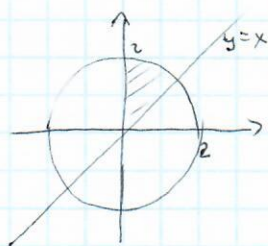


Zadanie 9

Obliczyć całkę podwójną

$$\iint_D \frac{1}{x^2+y^2+6} dx dy \quad \text{gdzie } D = \{(x,y) : x^2+y^2 \leq 4, 0 \leq x \leq y\}$$



$$\begin{aligned} & \int_0^2 \left[\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{(s \cos \varphi)^2 + (s \sin \varphi)^2 + 6} d\varphi s ds \right] = \\ &= \int_0^2 \left[\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{s}{s^2 + 6} d\varphi \right] ds = \int_0^2 \frac{s}{s^2 + 6} \varphi \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} ds = \\ &= \int_0^2 \left(\frac{s}{s^2 + 6} \right) \left(\frac{\pi}{2} - \frac{\pi}{4} \right) ds = \int_0^2 \frac{s}{s^2 + 6} \frac{\pi}{4} ds = \frac{\pi}{4} \int_0^2 \frac{s}{s^2 + 6} ds = \\ &= \frac{\pi}{4} \cdot \frac{1}{2} \int_0^2 \frac{2s}{s^2 + 6} ds = \left[\frac{\pi}{8} \ln |s^2 + 6| \right]_0^2 = \frac{\pi}{8} (\ln 10 - \ln 6) = \frac{\pi}{8} \ln \frac{10}{6} = \frac{\pi}{8} \ln \frac{5}{3} \end{aligned}$$