NTU UEE 2010

1.
(a) The function $f(x)$ is defined by,
$$f(x) = e^{2x}\sqrt{x^2 + 1}$$
Find $f'(x)$

(b) Evaluate the following integral
$$\int xe^{x+1}\, dx$$

(c) Evaluate the following integral
$$\int_0^1 \frac{dx}{3x^2 + 4x + 1}$$

(d) Evaluate the following integral
$$\int \frac{3x + 1}{1 - 2x - 3x^2}\, dx$$

2.
(a) The function $g(x)$ is an increasing function and $h(x)$ is a decreasing function in the interval $a \leq x \leq b$, where $g(x)$ and $h(x)$ is always positive. Another function $\theta(x)$ satisfies $L_1 \leq \theta(x) \leq L_2$. Find the value of L_1 and L_2 is the function $\theta(x)$ is defined by,

   i. $\theta(x) = g(x) \cdot h(x)$
   ii. $\theta(x) = g(x) - h(x)$
   iii. $\theta(x) = [g(x) - h(x)]^2$

(b) A piecewise function $q(x)$ is defined by,
$$q(x) := \begin{cases} \dfrac{7}{1 + 2x} & ; 0 \leq x \leq 3 \\ 3x^2 - 30x + 64 & ; 3 \leq x \leq 7 \end{cases}$$

Find the minimum and maximum value of the function $q(x)$.

3.
(a) Sketch the area that fulfills,
   I. $4x \geq y$
   II. $x + y \leq 5$
   III. $2x - y \geq 3$

(b) Find the maximum and minimum value of $2x + y$.

(c) Find the maximum and minimum value of xy.

4. Determine whether the following sequence converge or diverge. Hence, or otherwise, find the sum of the sequence.
(a)
$$\sum_{k=1}^{\infty} \frac{4^k}{5^{k+1}} = \cdots$$
(b)
$$\sum_{k=1}^{\infty} \frac{(-4)^{k+2}}{3^{k+1}} = \cdots$$