Applied Finance Project

IDIOSYNCRATIC VOLATILITY PUZZLE

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Table of Contents

1 Introduction

Ang et al. (2006) showed that high short-term total/ idiosyncratic volatility (calculated using last month's daily returns) is associated with low excess expected return. The effect persists in any market environment and cannot be explained by exposures to various risk factors such as size, book-to-market and liquidity. The result is counter-intuitive and not in alignment with modern finance theory. Specifically, investors should be compensated with higher expected return by taking higher systematic (or factor) risk. Thus, high total volatility (including variations of return from both systematic and idiosyncratic risks) should in general induce high expected return. Second, idiosyncratic risk is diversifiable thus should not be a determinant of portfolios' returns. The phenomenon remains a puzzle.

One hypothesis to explain this puzzle is that the anomaly is driven by short term speculative activities. The investors who invest in stocks of higher short term volatility would not be compensated with higher returns. Naturally, we wonder if the puzzle remains effective for long-term volatility (calculated using monthly or weekly returns) as short-term volatility and long-term volatility are economically and empirically different. We believe short-term volatility reflects speculative activity, excessive trading and investors' excitement on daily news, and long-term volatility reflects the rational analysis of a firm's past performance (e.g. financial report) and investors' updated views on firm's fundamentals. Statistically, ample evidence showed that daily returns are auto-correlated.

If higher long term volatilities (total or idiosyncratic) is justified with higher returns, the hypothesis may hold. On the contrary, if similar anomaly can be observed for long-term volatilities, they would serve as a better trading signal as they are more persistent.

The report is structured as follow. In Chapter 3, we first investigate the strength and regime characteristics of short-term volatility puzzle over an extended time horizon from 1965 to 2016 (Ang's original paper used data from 1963 to 2000). Next, in Chapter 4, we run an analysis (similar to that for short term volatility) to examine the existence of long-term volatility puzzle (higher long-term volatility, lower return or alpha). In Chapter 5, we characterize the nature of the long-term puzzle by investigating the relationship between long-term and short-term volatilities and their interactions in producing the anomalies. In Chapter 6, we examine the market frictions that could possibly cause and sustain the anomalies. To ensure the robustness of results, we conduct the cross-check with other well-known factors: size, reversal and beta in Chapter 7. We conclude in Chapter 8.

2 Data Source and Preprocessing

In this section, we describe our data source and its cleaning procedures. Key variables in our empirical exercise include stock returns and their volatilities. The latter variable is calculated from the stock returns, and the details of the calculation are described below. We describe our procedure below.

2.1 Stock Returns

The dataset we use is the daily U.S. equity total return data from the Center for Research in Security Prices (CRSP) from 1963 to 2016. This database includes the stocks with primary listings on the NYSE, NYSE MKT, NASDAQ and Arca exchanges (Center for Research in Security Prices, 2016). In this study, we focus on common stocks traded in the US. ETFs and REITS are excluded from our analysis.[1]

To handle outliers and negative prices, we perform the following data cleaning procedure:

1. We remove the stocks during the days when their total holding period return ('RET') appears as 'B'. 'B' refers to the case that the security is not traded on the exchange on that day (Center for Research in Security Prices);
2. When stocks' total holding period returns ('RET') appear to be 'C', we replace them with zero. 'C' refers to the case that there is valid current price but no valid previous price. Most of the times, the first-day return of a stock is shown as 'C';

[1] We filter stocks based on share code provided by CRSP; only share codes of 10, 11 and 12 are included in our stock universe.

3. If the price (PRC) of the stock is negative, we replace it with its absolute value. Negative price means that there is no actual closing price and the number shown is the average bid-ask average.

Regarding missing data points, we follow the rules below in selecting stocks to analyze within a certain time horizon:

1. Given a time window, stocks with more than 20% missing data points will be removed.
2. Stocks with more than 20% "Suspended" or "Halted" trading status are removed.
3. To make sure that the stocks prices are not stale, we remove stocks with more than 20% zero trading volume in any given period.
4. Stocks with average price below $2.5 are removed.

2.2 Volatility Calculation

As specified in Ang et al. (2006), a short-term total volatility is calculated using the standard deviation of daily returns over the past month $\sigma = \sqrt{\frac{1}{n-1}\sum_0^t (r-\bar{r})^2}$, and a short-term idiosyncratic volatility is defined from Fama-French 3 factor (FF-3) regression[2]:

$$R_t^e = \alpha + \beta_{MKT}\left(R_{MKT} - R_f\right) + \beta_{SMB}SMB_t + \beta_{HML}HML_t + \epsilon_t.$$

Specifically, an idiosyncratic volatility is the standard deviation of the residual returns in the above regression. Similarly, we define a long-term total volatility as the standard deviation of monthly returns over the past 24 months excluding the latest one month (i.e. from T-2 to T-25). We exclude the latest one month in long term volatility calculation to eliminate the

[2] For long only portfolio, the R_t^e is the raw return minus risk free rate. For long short portfolio, the R_t^e is simply the raw return without minus risk free rate.

impact of latest one-month volatility.[3] A long-term idiosyncratic volatility is calculated using the FF-3 residual return over the same sample.

We investigate both total volatility and idiosyncratic volatility to have a more robust analysis of the volatility puzzle. Total volatility incorporates systematic risks of the company, while idiosyncratic volatility excludes them.

2.3 Transaction Cost Calculation

Transaction cost is a key to explain the long existence of this volatility anomaly. Wagner and Glass (2001) estimated U.S. market transaction cost by taking into account opportunity cost, trade execution delay and market impact. We also consulted Professor Coldiron on the industrial estimation of transaction cost. The industrial estimation is based on live trading experience from 2003 to 2014. It considers explicit cost like commission and implicit costs such as market impact and opportunity cost for multi-day orders, which is consistent with the research by Plexus (Plexus Group, 1998).

To calculate the transaction cost for a portfolio, we also need to obtain the total turnover for the portfolio. Total turnover is calculated by summing up the absolute value of weighted changes in each stock. We use W_t^b to denote the weighting scheme for the portfolio at the beginning of month t. Due to market movement, value weighted holding at the month end will be different from what we had at the beginning. The end period holding of the portfolio is changed to $W_t^e = W_t^b \circ \frac{R_t + \mathbf{1}}{1 + R_t^T W_t^b}$, in which $\circ$ represents element-wise multiplication.

[3] We use monthly returns because it is less noisy and thus more suitable to gauge long-term variations in stock returns. Regarding a time horizon, we choose 24 months because it balances the number of observations and the possible impact of regime change. We also examine the results using weekly returns for the horizons of 24 weeks, 48 weeks and 72 weeks, and monthly returns for the horizons of 12 months and 18 months. These tests as shown in the appendix provide consistent patterns as the long-term volatility of 24 months.

Holding change is calculated as the sum of absolute value differences between W_t^e and W_{t+1}^b. It can be expressed as:

$$\mathbf{1}^T \left| W_t^e - W_{t+1}^b \right| = \mathbf{1}^T |W_t^b \circ \frac{R_t + \mathbf{1}}{1 + R_t^T W_t^b} - W_{t+1}^b|$$

The maximum holding change is 2. It can be reached when all positions in the t+1 portfolio are different from the holdings at t. Using the transaction cost λ we obtained, the impact of the transaction cost on portfolio return is:

$$-\lambda \mathbf{1}^T |W_t^b \circ \frac{R_t + \mathbf{1}}{1 + R_t^T W_t^b} - W_{t+1}^b|$$

2.4 Short Sale Fee

Another component for market friction besides transaction costs is the short sale fee. All US stock short sale fees are available from Markit. Unfortunately, they are only available from 2002 and the data are relatively sparse in comparison to the number of stocks in our calculation. In this study, if the short sale fee of any stock at any time is missing, the stock will not be taken into consideration.

We calculated the short fee as the weighted average short-fee of the stocks in the short leg. $\boldsymbol{C_t}$ is the cost vector measuring the short cost for each stock measured in bps. The total cost per month for shorting at the beginning of time t should be:

$$\boldsymbol{C_t} \times \mathbf{W_{short\ leg}}$$

3 Short Term Volatility Puzzle

Ang et al. showed that over the period from 1962 to 2000, on average high short-term total/ idiosyncratic volatility is associated with low return. Stocks with the highest short-term volatility also exhibit significantly negative alphas when the three Fama-French risk factors (market, size, B/M) are controlled. In this section, we first extend the original study over a longer period by including the data from 2000 and 2016. We aim to see if the recent market development affects the strength of the puzzle. Second, we construct the long-short volatility factor portfolio and study the factor's performance over time. This allows us to see whether the factor's behavior is time-specific, and what we should expect under the current environment.

3.1 Puzzle Effect from 1965 to 2016

We investigate the short-term volatility puzzle over the period from Feb 1965 to Dec 2016. At the end of each month, we apply the criteria from section 2.1 to select the investible universe. We construct five value-weighted quintile portfolios by sorting the stocks' short term total/idiosyncratic volatility calculated over the past one month. We measure the portfolios' returns over the next month and record their performance over time with monthly rebalancing.

Table 3-1 summarizes the result. Panel A shows that the mean returns of quintile 1 (low total volatility stock) to quintile 4 are about the same. Then the returns drop sharply to 0.33% for quintile 5 (high total volatility stock). We find monotonically decreasing CAPM and FF-3 alpha from quintile 1 to quintile 5. The FF-3 alpha for quintile 5, reported in the last column, is −0.88% per month. The FF-3 alpha spread between portfolio 5 and portfolio 1 is −0.94%, with a robust t-statistic of −4.71. We observe similar patterns in Panel B, where the portfolios are sorted on idiosyncratic volatility. The return spread is −0.526% per month and the FF-3 alphas spread is −0.89% per month, with a t-statistic of −5.42.

The post-formation volatilities of portfolio return match the past volatility rankings of the quintiles. The market capitalization of the portfolio decreases as the volatility increases. The market shares of the quintile portfolios sorted by total or idiosyncratic volatility also display distinct patterns. Stocks with low (high) volatility are generally large (small) stocks.

In general, our result is in line with the finding in Ang et al. (2006)' paper: The higher total or idiosyncratic volatility a quintile portfolio has, the lower its Jensen's alpha. Fig 3-1 shows the patterns of alphas for both total and idiosyncratic volatility sorted portfolios. Like in Ang et al. (2006), we find that the alphas of the first four quintile portfolios are not significantly different from zero and the effect of decreasing alphas with increasing volatility is not strong. The alpha spread (1-5) is mainly induced by the large negative alpha of the last quintile, which is mainly composed of small and illiquid stocks. Ang et al. conducted their study over the period from 1962 to 2000. Our result differs from Ang et al. (2006)'s in which we found that the abnormal signal is weakened when the past sixteen years' data is included.

3.2 Volatility Factor Returns

We constructed Fama-French style factor portfolio by going long the first quintile portfolio (lowest volatility) and short the last quintile portfolio (highest volatility). We denote this long short portfolio as [1-5]. Figure 3-2 shows the cumulative returns of the short-term volatility factors.

Over the investigated horizon, we find that volatility factors do not always generate positive risk premium (We define risk premium as expected factor return). During the periods of 1965-1969 and 1975-1980, the average factor returns are negative. During the periods of

1969-1975 and 1980-2004, the average returns are positive. After a 24-year long period of positive premium since 1980, volatility factors seem to cease generating positive premium anymore. The timing coincides with Ang's initial publication of his discovery of short-run volatility puzzle around the end of 2004. We also observe the negative effect of dot com bubble in 2000 and the positive effect of financial crisis in 2008 on the factor returns.

Figure 3-2 breaks down the factors' performances for two regimes: prior and post the initial publication of Ang et al. (2006) paper in 2004 (on SSRN). We find that the average factor returns for total and idiosyncratic volatility are 1.06% and 1.11% from 1980 to 2004, with a t-statistic of 2.33 and 2.8 respectively. After 2004, the average factor returns are not significantly different from zero. The factor returns are negatively correlated with the excess market return, which is expected as the factor portfolios go long low volatility stocks and short high volatility stocks. Because stocks with high (low) volatility are generally small (large) stocks and small (large) stocks on average have high (low) betas, it's possible that volatility is correlated with beta such that longing (shorting) low (high) volatility stocks is to some extent longing (shorting) low (high) beta stocks, giving the long-short factor portfolio an overall negative beta.

The betas for total and idiosyncratic volatility factor are about -0.95 and -0.75 respectively. Those numbers seem to be stable across regimes, as do the average excess market returns. The disappearance of positive factor returns after 2004 is mainly caused by the lowered alphas. The CAPM (FF-3) alpha of the total volatility factor portfolio is 1.67% (1.17%) and 0.95% (0.94%) before and after 2004 respectively. The CAPM (FF-3) alpha of the idiosyncratic volatility factor portfolio is 1.60% (1.25%) and 0.51% (0.51%) before and after 2004 respectively. Due to the different alphas after 2004, we saw a slow increase of the cumulative return of the total volatility factor versus a flat level of the cumulative return of the idiosyncratic volatility factor. Total volatility is not a pure measure of additional independent risk as it includes systematic beta risk, which explains why its factor portfolio is more correlated with the excess market return (more negative portfolio beta).

The diminishment of positive factor risk premium after Ang's first publication could be attributed to two possible reasons. First, long-short arbitragers stepped in so the anomaly is arbitrated away. Second, people started to realize that high volatility stocks are often overpriced. They viewed the stocks less attractive than before. The decreasing demand of high volatility stocks caused their prices to fall, and the return spread of the factor portfolio is reduced. The first explanation stands only if the long-short portfolio is profitable to trade (including the transaction cost and short fee) when the paper was published. Since it is statistical arbitrage, the traders also must be willing to bear the risk (shorting highly volatile and illiquid stocks) associated with the strategy. The second explanation needs less assumptions. We investigate those two possible explanations further in Chapter 6.

Rank	Mean	Std Dev.	% Mkt Share	CAPM Alpha	FF-3 Alpha
Panel A: Portfolio Sorted by Total Volatility of Daily Return					
1	0.866	3.6	36.2%	0.12	0.06
				[1.58]	[1.0]
2	0.95	4.5	27.4%	0.09	0.05
				[1.82]	[1.22]
3	1.009	5.5	18.4%	0.03	0
				[0.44]	[0.06]
4	0.919	6.9	11.9%	-0.17	-0.17
				[-1.21]	[-1.59]
5	0.33	8.5	6.1%	-0.86	-0.88
				[-4.1]	[-5.65]
5-1	-0.536			-0.98	-0.94
	[-1.71]			[-3.68]	[-4.71]
Panel B: Portfolio Sorted Idiosyncratic Volatility Relative to FF-3 of Daily Return					
1	0.891	3.8	43.5%	0.1	0.07
				[1.6]	[1.65]
2	0.950	4.8	25.1%	0.05	0.02
				[1.07]	[0.58]
3	0.951	5.7	15.6%	-0.04	-0.06
				[-0.54]	[-0.92]
4	0.878	6.8	10.3%	-0.2	-0.2
				[-1.51]	[-2.18]
5	0.365	8.1	5.5%	-0.78	-0.82
				[-3.93]	[-5.95]
5-1	-0.526			-0.88	-0.89
	[-1.89]			[-3.6]	[-5.42]

Table 3-1: Single Sort Volatility and Return

Note: At the beginning of each month, based on the past total volatility and idiosyncratic volatility, quintile portfolios are constructed from the entire investible stock universe. Rank is in the increasing order of volatilities. 5-1 describes the return of the 5th portfolio minus the 1st portfolio. Newey-West robust t-statistic is reported in the bracket with maximum of 12 lags. Mean and Std Dev. columns are calculated from absolute, not excessive, returns in the percentage term. CAPM Alpha and FF-3 Alpha are constant terms measured in percentage of CAPM regression and Fama-French 3-factor regression respectively. The sample period is from February 1965 to December 2016.

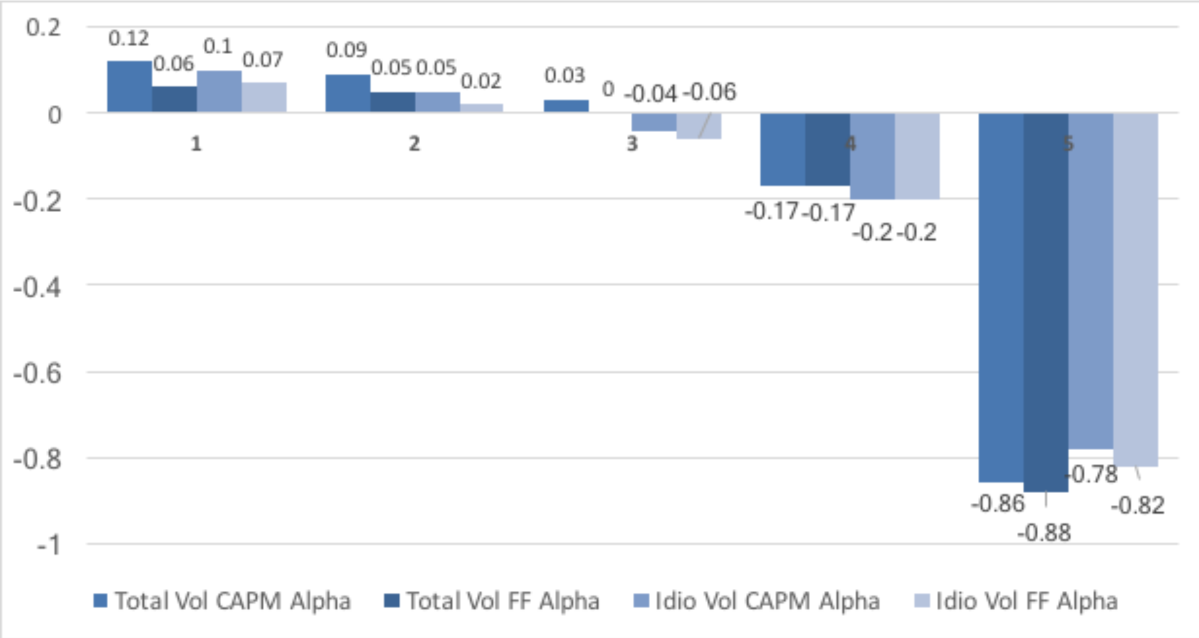

Figure 3-1: CAPM/FF-3 Alpha vs. Volatility

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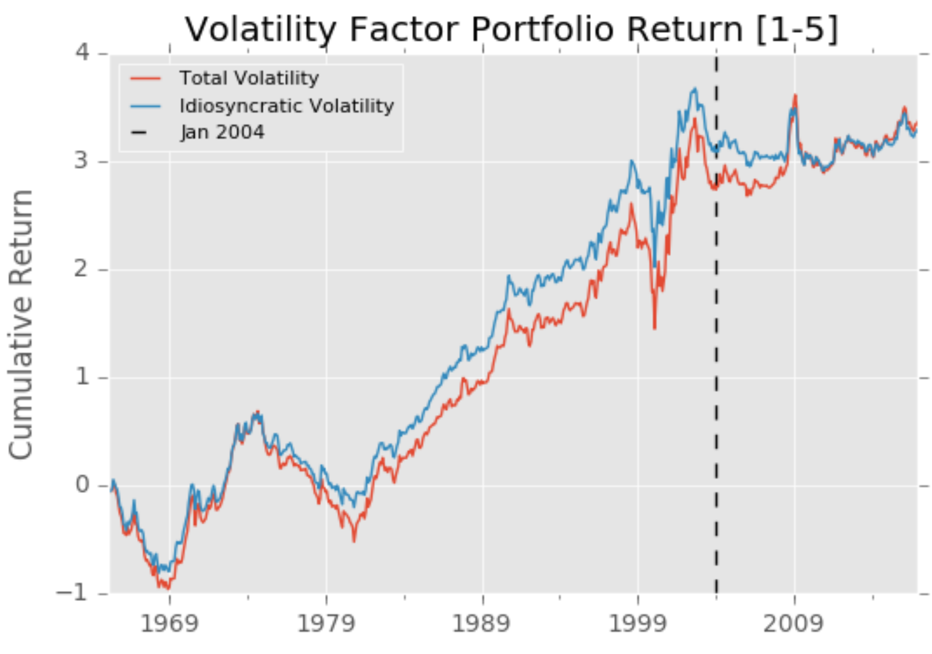

Figure 3-2: Volatility Factor Portfolio Return [1-5]

Period	Jan 1980 to Dec 2003	Jan 2004 to Dec 2016	Feb 1965 to Dec 2016
Panel A: Return of [1-5] Portfolio constructed based on 1-month daily total volatility			
Mean %	1.06	0.35	0.54
	[2.33]	[0.92]	[1.71]
CAPM alpha %	1.67	0.95	0.98
	[4.11]	[4.21]	[3.68]
CAPM beta	-0.96	-0.95	-0.9
	[-5.11]	[-6.58]	[-8.67]
FF3 alpha %	1.17	0.94	0.94
	[3.88]	[4.50]	[4.71]
Panel B: Return of [1-5] Portfolio constructed based on 1-month daily idiosyncratic			
Mean %	1.11	0.09	0.53
	[2.80]	[0.28]	[1.89]
CAPM alpha %	1.6	0.51	0.88
	[4.29]	[2.12]	[3.60]
CAPM beta	-0.77	-0.67	-0.73
	[-4.46]	[-5.12]	[-7.68]
FF3 alpha %	1.25	0.51	0.89
	[4.57]	[2.29]	[5.42]
Panel C: Excess return on market			
Mean %	0.64 (2.47)	0.63 (1.67)	0.49

Table 3-2: Volatility (Total/Idiosyncratic) vs. Alpha (CAMP/FF3)

Note: Excess return on market is defined as the “value-weight return of all CRSP firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ that have a CRSP share code of 10, 11 and 12 at the beginning of month t, good shares and price data at the beginning of t, and good return data for t minus the one-month Treasury bill rate (from Ibbotson Associates)” (Kenneth R. French Data Library, 2017). Newey-West robust t-statistic is reported in the bracket with maximum of 12 lags.

4 Long term volatility and expected returns

In the previous section, we confirm that the stocks with high short-term volatility are associated with low mean return and FF-3 alpha. One possible explanation is that high short-term volatility mainly reflects transient speculative activities of a company instead of its fundamental business risk. Thus, we would like to examine whether stocks with higher long term volatility, intuitively less contaminated by the noise from speculation, would have higher return. As discussed in Chapter 2.2, short-term volatility is calculated using daily returns over the past month, while long-term volatility is calculated using monthly returns over the past 24 months excluding the most recent month.

If investors demand a higher risk premium for long-term volatility ignoring temporary surges in short-term volatility, we might expect to observe no cross-sectional "volatility puzzle" with respect to long-term volatility. Empirically, there are at least two possibilities why short-term and long-term volatilities do not coincide. First, short-term volatility and long-term volatility are calculated using different frequency of returns. It has been shown that daily returns of stocks are not i.i.d., and they exhibit mean-reversion and auto-correlation (Ball & Kothari, 1989). Thus, volatilities vary depending on the frequency of returns. Second, short-term volatility and long-term volatility use different horizons of samples. The underlying instantaneous volatility of the continuous stochastic process of log stock price, which is unobserved, follows a non-trivial process and is changing over time. Due to the time varying property of this instantaneous volatility, the horizon of sample period would influence the estimate of volatility.

We repeat the same exercise of short-term volatility on long-term volatility. Again, we consider both the total volatility without controlling for systemic risk and the idiosyncratic volatility relative to CAPM and Fame-French 3-factor model. If volatility is a risk factor that is priced, the standard Fama-French 3-factor model should not fully explain the stock returns. Cross-sectionally, if alpha is persistently correlated with the residual volatility in FF3 regression, there is an indication of missing idiosyncratic volatility risk factor.

It is possible that Fama-French model performs plausibly in longer horizons whereas in short run it fails to capture short-term dynamics. In such a case, the anomaly should exist only in the short term and the mispricing should vanish over longer horizons. We would expect to see the “volatility puzzle” only with the short-term idiosyncratic volatility but not with the long-term idiosyncratic volatility.

We use the past 24-month estimate of long-term volatility (excluding the nearest month) to rank stocks and construct quintile portfolios to hold over the next month. The short-term and long-term volatilities are therefore obtained from non-overlapping sample periods.

In Table 4-1, we report the portfolio returns constructed from sorting by long-term total volatility and idiosyncratic volatility. We see that the long-term volatility sorting produces similar results compared to the short-term volatility sorting in Table 3-1, although the magnitude of alphas varies. For the first 4 quintiles, the degree of mispricing is not significant. The last quintiles still show relatively strong signs of mispricing though the magnitude compared to the short-term volatility results diminished substantially. The alpha spread (difference of alphas between the first and the last quintiles) is also significantly negative, largely resembling the previous result in short-term volatility.

Rank	Mean	Std Dev.	% Mkt Share	CAPM Alpha	FF-3 Alpha
Panel A: Portfolio Sorted by 24-Month Total Volatility					
1	0.888	3.7	41.6%	0.12	0.08
				[1.77]	[1.64]
2	0.930	4.9	25.7%	0.01	-0.03
				[0.24]	[-0.65]
3	0.967	6.0	19.3%	-0.05	-0.04
				[-0.59]	[-0.48]
4	0.927	7.3	8.9%	-0.19	-0.16
				[-1.17]	[-1.26]
5	0.675	8.9	4.6%	-0.55	-0.50
				[-2.40]	[-3.28]
5-1	-0.213			-0.67	-0.58
	[-0.667]			[-2.34]	[-3.03]
Panel B: Portfolio Sorted by 24-Month Idiosyncratic Volatility Relative to FF-3					
1	0.913	4.0	41.7%	0.10	0.06
				[1.78]	[1.39]
2	0.899	4.6	28.8%	0.01	-0.01
				[0.27]	[-0.14]
3	0.913	5.6	18.3%	0.07	-0.05
				[-0.87]	[-0.77]
4	0.939	6.7	7.6%	-0.13	-0.09
				[-0.88]	[-0.81]
5	0.776	8.4	3.7%	-0.40	-0.33
				[-1.89]	[-2.24]
5-1	-0.137			-0.50	-0.39
	[-0.48]			[-1.96]	[-2.22]

Table 4-1: Portfolio Sorted by Volatility

Note: At the beginning of each month, based on the past total volatility and idiosyncratic volatility, quintile portfolios are constructed from the entire investible stock universe. Rank is in the increasing order of volatilities. Newey-West robust t-statistic is reported in the bracket. To separate the effect of long-term (24-month volatility) from the previous month short-term volatility, 24-month total and idiosyncratic volatility are calculated from the data ending at one month prior to the portfolio formation with 24-month sample window. Mean and Std Dev. columns are calculated from absolute, not excessive, returns in the percentage term. CAPM Alpha and FF-3 Alpha are constant terms in CAPM regression and Fama-French 3-factor regression respectively. The sample period is from February 1965 to December 2016.

In Table 4-2, we examine the effect of the sample period on the FF-3 alphas of quintile portfolios. Specifically, quintile portfolios are constructed by sorting volatilities estimated from 12, 18 and 24 months' monthly returns excluding the nearest month with rebalancing. We observe that the alpha spread (5-1) obtained with 18 and 24-month sample period are close to each other. The alpha spread (5-1) obtained with 12- month sample period is slightly different, probably due to the limiting number of observations for the regression.

Sample Period (months)	Ret Freq	Ranking on Idiosyncratic Volatility					5-1
		1 Low	2	3	4	5 High	
12	Monthly	0.04	-0.02	-0.03	-0.13	-0.20	-0.24
		[0.84]	[-0.54]	[-0.55]	[-1.25]	[-1.44]	[-1.43]
18	Monthly	0.03	0.00	-0.04	-0.09	-0.38	-0.41
		[0.87]	[0.06]	[-0.66]	[-0.76]	[-2.55]	[-2.42]
24	Monthly	0.06	-0.01	-0.05	-0.09	-0.33	-0.39
		[1.39]	[-0.14]	[-0.77]	[-0.81]	[-2.24]	[-2.22]

Table 4-2: Quintile Portfolios of Idiosyncratic Volatility for Different Sample Periods

Note: The table reports alphas from Fama-French regression and its t-statistics in the bracket below the alphas. Newey-West robust t-statistic with a maximum of 12 lags is reported in the bracket. The column "5-1" shows the difference in FF-3 alphas between the highest portfolio (Rank 5) and the lowest portfolio (Rank 1) in terms of the volatility. Our sample period is February 1965 to December 2016. Idiosyncratic volatilities are calculated from the residuals of FF-3 regression of monthly returns.

5 Relationship between long term and short term volatility

5.1 Correlation

As shown in the previous two sections, the quintile portfolios constructed by sorting long term volatility show similar patterns of performance to those constructed by sorting short term volatility, though with less alpha spread. One natural explanation is that short term and long term volatilities are highly correlated.

To test this hypothesis, we calculate the cross-sectional correlation between short term and long term volatilities for each month since 1965. To get a complete picture, we add the results from an alternative definition of long term volatility using weekly returns. The calculation methodology is detailed in Chapter 2.1. We perform the analysis for both total return and FF-3 residual return.

The following histogram plots the distribution of correlations between short term volatility and the 6 long term volatilities. The number in the title refers to the time horizon and the character refers to the return frequency. E.g. 24W means the long-term volatility is calculated as the standard deviation of 1-month lagged 24-week weekly return. The orange area shows the total return volatility while the blue area is for FF-3 residual return volatility.

Generally, correlations are positive with only a few outliers. The distributions are different across various time horizons and return frequencies. We observe three noticeable patterns. First, long term volatilities calculated with higher frequency (weekly) returns has higher correlation with short-term volatilities than that calculated with lower frequency (monthly) returns. This is expected since short term volatility is calculated with daily returns. Long-term return of higher frequency should share more similarities with short-term return. Second, for long-term volatility calculated with monthly returns, the correlation between short-term total volatility and long-term total volatility is noticeably higher than that between short-term idiosyncratic volatility and long-term idiosyncratic volatility. Total volatility includes variances of other FF risk factors which daily return and monthly return have in common, whereas idiosyncratic volatility excludes the factor-induced common variation so that the correlation between daily and monthly returns should be reduced. We find this reduction of correlation for long-term volatility calculated using weekly returns, but with much smaller magnitude. The observations are in line with the summary statistics in Table 5-1. The large t-statistics indicate that the correlation between total long term and short term volatilities is unlikely to be the same as that of idiosyncratic volatilities. Third, for weekly returns, compared to that of 24-week sample period, the whole distributions of 48-week and 72-week sample period shift towards the lower end. The reason why this shift happens is worth further investigation. By contrast, the distributions calculated with monthly returns are relatively stable across different sample periods.

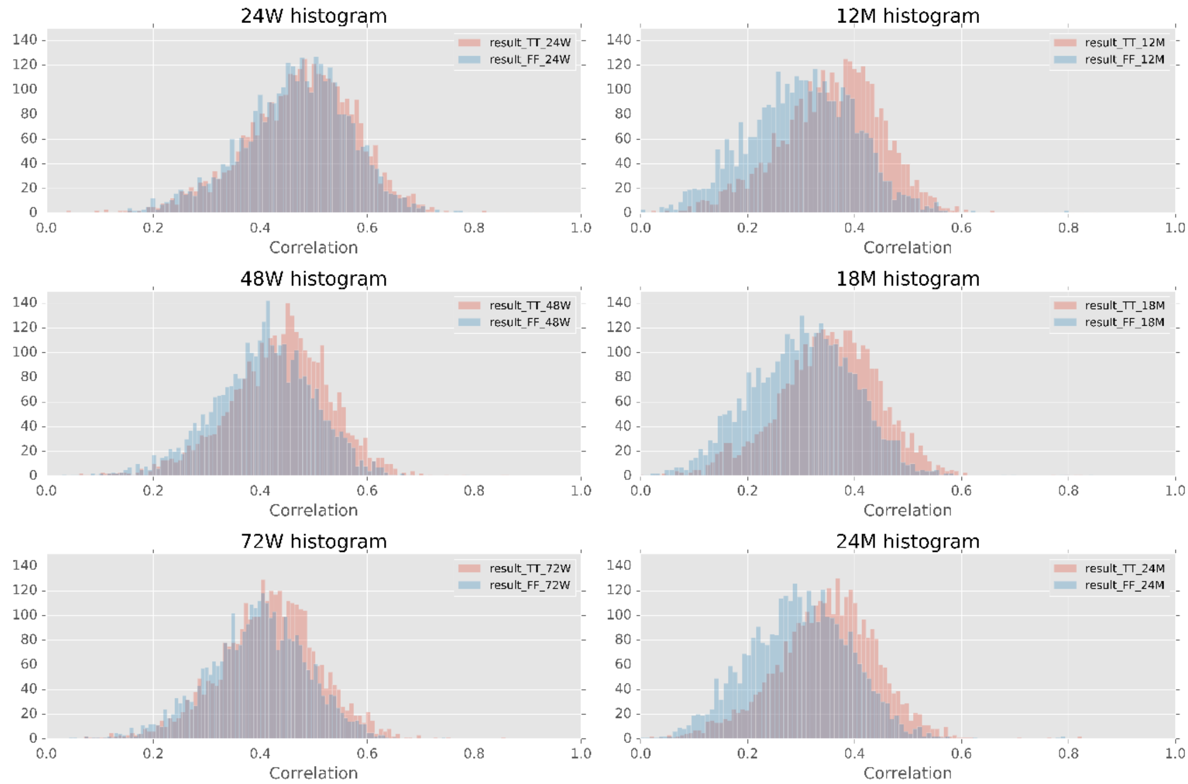

Figure 5-1: Histogram of correlations between various long term total/idiosyncratic volatility and 1-month daily total/idiosyncratic volatility

Volatility Type	Median	Mean	Std Dev	Mean Diff between TT and FF	t-statistics*
TT_24W	0.5094	0.5026	0.1032	0.0363	14.2606
FF_24W	0.4732	0.4663	0.1010		
TT_48W	0.4583	0.4515	0.0950	0.0460	19.5731
FF_48W	0.4103	0.4056	0.0931		
TT_72W	0.4192	0.4158	0.0927	0.0211	9.1909
FF_72W	0.3985	0.3947	0.0912		
TT_12M	0.3642	0.3570	0.0961	0.0556	22.5890
FF_12M	0.3039	0.3014	0.1010		
TT_18M	0.3557	0.3521	0.0940	0.0492	20.5877
FF_18M	0.3058	0.3029	0.0974		
TT_24M	0.3503	0.3458	0.0937	0.0495	20.8844
FF_24M	0.2977	0.2963	0.0954		

Table 5-1: Statistics of correlations between various long-term volatilities and short-term volatility. *t-statistics is for testing the null hypothesis that correlations calculated with total and idiosyncratic volatilities (for the same type of long-term volatility) have the same mean.

To see how the relationship between short-term volatility and long-term volatility evolves, we plot the time series of the correlation between 24 month long-term volatility and short-term volatility by portfolio in Figure 5-2. Other long-term volatility measures exhibit similar patterns so we do not include the results here.

Over time, the correlations fluctuate within a range of -0.1 to 0.6. During normal times, we expect a stable relationship between long-term and short-term volatilities resulting from the mean-reverting nature of daily returns. During crisis times, this relationship may break down in the sense that the short-term volatility could surge temporally whereas long-term volatility still stay relative insensitive to the drastic change of the nearest market condition. We observe this phenomenon around 2008 financial crisis when the correlation plunged.

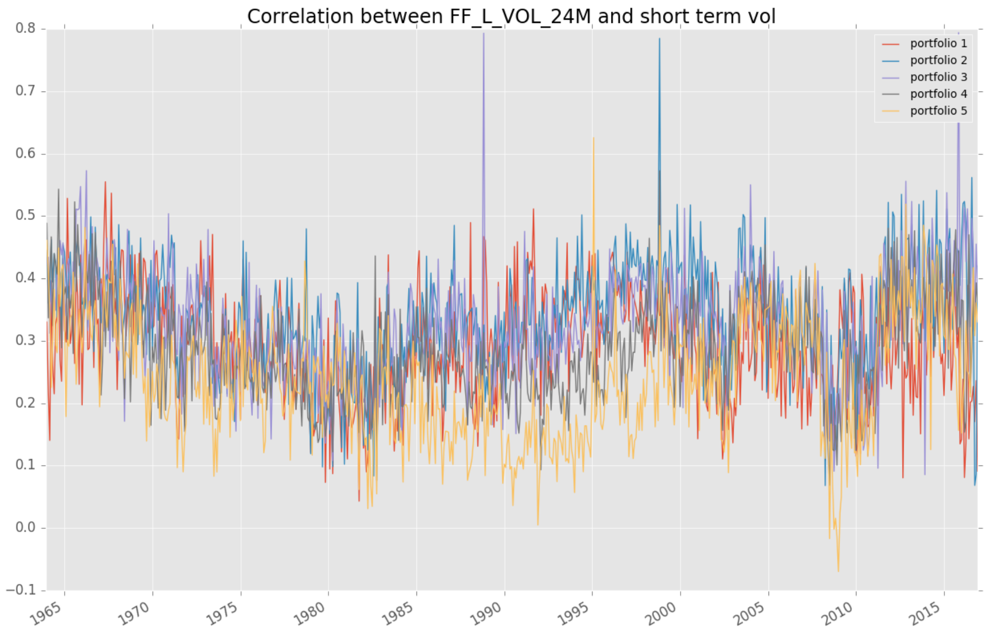

Figure 5-2: Cross-sectional correlation between short-term volatility and 24-month long term volatility over time by portfolio.

5.2 Double-sort: long term volatility vs. short term volatility

The results in the previous section prove that, though not perfectly correlated, there is positive and non-negligible correlation between long term and short term volatility. The volatility puzzle (higher total/idiosyncratic volatility, lower alpha) we find in long-term volatility could stem from its correlation with short-term volatility. Imaging that if volatility calculated using different return frequencies and sample horizons are perfectly correlated, such as in the case of i.i.d daily returns, we would observe the same anomaly in disguise of different measurement methodologies.

To see whether long-term volatility puzzle is independent of its short-term cousin, we double-sort stocks monthly according to their historical short-term and long-term volatilities (based on total or idiosyncratic risk). Specifically, we create short term/ long term quintile break points through single sort of short term and long term volatility respectively. We then form 5 x 5 quintile value-weighted portfolios and calculate their holding period returns next month. We run FF regression on the monthly returns of each portfolio to obtain their FF-3 alphas. All results are calculated based on 24-month long term volatility of monthly returns and 1-month short term volatility of daily returns. Other long term volatilities are also tested and exhibit similar patterns. Details are attached in the Appendix.

Table 5-2 records the average number of stocks within each double-sort category. Since long term and short term volatilities are positively correlated, most of the observations are concentrated around the diagonal and the matrix is roughly symmetric. Table 5-4 shows the FF-alphas of each double-sorted quintile portfolio and Table 5-5 gives the corresponding t-statistics.

In the previous sections, we find that volatility puzzle exists for both long-term measure and short-term measure, unconditioned on each other. Here we observe that short term puzzle persists regardless which long-term volatility quintile it is conditioned on, meaning for each row in Table 5-4, as short-term total or idiosyncratic volatility increases, FF-3 alpha for [5-1] portfolio of short-term volatility decreases. Alpha consistently changes from slightly positive or near zero to significantly negative when entering to the highest short-term volatility quintile, with robust t-statistics for each long-term category. This pattern is similar to those observed in the single sorting results. However, we cease to observe the existence of long-term volatility puzzle conditioned on short-term volatility. For each short-term category, the alpha spread ([Hi – Lo] or [5 - 1] in the previous sections) of quintile portfolio sorted by their long-term volatilities is not significantly different from zero.

Panel A: Portfolios sorted by 24 Month Monthly and 1 Month Daily Total Volatility						
		Short Term Volatility				
		Low	2	3	4	High
24-Month Long Term Volatility	Low	378.3	193.9	87.0	35.8	15.5
	2	183.2	225.0	165.8	92.1	44.1
	3	86.5	159.8	195.2	165.2	103.5
	4	41.5	90.5	163.7	215.3	199.1
	High	21.1	41.0	98.4	201.8	348.5
Panel B: Portfolios sorted by 24 Month Monthly and 1 Month Daily Idiosyncratic Volatility						
		Short Term Volatility				
		Low	2	3	4	High
24-Month Long Term Volatility	Low	330.1	184.9	102.2	56.5	31.5
	2	198.8	206.1	151.0	95.5	53.3
	3	102.9	164.4	180.1	152.4	104.8
	4	50.0	102.1	164.6	200.0	187.9
	High	23.3	47.2	106.8	200.2	327.8

Table 5-2: Average number of stocks in each double-sort category from 1965 Feb to 2016 Dec

Panel A: Portfolios sorted by 24 Month Monthly and 1 Month Daily Total Volatility						
		Short Term Volatility				
		Low	2	3	4	High
24-Month Long Term Volatility	Low	28%	12%	3%	1%	0%
	2	8%	9%	6%	2%	1%
	3	3%	6%	5%	3%	1%
	4	0%	1%	2%	2%	1%
	High	0%	0%	1%	1%	1%
Panel B: Portfolios sorted by 24 Month Monthly and 1 Month Daily Idiosyncratic Volatility						
		Short Term Volatility				
		Low	2	3	4	High
24-Month Long Term Volatility	Low	29%	10%	4%	1%	0%
	2	12%	9%	5%	2%	1%
	3	4%	5%	4%	3%	1%
	4	1%	1%	2%	2%	1%
	High	0%	0%	1%	1%	1%

Table 5-3: Percentage of market capitalization in each double-sort category from 1965 Feb to 2016 Dec

In fact, within the two lowest short-term volatility quintiles, driven by the large positive alphas (with relatively high t-statistics but still smaller than 2) of the portfolios with the highest long-term volatility, the alpha spreads are even slightly positive, coinciding with a normal expectation that higher long term volatilities should be rewarded with higher returns. Moving from the lowest short-term volatility quintile to the highest short-term volatility quintile, long term alpha spread changes from positive to negative. We do not intend to over-interpret the above observations as the t-statistics for the first four short-term quintiles are not high enough to redeem any robust conclusions. But we feel it is still worth pointing out the trend.

It is easy to see why the unconditional short term puzzle exists as it is just the value-weighted average of the consistent puzzle effects for each individual long-term volatility quintile. To explain why the unconditional long term puzzle exists (albeit it is weaker and less robust than short term puzzle but still statistically significant), we notice that the correlation between long term and short term volatilities plays an important role.

As we discussed before, for low short-term volatility quintiles, alphas are roughly zero or slightly positive while for high short-term volatility quintiles, alphas are roughly negative. In the absence of volatility correlation, for each long-term quintile, we would expect approximately the same number of firms in the low and high short-term volatility category. Thus, the above positive and negative alphas should offset each other to the same extent for both low and high long-term volatility quintiles, leaving little average alpha spread across the single-sorted long-term volatility quintiles.

In the presence of the correlation, the stocks cluster around the diagonal of the quantile grid in Table 5-1, so do the market capitalizations but with more weights in the upper left corner (Table 5-3). For the lowest long term volatility quantile (first row in Table 5-4), more firms with higher market capitalizations have zero or positive alphas (first two cells of low short-term volatility) than those who have negative alphas (last two cells of high short-term volatility). Therefore, on average, the lowest long-term quintile exhibits slightly positive alpha in Table 4-1. Similarly, for the highest long term volatility quantile (last row in Table 5-4), more firms with higher market capitalizations have negative alphas (last two cells of high short term volatility) than those who have positive alphas (first two cells of low short term volatility). Thus, on average, the highest long-term quintile exhibits significant negative alpha in Table 4-1. Overall, we end up with a significant negative FF-3 alpha spread [5-1] in Table 4-1, for both total and idiosyncratic volatility sorted portfolios. We thus conclude that the long-term volatility puzzle is mainly a reflection of the short-term puzzle.

Lastly, the top-left quintile portfolio sorted by idiosyncratic volatility shows a large positive FF-3 alpha (Panel B,Table 5-4) with a large t-statistics (Panel B,Table 5-5), suggesting that the most stable and isolated stocks (with lowest idiosyncratic volatility both in short term and long term) might be undervalued.

Panel A: Portfolios sorted by 24 Month Monthly and 1 Month Daily Total Volatility							
		Short Term Volatility					
		Low	2	3	4	High	Hi-Lo
Long Term Volatility	Low	0.079	0.09	-0.085	-0.159	-0.358	-0.437
	2	0.018	0.007	-0.014	-0.308	-0.974	-0.992
	3	0.19	-0.048	0.114	-0.158	-0.774	-0.964
	4	-0.1	0.158	0.123	-0.088	-0.981	-0.881
	High	0.294	0.155	-0.077	-0.234	-1.012	-1.307
	Hi - Lo	0.216	0.066	0.009	-0.075	-0.654	0
Panel B: Portfolios sorted by 24 Month Monthly and 1 Month Daily Idiosyncratic Volatility							
		Short Term Volatility					
		Low	2	3	4	High	Hi-Lo
Long Term Volatility	Low	0.119	0.015	-0.124	-0.207	-0.779	-0.898
	2	0.026	0.047	-0.096	-0.215	-0.812	-0.838
	3	0.037	-0.049	-0.002	-0.174	-0.884	-0.922
	4	0.053	0.187	-0.036	-0.153	-0.733	-0.786
	High	0.408	0.358	0.088	-0.404	-1.031	-1.439
	Hi - Lo	0.29	0.344	0.212	-0.197	-0.252	0

Table 5-4: Average FF-3 alpha for each category and 5th – 1st quintile portfolio

Panel A: Portfolios sorted by 24 Month Monthly and 1 Month Daily Total Volatility							
		Short Term Volatility					
		Low	2	3	4	High	Hi-Lo
Long Term Volatility	Low	1.187	1.39	-0.836	-1.02	-1.325	-1.543
	2	0.222	0.107	-0.183	-2.024	-4.398	-4.067
	3	1.901	-0.49	1.038	-1.29	-4.647	-4.642
	4	-0.632	1.152	0.931	-0.553	-5.237	-3.166
	High	1.387	0.759	-0.451	-1.418	-5.073	-4.818
	Hi - Lo	1.019	0.287	0.04	-0.297	-1.815	0
Panel B: Portfolios sorted by 24 Month Monthly and 1 Month Daily Idiosyncratic Volatility							
		Short Term Volatility					
		Low	**2**	**3**	**4**	**High**	**Hi-Lo**
Long Term Volatility	Low	2.038	0.241	-1.277	-1.451	-3.701	-4.048
	2	0.407	0.658	-1.019	-1.726	-3.767	-3.552
	3	0.385	-0.575	-0.019	-1.514	-4.903	-4.411
	4	0.306	1.294	-0.298	-1	-4.629	-3.163
	High	1.866	1.361	0.515	-2.611	-5.512	-5.451
	Hi - Lo	1.273	1.205	0.981	-0.999	-0.938	0

Table 5-5: T-statistics of FF-3 alpha for each category and 5th – 1st quintile portfolio

6 Market Frictions and Anomaly

There are several market frictions that may contribute to the existence of “volatility puzzle”, including shorting fees and transaction costs. In the following sections, we investigate the related empirical evidences.

6.1 Short sell

In Hong and Sraer’s 2016 paper ‘Speculative Beta’, the authors argued that if there is a short constraint (such as high shorting fee or ineligibility for shorting) on a stock, only the optimistic views can be sufficiently expressed. The lack of view-based shorting could cause the stock to be overpriced. Short constraint differs for different investors as well as for different stocks. We refrain ourselves to investigate the constraints only related to the stocks’ own characteristics.

In the previous sections, we show that the stocks with high total or idiosyncratic volatility are generally of small capitalization and illiquid to trade. It is very likely that the shorting fees for those stocks are also high. In addition, the high idiosyncratic volatility suggests that those stocks are more likely to be speculative targets (for their relatively unstable operational and financial status). The overpricing impact of shorting constraint could be magnified due to such excessive long-only speculations.

Granted that the anomaly appears, one may ask why it persistently exists. For a statistical arbitrager to step in, the trade needs to be profitable. In the simplest setting, the arbitrager needs to short the 5^{th} quintile portfolio (the portfolio with the highest volatility). If the shorting cost offsets the alpha spread, no trade will take place to close the anomaly. We will investigate the impact of short fees on long-short trading strategies in more details in the next section.

We use the indicative short fee on WRDS (Markit database) as a proxy of short constraint level. Indicative short fee is defined as the difference between Fed Fund Rate and indicative short rebate. Figure 6-1 plots the weighted-average short fee for each quintile portfolio sorted by short-term idiosyncratic volatility. The weights are the relative market capitalization of each stock in the portfolio. We observe that after 2005, the average short fee level for a portfolio is consistent with its volatility ranking. Particularly, the highest volatility quintile has the highest average short fee. The fee level is also most volatile and follows an upper trend for this quintile. Consequently, the stocks in this quintile are prone to be overpriced, leading to a negative portfolio return.

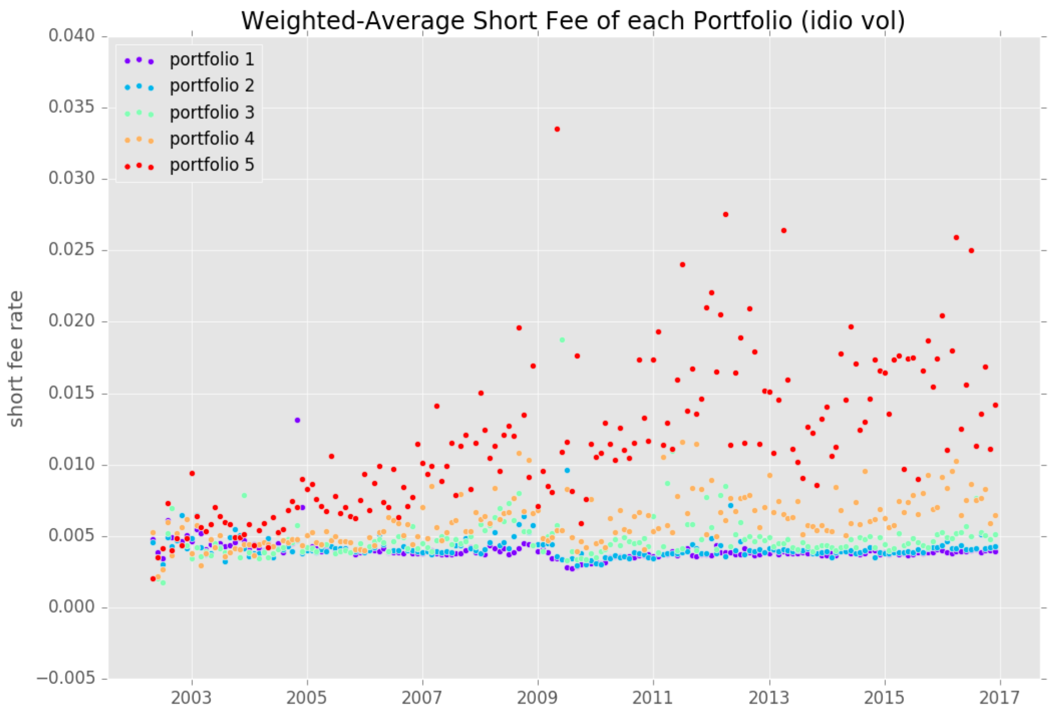

Figure 6-1: Value-weighted indicative short fee rate for each portfolio (constructed based on short term idiosyncratic volatility)

Figure 6-2 plots the total number of stocks with valid short fee data within each portfolio. Before 2004, since the short fee database was still under construction, the number of stocks whose short fee data are available is limited, resulting in much lower counts of firms at the beginning of the plot. Also, we observe a sharp dip of the count around 2008 because of the SEC short ban.

In general, the 5th quintile portfolio has the smallest number of available stocks, resulting from either more missing data or more stocks ineligible to short (infinity short fee). If the former, the sample size is still reasonably large to provide an accurate estimate of the average short fee. If the latter, it further proves that the stocks in portfolio 5 is harder to short than others.

According to Table 3-1, portfolio 5 consists of the stocks with smallest market cap. It is also possible that size factor, instead of short fee, drives the overpricing of the portfolio. To test this, we perform robustness checks on size factor in section 7. The results prove that size factor is unlikely to be the driving factor of the puzzle (overpricing anomaly).

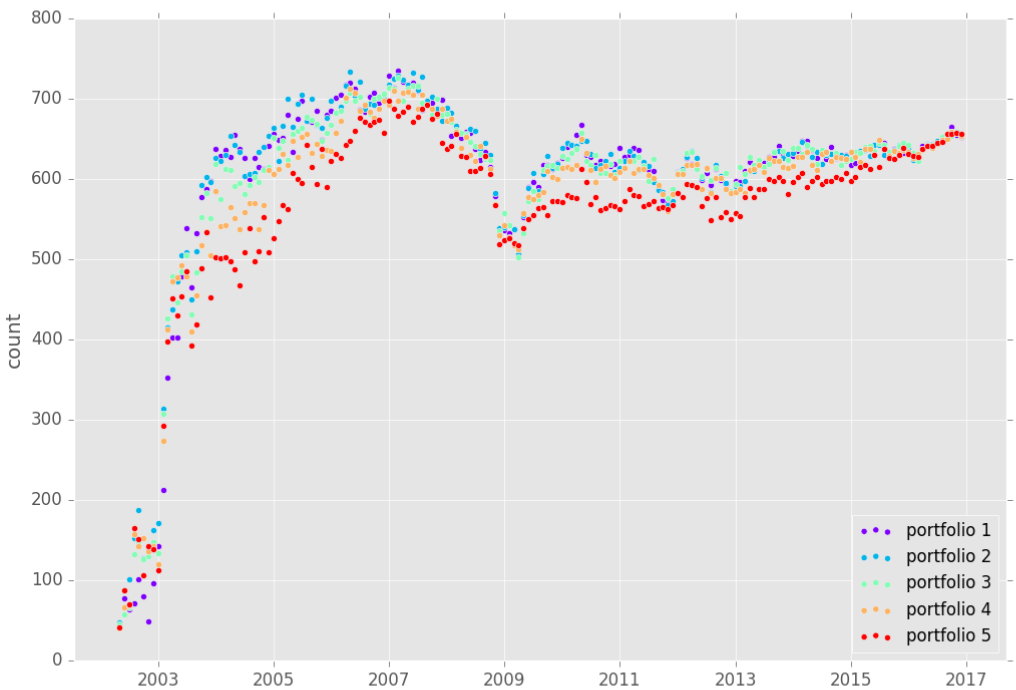

Figure 6-2: Number of stocks with short fee data available in Markit Database for each portfolio

6.2 Transaction cost

Like short fees, transaction costs are expected to be high for small stocks. If the total transaction cost for constructing the long-short portfolio [1-5] (going long the lowest volatility quintile and short the highest volatility quintile) is higher than its nominal return, then the anomaly is likely to persist. Transaction cost has four components: commission, market impact, manager delay cost and opportunity cost (Wagner & Glass, 2001). It is often hard to calculate the cost individually, especially for a big pool of stocks with diversified liquidity.

In this section, we do not aim to provide an accurate calculation of transaction costs. Instead, we would like to provide a conservative, yet sensible estimate to assess the market friction that potentially sustains the volatility puzzle. We estimate the cost based on two sources: Professor Coldiron observations and academic or industrial studies.

Professor Coldiron provide the following estimation based on live trading experience of US equities from 2003 – 2014. The results are based on a full implementation shortfall analysis including commissions and opportunity cost estimates for multi-day orders.

Russell 1000 Constituents	25 bps one way
Russell 2000 Constituents	40 bps one way
Russell Micro Cap	60 bps one way

On the academic side, most papers related to transaction costs were written before 2000, when the costs are relatively high. Based on a more recent estimate from Wagner's 2001 paper, as of June 2000, the average total transaction cost is around 101 bps excluding opportunity cost. The average commission is 12 bps. Opportunity cost, which represents the cost of not completing an order fully, is 16bps on average for 10% unexecuted trades. Another study done by Plexus in 1998 analyzed the transaction cost by management style. Table 6-1 shows their results (Plexus Group, 1998):

(bps)	Manager Timing Costs	Trading Desk Timing	Market Impact	Commission	Missed Trades	Total
Large Cap Value	1	13	8	15	28	65
Large Cap Growth	82	32	21	10	14	159
Small Cap Value	5	63	40	20	32	160
Small Cap Growth	136	72	57	18	29	312

Table 6-1: Transaction costs by management type (Plexus Group, 1998)

The average trading costs estimated from the academic studies are much higher than that from the market practitioner, suggesting that the transaction costs have been reduced in recent years. The lower bound of the average transaction cost across different firm sizes is around 40bps according to the market practitioner and the upper bound is around 170 bps according to Plexus' analysis. Within this range, we conduct scenario analysis assuming different levels of the average transaction cost.

The methods used to estimate the profit after transaction cost are detailed in the data section. Besides the level of transaction cost, another important input is turnover rate. Table 6-2 shows that stocks in portfolio 2-4 have very high turnover rate.

Portfolio	Total Volatility	Idiosyncratic
1	76%	60%
2	122%	130%
3	142%	147%
4	146%	152%
5	135%	141%

Table 6-2: Average monthly two-way turnover rate within each portfolio constructed based on short term volatility. E.g. 76% for total volatility portfolio 1 means that on average, 38% of the positions (in terms of capital) will be replaced. Maximum turnover rate is 200%, which means all positions in the portfolio are replaced.

The following table shows the after-transaction-cost return for long-short [1-5] portfolios constructed based on short-term total or idiosyncratic volatility. Last column shows the threshold rate of transaction cost that would make the portfolio return or alpha zero.

In both Panel A and B, the threshold cost to make arithmetic mean return breakeven is only 26 bps, which cannot cover even the lower bound of our transaction cost estimate of 40 bps. If one can effectively hedge the risk from market factor or FF-3 factors (for example, making the portfolio factor-neutral by shorting futures on factor index), CAPM and FF-3 alphas can tolerate higher transaction cost up to around 40 ~ 50 bps, which is at the edge of the lower bound of the cost estimate. Either way, [1-5] portfolio barely provides satisfactory post transaction cost returns.

Panel A: Return of [1-5] Portfolio constructed based on 1-month daily total volatility After transaction cost (1965 Feb to 2016 Dec)					
Transaction Cost One Way (bps)	**0**	**40**	**80**	**200**	**Threshold Cost**
Arithmetic Mean (%)	0.54	-0.31	-1.15	-3.67	0.26
t-stat	*1.71*	*-0.97*	*-3.64*	*-11.38*	
CAPM alpha (%)	0.98	0.14	-0.71	-3.23	0.47
t-stat	*3.68*	*0.51*	*-2.62*	*-11.57*	
FF-3 alpha (%)	0.94	0.1	-0.74	-3.28	0.45
t-stat	*4.71*	*0.49*	*-3.56*	*-14.46*	
Panel B: Return of [1-5] Portfolio constructed based on 1-month daily idiosyncratic volatility After transaction cost (1965 Feb to 2016 Dec)					
Transaction Cost One Way (bps)	**0**	**40**	**80**	**200**	**Threshold Cost**
Arithmetic Mean (%)	0.53	-0.28	-1.08	-3.5	0.26
t-stat	*1.89*	*-1.00*	*-3.87*	*-12.25*	
CAPM alpha (%)	0.88	0.08	-0.73	-3.14	0.44
t-stat	*3.60*	*0.31*	*-2.96*	*-12.57*	
FF-3 alpha (%)	0.89	0.09	-0.72	-3.14	0.44
t-stat	*5.42*	*0.52*	*-4.30*	*-17.80*	

Table 6-3: Return and t-statistics of portfolio constructed based on short term volatility after transaction cost before short fee (1965 Feb to 2016 Dec)

On top of the components mentioned, short fee adds as an additional cost for trading. The average annual short fee for portfolio 5 ranges from 0.5% to 3.5% (4 bps to 29 bps per month) during the period of 2002 – 2016. After adding this cost and estimating over a short and more recent horizon (2004 – 2016), the returns of [1-5] portfolio keep deteriorating, as shown in Table 6-2.

Panel A: Return of [1-5] Portfolio constructed based on 1-month daily total volatility After transaction cost and short fee (2004 Jan to 2016 Dec)					
Transaction Cost One Way (bps)	**0**	**40**	**80**	**200**	**Threshold Cost**
Arithmetic Mean (%)	0.25	-0.56	-1.38	-3.81	0.12
t-stat	*0.65*	*-1.48*	*-3.63*	*-10.04*	
CAPM alpha (%)	0.85	0.04	-0.78	-3.21	0.42
t-stat	*3.81*	*0.16*	*-3.45*	*-13.55*	
FF-3 alpha (%)	0.84	0.03	-0.78	-3.22	0.42
t-stat	*4.05*	*0.13*	*-3.60*	*-13.43*	

Panel B: Return of [1-5] Portfolio constructed based on 1-month daily idiosyncratic volatility After transaction cost and short fee (2004 Jan to 2016 Dec)					
Transaction Cost One Way (bps)	**0**	**40**	**80**	**200**	**Threshold Cost**
Arithmetic Mean (%)	-0.03	-0.88	-1.73	-4.28	-0.02
t-stat	*-0.10*	*-2.91*	*-5.73*	*-14.21*	
CAPM alpha (%)	0.39	-0.46	-1.31	-3.86	0.18
t-stat	*1.66*	*-1.92*	*-5.50*	*-16.13*	
FF-3 alpha (%)	0.39	-0.46	-1.31	-3.86	0.18
t-stat	*1.79*	*-2.11*	*-5.97*	*-17.29*	

Table 6-4: Portfolio return and t-statistics after transaction cost and short fee (2004 Jan to 2016 Dec)

One of the reasons we investigate long term volatility is that long term volatility could potentially be a more persistent alpha signal than short term volatility, thus requires less turnover for trading on it. We did the same profitability analysis on long term volatility signal.

Portfolio	Total Volatility	Idiosyncratic
1	10%	20%
2	27%	48%
3	32%	51%
4	32%	45%
5	24%	31%

Table 6-5: Average monthly turnover rate within each portfolio constructed based on long term volatility. E.g. 10% for total volatility portfolio 1 means that on average, 5% of the positions (in terms of capital) will be replaced. Maximum turnover rate is 200%, which means all positions in the portfolio is replaced.

According to Table 6-5, the turnover rate of trading on long term volatility is much lower than trading on short term volatility (the former is around 1/3 – 1/5 of the latter). The threshold costs are much larger. Trading on total volatility can tolerate 64 bps one-way transaction cost, higher than our lower bound estimation.

Panel A: Return of [1-5] Portfolio constructed based on 24-month monthly total volatility After transaction cost (1965 Feb to 2016 Dec)					
Transaction Cost One Way (bps)	**0**	**40**	**80**	**200**	**Threshold Cost**
Arithmetic Mean (%)	0.21	0.08	-0.06	-0.46	0.64
t-stat	*0.67*	*0.25*	*-0.18*	*-1.46*	
CAPM alpha (%)	0.67	0.53	0.4	-0.01	1.97
t-stat	*2.34*	*1.86*	*1.38*	*-0.04*	
FF-3 alpha (%)	0.58	0.44	0.31	-0.1	1.7
t-stat	*3.03*	*2.32*	*1.60*	*-0.53*	
Panel B: Return of [1-5] Portfolio constructed based on 24-month monthly idiosyncratic volatility After transaction cost (1965 Feb to 2016 Dec)					
Transaction Cost One Way (bps)	**0**	**40**	**80**	**200**	**Threshold Cost**
Arithmetic Mean (%)	0.14	-0.07	-0.27	-0.88	0.27
t-stat	*0.48*	*-0.23*	*-0.96*	*-3.18*	
CAPM alpha (%)	0.5	0.29	0.09	-0.52	0.97
t-stat	*1.96*	*1.16*	*0.35*	*-2.02*	
FF-3 alpha (%)	0.39	0.19	-0.02	-0.63	0.77
t-stat	*2.22*	*1.07*	*-0.09*	*-3.43*	

Table 6-6: Return and t-statistics of portfolio constructed based on long term volatility after transaction cost before short fee (1965 Feb to 2016 Dec)

Over the period of 2004 - 2016, though transaction cost (with the short fee included) is reduced significantly, the nominal excess return from trading on long term volatility before transaction cost is too low to keep the trade profitable. However, it might still be profitable to trade for alphas with factor neutralization, given the overall cost is less than 64 bps according to our rough estimate.

Panel A: Return of [1-5] Portfolio constructed based on 24-month total volatility After transaction cost and short fee (2004 Jan to 2016 Dec)					
Transaction Cost One Way (bps)	**0**	**40**	**80**	**200**	**Threshold Cost**
Arithmetic Mean (%)	-0.19	-0.32	-0.45	-0.83	-0.59
t-stat	*-0.50*	*-0.85*	*-1.20*	*-2.28*	
CAPM alpha (%)	0.34	0.21	0.08	-0.31	1.04
t-stat	*0.86*	*0.53*	*0.20*	*-0.77*	
FF-3 alpha (%)	0.34	0.21	0.08	-0.32	1.03
t-stat	*1.12*	*0.68*	*0.25*	*-1.03*	
Panel B: Return of [1-5] Portfolio constructed based on 24-month idiosyncratic volatility After transaction cost and short fee (2004 Jan to 2016 Dec)					
Transaction Cost One Way (bps)	**0**	**40**	**80**	**200**	**Threshold Cost**
Arithmetic Mean (%)	-0.09	-0.26	-0.42	-0.92	-0.22
t-stat	*-0.29*	*-0.84*	*-1.39*	*-2.97*	
CAPM alpha (%)	0.27	0.1	-0.07	-0.58	0.64
t-stat	*0.95*	*0.34*	*-0.24*	*-1.81*	
FF-3 alpha (%)	0.27	0.1	-0.07	-0.58	0.64
t-stat	*1.07*	*0.40*	*-0.27*	*-2.11*	

Table 6-7: Portfolio return and t-statistics after transaction cost and short fee (2004 Jan to 2016 Dec)

To conclude, turnover rate and market friction (transaction cost + short fee) are too high to allow the long-short portfolio profitable both historically and under the more recent market condition. Even if trading on long-term volatility could be potentially profitable given that the average transaction cost is below 60 bps and with factor neutralization, we still feel that the strategy entails a lot of liquidity risk as it requires shorting the most illiquid and highly volatile stocks. The tail risk plus margin call could easily wipe out investors' position. Our analysis partially explains the persistence of the volatility anomaly.

The conclusion here also helps us to understand the diminishment of positive return of the short-term volatility factor after 2004. As we mention in Chapter 3, there are two possible explanations. One explanation is that positive factor risk premium is arbitraged away.

Another is that people simply value the low volatility stocks more and the high volatility stocks much less upon the discovery of the puzzle. There are less long positions (or long-only speculations) of high volatility stocks. The reduced overpricing of the high volatility stocks results in the lower [1-5] return/alpha spread. Since the market friction is so high, it is unlikely that the long-short arbitrage is feasible in the first place. Therefore, the second explanation seems more plausible.

7 Robustness Check

In Chapter 3, we find evidence that short term volatility is correlated with other risk factors such as size and the market. The market capitalization of a quintile decreases monotonically with its volatility rank. The long-short [1-5] volatility factor portfolio has a stable negative beta over different periods, with a robust t-statistics. It is expected that small or young firms have higher idiosyncratic risks because of their immature business operations and weak financial positions. They are also prone to market risk as their sales are either marginal to those of the well-established firms, or of new services or products that are marginal to the established customer demand. Their business performance could largely depend on the aggregated demand of the economy, resulting large positive betas of their stocks.

In the following sections, we test whether short term volatility puzzle is genuine by controlling the effects of other risk factors including size and the market. We also investigate other short-term anomaly such as short-term return reversal, which could possibly be the cause of the short-term volatility puzzle.

7.1 Size

Our first robustness check is to test the existence of short term volatility puzzle while controlling firm size (market capitalization). Table 7-1 confirms the pure size effect (decreasing alpha with increasing size) with the results from single sort. The method we use to obtain this table is similar to the ones (short-term or long-term single sort) in the previous chapters, except with firm size as the sorting characteristic. We then control the size effect by first forming quintile portfolios ranked on market capitalization at the end of each month;

then generate 5 quintile portfolios within each market capitalization quintile based on total / idiosyncratic volatility.

Rank	Mean	Std Dev.	% Mkt Share	CAPM Alpha	FF-3 Alpha
1 Small Size	1.21	6.2	0.001	0.27	0
				[1.29]	[-0.03]
2	1.119	6.3	0.003	0.11	-0.16
				[0.65]	[-2.81]
3	1.131	6.1	0.007	0.12	-0.09
				[0.87]	[-2.4]
4	1.094	5.5	0.021	0.11	-0.03
				[1.19]	[-0.81]
5 Large Size	0.873	4.3	0.968	-0.02	0.01
				[1.19]	[0.66]
5-1	-0.003			-0.29	0.01
	[-1.337]			[-1.26]	[0.1]

Table 7-1: Portfolio Performance Sorted based on size

Table 7-2, Panel A shows the FF-3 alphas of 25 portfolios sorted by size and total volatility. Panel B shows the corresponding t-statistics. For all size quintiles, the alpha spread [Hi-Lo] induced by the spread in short term volatility is negative with robust t-statistics. In fact, we see a strong pattern of decreasing alpha with increasing volatility when size is controlled. Except for the last size quintile (largest market capitalization), alpha changes from significantly positive to significantly negative as volatility increases. Look along each column, we also observe size effect (decreasing alpha with increasing size) for the portfolios of similar volatility except for the ones with highest volatilities.

The magnitude of negative alpha for the portfolios with the highest volatilities is stable across the five size quintiles, with no obvious pattern when market capitalization increases. For those portfolios, the difference in alpha between the smallest capitalization and the largest capitalization is insignificant (only -0.003% with a t-test of -0.013). With those strong evidence, we conclude that condition on sizes, short-term volatility puzzle persists. In Table

7-3 where stocks are sorted by idiosyncratic risk and size, we obtained very similar results and arrive at the same conclusion.

Panel A: Portfolios Alpha (%) sorted by 1 Month Daily and Market Cap							
Portfolio Alpha		Total Volatility					
		Low	2	3	4	High	Hi-Lo
Market Cap	Small	0.441	0.603	0.549	0.237	-0.786	-1.228
	2	0.451	0.346	0.236	-0.08	-1.019	-1.47
	3	0.26	0.259	0.15	-0.207	-0.99	-1.249
	4	0.155	0.2	0.06	-0.257	-0.897	-1.052
	Large	0.055	0.031	-0.042	-0.176	-0.79	-0.844
	La - Sm	-0.387	-0.572	-0.59	-0.412	-0.003	0
Panel B: Portfolios Alpha T-test Result							
T-test		Total Volatility					
		Low	2	3	4	High	Hi-Lo
Market Cap	Small	3.432	4.581	4.556	2.285	-6.841	-8.479
	2	4.693	3.767	3.322	-1.077	-8.878	-8.562
	3	3.084	3.687	2.606	-3.089	-7.521	-6.554
	4	1.986	2.974	0.837	-2.821	-6.673	-5.541
	Large	0.9	0.757	-0.552	-1.293	-3.324	-3.11
	La - Sm	-3.048	-4.262	-4.17	-2.314	-0.013	0

Table 7-2: Portfolio Alpha Double Sort based on Market Cap and Total Volatility

Panel A: Portfolios Alpha sorted by 1 Month Volatility and Market Cap							
Portfolio Alpha		Idiosyncratic Volatility					
		Low	2	3	4	High	Hi-Lo
Market Cap	Small	0.482	0.603	0.604	0.255	-0.682	-1.164
	2	0.399	0.341	0.238	-0.071	-0.989	-1.388
	3	0.234	0.227	0.144	-0.201	-1.014	-1.248
	4	0.148	0.157	0.061	-0.267	-0.97	-1.118
	Large	0.063	-0.008	-0.098	-0.254	-0.686	-0.749
	La - Sm	-0.419	-0.611	-0.702	-0.509	-0.004	0
Panel B: Portfolios Alpha T-test Result							
T-test		Idiosyncratic Volatility					
		Low	2	3	4	High	Hi-Lo
Market Cap	Small	3.579	4.69	5.221	2.313	-6.578	-7.397
	2	4.195	3.626	3.275	-0.974	-9.131	-8.578
	3	3.229	3.291	2.231	-3.273	-8.208	-7.18
	4	2.151	2.371	0.886	-3.023	-7.943	-6.663
	Large	1.473	-0.182	-1.204	-2.085	-3.054	-3.056
	La - Sm	-3.139	-4.403	-4.766	-2.997	-0.018	0

Table 7-3: Portfolio Alpha Double Sort based on Market Cap and Idiosyncratic Volatility

7.2 Return Reversal

We test the existence of short term volatility puzzle while controlling the last month's return. It is well-known that the stock market exhibits short-term mean reversion. We construct 5 quintile portfolios by ranking stocks according to the previous month's returns. As we can see in Table 7-4, portfolio with low return in the previous month has high return in the next month.

Rank	Mean	Std Dev.	% Mkt Share	CAPM Alpha	FF-3 Alpha
1 Low Return	1.012	6.4	0.142	-0.04	-0.09
				[-0.33]	[-0.82]
2	1.05	4.9	0.223	0.12	0.1
				[2.35]	[1.96]
3	0.95	4.3	0.243	0.07	0.06
				[1.97]	[1.44]
4	0.875	4.3	0.232	0	0.01
				[0.08]	[0.15]
5 High Return	0.675	5.1	0.16	-0.24	-0.23
				[-2.86]	[-2.69]
5-1	-0.337			-0.2	-0.14
	[-2.033]			[-1.26]	[-0.8]

Table 7-4: Portfolio Performance Sorted based on Ex-Post Return

To check if the short-term volatility puzzle is caused or influenced by the short term mean reversion effect of the market. We control the stock's ex-post performance by constructing 5 quintile portfolios based on the previous month's returns. Within each return quintile, we further sort stocks into 5 quintile portfolios according to the last month's volatilities. Table 7-5 and Table 7-6 show FF-3 alphas of the portfolios sorted by total and idiosyncratic volatility, respectively.

The volatility puzzle still can be observed within each return quintile portfolio. Except for the highest return quintile, the alpha spread [Hi-Lo] is significantly negative with robust t-statistics. We also observe significant positive alphas for the portfolios with low past month returns and low short-term volatility, and significant negative alphas for the portfolios with highest short-term volatilities.

Unlike size, where the alpha spreads [Hi-Lo] caused by increasing short term volatility are even (at least have no pattern) across different size quintiles (in the last column), we observe increasing alpha spread with increasing last month return. This is caused by a rather interesting phenomenon: the portfolio return moves from short term reversal to short term momentum as short term volatility increases. For the stocks with the lowest total/idiosyncratic volatilities (first column in Table 7-5 and Table 7-6), the high-low portfolio has an alpha of -0.875% with a t-statistic of -4.29 and an alpha of -0.99% with a t-statistic of -4.75, for total and idiosyncratic volatility respectively. For the stocks with the highest total/idiosyncratic volatilities (last column in Table 7-5 and Table 7-6), the high-low portfolio has an alpha of 0.679 with a t-statistic of 2.49 and 0.486 with a t-statistic of 1.99, for total and idiosyncratic volatility respectively. The short-term reversal effect is largely reduced for the portfolios in the third and fourth volatility quintiles and shows up to be short term momentum in the fifth volatility quintile, through with less robust t-statistics than the reversal effect in the first two volatility quintiles.

Panel A: Portfolios Alpha sorted by 1 Month Volatility and ex-post Return							
Portfolio Alpha		Total Volatility					
		Low	2	3	4	High	Hi-Lo
Previous Month Return	Low	0.534	0.426	0.07	-0.097	-1.279	-1.813
	2	0.25	0.263	0.098	-0.027	-0.84	-1.09
	3	0.101	0.075	-0.122	-0.266	-1.022	-1.123
	4	-0.095	-0.066	0.132	-0.221	-0.993	-0.898
	High	-0.341	-0.417	-0.16	-0.205	-0.6	-0.259
	Hi-Lo	-0.875	-0.843	-0.23	-0.108	0.679	0
Panel B: Portfolios Alpha T-test Result							
T-test		Total Volatility					
		Low	2	3	4	High	Hi-Lo
Previous Month Return	Low	3.465	3.18	0.583	-0.543	-5.374	-5.669
	2	2.904	3.291	0.853	-0.187	-4.054	-4.37
	3	1.345	1.078	-1.433	-1.644	-5.141	-4.828
	4	-1.15	-0.992	1.359	-1.598	-5.564	-4.395
	High	-2.629	-3.85	-1.466	-1.544	-3.417	-1.129
	Hi-Lo	-4.29	-4.461	-1.264	-0.495	2.49	0

Table 7-5: Portfolio Performance Sorted based on Ex-Post Return and Total Volatility

Panel A: Portfolios Alpha sorted by 1 Month Volatility and ex-post Return							
Portfolio Alpha		Idiosyncratic Volatility					
		Low	2	3	4	High	Hi-Lo
Previous Month Return	Low	0.445	0.3	0.022	-0.245	-1.113	-1.558
	2	0.214	0.251	-0.029	0.046	-1.052	-1.267
	3	0.131	-0.037	-0.046	-0.316	-0.957	-1.088
	4	-0.049	-0.068	0.026	-0.173	-1.024	-0.975
	High	-0.545	-0.271	-0.172	-0.233	-0.627	-0.082
	Hi-Lo	-0.99	-0.571	-0.194	0.013	0.486	0
Panel B: Portfolios Alpha T-test Result							
T-test		Idiosyncratic Volatility					
		Low	2	3	4	High	Hi-Lo
Previous Month Return	Low	3.163	2.164	0.14	-1.369	-5.947	-6.407
	2	2.95	3.363	-0.269	0.321	-5.065	-5.197
	3	2.126	-0.557	-0.475	-2.71	-4.864	-4.848
	4	-0.715	-0.858	0.256	-1.518	-6.178	-5.367
	High	-4.135	-2.763	-1.45	-1.851	-3.456	-0.343
	Hi-Lo	-4.752	-3.027	-0.914	0.058	1.99	0

Table 7-6: Portfolio Performance Sorted based on Ex-Post Return and Idiosyncratic Volatility

7.3 Beta

The final robustness check is controlling the beta of the stocks. Beta puzzle is another well known puzzle, specifically stocks with high beta have low alpha (Frazzini & Pedersen, 2014). In this section, we would like to explore the relationship between volatility puzzle and the beta puzzle.

To construct beta sorted portfolios, we estimate beta following Frazzini & Pedersen, 2014, henceforce FP. We use the definition $\beta_i = \rho \frac{\sigma_i}{\sigma_m}$, where σ_i and σ_m are the estimated volatilities for the stock and the market respectively, and ρ is their correlation. For stock and market volatility, we use the sum of squared daily return for the past 120 days. Correlation is calculated using the overlapping three-day log return $r_t^{3d} = \sum_{k=0}^{2} \ln\,(1 + r_{t+k}^{i})$. We use the 750 trading day observations on overlapping log return to estimate the correlation factor.

Table 7-7 reports the returns of beta sorted portfolios. The risk-adjusted returns represented by CAPM and FF-3 alphas are all consistent with FP. Higher beta-stock portfolio delivers poorer risk-adjusted return. Interestingly, we observe that unadjusted portfolio returns are inversely related to betas; high beta portfolio exhibits low monthly returns on average. However, with a t-statistic of -1.66, the negative return spread is not significant at 90% confidence level. In FP, using US equities data from 1926 to 2012, they report that excess return of beta sorted decile portfolios are positively correlated with the levels of beta. The relationship between beta and returns (the security market line) have actually been documented to be regime dependent. Jylha found that tighter margin is associated with flatter security market line (Jylha, 2016). And Hong and Sraer empirically tested that the security market line can be significantly flatter conditional on high aggregate disagreement (Hong & Sraer, 2016). Our result, which is different to FP, is potentially due to a different sample period and data cleaning procedure.

Rank	Mean	Std Dev.	% Mkt Share	CAPM Alpha	FF-3 Alpha
1 Low	1.005	3.4	19.0%	0.34	0.19
				[3.07]	[1.94]
2	0.977	3.7	23.2%	0.24	0.13
				[2.41]	[1.54]
3	0.948	4.3	22.5%	0.12	0.03
				[1.56]	[0.44]
4	0.855	5.3	19.0%	-0.08	-0.17
				[-0.90]	[-2.09]
5 High	0.562	7.2	16.3%	-0.56	-0.57
				[-4.59]	[-4.78]
5-1	-0.442			-0.90	-0.75
	[-1.664]			[-4.51]	[-3.91]

Table 7-7: Portfolio Sorted by Beta

Note: At the beginning of each month, based on the past Beta, quintile portfolios are constructed from the entire investible stock universe. Mean and Std Dev. columns are calculated from absolute, not excessive, returns in the percentage term. CAPM Alpha and FF-3 Alpha are constant terms in CAPM regression and Fama-French 3-factor regression respectively. In the bracket, Newey-West robust t-statistic are reported with maximum lags of 12. The sample period is from February 1965 to December 2016.

To disentangle low returns associated with high beta stocks and high volatility stocks, Table 7-8 shows FF3 Alphas of 25 portfolios constructed by quintiles of short-term total volatility and market beta. Looking at the last row in Panel A and Panel B, we don't observe consistent beta effect (decreasing alpha with increasing beta) across all volatility quintiles. The second, fourth and fifth volatility quintiles show significant negative FF-3 alpha spreads (-0.833, -0.805 and -0.608 respectively) while the spreads of the rest two volatility quintiles are much less and with much weaker t-statistics, giving insufficient evidence of beta anomaly.

The last column shows the difference in FF-3 Alpha between highest volatility quintile and the lowest volatility quintile. For different beta quintiles, the alpha spreads are all significantly negative except the one with the second lowest beta. The spread is particularly wide (-1.119) for the highest beta quintile. The puzzle related to total volatility is thus most obvious for the highest beta quintile, suggesting a positive correlation between beta and total volatility.

Table 7-8 shows FF-3 alphas for 25 portfolios sorted by beta and idiosyncratic volatility. Unlike in Table 7-7, the alpha spread for beta (the last row in Panel A) is significantly negative across all five idiosyncratic volatility quintiles. The spread is relatively stable across different quintiles when compared with the results sorted by total volatility. This is because the idiosyncratic volatility portfolios are constructed after getting rid of the influence of the market factor.

The alpha spread for idiosyncratic volatility is significantly negative conditional on the 1st, 3rd and 5th beta quintiles but not so conditional on the 2nd and 4th beta quintiles. Though the alpha spread for idiosyncratic volatility reaches the minimum in the highest beta quintile, it is much smaller in scale than the alpha spread for total volatility (-1.119 for total volatility vs -

0.635 for idiosyncratic volatility). The alpha spread for total volatility might be enhanced by its positive correlation with beta.

In summary, we find that that the risk-adjusted low return of high volatility portfolio is partially related to high beta. Since total volatility includes market risk, it is not as good as idiosyncratic volatility for the purpose of disentangling the relationship between volatility puzzle and beta anomaly. It appears that beta puzzle is very persistent conditional on different idiosyncratic volatility while idiosyncratic volatility puzzle is influenced by the level of beta. Beta and idiosyncratic volatility seem to be correlated and their relationship is worth more investigation.

Panel A: FF-3 Alphas of Portfolios Sorted by Beta and 1 Month Total Volatility							
		Total Volatility					
		Low	2	3	4	High	Hi-Lo
Beta	Low	0.205	0.278	0.075	0.178	-0.296	-0.501
	2	0.095	0.247	0.242	0.162	-0.028	-0.122
	3	0.042	0.022	0.153	0.014	-0.351	-0.394
	4	-0.027	-0.131	-0.157	-0.05	-0.657	-0.63
	High	0.018	-0.554	-0.181	-0.429	-1.101	-1.119
	Hi-Lo	-0.187	-0.833	-0.256	-0.608	-0.805	

Panel B: FF-3 T-Test Portfolios Sorted by Beta and 1 Month Total Volatility							
		Total Volatility					
		Low	2	3	4	High	Hi-Lo
Beta	Low	2.09	2.355	0.483	0.969	-1.6	-2.737
	2	1.128	2.438	2.092	1.194	-0.129	-0.572
	3	0.485	0.275	1.61	0.081	-2.191	-2.016
	4	-0.232	-1.364	-1.411	-0.422	-4.355	-3.124
	High	0.084	-4.043	-1.454	-2.791	-6.076	-3.562
	Hi-Lo	-0.876	-3.961	-1.157	-2.356	-2.872	0

Table 7-8: FF-3 Alphas of Portfolios Sorted by Beta and Short-term Total Volatility

Panel A: FF-3 Alphas of Portfolios Sorted by Beta and 1 Month Idiosyncratic Volatility							
		Idiosyncratic Volatility					
		Low	2	3	4	High	Hi-Lo
Beta	Low	0.204	0.203	0.252	0.149	-0.276	-0.48
	2	0.103	0.2	0.278	0.205	-0.068	-0.17
	3	0.019	0.109	0.088	0.022	-0.349	-0.367
	4	-0.169	-0.126	-0.167	-0.066	-0.473	-0.304
	High	-0.449	-0.467	-0.288	-0.525	-1.084	-0.635
	Hi-Lo	-0.652	-0.669	-0.54	-0.674	-0.808	0

Panel B: FF-3 T-Test Portfolios Sorted by Beta and 1 Month Idiosyncratic Volatility							
		Idiosyncratic Volatility					
		Low	2	3	4	High	Hi-Lo
Beta	Low	2.088	1.796	1.805	0.896	-1.618	-2.663
	2	1.137	2.26	2.279	1.609	-0.377	-0.979
	3	0.256	1.164	0.865	0.165	-2.615	-2.473
	4	-1.461	-1.286	-1.532	-0.572	-3.243	-1.578
	High	-2.5	-3.577	-2.25	-3.413	-5.8	-2.938
	Hi-Lo	-3.098	-3.223	-2.426	-2.817	-3.124	0

Table 7-9: FF-3 Alphas of Portfolios Sorted by Beta and Short-term Idio. volatility

8 Conclusion

In this paper, we examine the empirical nature of short-term and long-term volatility puzzles, their relationship, possible causes for their existences, and their relationship with other related anomalies.

We find that short-term volatility puzzle is time dependent and diminishing after 2004 when Ang et al. first published their finding. With a single sort analysis, we find that a similar puzzle holds for long-term volatility, i.e. unconditionally, there are significant negative alphas for the portfolios with highest long term volatilities.

A detailed double sort analysis on long term and short term volatility shows that the root cause of the long-term anomaly is short-term volatility (through its high correlation with long-term volatility). The conditional returns of volatility-sorted portfolios are consistently determined by the quintiles of the short-term volatility but less so by the quintiles of the long-term volatility. This result fits with our hypothesis that the anomaly is mainly driven by short term speculative activities thus would not be compensated with higher returns.

We show with evidence that stocks of higher volatility usually have higher shorting fee. This may cause insufficient expression of negative views towards the firms and their stocks could be overpriced (thus lower returns). Another explanation of the puzzle is the limited arbitrage. To profit from the anomaly, arbitrageurs need to bear transaction costs and short fees when they go long the low volatility portfolio and short the high volatility portfolio. We show that after incorporating the costs, trading on volatility factors are no longer profitable.

To explain the narrowing return/alpha spread of volatility puzzle after 2004, we are inclined towards the story that the high volatility stocks became less favorable for long only investors rather than that the spreads are arbitraged away by long short strategies. As we have shown, the long-short trade is very unlikely profitable and also not well risk-adjusted even before 2004.

At last, we perform robustness checks on size, return reversal and beta factor through double-sorting with volatility. We prove that none of these three factors can fully explain the puzzle. In addition, we find a light evidence of short-term momentum effect for the stocks with the highest volatilities, as opposed to short term reversal effect for low volatility stocks. We believe that idiosyncratic volatility is a cleaner measure of volatility puzzle with less entanglement with the beta anomaly. The relationship between beta anomaly and idiosyncratic volatility puzzle is worth further study.

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10 Appendix:

Panel A: Portfolio constructed based on 24-Week Total Volatility						
Portfolio	**1**	**2**	**3**	**4**	**5**	**1-5**
Mthly Arithmetic Mean Return	*0.892*	*0.933*	*0.975*	*0.868*	*0.595*	*-0.297*
Mthly Geometric Mean Return	*0.829*	*0.824*	*0.809*	*0.605*	*0.198*	*-0.631*
CAPM alpha	*0.14*	*0.04*	*-0.02*	*-0.25*	*-0.62*	*-0.76*
t-stat	*2.16*	*0.85*	*-0.28*	*-1.79*	*-2.8*	*-2.78*
FF3 alpha	*0.09*	*0*	*-0.03*	*-0.23*	*-0.62*	*-0.71*
t-stat	*1.7*	*0.07*	*-0.42*	*-2.05*	*-4.23*	*-3.76*

Table 10-1 Portfolio Performance based on 24 Weekly Return Derived Volatility

Panel A: Portfolios Alpha sorted by 1 Month Total Volatility and 24-Week Total Volatility							
		1 Month Total Volatility					
		Low	2	3	4	High	Hi-Lo
24-Week Total Vol.	Low	0.058	0.079	0.086	0.214	-0.516	-0.574
	2	0.105	0.04	-0.112	-0.109	-0.475	-0.58
	3	-0.026	0.065	0.079	-0.271	-0.413	-0.387
	4	0.074	-0.216	-0.018	-0.158	-0.838	-0.912
	High	0.155	0.062	0.07	-0.374	-1.27	-1.425
	Hi-Lo	0.097	-0.017	-0.016	-0.588	-0.754	0
Panel B: Portfolios Alpha T-test Result							
		1 Month Total Volatility					
		Low	2	3	4	High	Hi-Lo
24-Week Total Vol.	Low	0.88	1.185	0.866	1.26	-2.269	-2.522
	2	1.372	0.716	-1.314	-0.666	-2.53	-2.858
	3	-0.275	0.714	0.791	-1.972	-2.422	-1.823
	4	0.414	-1.451	-0.14	-0.95	-4.963	-3.343
	High	0.718	0.319	0.425	-2.365	-6.247	-4.518
	Hi-Lo	0.473	-0.081	-0.085	-2.465	-2.35	0

Table 10-2 Portfolio Performance Sorted by 1 Month Total Vol. and 24-Week Total Vol.

Note: At the beginning of each month, quintile portfolios are constructed from the entire investible stock universe based on 24 weekly return calculated volatility. Each quintile portfolio is further sorted based on 1-month volatility into quintile portfolios. The Sample Period is Feb 1965 to Dec 2016. T-statistic is calculated using Newey-West HAC standard error with maximum lags of 12 periods.

Panel A: Portfolio constructed based on 48-Week Total Volatility						
Portfolio	**1**	**2**	**3**	**4**	**5**	**1-5**
Mthly Arithmetic Mean Return	*0.891*	*0.96*	*0.897*	*0.998*	*0.509*	*-0.382*
Mthly Geometric Mean Return	*0.826*	*0.844*	*0.712*	*0.715*	*0.088*	*-0.738*
CAPM alpha	*0.14*	*0.06*	*-0.13*	*-0.14*	*-0.72*	*-0.86*
t-stat	*1.97*	*0.97*	*-1.47*	*-0.83*	*-3.23*	*-3.07*
FF3 alpha	*0.09*	*0.02*	*-0.14*	*-0.1*	*-0.73*	*-0.82*
t-stat	*1.64*	*0.37*	*-1.84*	*-0.78*	*-4.72*	*-4.18*

Table 10-3 Portfolio Performance Based on 48 Weekly Return Derived Volatility

Panel A: Portfolios Alpha sorted by 1 Month Total Volatility and 48-Week Total Volatility							
		1 Month Total Volatility					
		Low	2	3	4	High	Hi-Lo
48-Week Total Vol.	Low	0.062	0.114	0.061	-0.09	-0.504	-0.566
	2	0.085	0.054	-0.058	-0.134	-0.407	-0.492
	3	0.026	-0.108	0.002	-0.271	-0.64	-0.666
	4	0.288	0.028	0.128	-0.049	-0.79	-1.079
	High	-0.297	0.243	0.017	-0.406	-1.273	-0.976
	Hi-Lo	-0.359	0.129	-0.044	-0.317	-0.77	0
Panel B: Portfolios Alpha T-test Result							
		1 Month Total Volatility					
		Low	2	3	4	High	Hi-Lo
48-Week Total Vol.	Low	0.905	1.811	0.601	-0.648	-2.171	-2.433
	2	1.094	1.024	-0.668	-0.949	-1.729	-1.997
	3	0.213	-1.102	0.026	-2.198	-4.083	-3.39
	4	1.89	0.183	0.91	-0.294	-4.502	-4.081
	High	-1.31	1.145	0.091	-2.416	-6.127	-2.963
	Hi-Lo	-1.692	0.558	-0.211	-1.381	-2.45	0

Table 10-4 Portfolio Performance Sorted by 1 Month Total Vol. and 48-Week Total Vol.

Note: At the beginning of each month, quintile portfolios are constructed from the entire investible stock universe based on 48 weekly return calculated volatility. Each quintile portfolio is further sorted based on 1-month volatility into quintile portfolios. The Sample Period is Feb 1965 to Dec 2016. T-statistic is calculated using Newey-West HAC standard error with maximum lags of 12 periods.

Panel A: Portfolio constructed based on 72-Week Total Volatility						
Portfolio	**1**	**2**	**3**	**4**	**5**	**1-5**
Mthly Arithmetic Mean Return	*0.895*	*0.984*	*0.928*	*0.912*	*0.508*	*-0.387*
Mthly Geometric Mean Return	*0.83*	*0.867*	*0.743*	*0.62*	*0.081*	*-0.749*
CAPM alpha	*0.14*	*0.08*	*-0.1*	*-0.23*	*-0.73*	*-0.87*
t-stat	*1.94*	*1.32*	*-1.09*	*-1.43*	*-3.21*	*-3.03*
FF3 alpha	*0.09*	*0.04*	*-0.09*	*-0.2*	*-0.75*	*-0.84*
t-stat	*1.71*	*0.86*	*-1.25*	*-1.56*	*-4.31*	*-3.88*

Table 10-5 Portfolio Performance Based on 72 Weekly Return Derived Volatility

Panel A: Portfolios Alpha sorted by 1 Month Total Volatility and 72-Week Total Volatility							
		1 Month Total Volatility					
		Low	2	3	4	High	Hi-Lo
72-Week Total Vol.	Low	0.078	0.126	0.073	-0.323	-0.508	-0.586
	2	0.084	0.06	-0.019	-0.09	-0.489	-0.573
	3	0.044	-0.115	-0.006	-0.164	-0.593	-0.637
	4	0.132	-0.112	0.106	-0.074	-0.827	-0.959
	High	-0.222	-0.127	-0.081	-0.396	-1.227	-1.005
	Hi-Lo	-0.3	-0.252	-0.154	-0.073	-0.719	0
Panel B: Portfolios Alpha T-test Result							
		1 Month Total Volatility					
		Low	2	3	4	High	Hi-Lo
72-Week Total Vol.	Low	1.136	2.078	0.651	-2.32	-2.206	-2.499
	2	0.967	1.032	-0.247	-0.631	-2.097	-2.318
	3	0.386	-1.092	-0.057	-1.36	-3.416	-2.866
	4	0.83	-0.759	0.792	-0.463	-5.188	-3.748
	High	-0.966	-0.623	-0.494	-2.089	-5.57	-2.819
	Hi-Lo	-1.439	-1.152	-0.882	-0.295	-2.269	0

Table 10-6 Portfolio Performance Sorted by 1 Month Total Vol. and 72-Week Total Vol.

Note: At the beginning of each month, quintile portfolios are constructed from the entire investible stock universe based on 72 weekly return calculated volatility. Each quintile portfolio is further sorted based on 1-month volatility into quintile portfolios. The Sample Period is Feb 1965 to Dec 2016. T-statistic is calculated using Newey-West HAC standard error with maximum lags of 12 periods.

Panel A: Portfolio constructed based on 12-Month Total Volatility						
Portfolio	**1**	**2**	**3**	**4**	**5**	**1-5**
Mthly Arithmetic Mean Return	*0.93*	*0.929*	*0.96*	*0.946*	*0.86*	*-0.07*
Mthly Geometric Mean Return	*0.865*	*0.836*	*0.834*	*0.776*	*0.571*	*-0.294*
CAPM alpha	*0.18*	*0.08*	*0.03*	*-0.06*	*-0.28*	*-0.46*
t-stat	*2.42*	*1.42*	*0.66*	*-0.74*	*-1.73*	*-2.07*
FF3 alpha	*0.14*	*0.03*	*0*	*-0.09*	*-0.27*	*-0.41*
t-stat	*2.34*	*0.79*	*0.02*	*-1.28*	*-2.25*	*-2.46*

Table 10-7 Portfolio Performance based on 12 Monthly Return Derived Volatility

Panel A: Portfolios Alpha sorted by 1 Month Total Volatility and 12-Month Total Volatility							
		1 Month Total Volatility					
		Low	2	3	4	High	Hi-Lo
12-Month Total Vol.	Low	0.062	0.172	0.031	-0.106	-0.391	-0.454
	2	0.015	0.056	-0.021	0.013	-0.102	-0.117
	3	0.089	-0.113	-0.062	-0.032	-0.343	-0.432
	4	0.044	0.161	-0.117	0.125	-0.521	-0.565
	High	0.035	-0.413	0.019	0.185	-0.734	-0.769
	Hi-Lo	-0.028	-0.585	-0.012	0.291	-0.343	0
Panel B: Portfolios Alpha T-test Result							
		1 Month Total Volatility					
		Low	2	3	4	High	Hi-Lo
12-Month Total Vol.	Low	0.807	3.347	0.325	-1.057	-2.753	-2.602
	2	0.138	0.916	-0.288	0.142	-0.835	-0.688
	3	0.829	-1.081	-0.629	-0.326	-2.37	-2.169
	4	0.268	0.961	-0.951	0.966	-3.753	-2.355
	High	0.155	-1.909	0.094	1.063	-3.795	-2.646
	Hi-Lo	-0.129	-2.575	-0.051	1.369	-1.458	0

Table 10-8 Portfolio Performance Sorted by 1 Month Total Vol. and 12-Month Total Vol.

Note: At the beginning of each month, quintile portfolios are constructed from the entire investible stock universe based on 12 Monthly calculated volatility. Each quintile portfolio is further sorted based on 1-month volatility into quintile portfolios. The Sample Period is Feb 1965 to Dec 2016. T-statistic is calculated using Newey-West HAC standard error with maximum lags of 12 periods.

Panel A: Portfolio constructed based on 18-Month Total Volatility						
Portfolio	**1**	**2**	**3**	**4**	**5**	**1-5**
Mthly Arithmetic Mean Return	*0.919*	*0.916*	*0.954*	*0.952*	*0.885*	*-0.034*
Mthly Geometric Mean Return	*0.854*	*0.824*	*0.824*	*0.772*	*0.584*	*-0.27*
CAPM alpha	*0.17*	*0.07*	*0.02*	*-0.06*	*-0.26*	*-0.43*
t-stat	*2.28*	*1.18*	*0.41*	*-0.77*	*-1.56*	*-1.89*
FF3 alpha	*0.13*	*0.02*	*-0.02*	*-0.08*	*-0.25*	*-0.38*
t-stat	*2.21*	*0.47*	*-0.34*	*-0.95*	*-2.02*	*-2.25*

Table 10-9 Portfolio Performance based on 18 Monthly Return Derived Volatility

Panel A: Portfolios Alpha sorted by 1 Month Total Volatility and 18-Month Total Volatility							
		1 Month Total Volatility					
		Low	2	3	4	High	Hi-Lo
18-Month Total Volatility	Low	0.061	0.162	0.014	-0.061	-0.366	-0.427
	2	0.023	0.022	0.002	0.021	-0.252	-0.276
	3	-0.068	0.058	-0.089	0.004	-0.402	-0.334
	4	0.228	-0.056	0.003	0.065	-0.459	-0.687
	High	0.018	-0.049	-0.156	0.183	-0.728	-0.746
	Hi-Lo	-0.042	-0.211	-0.17	0.244	-0.361	0
Panel B: Portfolios Alpha T-test Result							
		1 Month Total Volatility					
		Low	2	3	4	High	Hi-Lo
18-Month Total Volatility	Low	0.776	3.274	0.154	-0.498	-2.146	-2.129
	2	0.212	0.333	0.028	0.232	-2.134	-1.608
	3	-0.509	0.546	-0.8	0.043	-3.013	-1.617
	4	1.3	-0.341	0.023	0.505	-2.829	-2.532
	High	0.083	-0.19	-0.694	0.966	-4.094	-2.922
	Hi-Lo	-0.19	-0.789	-0.671	0.986	-1.468	0

Table 10-10 Portfolio Performance sorted by 1 Month Total Vol. and 18-Month Vol.

Note: At the beginning of each month, quintile portfolios are constructed from the entire investible stock universe based on 18 Monthly return calculated volatility. Each quintile portfolio is further sorted based on 1-month volatility into quintile portfolios. The Sample Period is Feb 1965 to Dec 2016. T-statistic is calculated using Newey-West HAC standard error with maximum lags of 12 periods.

Panel A: Portfolio constructed based on 24-Week Idiosyncratic Volatility						
Portfolio	**1**	**2**	**3**	**4**	**5**	**1-5**
Mthly Arithmetic Mean Return	*0.879*	*0.963*	*0.971*	*0.828*	*0.48*	*-0.399*
Mthly Geometric Mean Return	*0.805*	*0.847*	*0.794*	*0.564*	*0.118*	*-0.687*
CAPM alpha	*0.08*	*0.05*	*-0.04*	*-0.28*	*-0.68*	*-0.77*
t-stat	*1.75*	*1.4*	*-0.48*	*-1.92*	*-3.19*	*-3.02*
FF3 alpha	*0.07*	*0.02*	*-0.06*	*-0.26*	*-0.67*	*-0.74*
t-stat	*1.9*	*0.47*	*-0.83*	*-2.39*	*-5.01*	*-4.64*

Table 10-11 Portfolio Performance based on 24 Weekly Return Derived Idiosyncratic Vol

Panel A: Portfolios Alpha sorted by 1 Month Idiosyncratic Volatility and 24-Week Idiosyncratic Volatility							
		1 Month Idiosyncratic Volatility					
		Low	2	3	4	High	Hi-Lo
24-Week Idio. Vol.	Low	0.077	0.052	-0.143	0.006	-0.335	-0.412
	2	0.066	0.025	-0.019	-0.064	-0.761	-0.827
	3	0.036	-0.036	-0.037	-0.244	-0.382	-0.419
	4	0.087	0.058	-0.175	-0.24	-0.85	-0.936
	High	-0.003	-0.195	-0.114	-0.585	-1.247	-1.244
	Hi-Lo	-0.08	-0.247	0.029	-0.591	-0.912	0
Panel B: Portfolios Alpha T-test Result							
		1 Month Idiosyncratic Volatility					
		Low	2	3	4	High	Hi-Lo
24-Week Idio. Vol.	Low	1.525	0.926	-1.435	0.035	-1.675	-2.046
	2	1.071	0.391	-0.214	-0.587	-3.04	-3.122
	3	0.341	-0.371	-0.407	-2.107	-2.317	-2.118
	4	0.53	0.458	-1.25	-1.951	-5.086	-4.002
	High	-0.018	-0.802	-0.592	-3.669	-7.058	-5.031
	Hi-Lo	-0.432	-0.966	0.13	-2.464	-3.212	0

Table 10-12 Portfolio Performance Sorted by 1 Month Idio. Vol. and 24-Week Idio. Vol.

Note: At the beginning of each month, quintile portfolios are constructed from the entire investible stock universe based on FF3 residual volatility calculated using 24 weekly return. Each quintile portfolio is further sorted based on 1-month FF3 idiosyncratic volatility into quintile portfolios. The Sample Period is Feb 1965 to Dec 2016. T-statistic is calculated using Newey-West HAC standard error with maximum lags of 12 periods.

Panel A: Portfolio constructed based on 48-Week Idiosyncratic Volatility						
Portfolio	**1**	**2**	**3**	**4**	**5**	**1-5**
Mthly Arithmetic Mean Return	*0.88*	*0.952*	*0.962*	*0.909*	*0.558*	*-0.322*
Mthly Geometric Mean Return	*0.806*	*0.827*	*0.77*	*0.622*	*0.165*	*-0.641*
CAPM alpha	*0.08*	*0.02*	*-0.07*	*-0.21*	*-0.63*	*-0.72*
t-stat	*1.62*	*0.56*	*-0.79*	*-1.22*	*-2.85*	*-2.69*
FF3 alpha	*0.06*	*-0.01*	*-0.09*	*-0.18*	*-0.62*	*-0.68*
t-stat	*1.77*	*-0.27*	*-1.25*	*-1.39*	*-4.26*	*-3.98*

Table 10-13 Portfolio Performance based on 48-Week Return Derived Idiosyncratic Vol

Panel A: Portfolios Alpha sorted by 1 Month Idiosyncratic Volatility and 48-Week Idiosyncratic Volatility							
		1 Month Idiosyncratic Volatility					
		Low	2	3	4	High	Hi-Lo
48-Week Idio. Vol.	Low	0.082	0.079	-0.31	-0.026	-0.649	-0.731
	2	0.068	-0.105	0.026	-0.157	-0.394	-0.462
	3	-0.076	0.121	-0.065	-0.217	-0.7	-0.625
	4	0.294	0.175	0.081	-0.285	-0.881	-1.175
	High	0.151	0.259	-0.043	-0.441	-1.185	-1.336
	Hi-Lo	0.069	0.179	0.267	-0.415	-0.536	0
Panel B: Portfolios Alpha T-test Result							
		1 Month Idiosyncratic Volatility					
		Low	2	3	4	High	Hi-Lo
48-Week Idio. Vol.	Low	1.531	1.404	-3.468	-0.182	-3.124	-3.373
	2	1.01	-1.715	0.348	-1.477	-1.577	-1.757
	3	-0.675	1.365	-0.663	-2.061	-4.068	-3.208
	4	1.715	1.056	0.517	-2.062	-5.238	-4.97
	High	0.652	1.357	-0.248	-2.541	-6.289	-4.555
	Hi-Lo	0.291	0.871	1.243	-1.851	-1.852	0

Table 10-14 Portfolio Performance Sorted by 1 Month Idio. Vol. and 48-Week Idio. Vol.

Note: At the beginning of each month, quintile portfolios are constructed from the entire investible stock universe based on FF3 residual volatility calculated using 48 weekly returns. Each quintile portfolio is further sorted based on 1-month FF3 idiosyncratic volatility into quintile portfolios. The Sample Period is Feb 1965 to Dec 2016. T-statistic is calculated using Newey-West HAC standard error with maximum lags of 12 periods.

Panel A: Portfolio constructed based on 72-Week Idiosyncratic Volatility						
Portfolio	**1**	**2**	**3**	**4**	**5**	**1-5**
Mthly Arithmetic Mean Return	*0.884*	*0.933*	*1.007*	*0.893*	*0.521*	*-0.363*
Mthly Geometric Mean Return	*0.809*	*0.806*	*0.808*	*0.602*	*0.117*	*-0.692*
CAPM alpha	*0.08*	*0*	*-0.03*	*-0.24*	*-0.68*	*-0.76*
t-stat	*1.62*	*0.07*	*-0.3*	*-1.4*	*-2.88*	*-2.72*
FF3 alpha	*0.06*	*-0.03*	*-0.02*	*-0.22*	*-0.67*	*-0.73*
t-stat	*1.72*	*-0.66*	*-0.25*	*-1.86*	*-4.08*	*-3.83*

Table 10-15 Portfolio Performance based on 72 Weekly Returns Derived Idiosyncratic Vol

Panel A: Portfolios Alpha sorted by 1 Month Idiosyncratic Volatility and 72-Week Idiosyncratic Volatility							
		1 Month Idiosyncratic Volatility					
		Low	2	3	4	High	Hi-Lo
72-Week Idio. Vol.	Low	0.089	0.051	-0.246	0.03	-0.425	-0.514
	2	0.022	-0.054	-0.076	-0.255	-0.597	-0.619
	3	0.067	0.129	0.028	-0.109	-0.69	-0.757
	4	0.227	0.143	0.058	-0.323	-0.827	-1.054
	High	-0.074	0.034	-0.12	-0.42	-1.179	-1.105
	Hi-Lo	-0.163	-0.017	0.126	-0.45	-0.754	0
Panel B: Portfolios Alpha T-test Result							
		1 Month Idiosyncratic Volatility					
		Low	2	3	4	High	Hi-Lo
72-Week Idio. Vol.	Low	1.662	0.801	-2.409	0.184	-1.984	-2.256
	2	0.363	-0.906	-0.991	-2.17	-2.57	-2.633
	3	0.501	1.175	0.246	-0.964	-3.748	-3.176
	4	1.364	0.901	0.373	-2.448	-5.428	-4.528
	High	-0.271	0.145	-0.603	-2.295	-5.96	-3.335
	Hi-Lo	-0.577	-0.072	0.546	-1.735	-2.849	0

Table 10-16 Portfolio Performance Sorted by 1 Month Idio. Vol. and 72-Week Idio. Vol.

Note: At the beginning of each month, quintile portfolios are constructed from the entire investible stock universe based on FF3 residual volatility calculated using 72 weekly return. Each quintile portfolio is further sorted based on 1-month FF3 idiosyncratic volatility into quintile portfolios. The Sample Period is Feb 1965 to Dec 2016. T-statistic is calculated using Newey-West HAC standard error with maximum lags of 12 periods.

Panel A: Portfolio constructed based on 12-Month Idiosyncratic Volatility						
Portfolio	**1**	**2**	**3**	**4**	**5**	**1-5**
Mthly Arithmetic Mean Return	*0.884*	*0.876*	*0.933*	*0.897*	*0.893*	*0.009*
Mthly Geometric Mean Return	*0.803*	*0.778*	*0.786*	*0.687*	*0.562*	*-0.241*
CAPM alpha	*0.07*	*0.01*	*-0.03*	*-0.14*	*-0.25*	*-0.32*
t-stat	*1.15*	*0.19*	*-0.5*	*-1.16*	*-1.31*	*-1.35*
FF3 alpha	*0.04*	*-0.02*	*-0.03*	*-0.13*	*-0.2*	*-0.24*
t-stat	*0.84*	*-0.54*	*-0.55*	*-1.25*	*-1.44*	*-1.43*

Table 10-17 Portfolio Performance based on 12 Monthly Return Derived Idiosyncratic Vol

Panel A: Portfolios Alpha sorted by 1 Month Idiosyncratic Volatility and 12-Month Idiosyncratic Volatility							
		1 Month Idiosyncratic Volatility					
		Low	2	3	4	High	Hi-Lo
12-Month Idio. Vol.	Low	0.074	-0.015	-0.026	-0.123	-0.875	-0.948
	2	0.045	0.001	-0.093	-0.37	-0.579	-0.624
	3	0.015	0.046	-0.052	-0.184	-0.842	-0.857
	4	-0.036	0.165	-0.075	-0.196	-0.802	-0.766
	High	0.518	0.441	0.077	-0.227	-1.054	-1.572
	Hi-Lo	0.444	0.456	0.103	-0.104	-0.179	0
Panel B: Portfolios Alpha T-test Result							
		1 Month Idiosyncratic Volatility					
		Low	2	3	4	High	Hi-Lo
12-Month Idio. Vol.	Low	1.269	-0.224	-0.254	-0.819	-4.668	-4.892
	2	0.769	0.009	-1.175	-3.276	-2.774	-2.768
	3	0.187	0.454	-0.467	-1.402	-4.387	-4.147
	4	-0.258	1.211	-0.635	-1.425	-5.108	-3.3
	High	3.161	2.126	0.433	-1.446	-5.671	-6.851
	Hi-Lo	2.333	2.037	0.469	-0.496	-0.647	0

Table 10-18 Portfolio Performance Sorted by 1 Month Idio. Vol. and 12-Month Idio. Vol.

Note: At the beginning of each month, quintile portfolios are constructed from the entire investible stock universe based on FF3 residual volatility calculated using 12 Monthly return. Each quintile portfolio is further sorted based on 1-month FF3 idiosyncratic volatility into quintile portfolios. The Sample Period is Feb 1965 to Dec 2016. T-statistic is calculated using Newey-West HAC standard error with maximum lags of 12 periods.

Panel A: Portfolio constructed based on 18-Month Idiosyncratic Volatility						
Portfolio	**1**	**2**	**3**	**4**	**5**	**1-5**
Mthly Arithmetic Mean Return	*0.884*	*0.876*	*0.933*	*0.897*	*0.893*	*0.009*
Mthly Geometric Mean Return	*0.803*	*0.778*	*0.786*	*0.687*	*0.562*	*-0.241*
CAPM alpha	*0.07*	*0.01*	*-0.03*	*-0.14*	*-0.25*	*-0.32*
t-stat	*1.15*	*0.19*	*-0.5*	*-1.16*	*-1.31*	*-1.35*
FF3 alpha	*0.04*	*-0.02*	*-0.03*	*-0.13*	*-0.2*	*-0.24*
t-stat	*0.84*	*-0.54*	*-0.55*	*-1.25*	*-1.44*	*-1.43*

Table 10-19 Portfolio Performance based on 18 Monthly Return Derived Idiosyncratic Vol

Panel A: Portfolios Alpha sorted by 1 Month Idiosyncratic Volatility and 18-Month Idiosyncratic Volatility							
		1 Month Idiosyncratic Volatility					
		Low	2	3	4	High	Hi-Lo
18-Month Idio. Vol.	Low	0.08	0.002	-0.123	-0.052	-0.872	-0.951
	2	0.042	0.049	-0.108	-0.362	-0.672	-0.714
	3	-0.024	-0.003	0.042	-0.152	-0.705	-0.68
	4	0.298	0.09	0.02	-0.21	-0.791	-1.088
	High	0.228	0.401	-0.045	-0.369	-1.075	-1.304
	Hi-Lo	0.148	0.399	0.078	-0.317	-0.204	0
Panel B: Portfolios Alpha T-test Result							
		1 Month Idiosyncratic Volatility					
		Low	2	3	4	High	Hi-Lo
18-Month Idio. Vol.	Low	1.416	0.042	-1.417	-0.337	-4.053	-4.205
	2	0.67	0.817	-1.014	-3.175	-3.526	-3.536
	3	-0.298	-0.032	0.427	-1.425	-3.721	-3.091
	4	1.847	0.669	0.158	-1.52	-5.855	-5.202
	High	1.199	1.665	-0.254	-2.353	-5.632	-5.098
	Hi-Lo	0.724	1.548	0.385	-1.403	-0.753	0

Table 10-20 Portfolio Performance Sorted by 1 Month Idio. Vol. and 18-Month Idio. Vol.

Note: At the beginning of each month, quintile portfolios are constructed from the entire investible stock universe based on FF3 residual volatility calculated using 18 Monthly return. Each quintile portfolio is further sorted based on 1-month FF3 idiosyncratic volatility into quintile portfolios. The Sample Period is Feb 1965 to Dec 2016. T-statistic is calculated using Newey-West HAC standard error with maximum lags of 12 periods.