. Yanofsky

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## 2.1

### 1.

Let  $p, q$  and  $r$  be the following propositions:

$p =$  "it is raining,"

$q =$  "the sun is shining,"

$r =$  "there are clouds in the sky."

Translate the following into logical notation, using  $p, q, r$ , and logical connectives.

#### (a)

It is raining and the sun is shining.

$$p \wedge q$$

#### (b)

If It is raining, then there are clouds in the sky.

$$p \rightarrow r$$

#### (c)

If It is not raining, then the sun is not shining and there are clouds in the sky.

$$\neg p \rightarrow (\neg q \wedge r)$$

#### (d)

The sun is shining if and only if it is not raining.

$$q \longleftrightarrow \neg p$$

(e)

If there are no clouds in the sky, then the sun is shining.

$$\neg r \rightarrow q$$

**2.**

(d)

(e)

**3.**

(a)

Give truth values of the propositions in parts (a) to (e) of Example 1:

(a) Julius Caesar was president of the United States: False

(b)  $2 + 2 = 4$ : True

(c)  $2 + 3 = 7$ : False

(d) The number 4 is positive and the number 3 is negative: False

(e) If a set has  $n$  elements, then it has  $2^n$  subsets: True

(Bonus)

(f)  $2^n + n$  is a prime number for infinitely many  $n$ : don't know...

(g) Every even integer greater than 2 is the sum of two prime numbers: no one knows... see "Goldbach's conjecture"

(b)

Do the same for parts (a) and (b) of Example 2:

(a)  $x + y = y + x$  for all  $x, y \in \mathbb{R}$ : True commutative property

(b)  $2^n = n^2$  for some  $n \in \mathbb{N}$ : True for  $\{2, 4\}$

**9.**

(a)

Show that  $n = 3$  provides one possible counterexample to the assertion " $n^3 < 3^n \forall n \in \mathbb{N}$ ":

(b)

Can you find any other counterexamples?