Ruslans Babajans, 171REB152

Task 6

1. The distortion in the signal is a carrier phase difference, i.e., it is defined in the form:

$$x[n] = \tilde{x}[n]e^{j\tilde{\varphi}[n]}$$

where $\tilde{x}[n]$ is the original generated symbol, $\tilde{\varphi}[n]$ is a time-varying phase. Your task is to develop a dynamic system that compensates this distortion. Use x[n] as an input of the dynamic system. Denote an output of this system by y[n] and the current value of the recovered phase by ϕ[n].

• Define a correction application in the from y[n] = f(x[n], ϕ[n−1]).

$$y[n] = x[n]\cdot e^{-j\varphi[n-1]}$$

• Define a cost function J[n]. Assume you precisely know the value of the original symbol (you can use $\hat{y}[n] \equiv \tilde{x}[n]$ in this expression).

Hint: minimize square of the difference.

$$J[n] = \left(y[n] - \hat{y}[n]\right)^2$$

Or

$$J[n] = \left(y[n] - \tilde{x}[n]\right)^2$$

• Use the stochastic gradient approach to express the increment of the ϕ[n] on the next clock cycle.

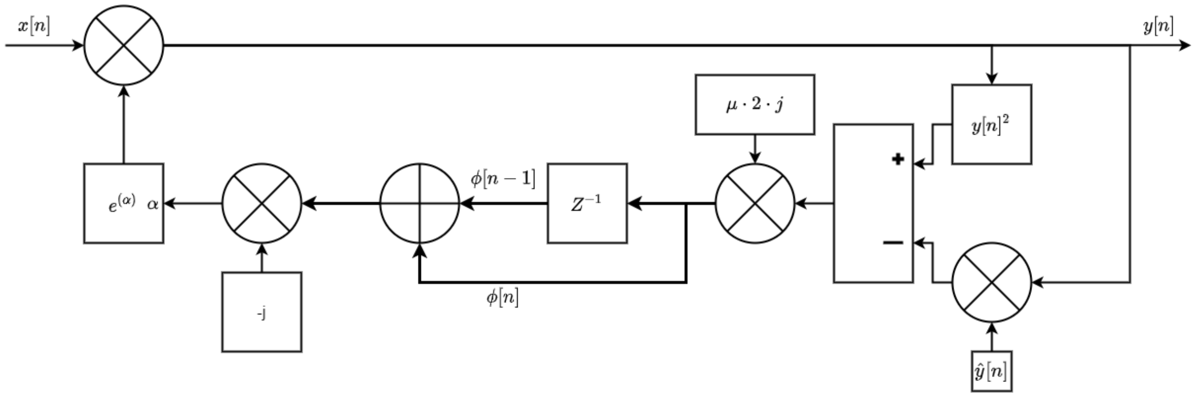
$$\varphi[n] = \varphi[n-1] - \mu\frac{\partial\left(y[n]-\hat{y}[n]\right)^2}{\partial\varphi[n-1]} = \varphi[n-1] - \mu\frac{\partial\left(x[n]\cdot e^{-j\varphi[n-1]} - \hat{y}[n]\right)^2}{\partial\varphi[n-1]} =$$

$$= \varphi[n-1] - \mu\frac{\partial\left(x[n]^2\cdot e^{-2j\varphi[n-1]} - 2\cdot x[n]\cdot e^{-j\varphi[n-1]}\cdot\hat{y}[n] + \hat{y}[n]^2\right)}{\partial\varphi[n-1]} =$$

$$= \varphi[n-1] - \mu\cdot\left(x[n]^2\cdot e^{-2j\varphi[n-1]}\cdot(-2j) - 2\cdot x[n]\cdot e^{-j\varphi[n-1]}\cdot(-j)\cdot\hat{y}[n] + 0\right) =$$

$$= \varphi[n-1] + \mu\cdot x[n]\cdot 2j\cdot\left(x[n]\cdot e^{-2j\varphi[n-1]} + e^{-j\varphi[n-1]}\cdot\hat{y}[n]\right)$$

• Draw block diagram of this dynamic system.



$x[n]$
$y[n]$
$\mu \cdot 2 \cdot j$
$y[n]^2$
$\phi[n-1]$
Z^{-1}
$e^{(\alpha)}$
α
+
−
-j
$\phi[n]$
$\hat{y}[n]$