Erratum 1

During Practical work 1, we have made some imprecisions. So, it is time to correct them:

1. To generate random integers from the set $\{-1; 1\}$, we have chosen a function $randi(k)$, which generates uniformly distributed integers from 1 to (including) k:

   ```
   kron(randi([-1 1],1,N),ones(1,4));
   ```

   Note that the number of generated elements is two; therefore, we have to choose $k = 2$. As the difference between elements of our generating set is 2, we have to multiply the output of the function by 2. Afterward, subtracting constant component from the resulting sequence, we obtain a final piece of code:

   ```
   kron(randi(2,1,N)*2-3,ones(1,4));
   ```

2. To emulate in MATLAB a crystal receiver, precisely a diode, we have used the $abs()$ function, which gives absolute value. I have agreed with you that it leads to the appearance of two new components in the spectral domain: at zero frequency and the double carrier frequency. In general, it is not valid.

   Consider a transfer function of the demodulator (volt-ampere characteristics of a diode) in its definition domain $x \in [-2, 2]$. An approximation of such characteristics with a polynomial of the 2nd, 4th, 6th, and 8th order is shown in Fig.1.

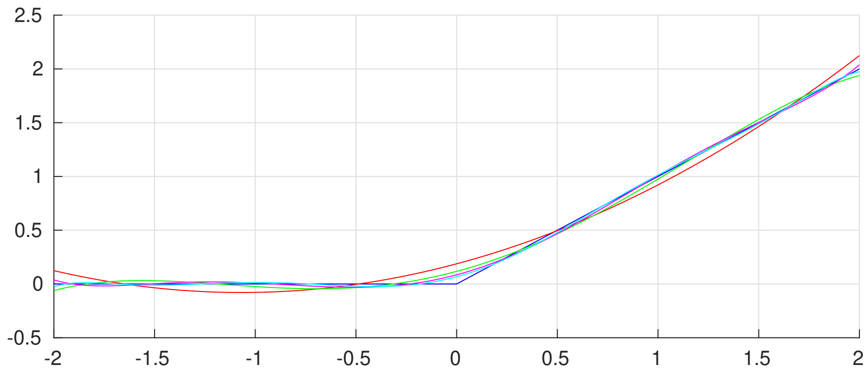

Figure 1: A polynomial approximation of the VAC of a diode

The script used to construct the VAC is given in Appendix. The results of the simulation allow us to use the following expression for approximation:

$$y(x) = 0.0855 + 0.5000x + 0.5762x^2 - 0.1757x^4 + 0.0228x^6. \qquad (1)$$

Passing a harmonic wave through this detector (or, in term of mathematics, substituting $s_{in}(t) = \sin(\omega t)$) yields to:

$$\begin{aligned} y(t) &= 0.0855 + 0.5000\sin(\omega t) + 0.5762 \cdot \sin^2(\omega t) - \\ &- 0.1757\sin^4(\omega t) + 0.0228\sin^6(\omega t) \end{aligned} \qquad (2)$$

Expressing sine powers as linear combinations of sine waves of double, triple, etc. frequencies and simplifying an expression, we can obtain:

$$\begin{aligned} y &= 3.1 \cdot 10^{-1} + 5.0 \cdot 10^{-1}\sin(x) - 2.1 \cdot 10^{-1}\cos(2x) - \\ &- 1.7 \cdot 10^{-2}\cos(4x) - 7.1 \cdot 10^{-4}\cos(6x) \end{aligned} \qquad (3)$$

Thus, we can see that demodulation with a diode leads to the appearance of new components at frequencies that are multiples of the carrier. The more precise approximation is used, the presence of the higher number of components in the output spectrum can be computationally proved.

Appendix

Script to approximate VAC of a diode:

```
% Generate transfer function (VAC of a diode)
x=linspace(-2,2,1000);
y=x; y(y<0)=0;

% Approximate by a polynomial
figure(1); hold on; grid on
plot(x,y, 'b-');

% Parameters of the approximation
C={   2,   4,   6,  8;
    'r', 'g', 'm', 'c'};

for k=1:size(C,2)
    p=polyfit(x,y,C{1,k})
    z=polyval(p,x);
    plot(x,z, C{2,k})
end
```