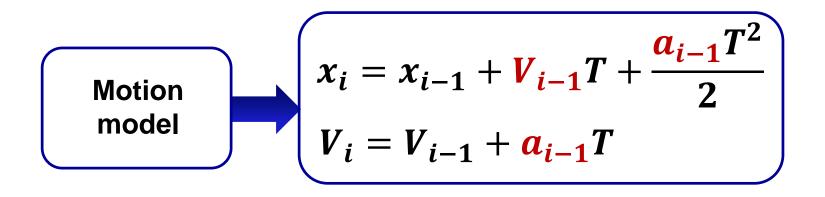


"Experimental Data Processing"

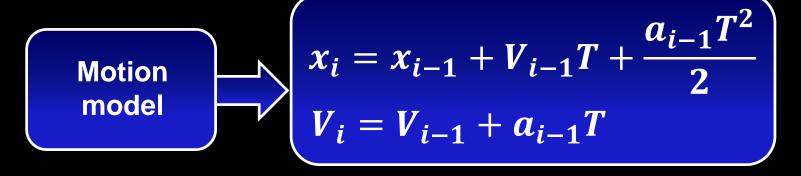
Laboratory work 7
Analysis of accuracy decrease of filtration in conditions of biased state and measurement noise

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Moving object which trajectory is disturbed by random acceleration



State equation

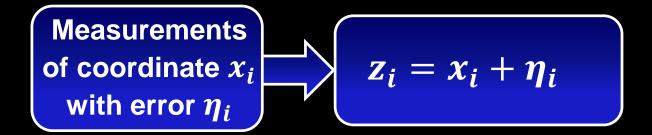


$$X_i = \begin{vmatrix} x_i \\ V_i \end{vmatrix}$$
 State information about the state of system at time i

State equation
$$X_i = \Phi X_{i-1} + G a_{i-1}$$

$$\Phi = \begin{vmatrix} 1 & T \\ 0 & 1 \end{vmatrix}$$
 Transition matrix $G = \begin{vmatrix} T^2/2 \\ T \end{vmatrix}$ Input matrix

Measurement equation



Measurement equation

$$z_i = HX_i + \eta_i$$

$$X_i = \begin{vmatrix} x_i \\ V_i \end{vmatrix}$$

Prediction procedure in Kalman filter

1 Prediction (extrapolation)

Prediction of state vector at time i using i-1 measurements

$$X_{i,i-1} = \Phi_{i,i-1} X_{i-1,i-1}$$

Prediction error covariance matrix

$$P_{i,i-1} = \Phi_{i,i-1}P_{i-1,i-1}\Phi_{i,i-1}^T + Q_i$$

$$P_{i,i-1} = E[(X_i - X_{i,i-1})(X_i - X_{i,i-1})^T]$$

 $X_{i,i-1}$

First subscript *i* denotes time on which the prediction is made

Second subscript i-1 represents the number of measurements to get $X_{i,i-1}$

Filtration procedure in Kalman filter

② Filtration Adjustment of predicted estimate

Improved estimate by incorporating a new measurement

$$X_{i,i} = X_{i,i-1} + K_i(z_i - HX_{i,i-1})$$
Residual

Filter gain, weight of residual

$$K_{i} = P_{i,i-1}H_{i}^{T}(H_{i}P_{i,i-1}H_{i}^{T} + R_{i})^{-1}$$

Filtration error covariance matrix

$$P_{i,i} = (I - K_i H_i) P_{i,i-1}$$

$$P_{i,i} = E[(X_i - X_{i,i})(X_i - X_{i,i})^T]$$

Standard Kalman filter provides optimal estimate

State noise and measurement noise are uncorrelated and unbiased

In practice these assumptions are often not true

Analysis and modifications of Kalman filter

Biased state noise



Random acceleration is biased
$$E[a_i] = q \neq 0$$

How to take bias of acceleration into account in Kalman filter algorithm?

Prediction procedure in Kalman filter

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Prediction procedure in Kalman filter taking into account bias of state noise

1 Prediction (extrapolation)

Prediction of state vector at time i using i-1 measurements

$$X_{i,i-1} = \Phi_{i,i-1}X_{i-1,i-1} + Gq$$

Prediction error covariance matrix

$$P_{i,i-1} = \Phi_{i,i-1}P_{i-1,i-1}\Phi_{i,i-1}^T + Q_i$$

$$P_{i,i-1} = E[(X_i - X_{i,i-1})(X_i - X_{i,i-1})^T]$$

 $X_{i,i-1}$

First subscript *i* denotes time on which the prediction is made

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Correlated state noise

In practice correlated noise is often presented as a Gauss-Markov first-order process

Random acceleration
$$a_i = e^{-\lambda T} a_{i-1} + \zeta_i$$

Uncorrelated noise with variance
$$\sigma_{\zeta}^2 = \sigma_a^2 (1 - e^{-2\lambda T})$$

Value that is inverse to correlation interval

$$\lambda = 1000$$
 a $_i$ - uncorrelated noise $\lambda = 0.1$ α_i - correlated noise

$$\sigma_a^2$$
 Variance of acceleration