5G WIRELESS TECHNOLOGIES

Basics of Quadrature Amplitude Modulation

Deniss Kolosovs
Deniss.Kolosovs@rtu.lv

Riga Technical university

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Arbitrary analog modulation

- A modulation process expands band occupied by the signal: $\Delta f = 2f_{\text{max}}$ for AM and $\Delta f = 2(f_{\text{max}} + f_{\text{dev}})$ for FM.
- How to enhance spectral efficiency? Change all parameters simultaneously!
- Instant frequency and instant phase of the signal are related by $f(t) = \frac{1}{2\pi}\frac{\mathrm{d}\varphi(t)}{\mathrm{d}t}$;
- Arbitrary modulated signal:

$$s_{\text{mod}}(t) = S_m(t)\cos\Big(2\pi f_0 t + \varphi(t)\Big).$$

- How to distinguish?

Quadrature components I

$$
\begin{aligned}
s_{\mathsf{mod}}(t) &= S_m(t)\cos\Big(2\pi f_0 t + \varphi(t)\Big) \\
&= S_m(t)\cos\varphi(t)\cos 2\pi f_0 t - S_m(t)\sin\varphi(t)\sin 2\pi f_0 t \\
&= I(t)\cos 2\pi f_0 t - Q(t)\sin 2\pi f_0 t
\end{aligned}
$$

Quadrature components:

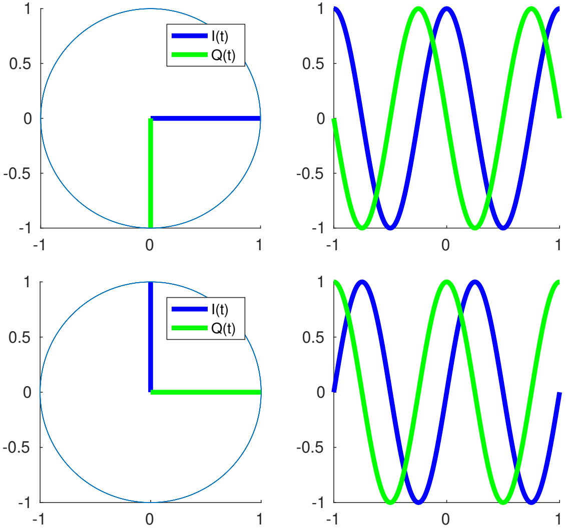
- $I(t) = S_m(t)\cos\varphi(t)$ — inphase component
- $Q(t) = S_m(t)\sin\varphi(t)$ — quadrature component

Minus or plus? Sine or cosine? Depends on which is the first!

Minus or plus? Sine or cosine?

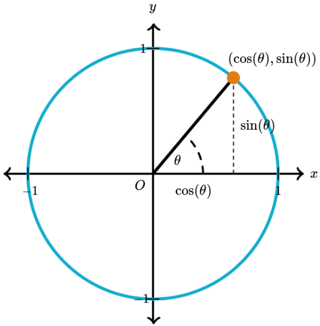
- The carrier of $Q(t)$ should precede the carrier of $I(t)$.
- Otherwise the spectral density of the $s(t)$ will be inverted.
- Initial phase does not matter.

Homework: prove these theses!



Quadrature components II

$$
\begin{aligned}
s_{\mathsf{mod}}(t) &= I(t)\cos 2\pi f_0 t - Q(t)\sin 2\pi f_0 t \\
&= \sqrt{I^2(t) + Q^2(t)}\cos\left(2\pi f_0 t - \mathrm{arctg}\frac{-Q(t)}{I(t)}\right) \\
&= S_m(t)\cos\Big(2\pi f_0 t + \varphi(t)\Big)
\end{aligned}
$$



So, we can express:

- $S_m(t) = \sqrt{I^2(t) + Q^2(t)}$ — instant amplitude;
- $\varphi(t) = \mathrm{arctg}\dfrac{Q(t)}{I(t)}$ — instant phase;
- instant frequency?

Conclusion: Each modulation can be univocally expressed by quadrature components.

Modulation and demodulation

Modulation:

$$s_{\mathsf{mod}}(t) = I(t)\cos 2\pi f_0 t - Q(t)\sin 2\pi f_0 t$$

Demodulation:

$$\begin{aligned}
s_{\mathsf{mod}}(t)\cos 2\pi f_0 t &= I(t)\cos^2 2\pi f_0 t - Q(t)\sin 2\pi f_0 t \cos 2\pi f_0 t = \\
&= \frac{I(t)}{2} + \frac{I(t)}{2}\cos 4\pi f_0 t - \frac{Q(t)}{2}\sin 4\pi f_0 t
\end{aligned}$$

$$\begin{aligned}
-s_{\mathsf{mod}}(t)\sin 2\pi f_0 t &= -I(t)\cos 2\pi f_0 t \sin 2\pi f_0 t + Q(t)\sin^2 2\pi f_0 t = \\
&= \frac{Q(t)}{2} - \frac{I(t)}{2}\sin 4\pi f_0 t - \frac{Q(t)}{2}\cos 4\pi f_0 t
\end{aligned}$$

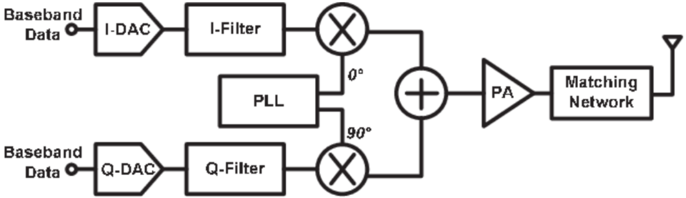
Reference material:

$$\sin x \cos x = \frac{1}{2}\sin 2x \qquad \cos^2 x = \frac{1}{2}(1 + \cos 2x) \qquad \sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

Demodulation with phase shifted waves φ_0?

QAM transmitter

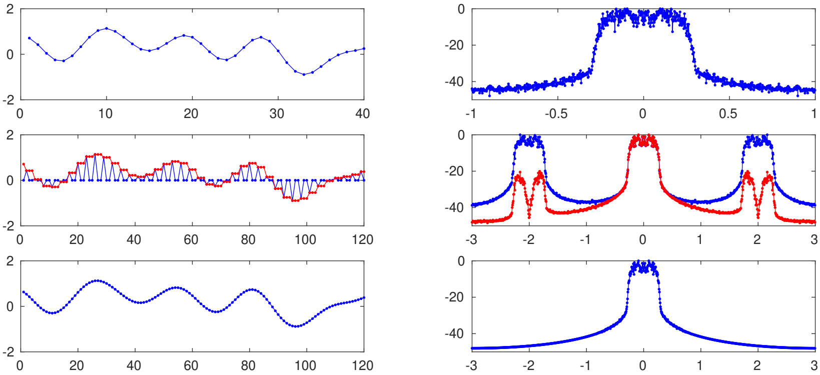
$$s_{\mathsf{mod}}(t) = I(t)\cos 2\pi f_0 t - Q(t)\sin 2\pi f_0 t$$



(a)

Why the I-filter and Q-filter are added?

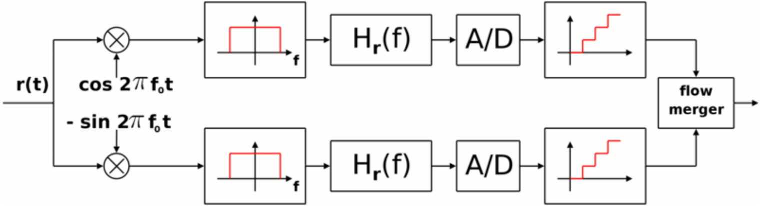
Smoothing filters after DAC



QAM receiver

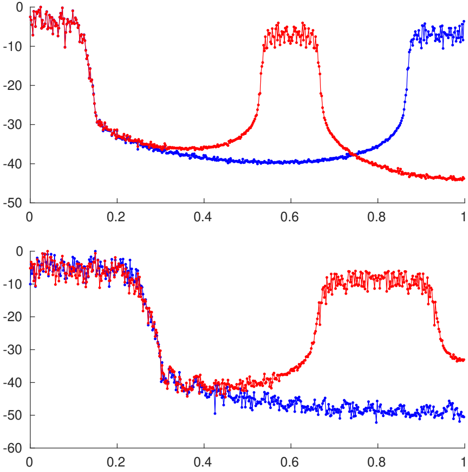
$$s_{\text{mod}}(t)\cos 2\pi f_0 t = \frac{I(t)}{2} - \frac{I(t)}{2}\cos 4\pi f_0 t + \frac{Q(t)}{2}\sin 4\pi f_0 t$$

$$-s_{\text{mod}}(t)\sin 2\pi f_0 t = \frac{Q(t)}{2} - \frac{I(t)}{2}\sin 4\pi f_0 t - \frac{Q(t)}{2}\cos 4\pi f_0 t$$



- LPF to get rid of $2f_0$ spectral component;
- Anti-aliasing filter.

Anti-aliasing filter



Where is the second spectral component?

Orthogonality

- The functions $\sin 2\pi f_0 t$ and $\cos 2\pi f_0 t$ are orthogonal, i.e.,

$$\int\limits_0^{1/f_0} \sin 2\pi f_0 t \cos 2\pi f_0 t \mathrm{d}t = 0.$$

- For signals whose maximal frequency $f_{\mathsf{max}} \ll f_0$ separation is guaranteed!
- Are there any restrictions on channels Q- and I- selection?
- Synchronous or not?

Spectral components

Symmetry property of the Fourier transform

Inverse Fourier transform

$$s(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \dot{S}(\omega)\, \mathrm{e}^{j\omega t} = 0.$$

Assume the spectral density is $\dot{S}(\omega) = S(\omega)\, \mathrm{e}^{j\Psi(\omega)}$.

$$s(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) \cos\Big(\omega t + \Psi(\omega)\Big) + \frac{j}{2\pi} \int_{-\infty}^{\infty} S(\omega) \sin\Big(\omega t + \Psi(\omega)\Big).$$

To obtain real $s(t)$, we have to restrict amplitude $S(\omega) = S(-\omega)$ to have even symmetry and phase $\Psi(\omega) = -\Psi(-\omega)$ to have odd symmetry.
Reference material:

$$\mathrm{e}^{jx} = \cos x + j \sin x$$

Spectral components

Quadrature components

Modulated signal:

$$s_{\mathsf{mod}}(t) = I(t)\cos 2\pi f_0 t - Q(t)\sin 2\pi f_0 t$$

Rewrite as following:

$$\begin{aligned} s_{\mathsf{mod}}(t) &= I(t)\frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2} - Q(t)\frac{e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}}{2j} = \\ &= \frac{I(t) + jQ(t)}{2} e^{j2\pi f_0 t} + \frac{I(t) - jQ(t)}{2} e^{-j2\pi f_0 t} \end{aligned}$$

Conclusions:

- Is it real?
- Spectrum at the carrier is the same as the spectrum of the signal $I(t) + jQ(t)$.
- Two spectra at the same frequency — better efficiency!

Reference material:

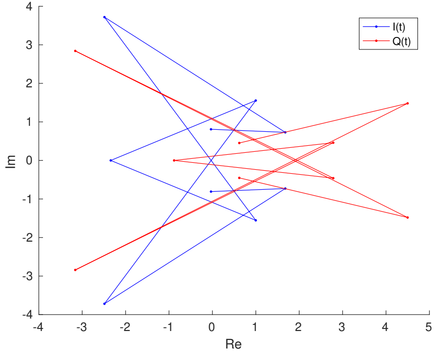
$$\mathrm{e}^{jx} = \cos x + j\sin x \quad \cos x = \frac{e^{jx} + e^{-jx}}{2} \quad \sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

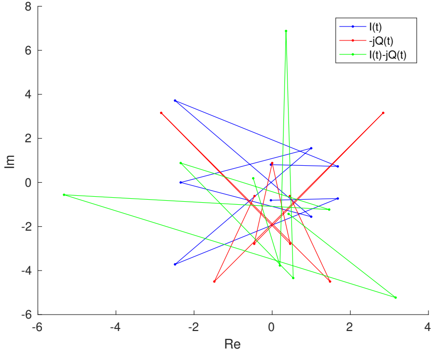
Spectral components

Spectrum of the complex signal

Modulated signal:

$$s_{\text{mod}}(t) = \frac{I(t) + jQ(t)}{2} e^{j2\pi f_0 t} + \frac{I(t) - jQ(t)}{2} e^{-j2\pi f_0 t}$$





Single-tone and dual-tone generation I

Why do we need it?

$$s_{\text{mod}}(t) = I(t)\cos 2\pi f_0 t - Q(t)\sin 2\pi f_0 t$$

- How to create single-tone at the carrier?
  If $I(t) = 1/\sqrt{2}$ and $Q(t) = 1/\sqrt{2}$, then

$$s_{\text{mod}}(t) = \cos 2\pi f_0 t - \sin 2\pi f_0 t = \cos(2\pi f_0 t + \pi/4)$$

- How to create single-tone at frequency $f_0 + f$?
  If $I(t) = \cos 2\pi f t$ and $Q(t) = \sin 2\pi f t$, then

$$s_{\text{mod}}(t) = \cos(2\pi f t)\cos(2\pi f_0 t) - \sin(2\pi f t)\sin(2\pi f_0 t) = \cos\Big(2\pi(f_0 + f)t\Big)$$

Reference material:

$$\begin{aligned}
\cos(\alpha + \beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta \\
\cos(\alpha - \beta) &= \cos\alpha\cos\beta + \sin\alpha\sin\beta \\
\sin(\alpha + \beta) &= \cos\alpha\sin\beta + \sin\alpha\cos\beta
\end{aligned}$$

Single-tone and dual-tone generation II

$$s_{\mathsf{mod}}(t) = I(t)\cos 2\pi f_0 t - Q(t)\sin 2\pi f_0 t$$

- How to create a single-tone at frequency $f_0 - f$?
  If $I(t) = \cos 2\pi f t$ and $Q(t) = \sin 2\pi f t$, then

$$s_{\mathsf{mod}}(t) = \cos(2\pi f t)\cos(2\pi f_0 t) + \sin(2\pi f t)\sin(2\pi f_0 t) = \cos\Big(2\pi(f_0 - f)t\Big)$$

- How to create a dual-tone at frequencies $f_0 - f$ and $f_0 + f$?

Reference material:

$$\begin{aligned}
\cos(\alpha+\beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta \\
\cos(\alpha-\beta) &= \cos\alpha\cos\beta + \sin\alpha\sin\beta \\
\sin(\alpha+\beta) &= \cos\alpha\sin\beta + \sin\alpha\cos\beta \\
\sin(\alpha-\beta) &= \sin\alpha\cos\beta - \cos\alpha\sin\beta
\end{aligned}$$

Dual-tone generation

$$s_{\mathsf{mod}}(t) = I(t)\cos 2\pi f_0 t - Q(t)\sin 2\pi f_0 t$$

- How to create a dual-tone at frequencies $f_0 - f$ and $f_0 + f$?
  If $I(t) = \cos(2\pi f t)$ and $Q(t) = \cos(2\pi f t)$, then

$$s_{\mathsf{mod}}(t) = \cos(2\pi f t)\cos(2\pi f_0 t) + \cos(2\pi f t)\sin(2\pi f_0 t) =?$$

$$\begin{aligned}
\cos(2\pi f t)\cos(2\pi f_0 t) &= \frac{1}{2}\cos(2\pi(f_0 + f)t) + \frac{1}{2}\cos(2\pi(f_0 - f)t) \\
\cos(2\pi f t)\sin(2\pi f_0 t) &= \frac{1}{2}\sin(2\pi(f_0 + f)t) + \frac{1}{2}\sin(2\pi(f_0 - f)t)
\end{aligned}$$

Reference material:

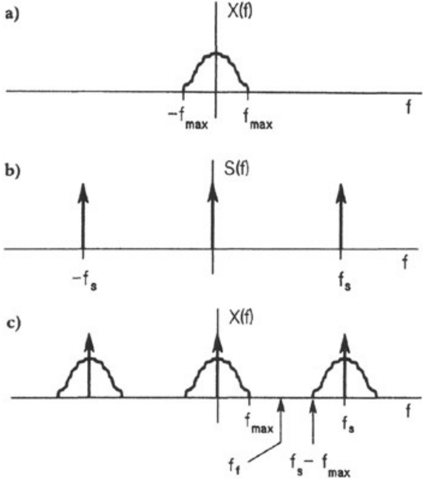
$$\begin{aligned}
\cos(\alpha + \beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta \\
\cos(\alpha - \beta) &= \cos\alpha\cos\beta + \sin\alpha\sin\beta \\
\sin(\alpha + \beta) &= \cos\alpha\sin\beta + \sin\alpha\cos\beta \\
\sin(\alpha - \beta) &= \sin\alpha\cos\beta - \cos\alpha\sin\beta
\end{aligned}$$

So, related or not?

- Until now — analog and absolute arbitrary Q- and I-channels.
- Transition from analog domain to digital domain.
- Introduce relation between quadrature channels.

Digital domain

- Sampling frequency or clock $f_s = 100\,\text{MHz}$.
- Nyquist frequency $f_N = f_s/2 \geqslant f_{\text{max}}$.
- Bandwidth of modulated signal $\Delta f = 25\,\text{MHz}$ is twice wider than in baseband.
- Symbol rate $R = 1/\tau_s$, where τ_s is a symbol length.
- Number of samples per symbol $K = \tau f_s = f_s/R$.



Symbol rate and bandwidth

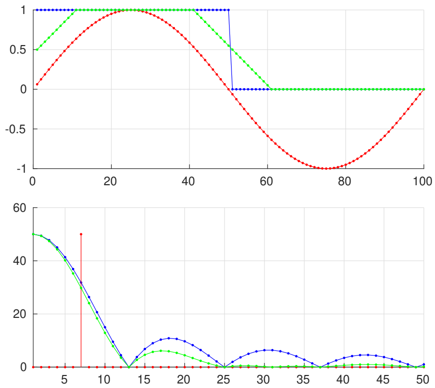
- Assume symbol length is τ_s, then its spectral density:

$$\dot{S}(f) = S_m \tau \mathrm{sinc}\left(\pi f \tau_s\right)$$

- Bandwidth:

$$\frac{\pi}{2} = \pi f \tau_s \Leftarrow R = \frac{1}{\tau_s} = 2f$$

- Another explanation...



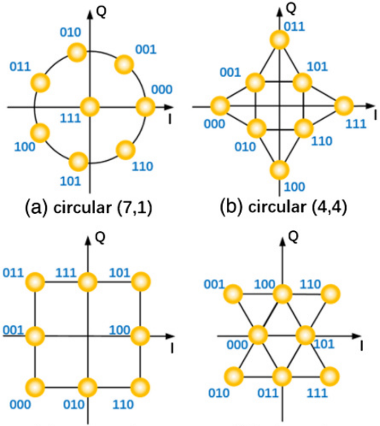
Problem

Sampling frequency of the system $f_s = 200$ MHz. Symbol rate to be ensured by the systen is $R = 20$ MBaud. Calculate:

- Bandwidth in the air Δf (20 MHz).
- Nyquist frequency f_N (100 MHz).
- Bandwidth in baseband (10 MHz).
- Number of samples per symbol (10 samples).
- Symbol length τ_s (50 ns).

Constellation I

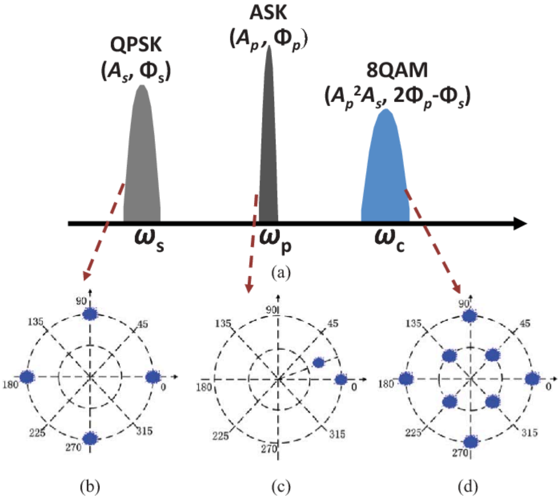
- Synchronize symbol instants for Digital systems: PAM+PAM.
- Is it possible to create constellation with $N \neq 2^n$?
- Criteria on point locations:
  - Maximal Euclidean distance;
  - Others?



(a) circular (7,1) (b) circular (4,4)

(c) rectangular (d) triangular

Constellation II



Capacity

- Capacity C shows the number of bits to be transmitted per second:

$$C = nR = R\log_2 N,$$

where N is number of points in the constellation and R is symbol rate.

- Capacity increment lowers at higher modulations.

			Channel Size 56MHz	
Bits per Symbol	Modulation		Capacity Mbps	Incremental % increase
8	256	QAM	370	
9	512	QAM	421	13.80%
10	1024	QAM	472	11.98%
11	2048	QAM	523	10.83%
12	4096	QAM	575	9.77%

Problem: Calculate capacity for 16-QAM modulation and symbol rate $R = 25\,\text{MBaud}$ (100Mbps).