1. The integer x is at least 3 and $n = x^6 - 1$. Let p be a prime and k be a positive integer such that p^k is a factor of n. Show that $p^{3k} < 8n$.

2. Let ABC be an acute-angled triangle with $AB > AC$ and $\angle BAC = 60^\circ$. Denote the circumcentre by O and the orthocentre by H and let OH meet AB at P and AC at Q. Prove that $PO = HQ$.

3. Given a and b distinct positive integers, show that the system of equations

$$\begin{aligned} xy + zw &= a \\ xz + yw &= b \end{aligned}$$

has only finitely many solutions in integers x, y, z, w.

4. Let S_n be the number of polygonal paths in the plane that start at $(0,0)$, end at (n,n), contain no points above the line $y = x$, and are composed of steps taken from the set $\{(0,1),(1,0),(1,1)\}$. For example, $S_0 = 1$, $S_1 = 2$, $S_2 = 6$, $S_3 = 22$, $S_4 = 90$. Prove that S_n is divisible by 3 for every positive even integer n.

5. Let $x_1, x_2, \ldots, x_n \in \mathbb{R}^+$ and $m = \min_i x_i$ and $M = \max_i x_i$. Let A and G be their arithmetic and geometric mean, respectively. Prove that

$$A - G \geq \frac{1}{n}(\sqrt{M} - \sqrt{m})^2$$

6. Misalkan m dan n bilangan asli. Jika

$$k = \frac{(m+n)^2}{4m(m-n)^2 + 4} \in \mathbb{Z},$$

buktikan k bilangan kuadrat.