ECS 132 Term Project

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Chapter 1

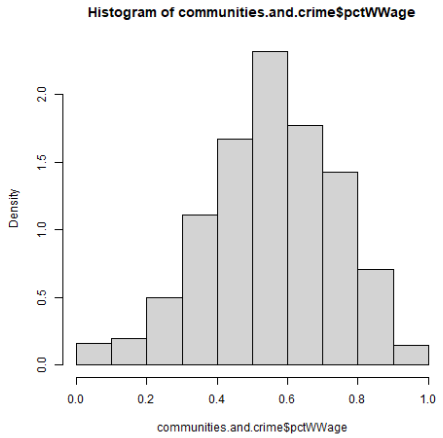
The Normal Family

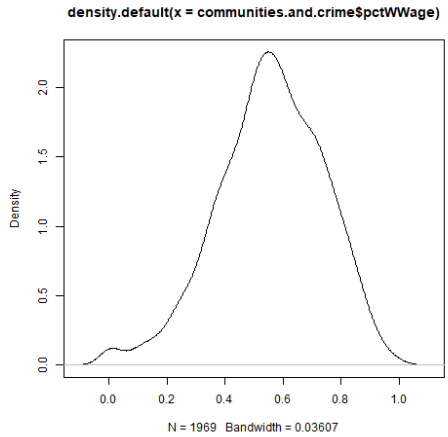
1.1 Communities and Crime: pctWWage

Our group observed that the variable **pctWWage** of the Communities and Crime dataset seemed well-approximated by the normal family of continuous distributions. According to the UCI Machine Learning Repository, **pctWWage** is described as the percentage of households within the United States with wage or salary income in 1989.

1.2 Histogram and Density

Below are the histogram and density plots of pctWWage using R's hist(), plot(), and density() functions.





1.3 Maximum Likelihood Estimation

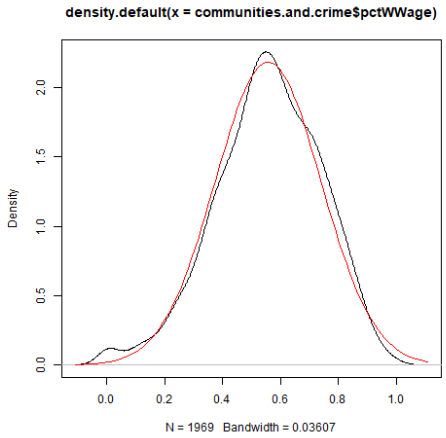
In order to find the MLE of pctWWage, we first had to define our log-likelihood function as

$$LL(\mu, \sigma^2) = -n \log(2\pi) + \frac{\log(\sigma^2)}{2} - \frac{\sum(x-\mu)^2}{2\sigma^2} \tag{1.1}$$

Using R's built-in mle() function, we use the *negative* log-likelihood to find the normal parameters of pctWWage.

```
z <- mle(minuslogl = ll, start = c(list(mean = 1),
  list(var = 1)))
```

Superimposing the resulting density on the kernel estimate plot results in

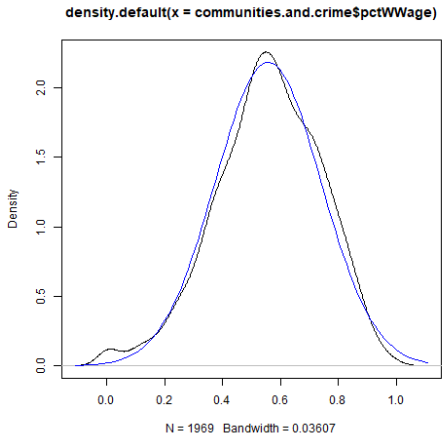


1.4 Method of Moments

To find the MM-estimated density of pctWWage, we used the following function to approximate μ and σ^2.

```
mm <- function(x) {
  mu <- mean(x)
  sigma <- sqrt(mean(x^2) - mu^2)
  return(c(mu, sigma))
}
```

Superimposing the resulting density grants us



1.5 Conclusion

As seen on the plots, the MLE and MM normal approximations appear to be a good fit for pctWWage's data. The density estimates obtained from both methods closely align with the density curve derived directly from the dataset. Thus, our group can confidently conclude that the variable pctWWage is well-approximated by the normal distribution family.

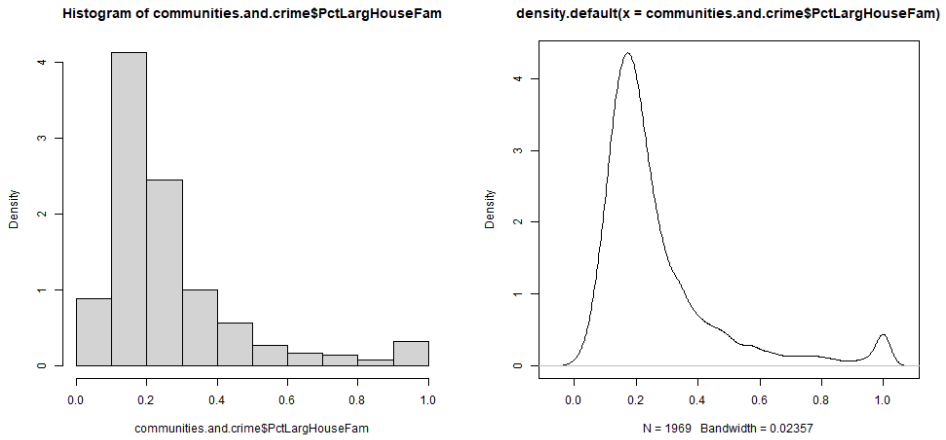
Chapter 2

The Exponential Family

2.1 Communities and Crime: PctLargHouseFam

For the exponential family of continuous distributions, we observed that the variable **PctLargHouseFam** was a suitable approximation. According to the UCI Machine Learning Repository, **PctLargHouseFam** is described as the percentage of family households with six or more family members.

2.2 Histogram and Density



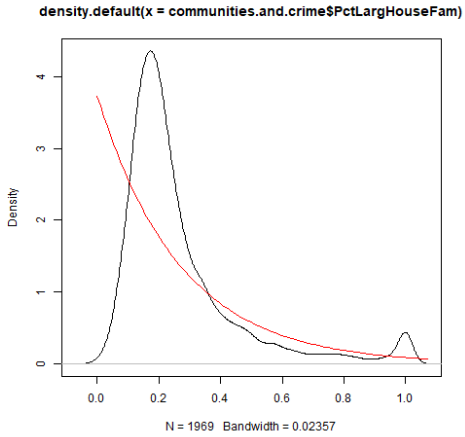
2.3 MLE and MM

To find the MLE of the exponential family, we first defined our log likelihood function:

$$L(\lambda) = n \log \lambda - \lambda \sum x \tag{2.1}$$

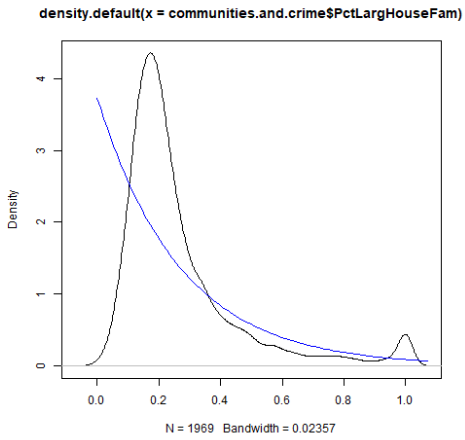
To find the MLE, we use R's built in mle() function with the negative log likelihood function.

```
z <- mle(minuslogl = ll, start = c(list(lambda =
    1)))
```



To find the MM of the exponential family, we used the following function to predict the value of λ:

```
mm <- function(x) {
  lambda <- 1 / mean(x)
  lambda
}
```



2.4 Conclusion

The exponential approximation shows a reasonable fit at the tail end of the density distribution, but it exhibits noticeable deviations from the actual data around $x = 0.2$. The exponential distribution assumes a constant and consistent decay rate, which does not accurately capture the characteristics of the data around its peak.

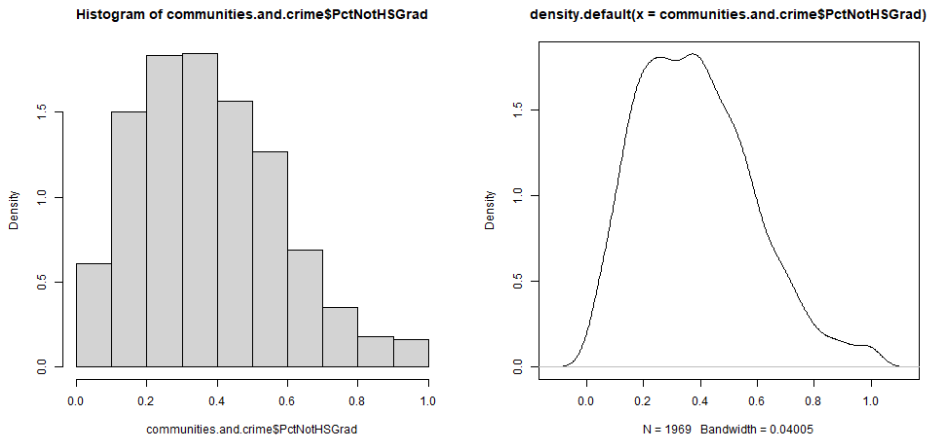
Chapter 3

The Gamma Family

3.1 Communities and Crime: PctNotHsGrad

We observed that the variable **PctNotHsGrad** of the Communities and Crime dataset seemed well-approximated by the gamma family of continuous distributions. According to the UCI Machine Learning Repository, **PctNotHsGrad** is described as the percentage of people 25 and over that are not high school graduates.

3.2 Histogram and Density



3.3 MLE and MM

To find the MLE of the gamma family, we first defined our log likelihood function:

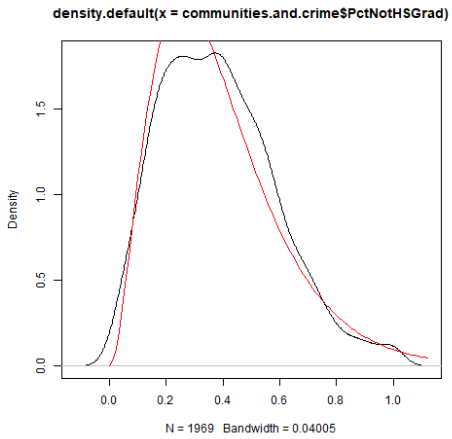
$$L(k,\theta) = (k-1)\sum(\log x) - \sum(\frac{x}{\theta}) - nk\log(\theta) - n\log(\Gamma(k)) \qquad (3.1)$$

We then had to scale our data to ensure the log likelihood function remained finite:

```
x[which(x == 0)] <- 0.0001
```

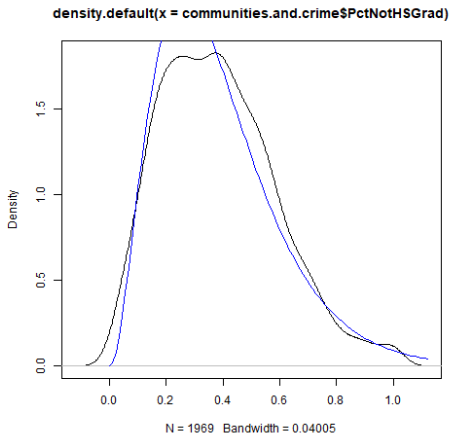
To find the MLE, we use R's built in mle() function with the negative log likelihood function.

```
z <- mle(minuslogl = ll, start = c(list(k = 1),
   list(theta = 1)))
```



To find the MM of the gamma family, we used the following function to predict the value of k and θ:

```
mm <- function(x) {
  mu <- mean(x)
  theta <- mean(x * log(x)) - mu * mean(log(x))
  k <- mu / theta
  return(c(k, theta))
}
```



3.4 Conclusion

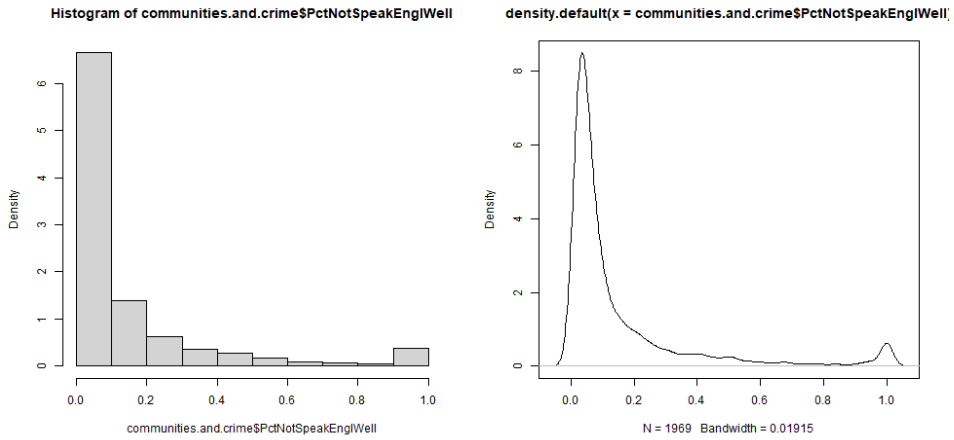
Chapter 4

The Beta Family

4.1 Communities and Crime: PctNotSpeakEnglWell

For the beta family of continuous distributions, we observed that the variable **PctNotSpeakEnglWell** was a suitable approximation. According to the UCI Machine Learning Repository, **PctNotSpeakEnglWell** is described as the percentage of people who do not speak English well.

4.2 Histogram and Density



4.3 MLE and MM

To find the MLE of the beta family, we first had to scale our data so it was within the support of $(0, 1)$:

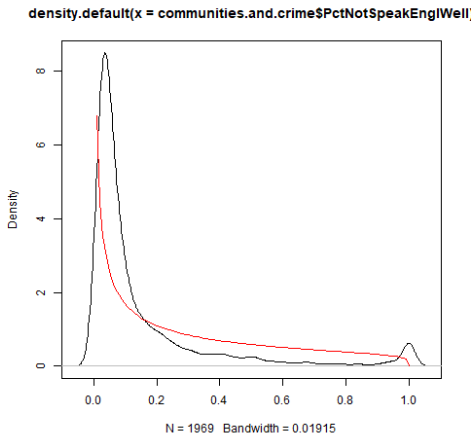
```
x[which(x == 0)] <- 0.0001
x[which(x == 1)] <- 0.9999
```

We then found the log likelihood function:

$$L(\alpha, \beta) = n \log\left(\Gamma(\alpha + \beta)\right) - n \log\left(\Gamma(\alpha)\right) - n \log\left(\Gamma(\beta)\right) + (\alpha - 1) \sum(\log(x)) + (\beta - 1) \sum(\log\left(1 - x\right)) \quad (4.1)$$

To find the MLE, we use R's built in mle() function with the negative log likelihood function.

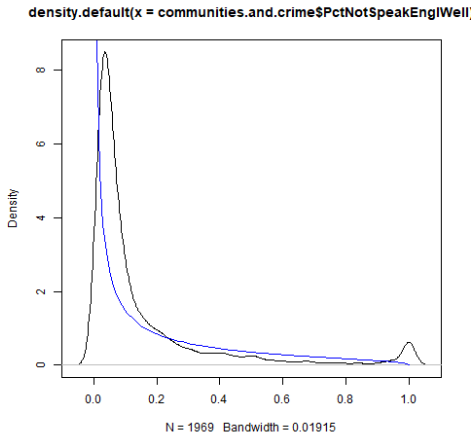
```
z <- mle(minuslogl = ll, start = c(list(alpha = 1),
  list(beta = 1)))
```



To find the MM of the beta family, we used the following function to estimate the α and β values:

```
mm <- function(x) {
  mu <- mean(x)
  var <- var(x)
  alpha <- mu * (mu * (1 - mu) / var - 1)
```

```
    beta <- (1 - mu) * (mu * (1 - mu) / var - 1)
    return(c(alpha, beta))
}
```



4.4 Conclusion

Chapter 5

Contributions