Logic - Stanford CS 221 Fall 2017-2018

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1 Problem 1: Propositional logic

2 Problem 2: First-order logic

3 Problem 3: Liar puzzle

4 Problem 4: Logical Inference

Having obtained some intuition on how to construct formulas, we will now perform logical inference to derive new formulas from old ones. Recall that: Modus ponens asserts that if we have two formulas, $A \implies B$ and A in our knowledge base, then we can derive B. Resolution asserts that if we have two formulas, $A \vee B$ and $\neg B \vee C$ in our knowledge base, then we can derive $A \vee C$. If $A \wedge B$ is in the knowledge base, then we can derive both A and B.

a Some inferences that might look like they're outside the scope of Modus ponens are actually within reach. Suppose the knowledge base contains the following two formulas: $KB = \{(A \vee B) \implies C, A\}$.

$(A \vee B) \implies C$

$\frac{(A \vee B) \implies C}{\neg(A \vee B) \vee C}$

$\frac{\neg(A \vee B) \vee C}{(\neg A \wedge \neg B) \vee C}$

$\frac{(\neg A \wedge \neg B) \vee C}{(\neg A \vee C) \wedge (\neg B \vee C)}$
$\neg A \vee C$ is the same as $A \implies C$ and $\neg B \vee C$ is the same as $B \implies C$.
Therefore $\frac{(\neg A \vee C) \wedge (\neg B \vee C)}{A \implies C \wedge B \implies C}$.

b Recall that Modus ponens is not complete, meaning that we can't use it to derive everything that's true. Suppose the knowledge base contains the following formulas: $KB = \{A \vee B, B \implies C, (A \vee C) \implies D\}$. In this example, Modus ponens cannot be used to derive D, even though $D$is entailed by the knowledge base. However, recall that the resolution rule is complete.

$KB = \{A \vee B, B \implies C, (A \vee C) \implies D\}$

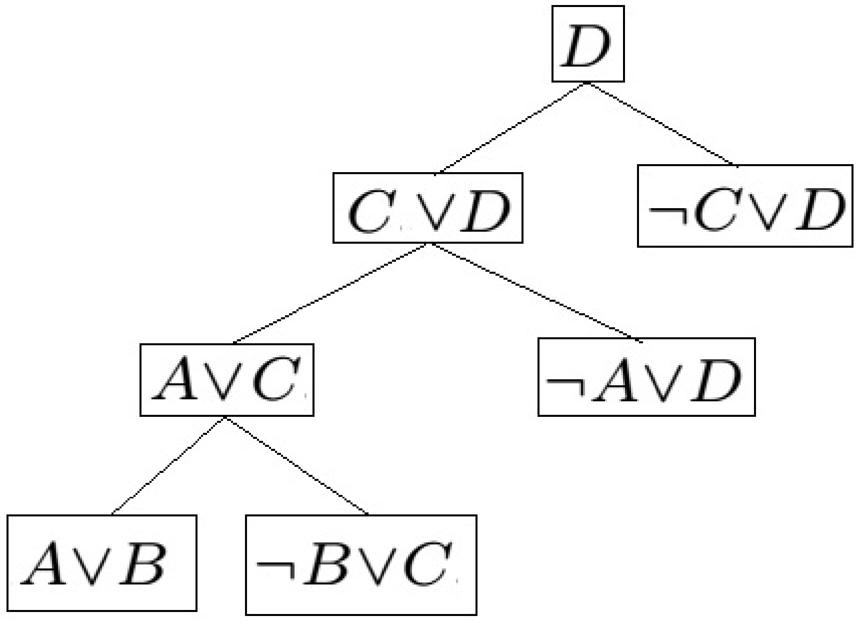

Figure 1: Derivation of D

$$= \frac{A \vee B, B \implies C, (A \vee C) \implies D}{A \vee B, \neg B \vee C, \neg(A \vee C) \vee D}$$

$$= \frac{A \vee B, \neg B \vee C, \neg(A \vee C) \vee D}{A \vee B, \neg B \vee C, (\neg A \wedge \neg C) \vee D}$$

$$= \frac{A \vee B, \neg B \vee C, (\neg A \wedge \neg C) \vee D}{A \vee B, \neg B \vee C, (\neg A \vee D) \wedge (\neg C \vee D)}$$

5 Problem 4: Odd and even integers

a

b Suppose we added another constraint: A number is not larger than itself. Prove that there is no finite, non-empty model for which the resulting set of 7 constraints is consistent. This means that if we try to prove this theorem by model checking only finite models, we will find that it is false, when in fact the theorem is true for a countably infinite model (where the objects in the model are the numbers).

The first condition states that each number x has exactly one successor which is not equal to x. So for a finite set to hold say we have numbers $\{x_1, \ldots, x_n\}$ we need to define successor $x_i = x_{i+1}$ and successor $x_n = x_1$. In this case with the first 6 constraints are consistent.
Note that larger than is an arbitrary assignment so the transitive property x_1 is larger than x_4 and x_4 is larger than x_n implies x_1 is larger than x_n holds.
Similarly since x_1 is the successor to x_n, the larger than property can hold for this as well.
However when we add the constraint "a number is not larger than itself" it breaks the transitive property.
For a finite set when we add the 7th constraint (a number is not larger than itself) we either break
Constraint 1 - A number has exactly one successor which is not itself
or
Constraint 6 - The transitive property
But this would hold for a countable infinite model where successor $x_n = x_{n+1}$.