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A MATLAB package for the EIDORS project to reconstruct two-dimensional EIT images

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Abstract
The EIDORS (electrical impedance and diffuse optical reconstruction software) project aims to produce a software system for reconstructing images from electrical or diffuse optical data. MATLAB is a software that is used in the EIDORS project for rapid prototyping, graphical user interface construction and image display. We have written a MATLAB package (http://venda.uku.fi/~mvauhkon/) which can be used for two-dimensional mesh generation, solving the forward problem and reconstructing and displaying the reconstructed images (resistivity or admittivity). In this paper we briefly describe the mathematical theory on which the codes are based on and also give some examples of the capabilities of the package.

Keywords: electrical impedance tomography, MATLAB, image reconstruction

1. Introduction

In electrical impedance tomography (EIT) electrodes are attached on the surface of the body and small alternating currents are injected through these electrodes. The corresponding voltages are measured and an estimate for the internal resistivity or more generally admittivity distribution is computed.

To reconstruct EIT images in general geometrical settings, we need a system for measuring the surface geometry, a mesh generator to make a mesh based on the surface information, code for computing the finite element approximation and an algorithm to solve the inverse problem. In addition we will need tools to display the results and also to analyse the images.

MATLAB[3] is a software for numerical computations with matrices. The program is suitable for rapid prototyping and data display and postprocessing but has its drawbacks due to the speed limitations. We have written a set of m-files (functions) in MATLAB for making

[3] MATLAB produced by the Math Works Inc, http://www.mathwoks.com/

two-dimensional (2D) meshes using a QMG mesh generator [4], solving the forward problem (complete electrode model), reconstructing resistivity or admittivity distributions and display of the results. It is possible to use whatever number of electrodes with different current patterns and measurement procedures.

In this paper we briefly describe the mathematical theory behind the functions and show some examples of the capabilities of the software. The package presented in the paper is part of the EIDORS (electrical impedance and diffuse optical reconstruction software) project, which aims to produce a software system for reconstructing images from electrical or diffuse optical data. For more on the project, see the article by Lionheart *et al* (1999).

2. Theory

2.1. Mesh generation

The first step in the EIT image reconstruction is the mesh generation. There are many mesh generators developed for complicated 2D meshing but three-dimensional (3D) situations are much more complicated and good 3D mesh generators are hard to find. We have utilized a quadtree mesh generator QMG written by S Vavasis in our programs (for more information, see http://www.cs.cornell.edu/home/vavasis/qmg1.0/meshgen.html and a book by George (1991)). With this mesh generator it is possible to make constrained meshes, which is useful if we have some *a priori* known internal structures, such as the skull layer. It would be possible to use QMG also for 3D mesh generation but so far this has not been utilized.

A drawback of the quadtree approach compared with the popular Delaunay triangulation is that the element quality in the Delaunay meshes is superior compared to the quadtree meshes. However, the properties of Delaunay meshes cannot be extended to 3D, which makes it difficult to apply the Delaunay approach in 3D. In some cases we have used mesh smoothing to produce better quality meshes (Field 1988).

2.2. Forward problem

There are different electrode models for EIT (Cheng *et al* 1989). We have used the complete electrode model and solved it with the finite element method (Vauhkonen *et al* 1998, 1999). The potential distribution can be solved for purely resistive domains or for domains that also have capacitive properties. The approximation of the solution (potential) can be either piecewise linear or piecewise quadratic, depending on the choice of the approximation basis.

2.3. Inverse problem

EIT image reconstruction is a nonlinear ill posed inverse problem, which makes it difficult to obtain stable and reliable results, especially in human experiments. To solve the inverse problem we have used an approach in which we minimize the functional

$$\Psi(\rho) = \|U - U(\rho)\|^2 + \alpha^2 \|L(\rho - \rho^*)\|^2 \quad (1)$$

with respect to the resistivity ρ. In (1) U is the vector of measured voltages and $U(\rho)$ the corresponding computed voltages, L is a regularization matrix, α is a regularization parameter and ρ^* is an *a priori* guess for ρ. The solution is sought iteratively by

$$\rho_{i+1} = \rho_i + \delta\rho_i \quad (2)$$

[4] http://www.cs.cornell.edu/home/vavasis/qmg-home.html

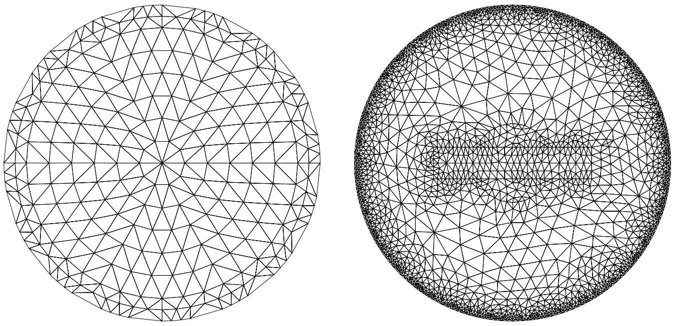

Figure 1. On the left there is a structured circular mesh and on the right there is a constrained circular mesh. Laplace smoothing has been applied on the constrained mesh to improve the element quality.

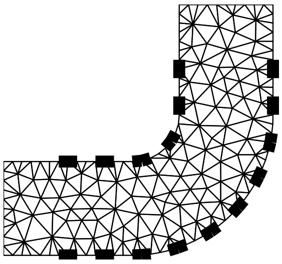

Figure 2. An unstructured mesh of a '2D pipe'. Black quadrilaterals denote electrodes on the boundary.

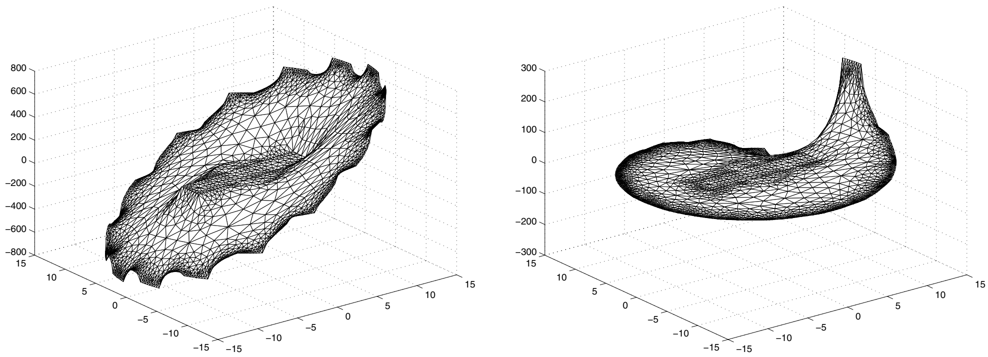

Figure 3. On the left there is a potential distribution corresponding to the first trigonometric current injection and on the right corresponding the adjacent injection.

where $\delta\rho_i$ is solved from

$$(J_i^{\mathrm{T}} J_i + \alpha^2 L^{\mathrm{T}} L)\delta\rho_i = J_i^{\mathrm{T}}(U - U(\rho_i)) - \alpha^2 L^{\mathrm{T}} L(\rho_i - \rho^*). \tag{3}$$

In 3D problems this type of approach is not recommendable due to the difficulty of solving equation (3). In 3D cases it is better to use nonlinear conjugate gradient or steepest-descent-type approaches. In these cases the iteration code can be written such that the Jacobian is only computed implicitly (Arridge and Schweiger 1998).

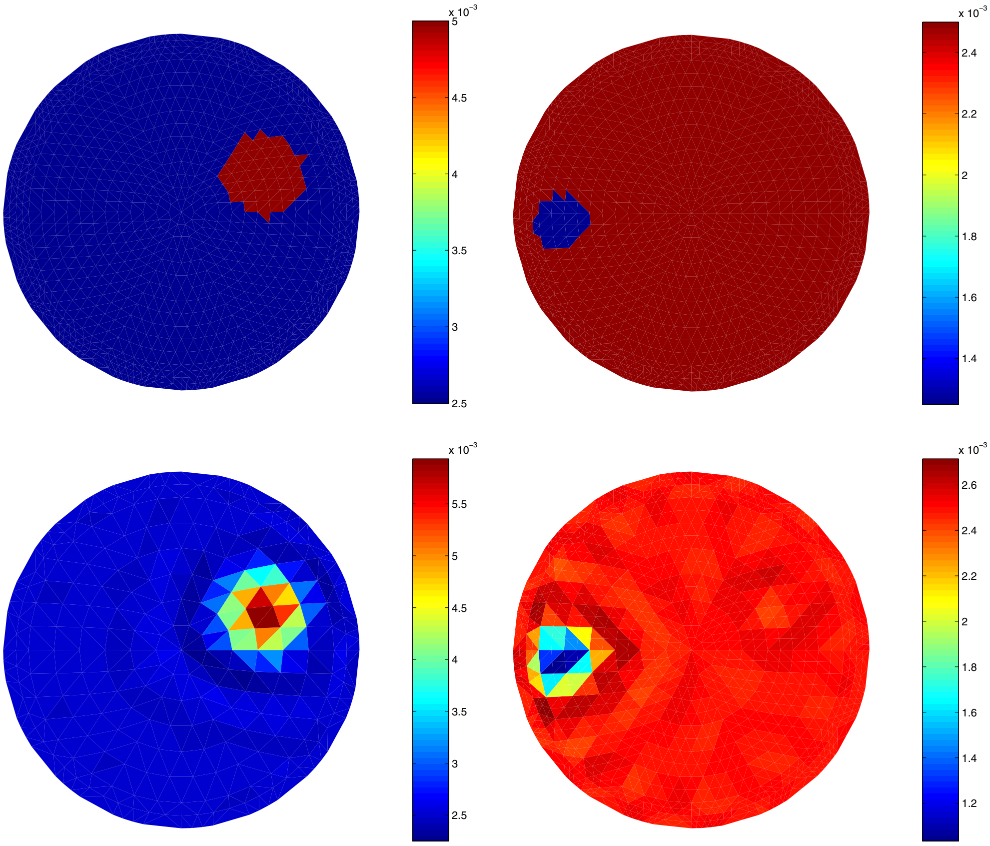

Figure 4. On the upper row, left, there is a true conductivity distribution and on the right the true scaled permittivity distribution. On the lower row, left, the reconstructed conductivity and on the right the reconstructed scaled permittivity after five iterations.

If the Jacobian is computed, the most efficient way is to use the so called *adjoint approach*. In this approach only forward solutions and two matrix products need to be calculated. For more details, see e.g. the article by Arridge and Schweiger (1998). This type of approach is also utilized in this software package.

3. Examples

Here we show some examples of the capabilities of the package to reconstruct EIT images in two dimensions.

3.1. Meshes and forward computations

Different types of 2D mesh can be constructed. Also meshes from external mesh generators can be easily utilized. Examples are shown in figures 1 and 2.

The forward solution includes the potential inside the domain and voltages on the electrodes. In figure 3 there are shown potential distributions computed using the mesh shown in figure 1 on the right. There was a highly conductive structure in the centre which makes the potential distribution flat near the centre. Also effects of the electrodes are clearly visible.

3.2. Reconstructions

In figure 4 there is an example of an admittivity reconstruction. Both the conductivity σ and scaled permittivity $w\epsilon$ were reconstructed based on the complex voltage data.

4. Discussion and conclusions

The existing set of MATLAB functions provides a platform for fast prototyping of different reconstruction schemes in EIT. Code modification is fairly easy and the results can be displayed and analysed quickly. A drawback of this type of approach is slow performance, especially in high-dimensional problems. For this purpose the tested approaches will be written in C/C++ for final applications.

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References

Arridge S R and Schweiger M 1998 A general framework for iterative reconstruction algorithms in optical tomography, using a finite element method *Computational Radiology and Imaging: Therapy and Diagnosis (IMA Volumes in Mathematics and its Applications vol 110)* ed C Borgers and F Natterer (IMA, Springer) pp 45–70

Cheng K-S, Isaacson D, Newell J C and Gisser D G 1989 Electrode models for electric current computed tomography *IEEE Trans. Biomed. Eng.* **36** 918–24

Field D A 1988 Laplacian smoothing and Delaunay triangulations *Commun. Appl. Numer. Methods* **4** 709–12

George P L 1991 *Automatic Mesh Generation, Application to Finite Element Methods* (Paris: Wiley)

Lionheart W R B, Arridge S R, Schweiger M, Vauhkonen M and Kaipio J P 1999 Electrical impedance and diffuse optical tomography reconstruction software *Proc. 1st World Congr. on Industrial Process Tomography* pp 474–7

Vauhkonen M, Vadász D, Karjalainen P A, Somersalo E and Kaipio J P 1998 Tikhonov regularization and prior information in electrical impedance tomography *IEEE Trans. Med. Imaging* **17** 285–93

Vauhkonen P J, Vauhkonen M, Savolainen T and Kaipio J P 1999 Three-dimensional electrical impedance tomography based on the complete electrode model *IEEE Trans. Biomed. Eng.* **46** 1150–60