

## Laboratory work 11

### Noise statistics identification to construct tracking filter of a moving object

Performance - Tuesday, October 17, 2017  
Due to submit a performance report – October 19, 2017

The objective of this laboratory work is to develop a tracking filter of a moving object on the basis of noise statistics identification which specification is crucial for optimal assimilation output. This will bring a deeper understanding of main difficulties of practical Kalman filter implementation and skills to overcome these difficulties.

This laboratory work is performed in the class by students as in teams of 2-4 on October 17 2017 and the team will submit one document reporting about the performance till October 19, 2017. Within your group, you may discuss all issues openly, and discuss and debate until you reach a consensus.

*Here is the recommended procedure:*

#### ***Part I. Noise statistics identification***

1. Generate a true trajectory  $X_i$  of an object motion disturbed by normally distributed **BIASED** random acceleration. Bias (mathematical expectation) of random noise  $q = 6$

$$a_{i-1}^{biased} = a_{i-1} + q$$
$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}^{biased}T^2}{2}$$
$$V_i = V_{i-1} + a_{i-1}^{biased}T$$

Size of trajectory is **500 000** points.

Initial conditions:  $x_1 = 5; V_1 = 1; T = 1$

Standard deviation of  $a_i, \sigma_a = 3$

2. Generate measurements  $z_i$  of the coordinate  $x_i$

$$z_i = x_i + \eta_i$$

$\eta_i$  –normally distributed random noise with zero mathematical expectation and standard deviation of  $\sigma_\eta = 10$ .

3. Identify bias  $q$ , standard deviation  $\sigma_a$ , and standard deviation  $\sigma_\eta$  using measurements  $z_i$ .

Consult page 45,

**Topic\_5\_Part\_I\_Model construction at state space under uncertainty.pdf**

#### ***Part II. Tracking filter of a moving object on the basis of noise statistics identification and sensitivity analysis of assimilation output to choice of noise statistics***

1. Generate again a true trajectory  $X_i$  and measurements  $z_i$  of a moving object.

The size of trajectory in this case is **200 points**.

Use the same noise statistics and initial conditions as in part I, items 1,2.

Bias (mathematical expectation) of random noise  $q = 6$

Variance of noise  $a_i, \sigma_a = 3$

Variance of noise  $\eta_i, \sigma_\eta = 10$

Initial conditions:  $x_1 = 5; V_1 = 1; T = 1$

2. Obtain estimates of state vector  $X = \begin{bmatrix} x \\ V \end{bmatrix}$  by Kalman filter and use identification results of  $q, \sigma_a, \sigma_\eta$  obtained in part I where it is needed in the estimation algorithm. Note that bias and covariance matrix of state noise  $Ga_i$  depends also on input matrix  $G$ .

Use initial conditions

Initial filtered estimate  $X_0 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

Initial filtration error covariance matrix

$$P_{0,0} = \begin{bmatrix} 10^{10} & 0 \\ 0 & 10^{10} \end{bmatrix}$$

3. Make  $M = 500$  runs of filter and compare true estimation error of coordinate  $x_i$  with errors of estimation  $P_{i,i}$  provided by Kalman filter algorithm. Plot results
4. Repeat item 3, but change the values of noise statistics and analyze sensitivity of Kalman filter output to these changes
  - (a) Instead of identified bias  $q$  of noise  $a_i$  use  $q = 0$ ,  
 $\sigma_a, \sigma_\eta$  – identified values in part I
  - (b) Instead of identified standard deviation  $\sigma_a$  use value 10 times greater  $\sigma_a = 10\sigma_a$ ,  
 $q$  and  $\sigma_\eta$  – identified values in part I
  - (c) Instead of identified standard deviation  $\sigma_a$  use value 10 times less  $\sigma_a = \sigma_a/10$   
 $q$  and  $\sigma_\eta$  – identified values in part I
  - (d) Instead of identified standard deviation  $\sigma_\eta$  use value 10 times greater  $\sigma_\eta = 10\sigma_\eta$ ,  
 $q$  and  $\sigma_a$  – identified values in part I
  - (e) Instead of identified standard deviation  $\sigma_\eta$  use value 10 times less  $\sigma_\eta = \sigma_\eta/10$ ,  
 $q$  and  $\sigma_a$  – identified values in part I

### ***Performance report***

1. Performance report should contain all the items listed
2. The code should be commented. It should include:
  - Title of the laboratory work, for example  
 % Converting a physical distance to a grid distance using least-square method
  - The names of a team, indication of Skoltech, and date, for example,  
 %Tatiana Podladchikova, Skoltech, 2017  
 Main procedures also should be commented, for example  
 %13-month running mean  
 ...here comes the code
3. If your report includes a plot, then it should contain: title, title of x axis, title of y axis, legend of lines on plot.