WARDAYA COLLEGE - TOP UNIVERSITIES CLASS

TEST 2

READ THESE INSTRUCTIONS FIRST

1. Write your name on all the work you hand in
2. Answer **all** the questions
3. No. 1 – 5 is worth 15. No. 6 – 8 is worth 10
4. The total number of marks for this paper is 105

1. The plane π has equation $r \bullet (i + 2j - 3k) = 5$

   The line l passes through the point P with position vector $7i + 4j - 6k$, and is parallel to $7i + 3j - 5k$

   (i) State the perpendicular distance from the origin to the plane π

   (ii) Find the acute angle between the line l and the plane π

   (iii) Find the position vector of the foot of the perpendicular from the point P to the plane π. Hence find the position vector of the reflection of the point P in π

2. The equation of the plane π is given by $x + 2y - 2z = 18$

   The position vectors of the points A and D are given by $-2i + j$ and $2i + 5j - 3k$ respectively

   The foot of the perpendicular from A to the plane π is B

   C is the point on BA produced such that BA : AC = 2 : 1

   Find

   (i) The position vector of B

   (ii) The perpendicular distance from A to the plane π

   (iii) The position vector of C

   (iv) The equation of the plane DBC in the form $r \bullet n = p$

   (v) The exact length of projection of the line BD on π

3. The plane π_1 has equation $r \bullet (i - 5j + 3k) = 1$

   The line l passes through the points A and B with position vectors $-i + 2j + 4k$ and $j + 5k$ respectively, and the point A lies on π_1. Find

   (a) A vector equation of the line l

   (b) The acute angle between π_1 and l

   (c) The exact perpendicular distance from B to π_1

   The plane π_2 contains l and is perpendicular to π_1. Find

   (d) The equation of π_2 in Cartesian form

   (e) A vector equation of the line which lies in both π_1 and π_2

4. Verify that $-2+3i$ is a root of the equation $z^3+5z^2+17z+13=0$ and determine the other roots of the equation
5. Solve the simultaneous equations

$$\begin{cases} iw+z=-1-i \\ 2z-(1+i)w=\dfrac{40}{6-2i} \end{cases}$$

Giving each answer in the form $a+bi$, where a and b are real

6. The cubiz equation P(z) = 0 has real coefficients. If two of the roots are – 3 and $-1-3i$
   (i) State the third root
   (ii) Find the cubic equation in the form $az^3+bz^2+cz+d=0$

Prove by Mathematical Induction this statement

7. $1^4+2^4+3^4+\ldots+n^4=\dfrac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$ for all n natural numbers
8. $7^n-(3n+4)4^{n-1}$ is divisible by 9

END OF PAPER