1. E	6. E	11. D
2. C	7. E	12. B
3. E	8. A	13. A
4. B	9. D	14. C
5. B	10. A	15. D

16. (Source: Australian Science Olympiads 2017, Section B Question 11)

Question 11

Suggested Time: 20 min

Niamh, the Milo enthusiast, is interested in the physics behind her favourite "energy food drink". To investigate, Niamh prepares to make a huge tank of chocolate milk. Niamh fills the tank of length L and height H with milk, then places a partition a distance L_V along the tank. She adds a large amount of chocolate powder to the milk to the left of the partition as shown in Figure 1 and mixes it thoroughly. The volume of milk to the left of the partition is V_C and the density of this chocolate milk is ρ_C. The remaining volume V_M of plain milk, which has density ρ_M, is to the right of the partition. The chocolate milk is denser than the plain milk.

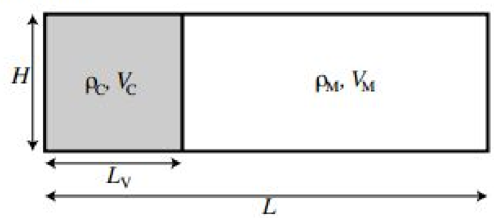
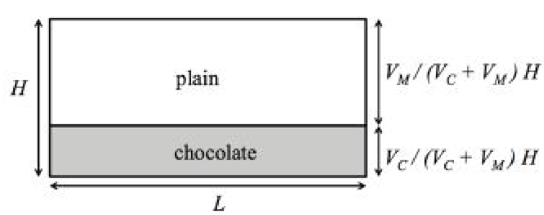


Figure 1: Tank containing chocolate milk and plain milk. View from side.

The partition is removed extremely quickly. **To begin with, it is assumed the chocolate milk does not mix with the plain milk.**

a) Draw a diagram of the contents of the tank a long time after the partition has been removed. Use the space provided on p. 2 of the Answer Booklet.

Solution:



Markers' Comments: For full credit, students needed to include labels and dimensions on their diagram. The dimensions can be obtained as the total volume of plain milk and of chocolate milk both remain constant. A very small layer of mixed milk between the chocolate milk and plain milk was accepted.

Upon removing the partition Niamh observes that the chocolate milk spreads through the tank like a wave. She constructs a basic model of the chocolate milk wave in order to find out how quickly it travels. She divides the chocolate milk behind the partition into two equal sections. When the partition is removed, the bottom section is displaced by the top section and begins to move with velocity U. This process is depicted in Figure 2.

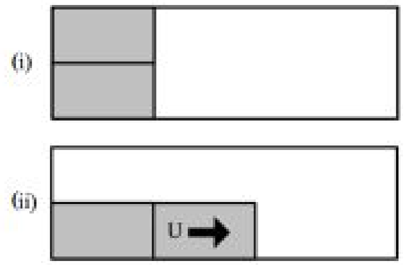
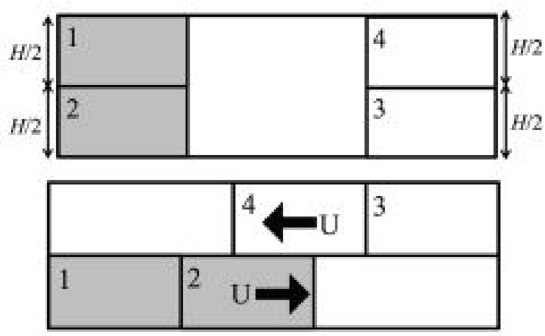


Figure 2: Niamh's model of the chocolate milk wave. (i) Before the partition is removed. (ii) after the partition is removed.

b) Find U.

Section 1 (top of choc milk) moves down and loses gravitational potential energy (GPE).
Section 2 (bottom of choc milk) moves across and gains kinetic energy.
Section 3 – plain milk of equal volume to section 2 = $V_C/2$ – is displaced upwards and gains gravitational potential energy.
Section 4 – plain milk of equal volume to section 2 = $V_C/2$ – is displaced in the opposite direction to section 2 and gains kinetic energy.



Statement of conservation of energy:

$$\mathrm{GPE}_1 = \mathrm{KE}_2 + \mathrm{GPE}_3 + \mathrm{KE}_4$$

Mass of section 1 = mass of section 2, $m_1 = m_2 = \rho_C V_C/2$
Mass of section 3 = mass of section 4, $m_3 = m_4 = \rho_M V_C/2$
The centre of section 1 moves down $H/2$, while the centre of section 3 moves up $H/2$.

$$\mathrm{GPE}_1 = m_1 g \Delta h_1 = \rho_C \frac{V_C}{2} g \left(\frac{H}{2}\right)$$

$$\text{GPE}_3 = m_3 g \Delta h_2 = \rho_C \frac{V_M}{2} g \left(\frac{H}{2}\right)$$

Expression for kinetic energies of section 2 and section 4:

$$\text{KE}_2 = \frac{1}{2} m_2 U^2 = \frac{1}{4} \rho_C V_C U^2$$

$$\text{KE}_4 = \frac{1}{2} m_4 U^2 = \frac{1}{4} \rho_M V_C U^2$$

Putting it all together:

$$\rho_C \frac{V_C}{4} g H = \frac{1}{4} \rho_C V_C U^2 + \rho_M \frac{V_C}{4} g H + \frac{1}{4} \rho_M V_C U^2 \tag{1}$$

$$\rho_C g H - \rho_M g H = (\rho_C + \rho_M) U^2 \tag{2}$$

$$g H \frac{\rho_C - \rho_M}{\rho_C + \rho_M} = U^2 \tag{3}$$

$$\therefore U = \sqrt{g H \frac{\rho_C - \rho_M}{\rho_C + \rho_M}} \tag{4}$$

Markers' Comments:

As this model includes many effects those students who included some gravitational potential energy and some kinetic energy were given most of the available credit. Students were also given credit if they identified qualitatively that the movement of the plain milk would affect the chocolate milk wave.

Some students attempted to apply equations for motion under constant acceleration. However, these equations do not apply in this situation.