1. Let α, β, γ, and τ be positive numbers such that for all x,

$$\sin \alpha x + \sin \beta x = \sin \gamma x + \sin \tau x$$

Prove that $\alpha = \gamma$ or $\alpha = \tau$.

2. Find all triangle such that a, b, c are all integers, and b is a prime, while $\angle C = 2\angle A$.

3. Let $A_1, A_2, \ldots, A_n$ $(n \geq 4)$ be points in the plane such that no three of them are collinear. Some pairs of distinct points among A_1, A_2, ..., A_n are connected by line segments in such a way that each point is connected to at least three others. Prove that there exists $k > 1$ and distinct points X_1, X_2, ..., X_{2k}] $\in \{A_1, \ldots, A_n\}$ such that for each $1 \leq i \leq 2k-1$, X_i is connected to X_{i+1} and X_{2k} is connected to X_1.

4. The real number x between 0 and 1 has decimal representation

$$0 \cdot a_1 a_2 a_3 a_4 \ldots$$

with the following property: the number of distinct blocks of the form

$$a_k a_{k+1} a_{k+2} \cdots a_{k+2019},$$

as k range through all positive integers, is less than or equal to 2020. Prove that x is rational.

5. Let $X = \{A_1, A_2, \ldots, A_n\}$ be a set of distinct 3-elements subset of $\{1, 2, \ldots, 36\}$ such that
(i) A_i and A_j have non-empty intersection for every i and j.
(ii) The intersection of all elements of X is the empty set.
Show that $n \leq 100$. How many such sets X are there when $n = 100$?

6. An exam consisting of six questions is sat by 2019 students. Each question is marked either right or wrong. Any three children have right answers to at least five of the six questions between them. Let N be the total number of right answers achieved by all the children (i.e the total number of question solved by student 1 + the total number of question solved by student 2 + $\cdots$ + the total number of question solved by student 2019). Find the least possible value of N.

7. Let ABC be a triangle with $AC > AB$. The point X lies on the side BA extended through A, and the point Y lies on the side CA in such a way that $BX = CA$ and $CY = BA$. The line XY meets the perpendicular bisector of side BC at P. Show that

$$\angle BPC + \angle BAC = 180^\circ$$

8. Let n be a positive integer. Prove that the set $\{1, 2, \ldots, n^2\}$ of the first n perfect squares can be partitioned into four subsets each having the same sum of elements if and only if $n = 8k$ or $n = 8k - 1$ for some integer $k \geq 2$.