1. What is the smallest natural numbers n, such that there exist integers a_1, a_2, ..., a_n such that the quadratic equation
$$x^2 - 2(a_1 + a_2 + \cdots + a_n)^2 x + (a_1^4 + a_2^4 + \cdots + a_n^4 + 1)$$
has at least one integer root?

2. Let $\mathscr{P}$ be the set of points on the plane. Let $f : \mathscr{P} \to \mathbb{R}$ be a function that, if M is the intersection of the median of triangle ABC, then we have $f(M) = f(A) + f(B) + f(C)$. Prove that $f(X) = 0$ for all $X \in \mathscr{P}$.

3. Let a, b, and c be real positive number such that $ab, bc, ca > 1$. Prove that
$$\frac{\sqrt{ab-1}}{b+c} + \frac{\sqrt{bc-1}}{c+a} + \frac{\sqrt{ca-1}}{a+b} \le \frac{a+b+c}{4}$$

4. Let G be a simple graph with 100 edges on 20 vertices. We can choose pairs of disjoint edges in 4050 ways. Prove that G is regular.

5. Peter plays a solitaire game with a deck of cards, some of which arc face-up while the others arc face-down. Peter loses if all the cards arc face-down. As long as at least one card is face up, Peter must choose a stack of consecutive cards from the deck, so that the top and the bottom cards of the stack arc face-up. They may be the same card. Then Peter turns the whole stack over and puts it back into the deck in exactly the same place as before. Prove that Peter always loses.

6. The numbers 1, 2,..., n are divided into two groups so that the sum of all numbers in one group is equal to that in the other. Is it true that for every such $n > 4$ one can remove two numbers from each group so that the sums of all numbers in each group are still the same?

7. Triangle ABC is given in a plane. Draw the bisectors of all three of its angles. Then draw the line that connects the points where the bisectors of angles $\angle ABC$ and $\angle ACB$ meet the sides AC and AB , respectively. Through the point of intersection of the bisector of angle BAG and the previously drawn line, draw another line, parallel to the side BC. Let this line intersect the sides AB and AC in points M and N. Prove that $2MN = BM + CN$.

8. If n is a natural number, prove that the number $(n+1)(n+2)\cdots(n+10)$ is not a perfect square.

9. For a map f of the plane into itself, it is known that it sends any two points A, B whose distance apart is 1 into 2 points at the same distance apart. Prove that for any natural n, $|A - B| = n$ implies $|f(A) - f(B)| = n$.

10. In the one-round soccer tournament, $n > 4$ teams were played. It was 3 points for a win, 1 for a draw, 0. for a loss. It turned out that all the teams scored equally.
    a) Prove that there are four teams that have equal wins, equal draws and equal losses.
    b) At what smallest n can there not be five such teams?