Lab 6. Equalization

Goal: to get familiar with the fading simulation techniques and obtain skills in the development of dynamic equalization systems.

1. Home task

1. Show analytically that an amplitude of the complex Gaussian process is Rayleigh distributed. A complex Gaussian process is a random signal whose real and imaginary parts are normally distributed.

2. Generate in Matlab a complex Gaussian process of length $N = 100\,000$ with the standard deviation $\sigma = 1.25$. Using Matlab `hist` or `histogram` function, construct probability density function of the real and imaginary parts and amplitude and phase of this process. Add to the same figure analytically calculated curves of these distributions.

3. **Optional** Show analytically that an amplitude of the complex Gaussian process with mean values μ_x and μ_y of its real and imaginary parts, respectively, is Rice distributed.
   Hint: use the same approach as for the Rayleigh distribution. Exploit the following definition of the zeroth-order modified Bessel function of the first kind:
$$I_0(r) = \frac{1}{\pi} \int\limits_0^\pi \mathrm{e}^{r\cos\varphi}\,\mathrm{d}\varphi.$$

4. Generate in Matlab a complex Gaussian process of length $N = 100\,000$ with the standard deviation $\sigma = 1.25$, real part mean value $\mu_x = 2$, and imaginary part mean value $\mu_y = 3$. Using Matlab `hist` or `histogram` function, construct probability density function of the real and imaginary parts and amplitude of this process. Add to the same figure analytically calculated curves of these distributions.

2. MATLAB simulations

For all simulation in this laboratory work, is not stated differently, use the following tips:

- Create baseband model made for the sampling frequency $f_s = 100$ MHz.
- Use for channel parameters calculation carrier frequency $f_0 = 1\,\mathrm{GHz}$.

- Implement 16-QAM modulated signal generation with value spacing equal to $\Delta V = 2$. The number of samples per symbol has to be chosen to ensure a 25 Mbaud high symbol rate.

- Use a single raised-cosine filter to form the signal.

1. Two-ray propagation channel model

- Create a two-ray channel model. The reflected ray is delayed by the time shift $\tau = 5.5\,\mathrm{ns}$. Channel should not change the level of the signal. The depth of the notch introduced by the channel is $L = 5\,\mathrm{dB}$.

- Calculate the position of the notch f_n in the spectrum analytically. Hint: calculate Fourier transform of the channel filter response, express its amplitude, and find its minimum. Plot the dependence of the position of this minimum on the time shift $\tau \in (4.5, 5.5)\,\mathrm{ns}$.

- Change channel parameters to obtain notch at following frequency values $f_n \in \{-0.25; -0.15; 0; 0.15; 0.25\}$. Add impulse responses and frequency responses of these channels into the report.

- Substitute reflected ray corresponding tap of the filter with the Rayleigh channel. Use constant delay $\tau = 5\,\mathrm{ns}$ Doppler spectrum band should be limited to the frequency $f_D = 0.01$! Standard deviation square σ^2 is a quarter of the line-of-sight ray power.

- Pass to the input of the channel a sequence of Kronecker pulses such that each 1000th sample is one and the others are zeros. Extract the outputs that correspond to 3 such pulses, plot impulse responses and corresponding frequency responses at these time moments.

2. Equalizer

- Set notch depth equal to $L = 5\,\mathrm{dB}$ and position equal to $f_n = 0.1$. Generate 16-QAM signal of the length $N = 100\,000$ symbols. Pass it through this channel. Do not add noise.

- Implement $K = 3$ tap long LMS equalizer and try to equalize the channel. Use three different step-size coefficient μ values to ensure equalizer convergence to the time moments $N_{\mathrm{cvg}} = \{5\,000; 25\,000; 75\,000\}$ symbols. Add to the report constellations (last 25 symbols), impulse responses of the equalizer at the end of the simulation, and common channel and equalizer impulse response at the end of the simulation. Construct frequency responses for the obtained impulse responses.

- Increase number of the equalizer filter taps to $K = 33$. Repeat operation of the previous paragraphs. Make conclusions on the equalizer length requirements.

- **Optional** Use step-size coefficient to ensure convergence at time moment $N_{\text{cvg}} = 25\,000$ symbols. Modify channel to ensure the following behavior. At the beginning of the simulation, the notch position is $f_n = 0.1$. After $N = 25\,000$ symbols, the notch starts to move to the direction of negative frequencies (change τ). Perform simulation and find maximal movement speed at which equalizer still is capable to track channel changes.

- Use constant channel with notch position $f_n = 0.1$. Add noise to the received signal to obtain $SNR = 20\,\text{dB}$ signal-to-noise ratio. Run simulation. Reduce SNR by 5 dB and rerun simulation. Keep reducing SNR till convergence is possible. Compare convergence curves. In conclusions, explain why convergence is no more possible for this noise level.

- Increase notch depth $L = 25\,\text{dB}$. Try to compensate it.

- Implement constant modulus algorithm (CMA) equalizer. Rerun previous task and try to compensate $L = 25\,\text{dB}$ deep notch. In conclusions, explain the difference between equalization algorithms and the reason for CMA convergence capability.

- Set notch depth $L = 5\,\text{dB}$. Equalize it using LMS and CMA algorithms, both of which should converge by $N = 50\,000$ symbols. Compare constellations at the end of the simulation. Calculate mean-square error of the last $N = 50\,000$ symbols. Describe your comparison result and its explanation in conclusions.

3. Task of increased complexity[1]

Find, describe and explain the necessity for the tap-leakage algorithm for the fractionally-spaced equalizers. Implement it and show experimentally, how it improves equalizer's performance.

4. Report structure

1. Home task.

[1] Necessary to get the highest mark

2. Block diagrams of the developed dynamic systems.
3. Channel and equalizer impulse responses and frequency responses
4. Constellations of the input and output signals of a dynamic system at the beginning of the adaptation and in the steady-state.
5. Listings.
6. Conclusions.