1. Let $n \geq 3$ be an integer. Each row in an $(n-2) \times n$ array consists of the numbers 1, 2, ..., n in some order, and the numbers in each column are all different. Prove that this array can be expanded into $n \times n$ array such that each row and each column consists of the numbers 1, 2, ..., n.

2. Each 1×1 square of an $n \times n$ table contains a different number. The smallest number in each row is marked, and these marked numbers are in different columns. Then the smallest number in each column is marked, and these marked numbers are in different rows. Prove that the two sets of marked numbers are identical.

3. There are $2n$ people, some of them are friend to each others. Suppose that each people know an even numbers of friend. Prove that there exists pair of student with even numbers of common friends.

4. We have 20×19 array, and we want to put a coin on some of 1×1 array. Pair of coins is called synchronised iff they are on the same column or on the same row and no other coins are placed between them.

   Find the maximum number of coins that can be put on the array so that each coins are synchronised to at most two other coins.

5. Several boxes are placed along a circle. Each box may contain any number of chips, including zero. A move consists of taking all the chips from some box and placing them in the subsequent boxes clockwise, one chip in every box, beginning from the next box in the clockwise direction.

   Suppose that in each move after the first one, one must take the chips from the box in which the last chip was placed on the previous move. Prove that after several moves, the initial distribution of the chips among the boxes will reappear.

6. On a large chessboard, $2n$ of its 1×1 squares have been marked such that the rook (which moves only horizontally or vertically) can visit all the marked squares without jumping over any unmarked ones. Prove that the figure consisting of all the marked squares can be cut into n rectangles.

7. A cake is prepared for a dinner party to which only p or q persons will come (p and q are given co-prime integers). Find the minimum number of pieces (not necessarily equal) into which the cake must be cut in advance so that the cake may be equally shared between the persons in either case.

8. A game is played on a 1×1000 board. There are n chips, all of which are initially in a box near the board. Two players move in turn. The first may choose 17 chips or less, from either on or off the board. She then puts them into unoccupied cells on the board so that there is no more than one chip in each of the cells. The second player may take off the board any number of chips occupying consecutive cells and put them back in the box. The first player wins if she can put all n chips on the board so that they occupy consecutive cells. For what maximal value of n can she win?

9. At first all rows of $2^n \times n$ array consists of all combination of $n-$tuple with component 1 or -1. Then some of the numbers are replaced with 0. Prove that one can choose a non-empty set of rows say r_1, r_2, ..., r_k so that

$$r_1 + r_2 + \cdots + r_k = (0, 0, 0, ..., 0)$$

   where the sum is taken component-wise.