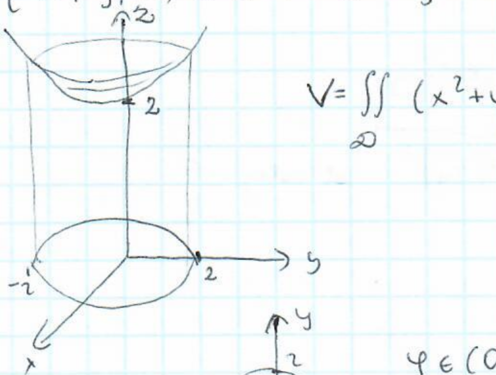
- Zadanie 1.

Obliczyć objętość bryły

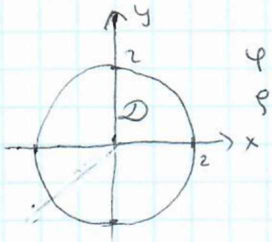
$$\{(x,y,z)\in\mathbb{R}^3:\ x^2+y^2\leq 4,\ 0\leq z\leq x^2+y^2+2\}$$



$$V=\iint_{D}(x^2+y^2+2)\,dx\,dy=\int_0^{2\pi}\left(\int_0^2\left((\varrho\cos\varphi)^2+(\varrho\sin\varphi)^2+2\right)\varrho\,d\varrho\,d\varphi=\right.$$

$$=\int_0^{2\pi}\left(\int_0^2(\varrho^2+2)\varrho\,d\varrho\right)d\varphi=$$

$$=\int_0^{2\pi}\int_0^2(\varrho^3+2\varrho)\,d\varrho\,d\varphi=$$



$\varphi\in(0,2\pi)$

$\varrho\in(0,2)$

$$=\int_0^{2\pi}\left[\frac{\varrho^4}{4}+2\frac{\varrho^2}{2}\right]_0^2 d\varphi=\int_0^{2\pi}\left(\frac{16}{4}+4\right)d\varphi=$$

$$=8\int_0^{2\pi}d\varphi=8\varphi\Big|_0^{2\pi}=8\cdot 2\pi=16\pi$$

- Zadanie 2

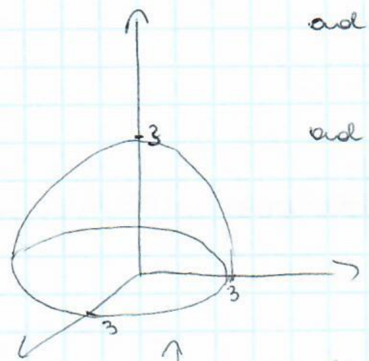
Obliczyć objętość bryły ograniczonej powierzchniami $z=\sqrt{9-x^2-y^2}$ i $z=3$.

a) korzystając ze wzoru

b) używając całki podwójnej

$(z=\sqrt{9-x^2-y^2}$ i $z=3)$

$z\geq 0$



ad a) $V=\frac{1}{2}V_{kuli}=\frac{1}{2}\frac{4}{3}\pi r^3\overset{r=3}{=}\frac{2}{3}\pi\cdot 27=\frac{54}{3}\pi=18\pi$

ad b) $$V=\iint_{D}\sqrt{9-x^2-y^2}\,dx\,dy=\int_0^{2\pi}\left(\int_0^3\sqrt{9-\varrho^2\cos^2\varphi-\varrho^2\sin^2\varphi}\,\varrho\,d\varrho\,d\varphi=\right.$$

$$=\int_0^{2\pi}\left(\int_0^3\sqrt{9-\varrho^2}\,\varrho\,d\varrho\right)d\varphi=$$

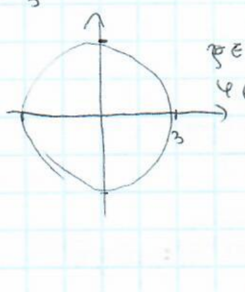
$t=9-\varrho^2$

$dt=-2\varrho\,d\varrho$

$\frac{dt}{-2}=\varrho\,d\varrho$

$\varrho=0\Rightarrow t=9$

$\varrho=3\Rightarrow t=0$



$\varrho\in(0,3)$

$\varphi\in(0,2\pi)$

$$=\int_0^{2\pi}\int_9^0\sqrt{t}\,\frac{dt}{-2}\,d\varphi=\int_0^{2\pi}-\frac{1}{2}\left(\frac{t^{3/2}}{3/2}\right)\Bigg|_9^0 dt\,d\varphi=$$

$$=\int_0^{2\pi}\left[-\frac{1}{2}\cdot\frac{2}{3}t^{3/2}\right]_9^0 dt\,d\varphi=\int_0^{2\pi}\left(-\frac{1}{3}\cdot 0+\frac{1}{3}\cdot 27\right)d\varphi=9\varphi\Big|_0^{2\pi}=2\pi\cdot 9=18\pi$$