1. Let $ABCD$ be a cyclic quadrilateral, O is the intersection of AC and BD. Let M, N, P, and Q be the midpoints of AB, BC, CD, and DA, respectively, and X, Y, Z, T be the projections of O on AB, BC, CD, and DA, respectively. Let U be the intersection of MP and YT, while V be the intersection of NQ and XZ. Prove that

$$UO \cdot BC \cdot DA = VO \cdot AB \cdot CD$$

2. Two quadrilaterals $ABCD$ and $A_1B_1C_1D_1$ are mutually symmetric with respect to the point P. It is known that A_1BCD, AB_1CD and ABC_1D are cyclic quadrilaterals. Prove that the quadrilateral $ABCD_1$ is also cyclic

3. The altitudes AH_1, BH_2, CH_3 of an acute-angled triangle ABC meet at point H. Points P and Q are the reflections of H_2 and H_3 with respect to H. The circumcircle of triangle PH_1Q meets for the second time BH_2 and CH_3 at points R and S. Prove that RS is a medial line of triangle ABC.

4. Given an integer number $n \geq 2$, a positive real number A, and $n+1$ distinct points in the plane, X_0, X_1, ..., X_n, show that the number of triangles $X_0X_iX_j$ or area A does not exceed $4n\sqrt{n}$.

5. Find all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ such that for all $x, y \in \mathbb{R}$ with $x > y$, we have

$$f\left(\frac{x}{x-y}\right) + f(xf(y)) = f(xf(x))$$

6. Given an integer number $n \geq 2$, show that there exists a function $f : \mathbb{R} \to \mathbb{R}$ such that $f(x) + f(2x) + \cdots + f(nx) = 0$, for all $x \in \mathbb{R}$, and $f(x) = 0$ if and only if $x = 0$.

7. Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(x)f(yf(x)-1) = x^2 f(y) - f(x)$$

for all $x, y \in \mathbb{R}$.

8. Determine the least real number c, such that for any integer $n \geq 1$ and any positive real numbers a_1, a_2, ..., a_n, the following holds

$$\sum_{k=1}^{n} \frac{k}{\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_k}} < c \sum_{k=1}^{n} a_k.$$

9. Let r be a positive integer and let N_r be the smallest positive integer such that the numbers

$$\frac{N_r}{n+r}\binom{2n}{n}, \qquad n = 0, 1, 2, \ldots,$$

are all integer. Show that

$$N_r = \frac{r}{2}\binom{2r}{r}.$$