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Electrical Impedance and Diffuse Optical Tomography Reconstruction Software

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Abstract *–The EIDORS (Electrical Impedance and Diffuse Optical Reconstruction Software) project aims to produce a robust portable software system for reconstructing images from electrical or diffuse optical data. At its heart are object oriented libraries for linear algebra and finite element method written in C++. An interface to the MATLAB interpreter will add the benefit of rapid prototyping and a platform independent user interface and display.*

Keywords : electrical impedance tomography, diffuse optical tomography, software, finite element methods

1. ELECTRICAL IMAGING

Electrical Impedance Tomography (EIT) is the commonly used misnomer for a technique which seeks to image the electrical conductivity and permittivity of a medium my applying external sources of alternating electric current and making surface measurement of the electric field.

A time harmonic electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$, with angular frequency omega, satisfy Maxwell's equations

$$\nabla\times\mathbf{E} = jwm\mathbf{H}$$
$$\nabla\times\mathbf{H} = (s - jwe)\mathbf{E} \qquad (1)$$

Here m the magnet permeability, e the electric permittivity and s the electrical conductivity are all considered as functions of space. The complex conductivity $s - jwe$ is also called the *admittivity*. It is in possible in principle to recover all three material parameters from a complete knowledge of tangential components of electric and magnet fields at the boundary [1]. Measurements can be taken at one frequency provided that it is not resonant.

Assuming sufficiently small permeability and frequency we make the approximation $\nabla\times\mathbf{E}=0$ and therefore $\mathbf{E}=\nabla f$ where f is the (complex valued) electric potential. Maxwell's equations (1) then reduce to the elliptic partial differential equation (PDE)

$$\nabla\cdot\big((s - jwe)\nabla f\big)=0 \qquad (2)$$

The known boundary data is then the electric field f and the normal component of the current density $\mathbf{n}\cdot\nabla f$ ($\mathbf{n}$ is the outward unit normal vector to the boundary). In practice one applies a number of known current patterns to the boundary and makes corresponding measurement of potentials. In medical applications, where the frequency used is typically 10 to 200 kHz both the real and imaginary components of the admittivity are significant. In some industrial process monitoring situations the real component may dominate in which case the technique is often called Electrical Resistance Tomography (ERT) and where the imaginary part dominates Electrical Capacitance Tomography (ECT). As the object is a continuum rather than an RC-network it would be more accurate to use the words resistivity and permittivity. Also it is not strictly a tomographic technique as one cannot independently reconstruct a slice. Perhaps the term Electrical Admittivity Imaging would have been better with hindsight.

2. OPTICAL TOMOGRAPHY

Optical tomography is the method of using light in a narrow-wavelength band in the near infra red (700 to 1000 nm) to transilluminate an object and using the resulting measurements of intensity on the tissue boundary to map the optical properties within the tissue. Applications of this technique so far have been in medical imaging. A general model for light transport is given by photon transport theory [2]. It is an integro-differential equation which can be simplified by expansion in spherical harmonics. The result is a set of $(N+1)^2$ coupled differential partial equations known as the P_N

approximation. For N odd such equations can be reduced to a single $(N+1)$ th order partial differential equation. For $N=1$ the four coupled partial differential equations can be reduced to a single parabolic PDE

$$\nabla \cdot \left(A\nabla\Phi\right) - B\Phi - C\P\Phi / \P t = q \qquad (3)$$

Here Φ is the isotropic photon density and A and B are positive functions of the spatial variable related to the scattering and absorption of the medium, C is the reciprocal of the speed of light in the medium and q is a function of space and time representing a source distribution. The source is concentrated near the boundary of the domain. The data we know are the boundary measurements $\mathbf{n} \cdot A\nabla\Phi$ for a variety of source terms q. Changing to the frequency domain we obtain the elliptic equation.

$$\nabla \cdot \left(A\nabla\hat{\Phi}\right) - (B + jwC)\hat{\Phi} = \hat{q} \qquad (4)$$

Typically one assumes that C is known and then one can recover A and B from complete data at the boundary, at two frequencies.

Uniqueness results for the inverse problem for Equations (2) (3) and (4) are explained in [3].

3. SOLVING THE INVERSE PROBLEM

Both these techniques reduce to the problem of determining unknown complex coefficients for elliptic PDEs. Such problems are nonlinear and ill-posed. The first step in solving such an inverse problem is to find a numerical method to solve the direct problem. That is to solve the elliptic equation given the coefficients (s and e in the electrical case and A , B and C in the optical case) and one set of boundary data (current density or source). The most flexible and most widely used approach is the Finite Element Method (FEM). One then linearizes the equations about a some assumed values for the unknown coefficients and solves a linear system to update the coefficients. This is repeated until the numerical model matches the measured data to adequate precision. Descriptions of this type of technique applied to the electrical problem can be found for example in [4],[5] and in the optical case [6]

Implementing a software system to solve the direct problem is itself a formidable challenge. The PDEs must be solved in three dimensions. The domains must be divided into meshes. The boundary conditions, which often create singularities in the fields, must be modelled accurately. The finite element code includes representations of different types of elements including shape functions and quadrature rules. There must also be a linear algebra library capable of manipulating matrices of a variety of structures (dense, random sparse, banded symmetric etc.) and solving systems of linear equations by both direct and iterative methods.

4. THE EIDORS PROJECT

Many groups have implemented their own software systems for solving the electrical and optical inverse problems. A variety of programming languages have been used including MATLAB[1], FORTRAN, C and C++. Each has their own merits, MATLAB for example facilitates rapid prototyping at the expense of execution speed, FORTRAN and C benefit from the availability of extensive stable libraries of numerical methods. The advantage of C++ is that it facilitates object oriented design. Finite element and matrix calculations can be naturally expressed in an object oriented paradigm. Object oriented programming results in a code which is generally easier to maintain and extend. As the task of programming a complete forward model and reconstruction code is large and it seems unfortunate that so many groups expend effort writing their own. It would be better to start from an existing code and extended that to provide the functionality needed. This way advances in reconstruction algorithms would be made more quickly. The problem is that it often seems harder to understand someone else's code that to write one's own. Assuming that that other person is willing to share their code.

The EIDORS (Electrical Impedance and Diffuse Optical Reconstruction Software) project aims provide a common software platform for forward modelling and reconstruction. The foundation of the project is the TOAST code [7] written at UCL (University College London) in C++. This incorporates a general matrix and linear algebra library (mathlib), a finite element library (felib) and a collection of inverse problem solvers for optical tomography (toastlib).

There are some attempts under way to provide general linear algebra libraries in C++, to replace the more traditional FORTRAN or C based libraries such as LAPACK. The Template Numerical Toolkit (TNT) [8] is a notable example, but this package is at an early stage of development. At present the UCL mathlib fits our purposes exactly. Should TNT, or an other freely available library, gain a dominant position we may later switch to avoid having to maintain and develop standard linear algebra routines ourselves. The situation with freely available

1 MATLAB produced bt The Math Works Inc, http://www.mathworks.com

FEM libraries is no better. One can find many good packages to solve two dimensional problems but we know of no freely available 3D FEM library which fits our requirements other than the UCL femlib.

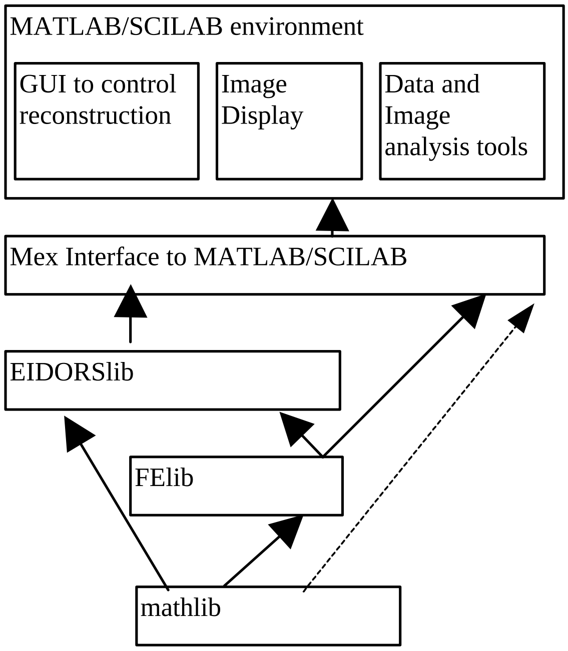

Figure 1: The structure of EIDORS. The arrow indicates the relation 'may be called by'

Work is underway to write C++ classes, initially as wrappers to the toastlib code, which implement solvers for general nonlinear inverse problems. This together with toastlib will become eidorslib.

To retain the advantage of rapid prototyping, and to provide flexibility in data analysis and display, an interface will be provided ('mex' files) so that eidorslib, as well as some of the basic mathlib and felib functions, can be called as functions from the MATLAB environment. MATLAB will thus serve as a scripting language for EIDORS. Furthermore graphical user interfaces (GUIs) can easily be constructed in MATLAB, including elements such as dialogue boxes and pull down menus. The structure of EIDORS is summarized in Figure 1.

We felt it important that if possible we create a system which could be used under Microsoft Windows as well as Linux/Unix X-windows. A GUI implemented in MATLAB, while perhaps slow to execute, is portable across a wide range of platforms. Users will have the choice to implement their own interfaces in high level languages and link in EIDORS code directly.

5. OPEN SOURCE

It is an important part of the strategy of the EIDORS project that the source code will be copyrighted but freely available. The cooperative open source model for software development has proved superior in many cases to closed source methods [9]. Indeed most of the software at the heart of the Internet was and still is developed in this way. We feel that it is important that users have the opportunity to fix bugs they may find, adapt the code to their own problems, add innovations of their own, and possibly port the code to different platforms.

We anticipate that the majority of users will not have a great interest in modifying eidorslib. C++ is a relatively hard language to learn and many scientists working in electrical or optical tomography are more interested the applications of these techniques than in the algorithms. They can uses the range of standard methods which will be implemented in EIDORS and accessible from menus. If they wish to implement variations on these without changing C++ code this may be possible from MATLAB. Useful innovations may later be implemented in C++ and contributed to the project.

As MATLAB is a proprietary, and quite expensive, product some users may prefer to use SCILAB[2] which is a very similar interpreted matrix oriented language. While not open source it is freely available and multi platform. If there proves to be a demand we may include a SCILAB interface to EIDORS.

6. FIRST PHASE OF DEVELOPMENT

Our goal in the first phase of development of the project is to produce a robust and flexible software system for reconstruction and display of both electrical and diffuse optical images. We hope to include:

- Adequate three dimensional meshing for irregular domains,
- Full electrode modelling for the electrical case.
- Complex admittivity
- The incorporation of known distribution of measurement error.
- Calculation and use of optimal patterns
- Fast linear solution from precomputed Jacobian
- Full non-linear solution
- Incorporation of prior information through basis constraints

2 SCILAB is produced by INRIA http://arikara.inria.fr/www-rocq.inria.fr/scilab/

7. FURTHER WORK

Work is in progress to implement

- Unknown internal boundaries
- Constrained anisotropic conductivity
- Translationally uniform conductivity from measurements in a plane
- Time series techniques to correct for variations in the image during the data collection cycle [10]
- Principal component analysis for time series of images
- Markov Chain Monte Carlo Methods to incorporate prior image information [11]
- Full transport model for optical tomography
- Infinite elements for semi infinite domains
- Imaging dielectric relaxation time distributions

At present these techniques have been implemented in experimental form in MATLAB or FORTRAN and we would hope to include them in the EIDORS project. The Markov Chain Monte Carlo techniques are the initiative of our collaborators in Auckland [11]. We have many ambitions for the project which are at present only on the horizon. The solution of the full Maxwell equations for the electrical case would be interesting. Also there is the possibility that the project might grow into a more general Inverse Problems Library for multidimensional nonlinear inverse problems.

8. CONCLUSIONS

While the main focus so far of the EIDORS project has been on medical imaging the software will be equally suited to industrial process monitoring, nondestructive testing or geophysical applications. We hope to gain the interest an involvement of the process tomography community in this project.

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