Multiple Choice Questions

1. The roots of the quadratic equation $x^2 - 3x + 1 = 0$ are α and β. Find a quadratic equation whose roots are $\alpha^2 + \alpha$ and $\beta^2 + \beta$.

   (A) $x^2 + 10x - 5 = 0$ (B) $x^2 - 10x + 5 = 0$ (C) $x^2 - 5x - 10 = 0$

   (D) $x^2 + 5x - 10 = 0$ (E) $x^2 - 5x - 5 = 0$

2. Simplify

$$\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}}.$$

   (A) $\sqrt{5}$ (B) $\sqrt{6}$ (C) $\sqrt{7}$ (D) $\sqrt{8}$ (E) $\sqrt{10}$

3. Which of the following is true?

   (A) $10^{30} > 40^{15} > 30^{20}$ (B) $30^{20} > 10^{30} > 40^{15}$ (C) $10^{30} > 30^{20} > 40^{15}$

   (D) $30^{20} > 40^{15} > 10^{30}$ (E) $40^{15} > 30^{20} > 10^{30}$

4. Which of the following is the largest?

   (A) $\log_5 7 - \log_5 6$ (B) $\log_6 8 - \log_6 7$ (C) $\log_7 9 - \log_7 8$

   (D) $\log_8 10 - \log_8 9$ (E) $\log_9 11 - \log_9 10$

5. In the expansion of $\left(x^2 + \frac{1}{x}\right)^9$, find the coefficient of x^{15}.

   (A) 9 (B) 36 (C) 84 (D) 126 (E) None of the above

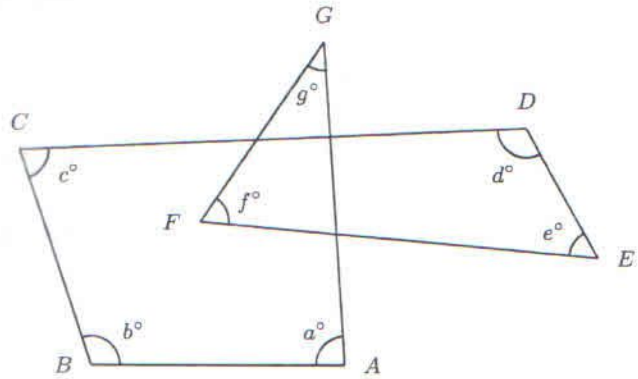
Short Questions

6. The line $y = 5x - 10$ meets the curve $x^2 - 4x + y^2 - 32 = 0$ at two points P and Q. Find the length of the line segment PQ in meters, assuming that both the x and y axis are measured in meters.

7. Suppose n is a positive integer, and x is measured in radians. If the equation $2\pi \sin nx = 2\pi - x$, where $0 \leq x < 2\pi$, has exactly 2018 different solutions, find n.

8. The polynomial $p(x) = x^3 + Ax^2 + Bx - 3$ has a factor of $x^2 + 7x + 1$. Find the remainder when the polynomial $p(x)$ is divided by $x - 4$.

9. In the figure below, the angles indicated at the vertex A, B, C, D, E, F, G are given by a°, b°, c°, d°, e°, f°, g° respectively. Find $a + b + c + d + e + f + g$.



10. Find x, where $0^\circ \leq x^\circ \leq 90^\circ$, such that $\cos x^\circ = \cos 49^\circ + \cos 71^\circ$.

11. Let x and y be real numbers. Find the maximum value of $2x^2 - 3xy - 2y^2$ subject to the condition that

$$25x^2 - 20xy + 40y^2 = 36.$$

12. Suppose $\sin 2x = \frac{7}{9}$. Find $108(\sin^6 x + \cos^6 x)$.

13. Find the sum of all the positive integers x satisfying

$$(4 \log_2(\log_{16} x))\,(3 \log_{16}(\log_2 x) - 1) = 1.$$

14. Consider the function $f(x) = ax^2 - c$, where a and c are some constants. Suppose that $-3 \leq f(1) \leq -2$ and $1 \leq f(2) \leq 6$. Find the largest possible value of $f(4)$.

15. Suppose a is the smallest number satisfying the inequality

$$\left| \frac{|x+9|}{10} - \frac{|x-9|}{10} \right| \leq x - 1.$$

If $a = \frac{m}{n}$, where m and n are positive integers having no common factors larger than 1, find the value of $m + n$.

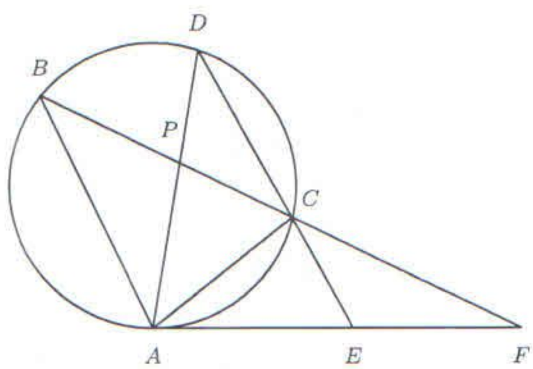
16. Let $P(x)$ be a polynomial of degree 4 such that $P(n) = \dfrac{120}{n}$ for $n = 1, 2, 3, 4, 5$. Determine the value of $P(6)$.

17. Let
$$L = \sum_{k=7}^{16} (1 + \tan(15k^\circ + 15^\circ)\tan 15k^\circ)\,.$$
Find the largest integer which is smaller than or equal to L.

18. Find the smallest three-digit positive integer whose square ends in the digits 129.

19. In the figure below, the points A, B, C and D lie on the circle such that the lines AD and BC intersect at the point P. The line AF is tangent to the circle. The point E lies on the line AF such that the lines DE and PF intersect at C.

If the line AP bisects the angle $\angle BAC$, and $\angle CEA = (22+y)^\circ$ where $y^\circ = \angle BAP$, find the angle $\angle APC$ (in $^\circ$).



20. Eleven identical boxes are arranged in a row. In how many ways can eight identical balls be put into the boxes if each box can hold at most one ball and no three empty boxes can appear consecutively next to each other?

21. Find the minimum positive integer N such that among any N distinct positive integers, there always exist two distinct positive integers such that either their sum or their difference is a multiple of 2018.

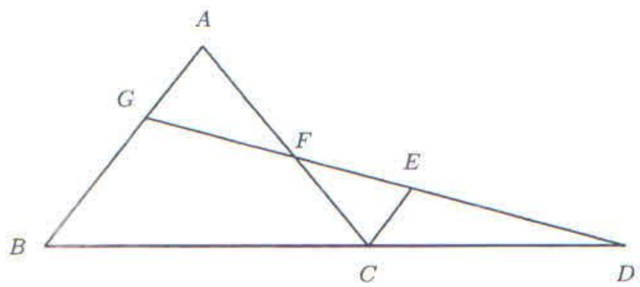
22. Suppose $f(x)$ is defined for all positive numbers x, and $2f(x - x^{-1}) + f(x^{-1} - x) = 3(x + x^{-1})^2$. Find $f(99)$.

23. Suppose x, y and z are positive real numbers satisfying the following system of equations:

$$\frac{\sqrt{xyz}}{x+y} = 3,$$
$$\frac{\sqrt{xyz}}{y+z} = \frac{5}{2},$$
$$\frac{\sqrt{xyz}}{z+x} = \frac{15}{7}.$$

If $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{N}{900}$, find N.

24. In the figure below, the lines AB and CE are parallel to each other. The point F is the midpoint of the line AC, and $AB = 3AG$. If the area of the triangle $\triangle ABC$ is 666 cm^2, find the area of the triangle $\triangle CDE$ (in cm^2).



25. For any real number x, let $\lfloor x \rfloor$ denote the largest integer smaller than or equal to x. For example, $\lfloor 3 \rfloor = 3$, $\lfloor 2.8 \rfloor = 2$, $\lfloor -2.3 \rfloor = -3$. Suppose that R is a real number such that

$$\left\lfloor R - \frac{1}{200} \right\rfloor + \left\lfloor R - \frac{2}{200} \right\rfloor + \left\lfloor R - \frac{3}{200} \right\rfloor + \cdots \left\lfloor R - \frac{99}{200} \right\rfloor = 2018.$$

Find $\lfloor 20R \rfloor$.