Practice Paper for SMO (Senior)

Multiple Choice Questions

1. (05/SAMO/S2/16) If $x+y+z=6$, $xy+xz+yz=11$ and $xyz=6$, then $\dfrac{x}{yz}+\dfrac{y}{xz}+\dfrac{z}{xy}$ equals
(A) $\dfrac{7}{3}$ (B) $\dfrac{13}{6}$ (C) $\dfrac{5}{3}$ (D) $\dfrac{6}{11}$ (E) $\dfrac{8}{3}$

2. (99a/USC/22) Let a, b and c be the three roots of
$$x^3-64x-14=0.$$
Find the value of $a^3+b^3+c^3$.
(A) -36 (B) 12 (C) 36 (D) 42 (E) 64

3. (00/Fermat/24) For the system of equations
$$x^2+x^2y^2+x^2y^4=525,$$
$$x+xy+xy^2=35,$$
the sum of the real y values that satisfy the equations is
(A) 20 (B) 2 (C) 5 (D) $\dfrac{55}{2}$ (E) $\dfrac{5}{2}$

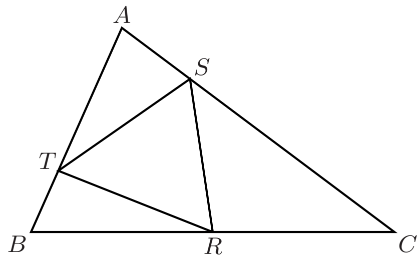
4. (10/AMC10B/22) Seven distinct pieces of candy are to be distributed among three bags. The red bag and the blue bag must each receive at least one piece of candy; the white bag may remain empty. How many arrangements are possible?
(A) 1930 (B) 1931 (C) 1932 (D) 1933 (E) 1934

5. (00/SAMO/S2/20) Consider the equation
$$2u+v+w+x+y+z=3.$$
How many solutions (u,v,w,x,y,z) of non-negative integers does this equation have?
(A) 27 (B) 25 (C) 30 (D) 40 (E) 35

6. (05a/USC/24) Suppose the roots of the quadratic equation $x^2+ax+b=0$ are $\sin 15^\circ$ and $\cos 15^\circ$. What is the value of a^4-b^2?
(A) -1 (B) 1 (C) $\dfrac{35}{16}$ (D) $1+\sqrt{2}$ (E) $3\sqrt{2}-1$

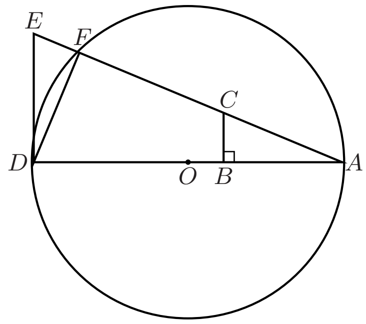
7. (05/AMC12B/21) A positive integer n has 60 divisors and $7n$ has 80 divisors. What is the greatest integer k such that 7^k divides n?
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

8. (97/Caley/25) In triangle ABC, $BR = RC$, $CS = 3SA$, and $AT : TB = p : q$. If the area of $\triangle RST$ is twice the area of $\triangle TBR$, then p/q is equal to



(A) $\frac{2}{1}$ (B) $\frac{8}{3}$ (C) $\frac{5}{2}$ (D) $\frac{7}{4}$ (E) $\frac{7}{3}$

9. (00/Caley/24) In the diagram shown, $\angle ABC = 90°$, $CB \parallel ED$, $AB = DF$, $AD = 24$, $AE = 25$ and O is the centre of the circle. Determine the perimeter of $CBDF$.



(A) 39 (B) 40 (C) 42 (D) 43 (E) 44

10. (97/USC/27) Given that $x > y > 0$ and $xy = 2$, find the smallest possible value of $\dfrac{x^2 + y^2}{x - y}$?
(A) 2 (B) $\frac{\sqrt{6}}{2}$ (C) $\frac{7}{2}$ (D) 4 (E) 5

Short Questions

11. Let $\mathrm{f}(n) = 3n^2 - 3n + 1$. Find the last four digits of

$$\mathrm{f}(1) + \mathrm{f}(2) + \cdots + \mathrm{f}(2012).$$

12. (00/AIME/II/1) The number

$$\frac{2}{\log_4 2000^6} + \frac{3}{\log_5 2000^6}$$

can be written as $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $m + n$.

13. (07/HKST/13) Let x_1, x_2, x_3, x_4, x_5 be non-negative real numbers whose sum is 300. Let M be the maximum of the four numbers

$$x_1 + x_2, \quad x_2 + x_3, \quad x_3 + x_4, \quad \text{and} \quad x_4 + x_5.$$

Find the least possible value of M.

14. (11/HKPSC/18) How many ways are there to arrange 10 identical red balls, 5 identical green balls and 5 identical blue balls in a row so that no two adjacent balls are of the same colour?

15. (07/HKST/9) The 3-digit number $\overline{abc}$ consists of three non-zero digits. The sum of the other five 3-digit numbers formed by rearranging a, b and c is 2017. Find $\overline{abc}$.

16. Let the polynomials $\mathrm{P}_0(x), \mathrm{P}_1(x), \mathrm{P}_2(x), \ldots$ be defined by

$$\begin{aligned} \mathrm{P}_0(x) &= x^3 - 20x^2 + 12x - 2012, \\ \mathrm{P}_n(x) &= \mathrm{P}_{n-1}(x - n) \qquad \text{for } n = 1, 2, 3, \ldots. \end{aligned}$$

What is the coefficient of x in $\mathrm{P}_{12}(x)$?

17. (00/HKPSC/9) Point B is in the exterior of the regular n-sided polygon $A_1A_2 \ldots A_n$ and A_1A_2B is an equilateral triangle. Find the largest value of n such that A_n, A_1 and B are consecutive vertices of a regular polygon.

18. (08/COMC/8) Determine the sum of all integer values of the parameter r for which the equation

$$x^3 - rx + r + 11 = 0$$

has at least one positive integer solution for x.

19. Find the value of

$$\frac{\sin 260^\circ}{\sin 30^\circ \sin 150^\circ \sin 340^\circ} - \frac{\cos 210^\circ}{\cos 60^\circ \cos 120^\circ \cos 350^\circ}.$$

20. (01/HKPSC/10) A certain number of unit cubes are stuck together to form a cuboid with each dimension being greater than 2. The six faces of the cuboid, none of which is a square, are painted. If x is the number of unit cubes with no face painted, y is the number of unit cubes with exactly 1 face painted and z is the number of unit cubes with exactly 2 faces painted, then $x - y + z = 2002$. Find the volume of the cuboid.

21. (01/HKPSC/16) Find the number of distinct (a, b, c, d) such that a, b, c, d are integers, $1 \leq a < b < c < d \leq 30$ and $a + d = b + c$.

22. (01/HKPSC/11) In $\triangle ABC$, M and N are two points on BC such that $BM < BN$, $BM = NC = 4$ and $MN = 3$. If $\angle BAM = \angle MAN = \angle NAC$, find the length of AC.

23. (11/HKPSC/10) In $\triangle ABC$, $AB = 9$, $BC = 8$ and $AC = 7$. The bisector of $\angle A$ meets BC at D. The circle passing through A and tangent to BC at D cuts AB and AC at M and N respectively. Find MN.

24. (12/AIME/II/10) Find the number of positive integers n less than 1000 for which there exists a positive real number x such that $n = x\lfloor x \rfloor$.

25. A circle is inscribed in $\triangle ABC$. D and E are points on AB and AC respectively, such that DE is parallel to BC and is tangent to the circle. If the perimeter of $\triangle ABC$ is 3600, find the maximum length of DE.

Answers

1. A 2. D 3. E 4. C 5. D 6. C 7. C 8. E 9. C 10. D
11. 5728 12. 7 13. 100 14. 1764 15. 425 16. 21 384 17. 42
18. 321 19. 8 20. 30 030 21. 1925 22. 8 23. 6 24. 496 25. 450