1. Find all function $f : \mathbb{Q}^+ \to \mathbb{Q}^+$ such that:
   i) $f(x)$ is an integer if and only if x is an integer.
   ii) $f(f(xf(y))+x) = yf(x)+x$ for all $x, y \in \mathbb{Q}^+$.

2. Let $f(x)$ be a monic polynomial of degree 1991 with integer coefficients. Define $g(x) = f^2(x) - 9$. Show that the number of distinct integer solutions of $g(x) = 0$ cannot exceed 1991.

3. For all positive real numbers a, b, c satisfying $a+b+c = 3$, prove that

$$\sum_{cyc} \frac{a^2+3b^2}{ab^2(4-ab)} \geq 4$$

4. Let a_0, a_1, a_2, ... be a sequence of nonnegative integers satisfying the conditions:
   (1) $a_{n+1} = 3a_n - 3a_{n-1} + a_{n-2}$ for $n > 1$
   (2) $2a_1 = a_0 + a_2 - 2$
   (3) For every positive integer m, in the sequence a_0, a_1, a_2, ... there exists m terms a_k, a_{k+1}, ..., a_{k+m-1} which are perfect squares.

   Prove that a_i is a perfect square for all $i \geq 0$.

5. Let n be a positive integer and let $p(x)$ be a polynomial with real coefficients on the interval $[0,n]$ such that $p(0) = p(n)$. Prove that there are n distinct ordered pairs (a_i, b_i) with $i = 1,2,...,n$ such that $0 \leq a_i < b_i \leq n$, $b_i - a_i$ is an integer and $p(a_i) = p(b_i)$.

6. Find all the functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(x - f(y)) = f(f(y)) + xf(y) + f(x) - 1$$

   for all $x, y \in \mathbb{R}$.

7. Let α be a rational number with $0 < \alpha < 1$ and $\cos(3\pi\alpha) + 2\cos(2\pi\alpha) = 0$. Prove that $\alpha = \frac{2}{3}$.

8. A polynomial $P(x) \in \mathbb{Q}[x]$ is called *integer-valued* iff $P(a) \in \mathbb{Z}$ whenever $a \in \mathbb{Z}$. Define the polynomial $P_k(z) = \binom{z}{k}$. Prove that $f(x)$ is integer-valued polynomial of degree n iff

$$f(x) = \sum_{k=0}^{n} a_k P_k(x)$$

   for some integers a_1, a_2, ..., a_n.

9. Leta, b, c be real positive numbers with $abc = 1$ Prove that

$$a^3 + b^3 + c^3 + 9 \geq 4(ab + ac + bc)$$

10. Let P and Q be monic polynomials with integer coefficient. Suppose that $P(a) = Q(b) = 0$. Prove that there exists monic polynomial with integer coefficient T such that $T(a+b) = 0$.