Seleksi Tes IMC Indonesia 2019

Bogor, 14-15 Juni 2019

Hari Pertama (300 menit)

1. For $1 \leq r \leq k$, prove combinatorially that :

$$\binom{n}{k} = \sum_{j=r}^{n+r-k} \binom{j-1}{r-1}\binom{n-j}{k-r}$$

2. Misalkan A dan B matriks berukuran $n \times n$ yang memenuhi $AB = BA$. Buktikan bahwa terdapat y yang merupakan vektor eigen matriks A dan B.
3. Suppose $(\boldsymbol{A}, +, \cdot)$ is a ring in which $x^2 = 0$ only for $x = 0$. Let $B = \{a \in A |\ a^2 = 1\}$. Prove that :
   (a) For all $a \in A$ and $b \in B$ we have $ab - ba = bab - a$.
   (b) $(b, \cdot)$ is a group.
4. Let $f(n)$ be a nonnegative real-valued function defined on all nonnegative integers and satisfying

$$f(n+m) \leq f(n) + f(m)$$

   Prove that $\dfrac{f(n)}{n}$ converges as $n \to \infty$ and $\lim\limits_{n\to\infty} \dfrac{f(n)}{n} = \inf\limits_{n\geq 1}\left\{\dfrac{f(n)}{n}\right\}$
5. Let R be a positive real number such that the function $f: D \to \mathbb{C}$ that satisfies the functional equation $f(z^2) = f(z) - z$ is analytic on $D = \{z \in \mathbb{C} : |z| < R\}$.
   (a) Show that $R \leq 1$.
   (b) For $R = 1$, can you find an open connected set $E \supseteq D$ such that there exists an analytic function $g: E \to \mathbb{C}$ with the condition $g(z) = f(z), \forall z \in D$? (Justify your answer)

Hari Kedua (300 menit)

1. If $z \in \mathbb{C}$ satisfying $|z^2 + 2019| < 2019$, show that
$$\left|z + \sqrt{2019}\right| > 31$$
2. We fix $\bar{a}$ and $\bar{b}$ vectors in $\mathbb{R}^3$ where $\bar{a} \cdot \bar{b} \neq 0$. Define a linear trasformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ by:
$$T(\bar{v}) = (\bar{a} \times \bar{v}) \times \bar{b} \text{ where } \times \text{ is cross} - \text{product}$$
For all $\bar{v}$ in $\mathbb{R}^3$. Determine the eigenvalue of T.
3. Let $f: [0, \infty) \to \mathbb{R}$ be a strictly decreasing continuous function such that $\lim_{x \to \infty} f(x) = 0$. Prove that :
$$\int_0^{\infty} \frac{f(x) - f(x+1)}{f(x)} dx \text{ diverges.}$$
4. Suatu Ring R dikatakan "*cantik*" jika R memenuhi :
   1. R memiliki elemen 1 (identitas perkalian).
   2. R bukan lapangan.
   3. Setiap $r \in R$ bisa dinyatakan secara tunggal sebagai jumlahan suatu unsur yang punya invers dan suatu unsur yang tak punya invers.
      (a) Berikan contoh ring R yang "*cantik*" yang sedikitnya terdiri dari dua elemen.
      (b) Tentukan semua karakteristik bagi ring *cantik*.
5. Let G be a simple graph on a n vertices $\{v_1, v_2, \dots, v_n\}$ and m edges $\{e_1, e_2, \dots, e_m\}$. The vertex-edge incidence matrix $B = (b_{ij})$ of the graph G is an n by m matrix such that $b_{ij} = 1$ if the vertex v_i is an end vertex of the edge e_j, and $b_{ij} = 0$ otherwise. For $i = 1,2, \dots, n$. Let $B(i, :)$ be the i th row of B corresponds to the vertex v_i.
   For any two rows $B(i, :)$ and $B(j, :)$ of B we define the Hamming distance of $B(i, :)$ and $B(j, :)$, denoted by : $Hd(B(i, :), B(j, :))$ to be the number of columns k such that $B(i, k) \neq B(j, k)$. Define :
$$H(G) = \sum_{1 \leq i \leq j \leq n} Hd(B(i, :), B(j, :))$$
   Prove that $H(G) = 2(n-2)m$.

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