Bank Soal

Statistics 1

1. The letters in the word WARDAYA are to be arranged in such a way that no two vowels come together. Find the number of such arrangements. You may leave the answer as either a number or factorial terms, but you should give a careful explanation of your method.

2. (Pre-Top Mathematics Test June 2019)
The letters in the word MATHEMATICS are to be arranged in an order in such a way that the letters H, C and S come together and no two vowels come together. Find the number of such arrangements. You may leave the answer as either a number or factorial terms, but you should give a careful explanation of your method.

3. Seven persons are to be seated at three tables holding 2, 2 and 3 persons respectively. In how many ways can the group sitting at the tables be selected, assuming that the order of sitting at the tables does not matter?

4. Each of the digits 1, 2, 2, 3, 4, 4 is written on a separate card. The six cards are then laid out in a row to form a 6-digit number.
   (a) How many distinct 6-digit numbers are there?
   (b) How many of these 7-digit numbers start and end with the same digit?

5. A committee of three people is to be selected from four women and five men. The first rule states that there must be at least one man and one woman on the committee. Suppose one of the men and one of the women marry each other. The second rule states that a married couple may not both serve on the committee. A person is assigned to select the committee members, without knowing any rule. What is the probability that his selection obeys both the rules?

6. A box contains twelve lightbulbs. It is known that four of them are broken and the rest are perfect. Three men are going to pick exactly one lightbulb in turns. What is the probability that the third man gets a perfect lightbulb?

7. (a) The probability that an event A occurs is $\mathbb{P}(A) = 0.4$. It is known that $\mathbb{P}$(A or B or both) $= 0.7$ and $\mathbb{P}(B) = 0.5$. Is B an event independent of A or not?
   (b) C and D are two events such that $\mathbb{P}(C|D) = \frac{1}{3}$ and $\mathbb{P}(D|C) = \frac{1}{4}$. It is given that $\mathbb{P}$(C or D or both occur) $= \frac{1}{30}$. Find $\mathbb{P}$(C and D occur).

8. (Pre-Top Mathematics Test June 2019)
Let A and B be two events. the events A' and B' denote the complement of A and B respectively. Given that $\mathbb{P}(A|B) = 0.4$, $\mathbb{P}(B'|A') = 0.3$ and $\mathbb{P}$(A or B') $= 0.58$. Find the probability $\mathbb{P}$(A and B').
Hence or otherwise, determine whether A and B are independent or not.

9. A weather forecaster classifies all days as wet or dry. She estimates that the probability that 1 July next year is wet is 0.3. If any particular day in July is wet, the probability that the next day is wet is 0.8; otherwise the probability that the next day is wet is 0.2. Find the probability that, next year,

(a) the first two days of July are both wet

(b) July 2nd is wet

(c) at least one of the first three days of July is wet

10. A bag contains five red and four green counters. Three counters are drawn randomly from the bag, without replacement. Tabulate the probability distribution of the number of green counters obtained.

11. In the following distribution, q is a constant. Find the value of $2q - 1$.

x	0	1	2	3	4
$\mathbb{P}(X = x)$	q	0.2	0.3	$3q - 0.25$	$q - 0.15$

12. A cubical dice is biased so that the probability of any particular score between 1 and 6 (inclusive) being obtained is proportional to the square of that score. Find the probability of scoring a 3.

13. A random variable S has a probability distribution given by the following formula.

$$\mathbb{P}(S = s) = \begin{cases} 0.2 \times (0.6)^s & \text{for } s = 1, 2, 3, 4, 5, \\ x & \text{for } s = 6, 7, \\ 0 & \text{otherwise.} \end{cases}$$

Find the value of x, and find the expected frequency of the result S = 2 when 10000 independent observations of S are made.

14. The following table gives the probability that a firm initially rated A or B will default (go bankrupt) within the next five years. (Note: If X denote the number of years after today when the firm defaults, then the following table is the cumulative probability distribution of X for a given rating.)

Rating	1	2	3	4
A	0.008	0.017	0.039	0.105
B	0.293	0.431	0.726	0.899

Based on the above table,

(a) find the probability that a firm initially rated A will default within the next two years.

(b) find the probability that a firm initially rated A will default in the second year.

(c) find the probability that a firm initially rated B will default in the second year or in the third year.

15. The table below gives the cumulative probability distribution for a random variable T. 'Cumulative' means that the probability given is $\mathbb{P}(T \leq t)$, not $\mathbb{P}(T = t)$.

t	0	1	2	3	4	5	6
$\mathbb{P}(T \leq t)$	0.0089	0.0356	0.0927	0.1234	0.4575	0.7891	1.000

One thousand observations of R are made. Calculate the expected frequencies of each outcome, giving each answer to the nearest whole number.

16. Given that C $\sim$ B(10, 0.3). Find

    (a) $\mathbb{P}(C = 2)$

    (b) $\mathbb{P}(C = 5 \text{ or } 6)$

    (c) $\mathbb{P}(C \leq 3)$

    (d) $\mathbb{P}(7 \leq C < 10)$

17. (Pre-Top Mathematics Test June 2019)
    It is known that in a particular year, the probability that there is a road accident in any given month can be taken as 35%.

    (a) Use binomial distribution to compute the probability that there is a road accident in more than one month of the year.

    (b) Find the expectation and standard deviation of the number of months of the year in which there is a road accident.

    (c) State one assumption (in context) needed for a binomial distribution to be a good model in this case.

18. On a particular island, the probability that there is a sandstorm in any given week can be taken to be 0.12. Use a binomial distribution to calculate the probability that there is a sandstorm in more than two weeks of a year, correct to 5 significant figures. State two assumptions needed for a binomial distribution to be a good model. Why may one of the assumptions not be valid?

19. Based on the data recieved from PT Pegi Jalan Jalan travel bureau, specialized in international tourists, 25% of the tourists state a very high level of satisfaction when visiting Indonesia, 50% state a high level of satisfaction, 20% state a medium level, and the others state an unsatisfaction. Assume we meet 6 of the tourists visiting Indonesia, find the probability that

    (a) exactly 3 of them state a medium level of satisfaction.

    (b) at least one of them states an unsatisfaction.

20. An insurance seller sells an insurance policy to five men, all having the same age and in a good health condition. According to the insurance's table formula, the probability that a man in this age will live within the next 25 years is $\frac{3}{4}$. Find the probability that in the next 25 years,

    (a) all the men will remain alive.

    (b) at most 2 men will live.

21. The random variable T has the probability distribution given in the following table.

t	1	2	3	4	5
$\mathbb{P}(T = t)$	0.1	a	0.3	0.05 + b	0.2

If $a = 6b$, find $\mathbb{E}(T)$ and the standard deviation of T.

22. The random variable D has a binomial distribution with mean 7.5 and standard deviation 2.25. Find

    (a) $\mathbb{P}(D = 10)$, correct to 4 significant figures.

    (b) $\mathbb{P}(D \geq 3)$, correct to 4 significant figures.

23. All apples sold in a minimarket are packed in boxes of a dozen. For any apple, the probability that it is not ripe is 0.3, independently of all other apples. A shelf contains 100 of these boxes. Calculate the expected value of the number of boxes on the shelf which contain at least three raw apples.

24. The number of apples, X, supplied to a store in a day has the probability distribution given in the following table.

x	2	3	4	5
$\mathbb{P}(X = x)$	0.1	0.2	0.3	0.4

For any apple, the probability that it is bought is 0.5, independently of all other apples. Let Y denote the number of bought apples in a randomly chosen basket.

    (a) Obtain the probability distribution of Y.

    (b) Find $\mathbb{E}(Y)$ and Var(Y).

25. Given that X $\sim$ N(3,2.25). Find the values of s,t,u,v, correct to 2 decimal places, such that

    (a) $\mathbb{P}(X < s) = 0.47$

    (b) $\mathbb{P}(-t < X < t) = 0.95$

    (c) $\mathbb{P}(X > u) = 0.32$

    (d) $\mathbb{P}(X < -v) = 0.33$

26. In a statistics examination, 10% of the candidates scored more than 70 marks and 25% of the candidates scored less than 45 marks. Assuming that the marks were distributed normally, find the mean mark and the standard deviation.

27. The random variable X is normally distributed with mean and standard deviation both equal to a. Given $\mathbb{P}(X < 2) = 0.3$, find a, correct to 2 decimal places.

28. (Pre-Top Mathematics Test June 2019)
    The weights of rabbits are normally distributed with mean 860 g and standard deviation 44 g.

    (a) Find the probability that a randomly chosen rabbit weighs between 800 g and 900 g.

    (b) Suppose that 50 rabbits are independently chosen at random. Using a suitable approximation, find the probability that at least 31 will weigh between 800 g and 900 g. Explain why your approximation is valid.

29. (Pre-Top Mathematics Test June 2019)
A university classifies grades for each course as A, B, C, D, Pass and Fail. Grades are awarded on the basis of marks which may be taken as continuous and modelled by a normal distribution with mean μ and standard deviation σ where μ and σ are constant. In a particular course, the lowest mark for an A grade is 84.82, the lowest mark for a B grade is 80.42 and the lowest mark required to pass is 50. It is known that 10% of the students taking the course obtained an A grade and 10% of the students obtained a B grade.

   (a) Show that $\mu + 1.282\sigma = 84.82$ and $\mu + 0.842\sigma = 80.42$.

   (b) Solve the equations in part (i) to find the exact values of μ and σ.

   (c) Find the percentage of students who failed the course, correct to 3 significant figures.

30. (Pre-Top Mathematics Test June 2019)
Ace and Joker are playing a game using a fair standard dice. Ace wins the game when he obtains a six. Joker wins the game when he obtains either a five or a six. They roll the dice alternately, until either one of them wins. Let A be the event that Ace wins the game given that Ace rolls the dice first. Let B be the event that Ace wins the game given that Joker rolls the dice first.

   (a) Without computing $\mathbb{P}(A)$ and $\mathbb{P}(B)$, show that $\mathbb{P}(A) = \frac{1}{6} + \frac{5}{6}\mathbb{P}(B)$ and $\mathbb{P}(B) = \frac{2}{3}\mathbb{P}(A)$.

   (b) Suppose that Ace rolls the dice first, find the probability that he wins the game.

   A biased two-sided coin is tossed once before the game to determine who rolls the dice first. Ace rolls the dice first if a head comes up and Joker rolls the dice first if a tail comes up. On any toss, the probability that a head coming up is twice the probability that a tail coming up.

   (c) Find the probability that Ace wins the game.

   (d) Given that Ace wins the game, find the probability that he rolls the dice first.

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