1. Consider the set of positive integers which, when written in binary, have exactly 2019 digits and more 0s than 1s. Let n be the number of such integers and let s be their sum. Prove that, when written in binary, $n+s$ has more 0s than 1s.

2. The point P lies inside triangle ABC so that $\angle ABP = \angle PCA$. The point Q is such that $PBQC$ is a parallelogram. Prove that $\angle QAB = \angle CAP$.

3. A line through a point A intersects a circle in two points, B and C, in such a way that B lies between A and C. From the point A draw the two tangents to the circle, meeting the circle at points S and T. Let P be the intersection of the lines ST and AC. Show that $AP/PC = 2AB/BC$.

4. Given a positive integer n, let $b(n)$ denote the number of positive integers whose binary representations occur as blocks of consecutive integers in the binary expansion of n. For example $b(13) = 6$ because $13 = 1101_2$, which contains as consecutive blocks the binary representations of $13 = 1101_2$, $6 = 110_2$, $5 = 101_2$, $3 = 11_2$, $2 = 10_2$, and $1 = 1_2$.

   Show that if $n \leq 2500$, then $b(n) \leq 39$, and determine the values of n for which equality holds.

5. The sequence $\{a_n\}$ is defined by $a_0 = 2$, $a_1 = 1$, and $a_{n+1} = a_n + a_{n-1}$ for $n \geq 1$. Show that if p is a prime factor of $a_{2k} - 2$, then p is a factor of $a_{2k+1} - 1$

6. Let $n > 1$ be an odd natural number. Let $S = \{1, 2, \cdots, n^2\}$ and define a permutation $f : S \to S$ as follows. Taken n^2 cards numbered from 1 to n^2 and lay them in a square array, with the i-th row containing cards $(i-1)n+1, (i-1)n+2, \ldots, in$ in order from left to right. Pick up the cards along rising diagonals, starting with the upper left-hand corner. If card j is the k-th card picked up, put $f(j) = k$. For example, if $n = 3$, then $f(1) = 1$, $f(4) = 2$, $f(2) = 3$, $f(7) = 4$, $f(5) = 5$, $f(3) = 6$, $f(8) = 7$, $f(6) = 8$, $f(9) = 9$.
   Prove that the permutation f has fixed point not in the set $\{1, n^2, (n^2+1)/2\}$ iff n has at least two different prime factors.

7. For a positive integer k, let $n = (2^k)!$ and let $\sigma(n)$ denote the sum of all positive divisors of n. Prove that $\sigma(n)$ has at least one prime divisor larger than 2^k.

8. All vertices of a polygon P lie at points with integer coordinates in the plane (that is to say, both their co-ordinates are integers), and all sides of P have integer lengths. Prove that the perimeter of P must be even.

9. A "labyrinth" is an 8×8 chessboard with walls between some neigh- boring squares. If a rook can traverse the entire board without jumping over the walls, the labyrinth is "good"; otherwise it is "bad". Are there more good labyrinths or bad labyrinths?

10. Given a triangle ABC with circumcircle Γ, let circle Γ' cented on the line BC intersect Γ at D and D'. Denote by Q and Q' the projections of D and D' on the line AB, and by R and R' their projections on AC; assume that none of these projections coincide with a vertex of the triangle.

    Show that if Γ' is orthogonal to Γ, then $\frac{BQ}{BQ'} = \frac{CR}{CR'}$

11. Each of n gangsters belongs to several gangs. There are no two gangs with the same roster. Gangsters from the same gang are all allies. If a gangster does not belong to a gang, he has at least one enemy in this gang. Find the formula for $B(n)$, the greatest possible number of gangs.