

$\alpha \rightarrow \beta \rightarrow \alpha$   
 $(\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \beta \rightarrow \gamma) \rightarrow (\alpha \rightarrow \gamma)$   
 $\alpha \rightarrow \beta \rightarrow \alpha \& \beta$   
 $\alpha \& \beta \rightarrow \alpha$   
 $\alpha \& \beta \rightarrow \beta$   
 $\alpha \rightarrow \alpha \vee \beta$   
 $\beta \rightarrow \alpha \vee \beta$   
 $(\alpha \rightarrow \gamma) \rightarrow (\beta \rightarrow \gamma) \rightarrow (\alpha \vee \beta \rightarrow \gamma)$   
 $(\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \neg \beta) \rightarrow \neg \alpha$   
 $\alpha \rightarrow \neg \alpha \rightarrow \beta$

$$\frac{\Gamma \vdash \varphi \rightarrow \psi \quad \Gamma \vdash \varphi}{\Gamma \vdash \psi}$$

MP

2

1

$$\frac{\alpha, \beta \vdash \alpha \quad \alpha \vdash \beta \rightarrow \alpha \quad \Gamma \vdash \alpha \quad \Gamma \vdash \alpha \rightarrow \beta}{\Gamma \vdash \beta \rightarrow \gamma \quad \Gamma \vdash \beta \quad \Gamma \vdash \alpha \rightarrow \beta \quad \Gamma \vdash \alpha \rightarrow \gamma}{\Gamma \vdash \alpha \rightarrow \gamma}$$

$$\frac{\alpha \rightarrow \beta, \alpha \rightarrow \beta \rightarrow \gamma, \alpha \vdash \gamma}{\alpha \rightarrow \beta, \alpha \rightarrow \beta \rightarrow \gamma \vdash \alpha \rightarrow \gamma}$$
$$\frac{\alpha \rightarrow \beta \vdash (\alpha \rightarrow \beta \rightarrow \gamma) \rightarrow (\alpha \rightarrow \gamma)}{\vdash (\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \beta \rightarrow \gamma) \rightarrow (\alpha \rightarrow \gamma)}$$

3

$$\frac{\alpha, \beta \vdash \alpha \quad \alpha, \beta \vdash \beta}{\alpha, \beta \vdash \alpha \& \beta} \quad \frac{\alpha \vdash \beta \rightarrow \alpha \& \beta}{\vdash \alpha \rightarrow \beta \rightarrow \alpha \& \beta}$$

4

$$\frac{\alpha \& \beta \vdash \alpha \& 0}{\alpha \& \beta \vdash \alpha} \quad \vdash \alpha \& \beta \rightarrow \alpha$$

5

$$\frac{\alpha \& \beta \vdash \alpha \& \beta}{\alpha \& \beta \vdash \beta} \quad \vdash \alpha \& \beta \rightarrow \beta$$

6

$$\frac{\alpha \vdash \alpha}{\alpha \vdash \alpha \vee \beta} \quad \vdash \alpha \rightarrow \alpha \vee \beta$$

7

$$\frac{\beta \rightarrow \beta}{\beta \vdash \alpha \vee \beta} \quad \vdash \beta \rightarrow \alpha \vee \beta$$

8

$$\frac{\Gamma, \alpha \vdash \alpha \quad \Gamma, \alpha \vdash \alpha \quad \Gamma, \beta \vdash \beta \rightarrow \gamma \quad \Gamma, \beta \vdash \beta}{\Gamma, \alpha \vdash \gamma \quad \Gamma, \beta \vdash \gamma \quad \Gamma \vdash \alpha \vee \beta} \quad \frac{\alpha \rightarrow \gamma, \beta \rightarrow \gamma, \alpha \vee \beta \vdash \gamma}{\alpha \rightarrow \gamma, \beta \rightarrow \gamma \vdash \alpha \vee \beta \rightarrow \gamma} \quad \frac{\alpha \rightarrow \gamma \vdash (\beta \rightarrow \gamma) \rightarrow (\alpha \vee \beta \rightarrow \gamma)}{\vdash (\alpha \rightarrow \gamma) \rightarrow (\beta \rightarrow \gamma) \rightarrow (\alpha \vee \beta \rightarrow \gamma)}$$

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$$\frac{\Gamma \vdash \alpha \rightarrow \beta \rightarrow \perp \quad \Gamma \vdash \alpha}{\Gamma \vdash \beta \rightarrow \perp} \quad \frac{\Gamma \vdash \alpha \rightarrow \beta \quad \Gamma \vdash \alpha}{\Gamma \vdash \beta} \quad \frac{\alpha \rightarrow \beta, \alpha \rightarrow \beta \rightarrow \perp, \alpha \vdash \perp}{\alpha \rightarrow \beta, \alpha \rightarrow \beta \rightarrow \perp \vdash \alpha \rightarrow \perp} \quad \frac{\alpha \rightarrow \beta \vdash (\alpha \rightarrow \beta \rightarrow \perp) \rightarrow \alpha \rightarrow \perp}{\vdash (\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \beta \rightarrow \perp) \rightarrow \alpha \rightarrow \perp}$$

10)  $\alpha \rightarrow (\alpha \rightarrow \perp) \rightarrow \beta$

$$\frac{\alpha, \alpha \rightarrow \perp \vdash \alpha \quad \alpha, \alpha \rightarrow \perp \vdash \alpha \rightarrow \perp}{\alpha, \alpha \rightarrow \perp \vdash \perp} \quad \frac{\alpha, \alpha \rightarrow \perp \vdash \perp}{\alpha, \alpha \rightarrow \perp \vdash \beta} \quad \frac{\alpha \vdash (\alpha \rightarrow \perp) \rightarrow \beta}{\vdash \alpha \rightarrow (\alpha \rightarrow \perp) \rightarrow \beta}$$

$\langle \text{Выражение} \rangle ::= \langle \text{Дизъюнкция} \rangle \mid \langle \text{Дизъюнкция} \rangle \text{'->'} \langle \text{Выражение} \rangle$   
 $\langle \text{Дизъюнкция} \rangle ::= \langle \text{Конъюнкция} \rangle \mid \langle \text{Дизъюнкция} \rangle \text{'\&'} \langle \text{Конъюнкция} \rangle$   
 $\langle \text{Конъюнкция} \rangle ::= \langle \text{Отрицание} \rangle \mid \langle \text{Конъюнкция} \rangle \text{'\&'} \langle \text{Отрицание} \rangle$   
 $\langle \text{Отрицание} \rangle ::= \text{'!'} \langle \text{Отрицание} \rangle \mid \langle \text{Переменная} \rangle \mid \text{'('} \langle \text{Выражение} \rangle \text{'\>'}$   
 $\langle \text{Переменная} \rangle ::= \text{'A' ... 'Z'} \{ \text{'A' ... 'Z'} \mid \text{'0' ... '9'} \mid \text{'.'} \}^*$

