PAPER CODE	EXAMINER	DEPARTMENT	TEL
CPT 107	K.L. Man & Gabriela Mogos	CPT	1509

2023/24 SEMESTER 1 – Assessment I

BACHELOR DEGREE – Year 2

Discrete Mathematics and Statistics

INSTRUCTIONS TO CANDIDATES

1. **Total marks available are 100, accounting for 10% of the overall module marks.**
2. **The number in the column on the right indicates the marks for each question.**
3. **Answer all questions.**
4. **Answers should be written in English.**
5. **Relevant and clear steps should be included in your answers.**

Notes:

- **To obtain full marks for each question, relevant and clear steps need to be included in the answers.**
- **Partial marks may be awarded depending on the degree of completeness and clarity.**

Question 1: Proof Techniques **[34 marks]**

(a) Use proof by contradiction to show the following statement: if a and b are rational numbers and $b \neq 0$, then $a - b\sqrt{3}$ is irrational.

(8 marks)

(b) $\sqrt{75}$ is irrational. If you think that it is true, prove it. If not, explain why.

(12 marks)

(c) For every natural number n, use proof by induction to show that $n^3 - n$ is divisible by 3.

(8 marks)

(d) Let x $\in \mathbb{Z}$ and $0 \leq$ x, use proof by induction to show that:

$$2(1 + 2 + 4 + \cdots + 2^x) = 2^{x+2} - 2.$$

(6 marks)

Question 2: Set Theory **[22 marks]**

(a) Let A, B and C be non-empty sets, then $A - C = (A - C) \cup (B - C)$. If you think that it is true, prove it. Otherwise, disprove it.

(6 marks)

(b) Let X and Y be sets, then $X \subseteq Y$ if and only if $X \cap Y = X$. If you think that it is true, prove it. Otherwise, give a counterexample to show that it is false.

(10 marks)

(c) The football cup Liverpool F.C. has 35 players, 29 players play in attack, 16 players play in midfield and 10 players play in both attack and midfield. Find the number of players who play:

1. in attack only.
2. in midfield only.
3. in neither attack nor midfield.

(6 marks)

Question 3: Relations **[44 marks]**

(a) Let $X = \{a, b, c, d, e, f\}$ and let S be a relation on X such that $S = \{(a, b), (d, e), (e, f), (b, a), (c, d)\}$. Find the transitive closure of the relation S on X.

(10 marks)

(b) Let $X = \{a, b, c, d, e\}$ and let S be a relation on X such that:
$S = \{(a, a), (a, c), (a, d), (a, e), (b, b), (b, c), (b, d), (b, e), (c, c), (c, d), (c, e), (d, d), (e, e)\}$.
Prove or disprove that S is a partial order. If S is a partial order, draw its Hasse diagram.

(12 marks)

(c) Let A and B be both transitive relations on the set S. Prove or disprove the following:

1. $A \cap B$ is also transitive.
2. $A \circ B$ is also transitive.
3. "$A \circ B$ is also transitive" implies "$A \cap B$ is also transitive".

(12 marks)

(d) If A is an equivalence relation over a set S, then A^{-1} is an equivalence relation over S. If you think that it is true, prove it. If not, give a counter-example.

(10 marks)

END OF ASSESSMENT I PAPER