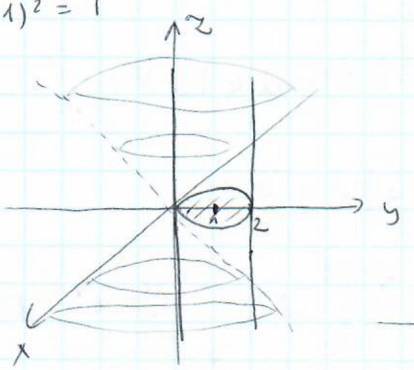
Zadanie 3

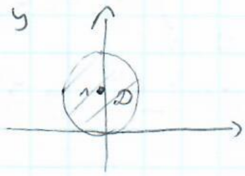
Obliczyć pole powierzchni części stożka $z^2 = x^2 + y^2$ leżącej wewnątrz walca $x^2 + y^2 = 2y$

$x^2 + y^2 = 2y$

$x^2 + y^2 - 2y = 0$

$x^2 + (y-1)^2 = 1$





bo płat górny i dolny

$$|S| = 2 \iint_D \sqrt{1 + (z'_x)^2 + (z'_y)^2}\, dx\,dy \overset{(*)}{=}$$

$$z = \sqrt{x^2 + y^2}$$

$$z'_x = \frac{1}{\cancel{2}\sqrt{x^2+y^2}} \cdot \cancel{2}x = \frac{x}{\sqrt{x^2+y^2}}$$

$$z'_y = \frac{1}{2\sqrt{x^2+y^2}}\, 2y = \frac{y}{\sqrt{x^2+y^2}}$$

A zatem:

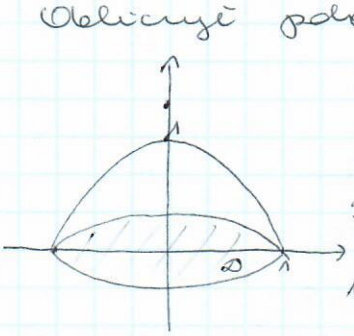
$$\sqrt{1 + (z'_x)^2 + (z'_y)^2} = \sqrt{1 + \frac{x^2}{x^2+y^2} + \frac{y^2}{x^2+y^2}} = \sqrt{1 + \frac{x^2+y^2}{x^2+y^2}} = \sqrt{2}$$

$$\overset{(*)}{=} 2 \iint_D \sqrt{2}\, dx\,dy = 2\sqrt{2} \iint_D dx\,dy = 2\sqrt{2}\,(\text{pole } D) = 2\sqrt{2} \cdot \pi \cdot 1^2 = 2\sqrt{2}\,\pi$$

NIE LICZYĆ TEJ CAŁKI !!!

- Zadanie 4

Obliczyć pole części paraboloidy $z = 1 - x^2 - y^2$ odciętej płaszczyzną $z=0$



$z = 1 - x^2 - y^2$

$z'_x = -2x$

$z'_y = -2y$

$$\sqrt{1 + (z'_x)^2 + (z'_y)^2} = \sqrt{1 + (2x)^2 + (2y)^2} = \sqrt{1 + 4x^2 + 4y^2}$$

$$|S| = \iint_D \sqrt{1 + 4x^2 + 4y^2}\, dx\,dy = \int_0^1 \left[\int_0^{2\pi} \sqrt{1 + 4(\rho\cos\varphi)^2 + 4(\rho\sin\varphi)^2}\, d\varphi \right] \rho\, d\rho =$$

$$= \int_0^1 \left[\int_0^{2\pi} \sqrt{1 + 4\rho^2}\, d\varphi \right] \rho\, d\rho = 2\pi \int_0^1 \sqrt{1+4\rho^2}\, \rho\, d\rho = 2\pi \int_1^5 t^{\frac{1}{2}} \frac{dt}{8} = \frac{2\pi}{8} \int_1^5 t^{\frac{1}{2}} dt = \frac{\pi}{4} \cdot \frac{2}{3} t^{3/2} \Big|_1^5 =$$

$$\int_0^{2\pi} d\varphi = \varphi \Big|_0^{2\pi} = 2\pi$$

$$\left\{ \begin{aligned} t &= 1 + 4\rho^2 \\ dt &= 8\rho\, d\rho \\ \frac{dt}{8} &= \rho\, d\rho \\ \rho = 0 &\Rightarrow t = 1 \\ \rho = 1 &\Rightarrow t = 5 \end{aligned} \right\}$$

$$= \frac{\pi}{6} \left(5\sqrt{5} - 1 \right)$$