5G WIRELESS TECHNOLOGIES

Equalization

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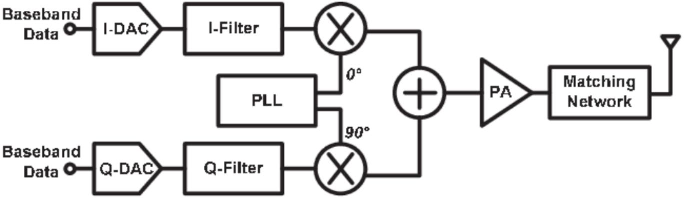
Operations to perform in analog domain

Transmitter

Tasks for the transmitter:

- Digital-analog conversion;
- Smoothing-filter application;
- Quadrature modulation;
- Upconversion;
- Power amplification.



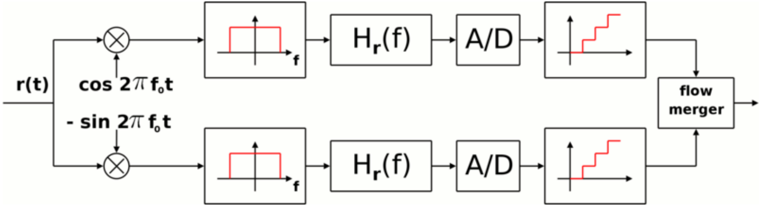
(a)

Operations to perform in analog domain

Receiver

Tasks for the receiver:

- Low-noise amplification;
- Automatic gain control;
- Image channel suppression and mixing (if necessary);
- Quadrature demodulation;
- Double-frequency component suppression;
- Anti-aliasing filter application;
- Analog-to-digital conversion.

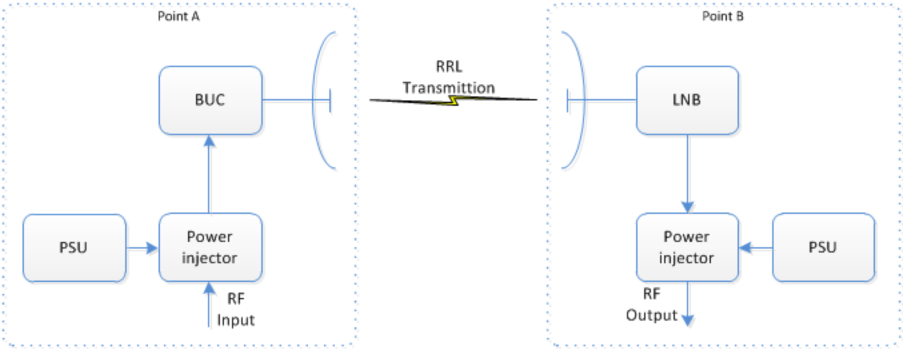


Operations to perform in digital domain

- Tasks for the transmitter:
  - User data mapping to symbols and zeros insertion;
  - Pulse-shaping filtration;
  - Pre-distortions.
- Tasks for the receiver:
  - Gain control;
  - IQ-impairment post-compensation;
  - Timing recovery;
  - Equalization;
  - Carrier recovery;
  - Matched filtration;
  - Detection.

Radiowave propagation

System structure



- BUC (Block Upconverter); LNB (Low Noise Block Converter—downcoverter).
- Unlike all previously discussed effects, let us consider signal distortion during transmission.

Fading

Definition and reasons

- Fading is an attenuation of the signal that is dependent on radio wave propagation parameters: time, frequency, and wave number effects.
- In our course, we will observe time and frequency varying effects.
- The main reason—multipath propagation; however, other effects can also induce fading, e.g., rain, mist, shadowing, etc.
- Multipath phenomenon:

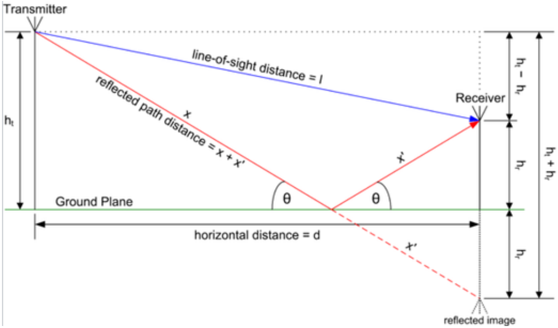
$$\sin 2\pi f_0 t \to \frac{A}{\sqrt{A^2 + (1+A)^2}} \sin 2\pi f_0 t + \frac{1-A}{\sqrt{A^2 + (1+A)^2}} \sin(2\pi f_0 t - 2\pi f_0 \tau)$$

- Constructive and destructive interference.

Two-path propagation

Model description

- Line-of-sight path and reflected path.
- Fresnel ellipsoid for precise calculation.
- For high frequencies, it is sufficiently narrow to ignore it.
- Assume the distance between the towers is $d = 10\,\mathrm{km}$, the heights of the towers are $h_1 = 50\,\mathrm{m}$ and $h_2 = 30\,\mathrm{m}$. Calculate the reflected path delay!



Two-path propagation

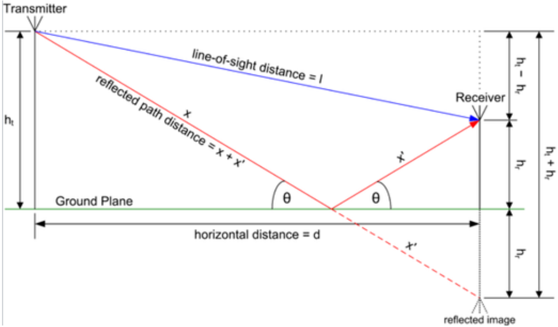
Propagation distance difference I

- Line-of-sight path length:

$$l_s = \sqrt{d^2 + (h_1 - h_2)^2}$$

- Only one reflected ray goes to the receiving point.
- Assume d_1 is a distance between the first tower and the reflection point.
- Then θ angle can be expressed:

$$\theta = \arctan\frac{h_1}{d_1} = \arctan\frac{h_2}{d - d_1}.$$



Two-path propagation

Propagation distance difference II

- Angle θ was expressed:

$$\theta = \arctan \frac{h_1}{d_1} = \arctan \frac{h_2}{d - d_1}.$$

- Assuming that two expression are equal, obtain:

$$\frac{d_1}{h_1} = \frac{d}{h_2} - \frac{d_1}{h_2} \quad \Longrightarrow \quad d_1 \left(\frac{1}{h_1} + \frac{1}{h_2} \right) = \frac{d}{h_2} \quad \Longrightarrow \quad d_1 = d \frac{h_1}{h_1 + h_2}.$$

- Substituting d_1 into the θ expression:

$$\theta = \arctan \frac{h_1}{d_1} = \arctan \frac{h_1 + h_2}{d}.$$

Two-path propagation

Propagation distance difference III

- Reflected path length:

$$l_r = \frac{h_1}{\sin\theta} + \frac{h_2}{\sin\theta} = \frac{h_1 + h_2}{\sin\theta} = \sqrt{d^2 + (h_1 + h_2)^2}$$

- Thus, difference between two paths is:

$$\Delta l = l_r - l_s = \sqrt{d^2 + (h_1 + h_2)^2} - \sqrt{d^2 + (h_1 - h_2)^2}$$

- Substituting values into the expressions, obtain the following results:
  - Angle between ground plane and incident ray $\theta = 0.4584^\circ$;
  - Line-of-sight path length $l_s = 10\,000.02$ m;
  - Reflected path length $l_r = 10\,000.31$ m;
  - Two rays differences $\Delta l = l_r - l_s = 30$ cm.
- Assuming light speed $c = 3 \cdot 10^8$ m/s, this time difference implies $\tau = \dfrac{\Delta l}{c} = 10^{-9}$s, or 1 ns, time delay (comparable with the sampling step for $R = 25$ Mbaud symbol rate).

Two-path propagation

Baseband channel model I

- Assume we have modulated QAM signal:

$$s_{tx}(t) = S(t)\cos\Big(2\pi f_0 t + \varphi(t)\Big).$$

- According to the described model, we receive the following signal:

$$s_{rx}(t) = AS(t)\cos\Big(2\pi f_0 t + \varphi(t)\Big) + BS(t-\tau)\cos\Big(2\pi f_0 t + \varphi(t-\tau) - 2\pi f_0 \tau\Big),$$

  where $2\pi f_0 \tau$ is phase shift of the reflected ray in the channel; A and B are the rays power controlling coefficients.
- In Hilbert transform:
  - Analytical signal $\dot{s}_{tx}(t) = s_{tx}(t) + j\mathcal{H}\{s_{tx}(t)\}$;
  - Narrow-band signal image $\mathcal{H}\{S(t)\} = S(t)$;
  - Image of the cosine function: $\mathcal{H}\{\cos(2\pi f_0 t + \varphi(t))\} = \sin(2\pi f_0 t + \varphi(t))$.

Two-path propagation

Baseband channel model II

- Taking Hilbert transform of the $s_{tx}(t)$, obtain analytical signal:

$$\dot{s}_{tx}(t) = S(t)\cos\Big(2\pi f_0 t{+}\varphi(t)\Big){+}jS(t)\sin\Big(2\pi f_0 t{+}\varphi(t)\Big) = S(t)\,\mathrm{e}^{j2\pi f_0 t + j\varphi(t)} = \dot{S}(t)\,\mathrm{e}^{j2\pi f_0 t}\,.$$

- The real received signal was:

$$s_{rx}(t) = AS(t)\cos\Big(2\pi f_0 t + \varphi(t)\Big) + BS(t-\tau)\cos\Big(2\pi f_0 t + \varphi(t-\tau) - 2\pi f_0 \tau\Big).$$

- Corresponding analytical signal can be expressed as:

$$\dot{s}_{rx}(t) = A\dot{S}(t)\,\mathrm{e}^{j2\pi f_0 t} {+} B\dot{S}(t{-}\tau)\,\mathrm{e}^{-j2\pi f_0 \tau}\,\mathrm{e}^{j2\pi f_0 t} = \Big(A\dot{S}(t) + B\dot{S}(t-\tau)\,\mathrm{e}^{-j2\pi f_0 \tau}\Big)\,\mathrm{e}^{j2\pi f_0 t}\,.$$

Two-path propagation

Baseband channel model III

- Dropping complex exponents, transmitted signal was:

$$\dot{s}_{tx}(t) = \dot{S}(t).$$

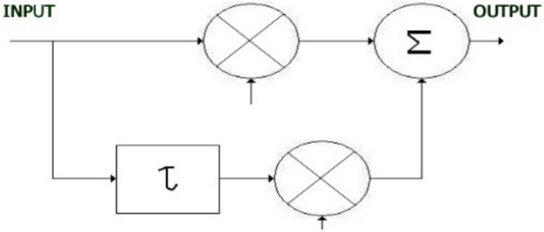
- The received signal in this case is:

$$\dot{s}_{rx}(t) = A\dot{S}(t) + B\dot{S}(t-\tau)\,\mathrm{e}^{-j2\pi f_0 \tau}\,.$$

- In this way, a channel is a FIR filter with the impulse response:

$$h(t) = A\delta(t) + B\delta(t-\tau)\,\mathrm{e}^{-j2\pi f_0 \tau}\,.$$

- Complex coefficients—filter's frequency response is asymmetric.



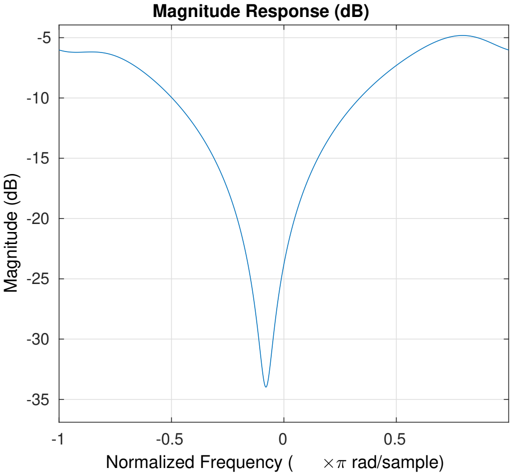
Two-path propagation

Interference concept

- The received signal is:

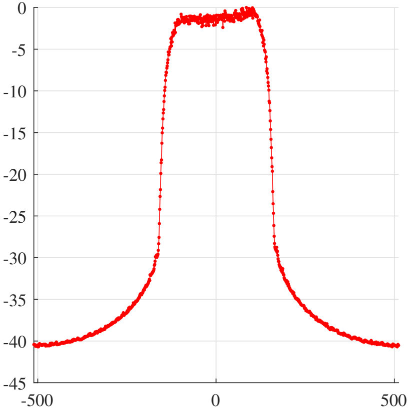
$$\dot{s}_{rx}(t) = A\dot{S}(t) + B\dot{S}(t-\tau)\,\mathrm{e}^{-j2\pi f_0\tau}\,.$$

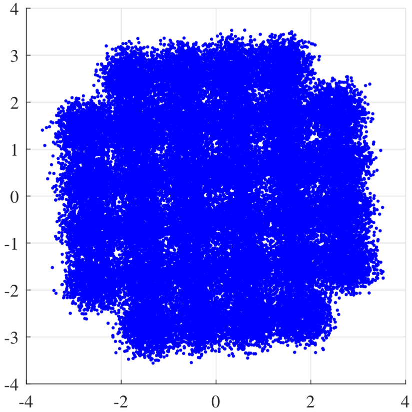
- Assume that τ is constant.
- Observe the frequencies for which $f_0\tau = n$ ($n \in \mathbb{Z}$ is integer). For them complex exponent is equal to one, and signal becomes amplified—constructive interference.
- Observe the frequencies for which $f_0\tau = n + 0.5$ ($n \in \mathbb{Z}$ is integer). For them complex exponent is equal to minus one, and signal becomes attenuated—destructive interference.



Fading

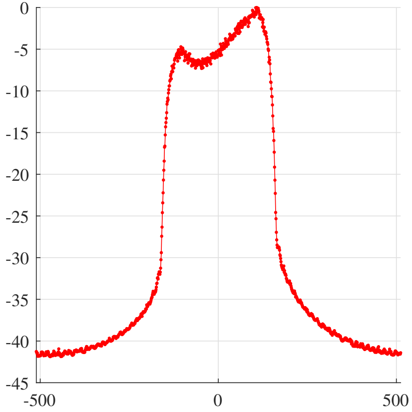
Two-ray channel example 2dB

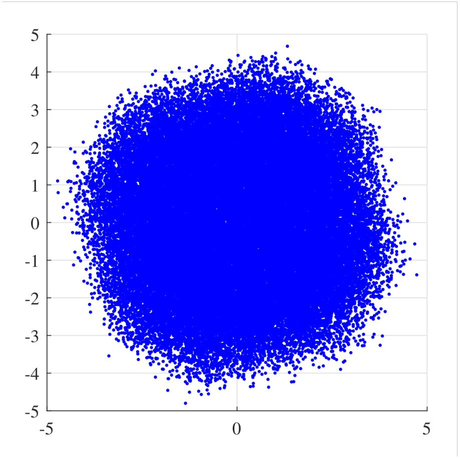




Fading

Two-ray channel example 8dB





Probability density functions

Rayleigh distribution

- Gaussian probability density function (all real-world processes according to the central limit theorem):

$$\rho(x) = \frac{1}{\sigma\sqrt{2\pi}}\,\mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}.$$

- Let us calculate probability distribution in the case of two normally distributed orthogonal variables with zero mean ($\mu = 0$):

$$P(x,y) = \iint \frac{1}{\sigma^2 2\pi}\,\mathrm{e}^{-\dfrac{x^2+y^2}{2\sigma^2}}\,\mathrm{d}x\,\mathrm{d}y = \int_0^\infty\int_0^{2\pi} \frac{r}{\sigma^2 2\pi}\,\mathrm{e}^{-\dfrac{r^2}{2\sigma^2}}\,\mathrm{d}r\,\mathrm{d}\varphi.$$

- Thus, the amplitude is Rayleigh distributed, and the phase is uniformly distributed:

$$\rho(r) = \frac{r}{\sigma^2}\,\mathrm{e}^{-\dfrac{r^2}{2\sigma^2}}; \quad \rho(\varphi) = \frac{1}{2\pi}.$$

Probability density functions

Rice distribution

- Gaussian and Rayleigh probability density functions:

$$\rho(x) = \frac{1}{\sigma\sqrt{2\pi}}\,\mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}; \quad \rho(r) = \frac{r}{\sigma^2}\,\mathrm{e}^{-\frac{r^2}{2\sigma^2}}.$$

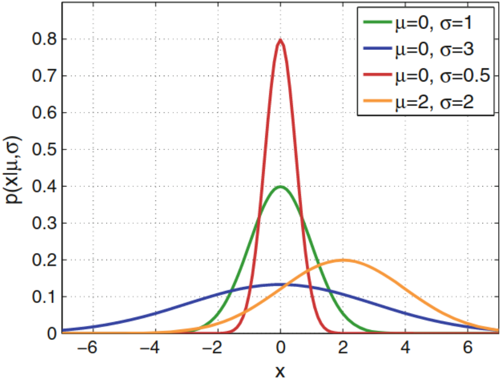
- If one calculate probability distribution in case of two normally distributed orthogonal variables with nonzero mean ($\mu \neq 0$), the Rice distribution will be obtained—a distribution of amplitudes if channels have mean value:

$$\rho(r) = \frac{r}{\sigma^2} I_0\left(\frac{\mu r}{\sigma^2}\right)\mathrm{e}^{-\frac{r^2+\mu^2}{2\sigma^2}},$$

  where $I_0(\cdot)$ is zeroth-order modified Bessel function of the first kind.

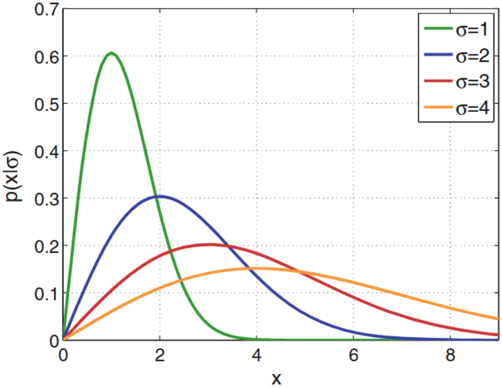
Normal distribution

$$\rho(x) = \frac{1}{\sigma\sqrt{2\pi}}\,\mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$



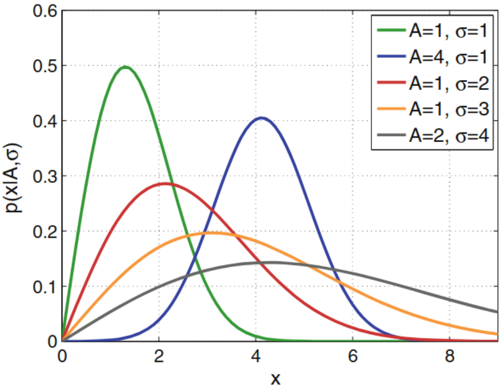
Rayleigh distribution

$$\rho(r) = \frac{r}{\sigma^2}\, \mathrm{e}^{-\frac{r^2}{2\sigma^2}}$$



Rice distribution

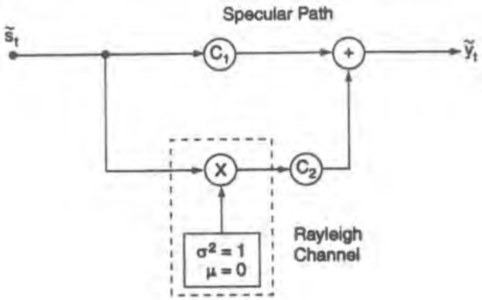
$$\rho(r) = \frac{r}{\sigma^2} I_0\left(\frac{\mu r}{\sigma^2}\right) \mathrm{e}^{-\frac{r^2+\mu^2}{2\sigma^2}}$$



Ricean channel

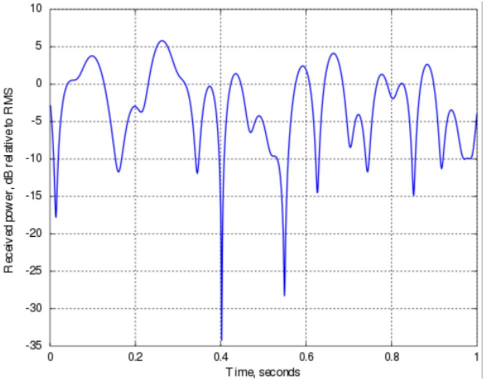
- Assume $B(t)$ and $\tau(t)$ in the two-ray propagation model are time-varying.
- One can express $B(t)\,\mathrm{e}^{-j2\pi f_0 \tau(t)} = \beta_r(t) + j\beta_i(t)$.
- According to the central limit theorem (multiple scatterers), both $\beta_r(t)$ and $\beta_i(t)$ are randomly distributed.
- Then amplitude of the side path signal is Rayleigh distributed and output amplitude—Rice distributed.
- Path coefficients then are:

$$C_1 = \frac{\mu}{\sqrt{\sigma^2 + \mu^2}}; \quad C_2 = \frac{\sigma^2}{\sqrt{\sigma^2 + \mu^2}}$$



Rayleigh channel

- No line-of-sight component, i.e., there is no evident single pulse.
- Both $\beta_r(t)$ and $\beta_i(t)$ are Gaussian distributed, and the amplitude has Rayleigh distribution.
- Used for urban environment simulation.
- As a rule, characterizes flat fading.
- In the figure, Rayleigh fading level for $10\,$Hz Doppler shift.



Fading

Multipath propagation

- Can be expressed as time-varying, but linear system:

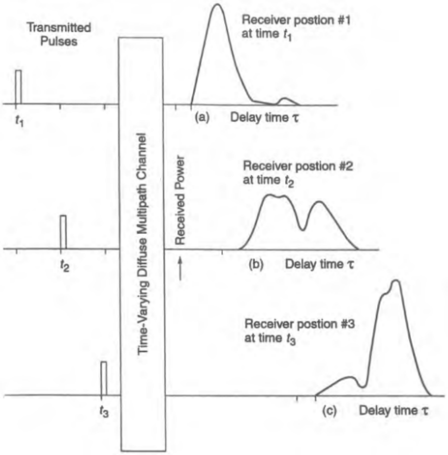
$$h(\tau_n(t), t) = \sum_{n=0}^{N-1} a_n(t) e^{j\varphi_n(t)} \delta(t - \tau_n(t))$$

- Taking Fourier transform

$$H(f) = \int_{-\infty}^{+\infty} h(t) e^{-j2\pi f t} dt,$$

results in (approximately):

$$H(\tau_n(t), f) = \sum_{n=0}^{N-1} a_n(\tau_n(t)) e^{j\varphi_n(\tau_n(t))} e^{-j2\pi f \tau_n(t)}$$



Fading

Channel time-frequency characteristics

- As the impulse response is time-varying and random, observe its autocorrelation function:

$$B_h(\tau_1, \tau_2, t_1, t_2) = E[h(\tau_1, t_1)h(\tau_2, t_2)].$$

- Assume the process is wide-sense stationary:

$$B_h(\tau_1, \tau_2, \Delta t) = E[h(\tau_1, t)h(\tau_2, t + \Delta t)].$$

- Attenuation and phase for different τ are uncorrelated (uncorrelated scattering):

$$B_h(\tau_1, \tau_2, \Delta t) = E[h(\tau_1, t)h(\tau_1, t + \Delta t)]\delta(\tau_1 - \tau_2) = B_h(\tau, \Delta t)\delta(\tau_1 - \tau_2).$$

- Taking Fourier transform of the auto correlation function, we obtain scattering function:

$$S_h(\tau, f) = \int_{-\infty}^{\infty} B_h(\tau, \Delta t)\, \mathrm{e}^{-j2\pi f \Delta t}\, \mathrm{d}\Delta t.$$

Fading

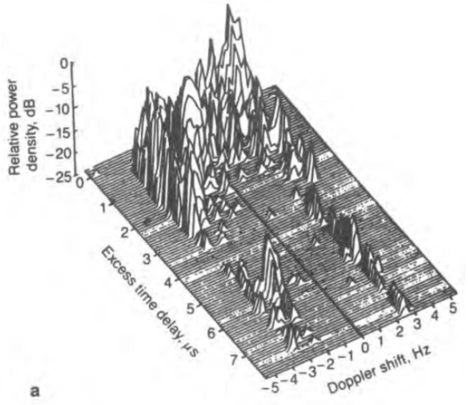
Scattering function

- Scattering function $S(\tau, f)$ shows spectral behavior for different taps.
- Variable f is called *Doppler shift*.
- $S(\tau, f)$ mean value with respect to f is *delay-power profile* of the channel:

$$p(\tau) = \int_{-\infty}^{\infty} S(\tau, f)\, \mathrm{d}f.$$

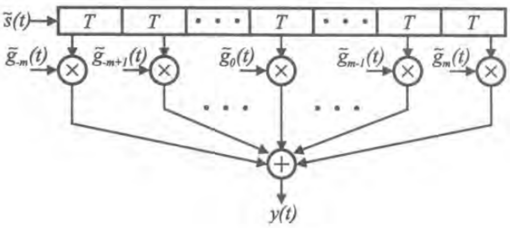
- $S(\tau, f)$ mean value with respect to τ is *Doppler power spectral density* of the channel:

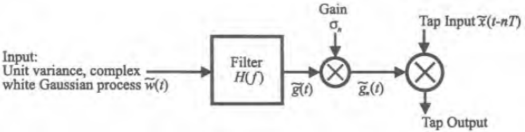
$$S(f) = \int_{-\infty}^{\infty} S(\tau, f)\, \mathrm{d}\tau.$$



Fading

Discrete channel generation example

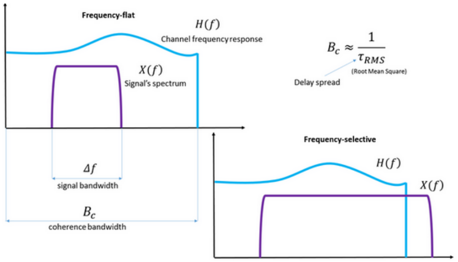




Fading

Flat and frequency selective fading

- Coherence bandwidth B_c.
- Flat fading. Compensated with AGC.
- Frequency-selective fading. Deep at certain frequency.
- Dispersive effects. Equalizers.



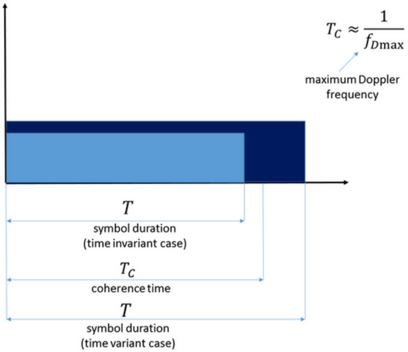
Fading

Slow and fast fading

- Coherence time T_c from AKF.
- For slow fading, a symbol duration is smaller, than the coherence time.
- Fast fading—can not be compensated. Deep fading.
- Doppler shift:

$$T_c = \frac{1}{D_s}$$

- Doppler spectrum.



Main dynamic system construction stages

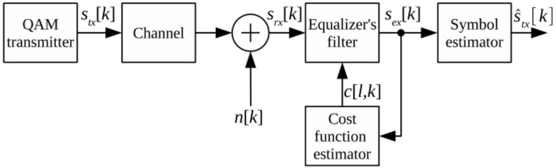
The following requirements must be fulfilled to create a dynamical system:

- Define a way how to compensate a distortion if its amount in the signal is static precisely known—compensation application;
- Understand how distortion affects a signal and find a mathematical expression that shows the consequences of its presence—cost function;
- Find an algorithm how to calculate an amount of compensation from the cost function—compensation algorithm.

Equalizer as a filter

System description

- Requirement to compensate linear distortion of the channel.
- Compensation also has to be linear.
- Finite (FIR) or infinite (IIR) impulse response filters with time-varying coefficients.
- Fractionally-spaced (all samples) and symbol-spaced equalizers.



Equalizer as a filter

Rigorous output goal formulation

- In the channel, the transmitted signal $s_{\text{tx}}[k]$ is distorted as follows:

$$s_{in}[k] = \sum_{l_h=0}^{L_h-1} s_{tx}[k-l_h]h[l_h].$$

- The equalizer, being a filter with a set of coefficients $c[l_c,k]$, forms output:

$$s_{ex}[k] = \sum_{l_c=0}^{L_c-1} s_{in}[k-l_c]c[l_c,k].$$

- Thus, the output of the equalizer should be:

$$s_{ex}[k] = \sum_{\substack{0\leqslant l_h<L_h \\ 0\leqslant l_c<L_c}} s_{tx}[k-l_h-l_c]h[l_h]c[l_c,k] = s_{tx}[k-l_\delta] + \sum_{\substack{0\leqslant l<L_h+L_c \\ l\neq l_\delta}} s_{tx}[k-l] \sum_{0\leqslant l_c<l} h[l-l_c]c[l_c,k].$$

- Introduces a delay l_δ and tries to minimize $\sum_{0\leqslant l_c<l} h[l-l_c]c[l_c,k]$.

Equalizer as a filter

Short FIR example I

- Assume the transmitted signal is single pulse $x_{tx}[n] = \delta_n$.
- An impulse response of the channel is $h[n] = a_1[n] + a_2[n-1]$.
- The signal at the receiver will be $x_{rx}[n] = a_1[n] + a_2[n-1]$.
- Assume impulse response of the equalizer is $c[n] = b_1[n] + b_2[n-1] + b_3[n-3]$;
- The output of the equalizer will be calculated as a discrete convolution:

$$\begin{array}{ccccc} & & a_1 & a_2 & \\ b_3 & b_2 & b_1 & & \end{array} \implies a_1 b_1 \qquad \begin{array}{ccc} a_1 & a_2 & \\ b_3 & b_2 & b_1 \end{array} \implies a_2 b_2 + a_1 b_3$$

$$\begin{array}{cccc} & a_1 & a_2 \\ b_3 & b_2 & b_1 \end{array} \implies a_2 b_1 + a_1 b_2 \qquad \begin{array}{cccc} a_1 & a_2 & & \\ & b_3 & b_2 & b_1 \end{array} \implies a_2 b_3$$

Equalizer as a filter

Short FIR example II

- The result has to have only one pulse, in the ideal case:

$$\begin{cases} a_1b_1 & & & & & = & 0 \\ a_2b_1 & + & a_1b_2 & & & = & 0 \\ & & a_2b_2 & + & a_1b_3 & = & 1 \\ & & & & a_2b_3 & = & 0 \end{cases}$$

- The system can be rewritten in the matrix form $\mathbf{AB} = \mathbf{C}$:

$$\begin{bmatrix} a_1 & 0 & 0 \\ a_2 & a_1 & 0 \\ 0 & a_2 & a_1 \\ 0 & 0 & a_2 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix},$$

  where $\mathbf{A}$ is a lower triangle of the Toeplitz matrix.

- The system is overdefined (number of equations higher than number of the variables); it can be solved using LMS (backslash operator in Matlab).

Stochastic gradient

- We have a set of parameters $\mathbf{a}$ to calculate from cost function. Best solution—LMS.
- Alternatively, calculate increment for $\mathbf{a}$. We need to keep current values $\mathbf{a}[n]$.
- A cost function $J[n]$ is $y[n] = f(x[n], \mathbf{a})$ dependent, possibly, with additional operations.
- To find necessary direction, where to shift parameters to lower cost function, use gradient:

$$\nabla = \frac{\partial}{\partial a_0}\overrightarrow{a_0} + \frac{\partial}{\partial a_1}\overrightarrow{a_1} + \frac{\partial}{\partial a_2}\overrightarrow{a_2} + \cdots ,$$

  which shows the direction of the steepest rise of the function.
- Therefore, to obtain an update of the current parameter set,

$$\mathbf{a}[n+1] = \mathbf{a}[n] - \mu \nabla J[n],$$

  where μ is a step-size coefficient.

Equalizer—LMS

Derivation

- Application of the equalizer correction is

$$f : y[n] = \sum_{k=0}^{k-1} x[n-k]h[n-1,k],$$

  where $\mathbf{a}[n] \equiv \mathbf{h}[n-1]$ is an impulse response of the equalizer.

- A cost function for the equalizer (LMS):

$$J[n] = (y[n] - \hat{y}[n])^2 = \left(\sum_{k=0}^{k-1} x[n-k]h[n-1,k] - \hat{y}[n] \right)^2 .$$

- Calculating a gradient:

$$h[n,k] = h[n-1,k] - \mu \frac{\partial (y[n] - \hat{y}[n])^2}{\partial h[n-1,k]} = h[n-1,k] - 2\mu(y[n] - \hat{y}[n])x[n-k].$$

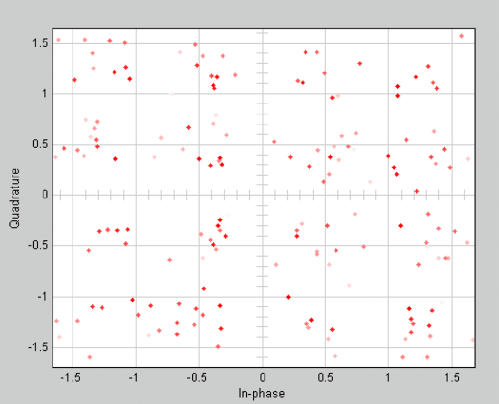
Equalizer—LMS

Properties

- The increment of the kth coefficient:

  $$h[n,k] = h[n-1,k] - 2\mu(y[n] - \hat{y}[n])x[n-k].$$

- Advantage: precise adaptation and zero correction for ideal equalization.
- Drawback: decision-directed, incorrect adjustment in the case of wrong decision.
- Symbol phase dependent algorithm.
- Effective in the tracking mode.



Constant Modulus Algorithm

Derivation

- Goals:
  - An algorithm without decision making.
  - Symbol phase independent algorithm.
- A cost function for the equalizer is minimization of the **dispersion**:

$$J[n] = (y^2[n] - R)^2 = \left(\left(\sum_{k=0}^{k-1} x[n-k]h[n-1,k] - \hat{y}[n] \right)^2 - R \right)^2 ,$$

  where R is a dispersion constant.
- Calculating a gradient

$$h[n,k] = h[n-1,k] - \mu \frac{\partial (y^2[n] - R)^2}{\partial h[n-1,k]} = h[n-1,k] - 4\mu (y^2[n] - R) y[n] x[n-k].$$

Constant Modulus Algorithm

Dispersion constant

- Increment of the equalizer coefficient:

$$h[n,k] = h[n-1,k] - 4\mu(y^2[n] - R)y[n]x[n-k].$$

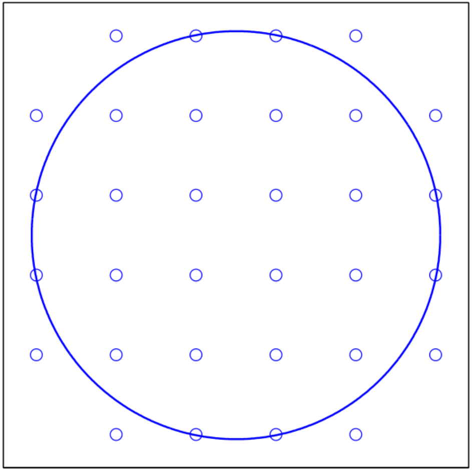
- For the absence of multipath $x[k] = y[k]$, mean increment should be equal to zero:

$$E\left[(x^2[n] - R)x[n]x[n-k]\right] = 0$$

- Expressing R, we obtain:

$$R = \frac{E\left[x^4[n]\right]}{E\left[x^2[n]\right]}.$$

- Thus, the algorithm tries to pull all the point to the single radius.



QAM equalization

- Squared absolute value of a complex signal can be calculated as $|x|^2 = x \cdot x^*$.
- For the LMS algorithm, $(y[n] - \hat{y}[n])^2 \to (y[n] - \hat{y}[n]) \cdot (y[n] - \hat{y}[n])^*$.
- Increments for the coefficients are:

$$h[n, k] = h[n-1, k] - 2\mu(y[n] - \hat{y}[n])x^*[n-k].$$

- For the CMA algorithm, $(y^2[n] - R)^2 \to (y[n]y^*[n] - R)^2$.
- Increments for the coefficients are:

$$h[n, k] = h[n-1, k] - 4\mu(y^2[n] - R)y[n]x^*[n-k].$$