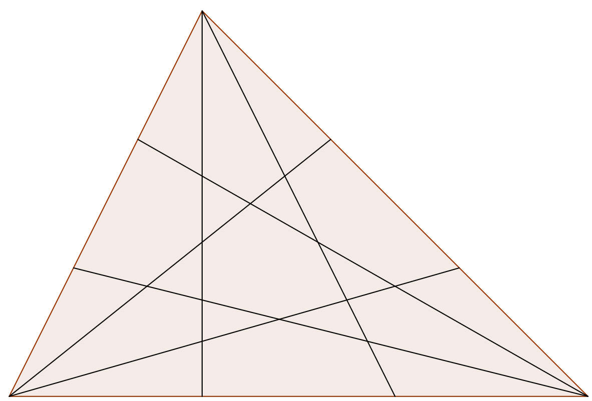
1. Suppose that all vertices of a regular n-gon are lattice points. Prove that $n = 4$.

2. Suppose that $ABCD$ is a square and that P is a point which is on the circle inscribed in the square. Determine whether or not it is possible that PA, PB, PC, PD and AB are all integers.

3. Let A, B, and C be lattice points such that all angles of a triangles ABC are rational multiples of π. Prove that ABC is right triangle and isosceles.

4. Let p be an odd prime number. Divide each side of a triangle into p equal parts by using $p-1$ points on each side, and call each of the $3(p-1)$ points as division points. For each side, join all the division points on that side to the vertex of triangle which is on the opposite of those division points. Find the maximum number of disjoint regions in the interior of the triangle. (The diagram below shows a possible configuration for $p = 3$)



5. Let a, b, c be three positive integers such that

$$\operatorname{lcm}(a,b) \cdot \operatorname{lcm}(b,c) \cdot \operatorname{lcm}(c,a) = abc \cdot \gcd(a,b,c).$$

Given that no quotient of any two of a, b, c is an integer. Find the minimum possible value of $a+b+c$.

6. Let b be an integer such that $b \geq 2$, and let a be real numbers such that $\frac{1}{a} + \frac{1}{b} > 1$. Prove that the sequence

$$\lfloor a \rfloor, \lfloor 2a \rfloor, \lfloor 3a \rfloor, \ldots$$

contains infinitely many integral powers of b.

7. Let $b_1 < b_2 < b_3 < \ldots$ be distinct positive integers expressible as sum of two squares of integers. Prove that for any given positive integers d, the equality $b_{n+1} - b_n = d$ holds for infinitely many n

8. Let $a, b \in \mathbb{N}$ with $gcd(a,b) = 1$. Let $p_1 < p_2 < p_3 < \ldots$ be the set of primes in the progression $\{ak+b\}_{k=0}^{\infty}$. Consider

$$\alpha = 0.p_1 p_2 p_3 \cdots ,$$

where the digits of the prime numbers p_1, p_2, ... placed side by side form the digits of α. Prove that α is irrational.