Art of Problem Solving
2017 Japan MO Finals

Japan MO Finals 2017

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1 Let a, b, c be positive integers. Prove that $lcm(a, b) \neq lcm(a + c, b + c)$.

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2 Let N be a positive integer. There are positive integers $a_1, a_2, \cdots, a_N$ and all of them are not multiples of 2^{N+1}. For each integer $n \geq N + 1$, set a_n as below:

If the remainder of a_k divided by 2^n is the smallest of the remainder of $a_1, \cdots, a_{n-1}$ divided by 2^n, set $a_n = 2a_k$. If there are several integers k which satisfy the above condition, put the biggest one.

Prove the existence of a positive integer M which satisfies $a_n = a_M$ for $n \geq M$.

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3 Let ABC be an acute-angled triangle with the circumcenter O. Let D, E and F be the feet of the altitudes from A, B and C, respectively, and let M be the midpoint of BC. AD and EF meet at X, AO and BC meet at Y, and let Z be the midpoint of XY. Prove that A, Z, M are collinear.

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4 Let $n \geq 3$ be an integer. There are n people, and a meeting which at least 3 people attend is held everyday. Each attendant shake hands with the rest attendants at every meeting. After the nth meeting, every pair of the n people shook hands exactly once. Prove that every meeting was attended by the same number of attendants.

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5 Let $x_1, x_2, \cdots, x_{1000}$ be integers, and $\sum_{i=1}^{1000} x_i^k$ are all multiples of 2017 for any positive integers $k \leq 672$. Prove that $x_1, x_2, \cdots, x_{1000}$ are all multiples of 2017. Note: 2017 is a prime number.
