5G WIRELESS TECHNOLOGIES

Automatic Gain Control

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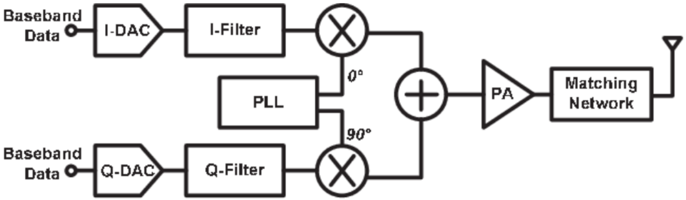
Operations to perform in analog domain

Transmitter

Tasks for the transmitter:

- Digital-analog conversion;
- Smoothing-filter application;
- Quadrature modulation;
- Power amplification.



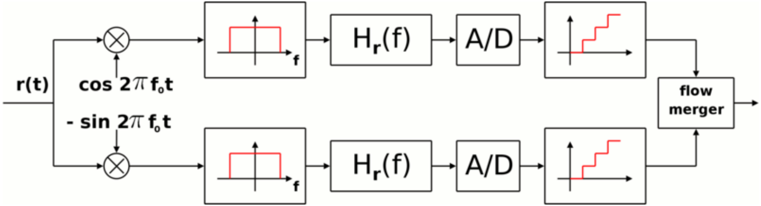
(a)

Operations to perform in analog domain

Receiver

Tasks for the receiver:

- Low-noise amplification;
- Automatic gain control;
- Image channel suppression and mixig (if necessary);
- Quadrature demodulation;
- Double-frequency component suppression;
- Anti-aliasing filter application;
- Analog-to-digital conversion.



Operations to perform in digital domain

- Tasks for the transmitter:
  - User data mapping to symbols and zeros insertion;
  - Pulse-shaping filtration;
  - Pre-distortions.
- Tasks for the receiver:
  - Distorting effect mitigation;
  - Timing and carrier recovery;
  - Matched filtration;
  - Detection.

Distorting effects in the modem

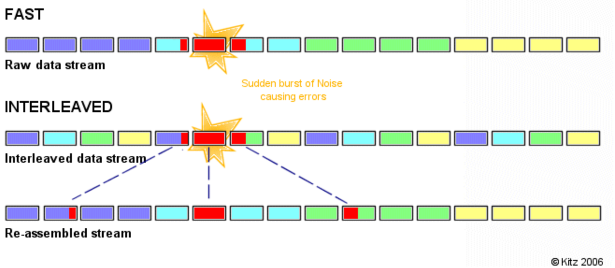
- Desynchronization based effects:
  - Digital scheme clock frequency and phase difference;
  - Carrier frequency and phase difference.
- Circuitry mismatch effects:
  - Non-corresponding levels;
  - Transmitter/receiver IQ-imbalances (gain, phase, DC offset and delay);
  - Echoing in the cable.
- Non-ideal transmission impact:
  - Frequency-selective fading (multipath);
  - Nonlinear distortions.
- Additive, phase and radial (amplitude) noises.

Distorting effects combatting

- Compensable effects:
  - Synchronization establishment for the desynchronization based effects (only receiver);
  - Calibration for static hardware mismatch (both transmitter and receiver);
  - Adaptation for dynamical hardware mismatch (only receiver);
  - Dynamical compensation of time-varying predictable effects (only receiver);
  - Static or dynamical pre-distortion (only transmitter).
- Non-compensable effects:
  - Alternative parallel transmission—time and frequency diversity;
  - Error correction codes and interleaving for burst errors;
  - Change transmission conditions to lower effect, e.g., space diversity.

FEC with interleaving

- Forward error correction (FEC) codes sacrifice the capacity to be able correct errors;
- Burst errors—multiple successive erroneous symbols;
- Interleaving process to distribute among multiple FEC frames.

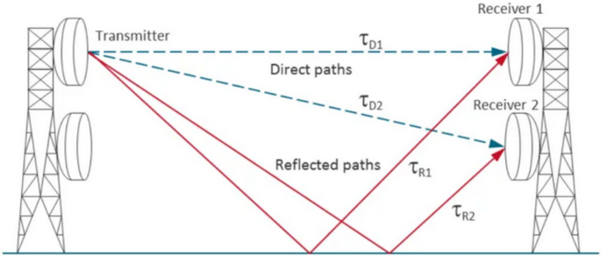


FEC coding $\rightarrow$ Interleaving $\rightarrow$ Transmission $\rightarrow$ Deinterleaving $\rightarrow$ FEC decoding.

Space diversity

- Necessary for very deep fading (both flat and frequency-selective);
- Requires either two receivers (Rx diversity) or two transmitters (Tx diversity);
- In case of the receiver diversity, the best of two signals or processing of their sum;
- In case of the transmitter diversity, the sum of two signal does not have notches deeper than 3 dB:

$$s_{\text{rx}} = \frac{1}{2}(s_{\text{tx1}} + s_{\text{tx2}}).$$



Effects to be compensated using dynamical adaptation

Compensable effects:

- Synchronization establishment for the desynchronization based effects (only receiver);
  - Digital scheme clock frequency and phase difference;
  - Carrier frequency and phase difference.
- ~~Calibration for static hardware mismatch (both transmitter and receiver)~~;
- Adaptation for dynamical hardware mismatch (only receiver);
  - Non-corresponding levels;
  - Only receiver IQ-imbalances (gain, phase, DC offset and delay);
  - Echoing in the cable.
- Dynamical compensation of time-varying predictable effects (only receiver);
  - Frequency-selective fading (multipath).
- ~~Static or dynamical pre-distortion (only transmitter)~~.

~~Non-compensable effects~~.

Main dynamic system construction stages

The following requirements must be fulfilled to create a dynamical system:

- Define a way how to compensate a distortion if its amount in the signal is static precisely known—compensation application;
- Understand how distortion affects a signal and find a mathematical expression that shows the consequences of its presence—cost function;
- Find an algorithm how to calculate an amount of compensation from the cost function—compensation algorithm.

Requirements for the signal:

- Wide-sense stationary and ergodic;
- All constellation points equally probable and pseudo-random.

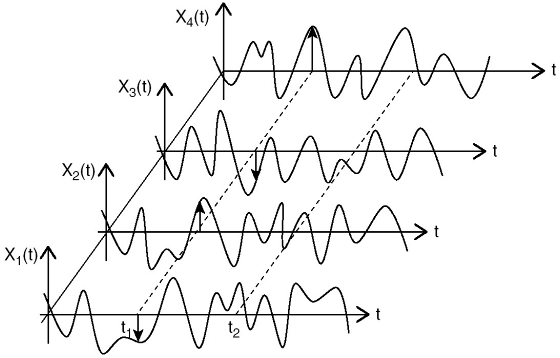
Stationary processes I

- The set of N possible trials is called ensemble;
- Mean value:

$$\overline{s(t)} = E[s(t)] = \frac{1}{N}\sum_{n=0}^{N-1} s_n(t)$$

- Autocorrelation function:

$$\begin{aligned} K(t_1, t_2) &= E[s(t_1)s(t_2)] \\ &= \frac{1}{N}\sum_{n=0}^{N-1} s_n(t_1)s_n(t_2) \end{aligned}$$



Stationary processes II

Wide-sense stationarity requirements:

- Mean value is time independent:

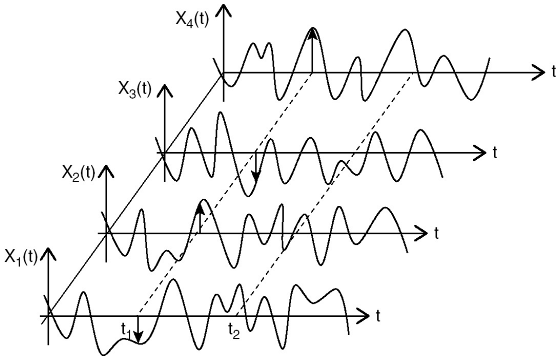
$$\overline{s(t_1)} = \overline{s(t_2)} \quad \forall t_1, t_2$$

- Autocorrelation function is only time shift dependent:

$$K(t_1, t_2) = K(t_1 - t_2) \quad \forall t_1, t_2$$

Ergodicity allows averaging through time:

$$\overline{s(t)} = \frac{1}{T} \int_0^T s(t)\,\mathrm{d}t$$



Cost function

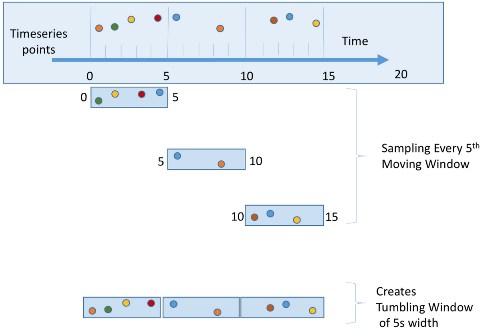
- Cost function or target function (mērķa funkcija) shows an amount of the distortion in the signal.
- Must not be obligatory equal to zero in the case of the distortion absence, e.g., it can obtain minimal value or aim to the predefined level.
- In practical applications, all expectations are calculated assuming ergodicity property of the signal.

Examples:

- DC offset canceller cost function $E[s(t)]$ must be zero.
- Automatic gain control cost function $E[s^2(t)]$ must be equal to the necessary signal power level (in wats).
- Multipath mitigation cost function $E[(s(t) - \hat{s}(t))^2]$ tries to minimize squared error.

Jumping window averaging

- As you could notice, all cost fuctions contained mathematical expectation $E[]$.
- Use time averaging because of impossiblity of ensemble averaging during real-time adaptation (must be ergodic).
- Averaging within window length N, afterward new accumulation process;
- Advantage: simple implementation;
- Disadvantage: mean value updates once per N samples.



Sliding window averaging

Idea

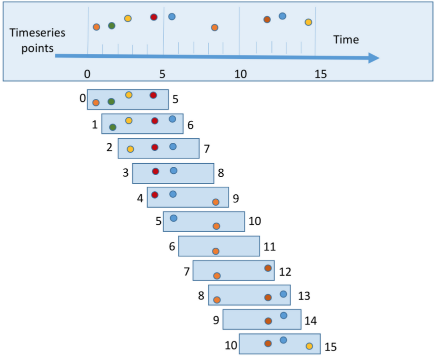
- In sliding window approach, each clock cycle the average of last N samples is recalculated:

$$\overline{s}[k] = \frac{1}{N}\sum_{n=0}^{N-1} s[k-n];$$

- Recources highly consuming implementation (non-optimized);
- Possible optimization:

$$\overline{s}[k] = \overline{s}[k-1] + \frac{1}{N}\big(s[k] - s[k-N+1]\big);$$

- The necessity to keep last N samples of the averaging signal.



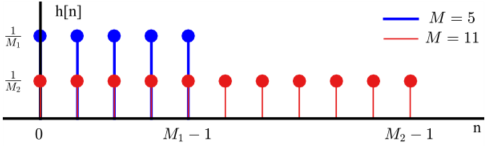
Sliding window averaging

Filter representation—impulse response

- In sliding window averaging,

$$\overline{s}[k] = \frac{1}{N}\sum_{n=0}^{N-1} s[k-n]$$

  can be considered as a convolution wight N–clock cycles wide and $1/N$ high impulse.



- Finite impuse response filter with square pulse impulse response;
- The Fourier transform of the square pulse:

$$H(f) = S_m \tau \,\mathrm{sinc}\,(\pi f \tau)\,.$$

- Assume amplitude $S_m = 1/N$, and impulse length $\tau = N/f_d$ for sampling frequency $f_d = 2$.

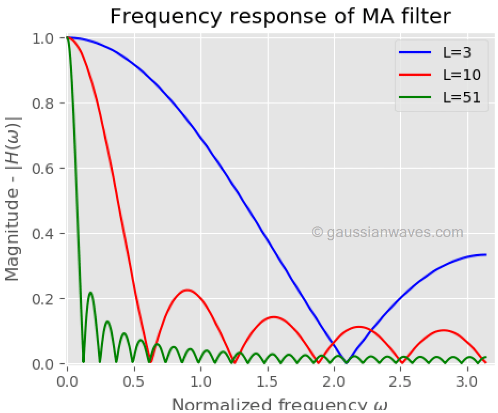
Sliding window averaging

Filter representation—frequency response

- The substitution of S_m and τ into transfer function yields to

$$H(f) = \frac{1}{2}\,\mathrm{sinc}\left(\frac{\pi f N}{2}\right);$$

- **Conclusion:** sliding window averaging is a low-pass filter;
- The window length N defines the cut-off frequnecy of the filter.
- The first zero of $\sin(\pi f N/2)$ is at $\pi/2$; thus, from $fN = 1$ cut-off frequency is $f = 1/N$.



Exponential window averaging

- Infinite impulse response filters are less resource consuming for the same impulse response lengh;
- The simplest low-pas IIR filter

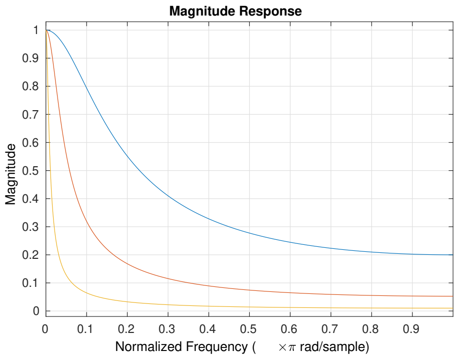
$$y[k] = (1 - 1/N)y[k-1] + 1/Nx[k];$$

- Transfer function:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1/N}{1-(1-1/N)z^{-1}};$$

- Impulse response:

$$h[n] = \frac{1/N}{(1-1/N)^n}.$$



Compensation application

- Assume the input of the compensating block is $x[n]$, and its output is $y[n]$.
- The block has to implement compensation application function:

$$y[n] = f(x[n]);$$

- Each time moment, there is a set of parameters $\mathbf{a}$ in the module that establishes locally static compensation (probably, explicitly given).

Examples:

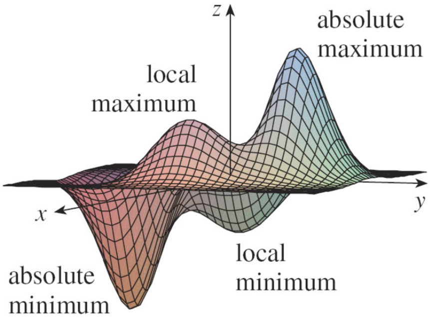
- DC offset canceller compensation application function $f : y[n] = x[n] - \overline{x}$, where $\overline{x}$ is calculated DC-offset value.
- Automatic gain control compensation application function $f : y[n] = ax[n]$, where a is a gaining coefficient.
- Equalizer compensation application function:

$$f : y[n] = \sum_{k=0}^{K} x[n-k]h[k],$$

where $h[k]$ is equalizer's impulse response.

Compensation algorithm

- Defines, how to obtain a set of locally static compensation parameters from the cost function.
- Can be given explicitly (DC-offset in previous examples). In this case, straight application.
- If explicit definition is not available, find a set of parameters to minimize the cost function—least mean square algorithm.
- LMS is very resource consuming approach; updates are rare.
- Approximation of the cost function minimization process—stochastic gradient.



Stochastic gradient

- We have a set of parameters $\mathbf{a}$ to calculate from cost function. Best solution—LMS.
- Alternatively, calculate increment for $\mathbf{a}$. We need to keep current values $\mathbf{a}[n]$.
- A cost function $J[n]$ is $y[n] = f(x[n], \mathbf{a})$ dependent, possibly, with additional operations.
- To find necessary direction, where to shift parameters to lower cost function, use gradient:

$$\nabla = \frac{\partial}{\partial a_0}\overrightarrow{a_0} + \frac{\partial}{\partial a_1}\overrightarrow{a_1} + \frac{\partial}{\partial a_2}\overrightarrow{a_2} + \cdots ,$$

  which shows the direction of the steepest rise of the function.
- Therefore, to obtain an update of the current parameter set,

$$\mathbf{a}[n+1] = \mathbf{a}[n] - \mu \nabla J[n],$$

  where μ is a step size coefficient.

Stochastic gradient application examples

DC canceller

- Application of the DC canceller correction is

$$f : y[n] = x[n] - \overline{x}[n-1],$$

where $a[n] \equiv \overline{x}[n]$ is a DC-offset value, single parameter of the block.

- A cost function for the DC canceller:

$$J[n] = y^2[n] = (x[n] - \overline{x}[n-1])^2.$$

- Calculating a gradient

$$\overline{x}[n] = \overline{x}[n-1] - \mu \frac{\partial}{\partial \overline{x}[n-1]} (x[n] - \overline{x}[n-1])^2 = \overline{x}[n-1] + 2\mu(x[n] - \overline{x}[n-1]) = \overline{x}[n-1] + 2\mu y[n].$$

- Thus, we just have to average the output of the DC canceller block.

Stochastic gradient application examples

Automatic gain control

- Application of the AGC correction is

$$f : y[n] = x[n]a[n-1],$$

where $a[n]$ is a gain value, also single parameter of the block.

- A cost function for the AGC:

$$J[n] = (y^2[n] - PW)^2 = (a[n-1]^2 x^2[n] - PW)^2,$$

where PW is necessary power level, not an adaptation parameter.

- Calculating a gradient

$$a[n] = a[n-1] - \mu \frac{\partial (y^2[n] - PW)^2}{\partial a[n-1]} = a[n-1] - 4\mu (y^2[n] - PW) a[n-1] x^2[n].$$

Stochastic gradient application examples

Equalizer—LMS

- Application of the equalizer correction is

$$f : y[n] = \sum_{k=0}^{k-1} x[n-k]h[n-1,k],$$

where $\mathbf{a}[n] \equiv \mathbf{h}[n-1]$ is a impulse response of th equalizer.

- A cost function for the equalizer (LMS):

$$J[n] = (y[n] - \hat{y}[n])^2 = (\sum_{k=0}^{k-1} x[n-k]h[n-1,k] - \hat{y}[n])^2.$$

- Calculating a gradient

$$h[n,k] = h[n-1,k] - \mu \frac{\partial (y[n] - \hat{y}[n])^2}{\partial h[n-1,k]} = h[n-1,k] - 2\mu(y[n] - \hat{y}[n])x[n-k].$$

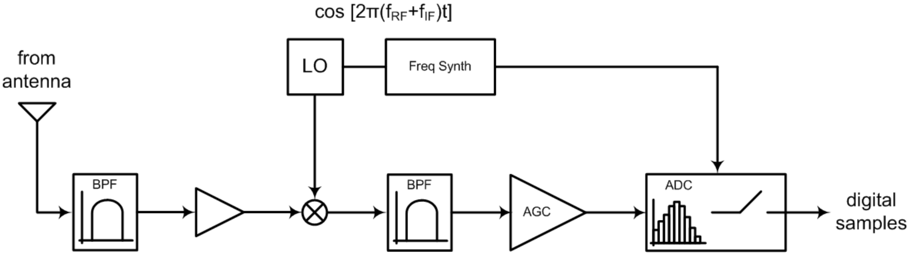
Automatic gain control—objectives

Analog domain AGC

- Imagine, we have an amplifier with the constant gain G. After analog processing and demodulation, the signal is passed to the ADC with input voltage range from -1 V to 1 V.
- Let us calculate a gain for this system.
- Assume, input signal is in the range from $-100\,\mu$V to $100\,\mu$V. Than gain should be $G = 10000$!
- For the input signal is in the range from -10 mV to 10 mV, the same gain $G = 10000$ leads to ADC input voltage range from -100 V to 100 V.
- Solution is automatic gain control (AGC) in the analog domain.

Automatic gain control—objectives

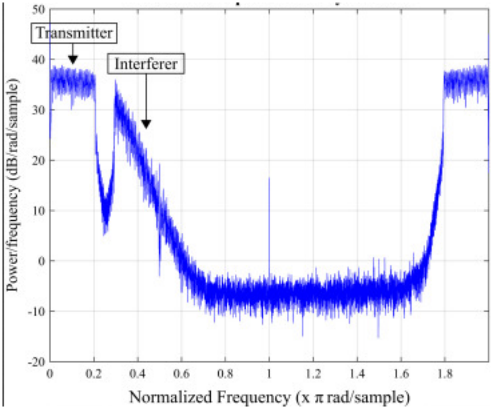
Analog domain AGC placement



Automatic gain control—objectives

Digital domain AGC

- Slight level misadjustemnt at the input of the digital part (prior to ADC);
- Power changes after filtration (adjacent channel or high noise level).
- Where to place AGC in the digital domain?



Simple digital automatic gain control

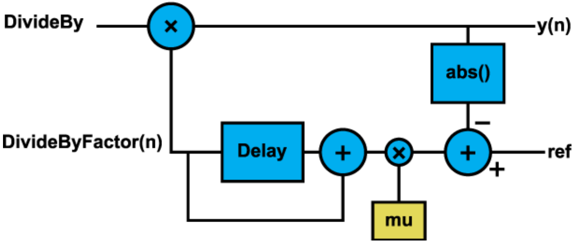
- Has to perform the following operation:

$$y[n] = x[n]a[n-1],$$

  where $a[n]$ is a gain value, also single parameter of the block.
- Needs a power level PW to be defined;
- Gain increment is expressed as

$$a[n] = a[n-1] - 4\mu(y^2[n] - PW)a[n-1]x^2[n]$$

- Possible optimizations?



Noise and signal level

- Correction application:

$$y[n] = x[n]a[n-1];$$

- In the case of ideal $a[n]$, becomes:

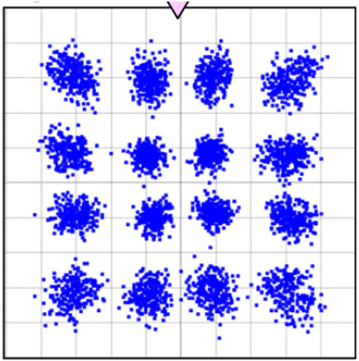
$$a[n] = a[n-1] - w[n],$$

  where $w[n]$ is noise;

- Substituting in $y[n]$,

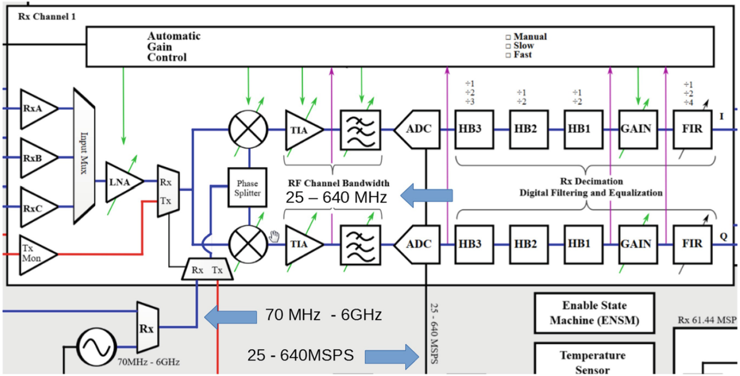
$$y[n] = x[n]a[n-1] + x[n]w[n-1];$$

- AGCs introduce amplitude noises;
- Choose step-size coefficient μ as small as possible.



System of multiple AGCs

- Should be placed after each filtration block:
  1. Matched filter;
  2. Digital anti-aliasing filter.
- Requirement:
  1. Use whole amplitude range, e.g., from -1024 to 1023 for 11-bit signals;
  2. Prevent signal clipping.
- Each next AGC step-size coefficient should be higher.



Let's try now!