Eighth Edition

GATE

ELECTRONICS & COMMUNICATION

Electromagnetics

Vol 10 of 10

RK Kanodia
Ashish Murolia

NODIA & COMPANY

GATE Electronics & Communication Vol 10, 8e
Electromagnetics
RK Kanodia & Ashish Murolia

MRP 590.00

NODIA & COMPANY
B – 8, Dhanshree Ist, Central Spine, Vidyadhar Nagar, Jaipur – 302039
Ph : +91 – 141 – 2101150,
www.nodia.co.in
email : enquiry@nodia.co.in

Printed by Nodia and Company, Jaipur

To Our Parents

Preface to the Series

For almost a decade, we have been receiving tremendous responses from GATE aspirants for our earlier books: GATE Multiple Choice Questions, GATE Guide, and the GATE Cloud series. Our first book, GATE Multiple Choice Questions (MCQ), was a compilation of objective questions and solutions for all subjects of GATE Electronics & Communication Engineering in one book. The idea behind the book was that Gate aspirants who had just completed or about to finish their last semester to achieve his or her B.E/B.Tech need only to practice answering questions to crack GATE. The solutions in the book were presented in such a manner that a student needs to know fundamental concepts to understand them. We assumed that students have learned enough of the fundamentals by his or her graduation. The book was a great success, but still there were a large ratio of aspirants who needed more preparatory materials beyond just problems and solutions. This large ratio mainly included average students.

Later, we perceived that many aspirants couldn't develop a good problem solving approach in their B.E/B.Tech. Some of them lacked the fundamentals of a subject and had difficulty understanding simple solutions. Now, we have an idea to enhance our content and present two separate books for each subject: one for theory, which contains brief theory, problem solving methods, fundamental concepts, and points-to-remember. The second book is about problems, including a vast collection of problems with descriptive and step-by-step solutions that can be understood by an average student. This was the origin of *GATE Guide* (the theory book) and *GATE Cloud* (the problem bank) series: two books for each subject. *GATE Guide* and *GATE Cloud* were published in three subjects only.

Thereafter we received an immense number of emails from our readers looking for a complete study package for all subjects and a book that combines both *GATE Guide* and *GATE Cloud*. This encouraged us to present GATE Study Package (a set of 10 books: one for each subject) for GATE Electronic and Communication Engineering. Each book in this package is adequate for the purpose of qualifying GATE for an average student. Each book contains brief theory, fundamental concepts, problem solving methodology, summary of formulae, and a solved question bank. The question bank has three exercises for each chapter: 1) Theoretical MCQs, 2) Numerical MCQs, and 3) Numerical Type Questions (based on the new GATE pattern). Solutions are presented in a descriptive and step-by-step manner, which are easy to understand for all aspirants.

We believe that each book of GATE Study Package helps a student learn fundamental concepts and develop problem solving skills for a subject, which are key essentials to crack GATE. Although we have put a vigorous effort in preparing this book, some errors may have crept in. We shall appreciate and greatly acknowledge all constructive comments, criticisms, and suggestions from the users of this book. You may write to us at rajkumar.kanodia@gmail.com and ashish.murolia@gmail.com.

Acknowledgements

We would like to express our sincere thanks to all the co-authors, editors, and reviewers for their efforts in making this project successful. We would also like to thank Team NODIA for providing professional support for this project through all phases of its development. At last, we express our gratitude to God and our Family for providing moral support and motivation.

We wish you good luck !
R. K. Kanodia
Ashish Murolia

SYLLABUS

GATE Electronics & Communications:

Electromagnetics :

Elements of vector calculus: divergence and curl; Gauss' and Stokes' theorems, Maxwell's equations: differential and integral forms. Wave equation, Poynting vector. Plane waves: propagation through various media; reflection and refraction; phase and group velocity; skin depth. Transmission lines: characteristic impedance; impedance transformation; Smith chart; impedance matching; S parameters, pulse excitation. Waveguides: modes in rectangular waveguides; boundary conditions; cut-off frequencies; dispersion relations. Basics of propagation in dielectric waveguide and optical fibers. Basics of Antennas: Dipole antennas; radiation pattern; antenna gain.

IES Electronics & Telecommunication

Electromagnetic Theory

Analysis of electrostatic and magnetostatic fields; Laplace's and Poisson's equations; Boundary value problems and their solutions; Maxwell's equations; application to wave propagation in bounded and unbounded media; Transmission lines : basic theory, standing waves, matching applications, microstrip lines; Basics of wave guides and resonators; Elements of antenna theory.

IES Electrical

EM Theory

Electric and magnetic fields. Gauss's Law and Amperes Law. Fields in dielectrics, conductors and magnetic materials. Maxwell's equations. Time varying fields. Plane-Wave propagating in dielectric and conducting media. Transmission lines.

****

CONTENTS

CHAPTER 4 MAGNETOSTATIC FIELDS

CHAPTER 5 MAGNETIC FIELDS IN MATTER

****

CHAPTER 6

TIME VARYING FIELDS AND MAXWELL EQUATIONS

6.1 INTRODUCTION

Maxwell's equations are very popular and they are known as Electromagnetic Field Equations. The main aim of this chapter is to provide sufficient background and concepts on Maxwell's equations. They include:

- Faraday's law of electromagnetic induction for three different cases: time-varying magnetic field, moving conductor with static magnetic field, and the general case of moving conductor with time-varying magnetic field.
- Lenz's law which gives direction of the induced current in the loop associated with magnetic flux change.
- Concept of self and mutual inductance
- Maxwell's equations for static and time varying fields in free space and conductive media in differential and integral form

6.2 FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

According to Faraday's law of electromagnetic induction, emf induced in a conductor is equal to the rate of change of flux linkage in it. Here, we will denote the induced emf by V_{emf}. Mathematically, the induced emf in a closed loop is given as

$$V_{emf} = \frac{-d\Phi}{dt} = -\frac{d}{dt}\int_S B \cdot dS \qquad ...(6.1)$$

where Φ is the total magnetic flux through the closed loop, B is the magnetic flux density through the loop and S is the surface area of the loop. If the closed path is taken by an N-turn filamentary conductor, the induced emf becomes

$$V_{emf} = -N\frac{d\Phi}{dt}$$

6.2.1 Integral Form of Faraday's Law

We know that the induced emf in the closed loop can be written in terms of electric field as

$$V_{emf} = \oint_L E \cdot dL \qquad ...(6.2)$$

From equations (6.1) and (6.2), we get

$$\oint_L E \cdot dL = -\frac{d}{dt}\int_S B \cdot dS \qquad ...(6.3)$$

This equation is termed as the integral form of Faraday's law.

6.2.2 Differential Form of Faraday's Law

Applying Stoke's theorem to equation (6.3), we obtain

$$\int_S (\nabla \times E) \cdot dS = -\frac{d}{dt}\int_S B \cdot dS$$

Thus, equating the integrands in above equation, we get

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

This is the differential form of Faraday's law.

6.3 LENZ'S LAW

The negative sign in Faraday equation is due to Lenz's law which states that the direction of emf induced opposes the cause producing it. To understand the Lenz's law, consider the two conducting loops placed in magnetic fields with increasing and decreasing flux densities respectively as shown in Figure 6.1.

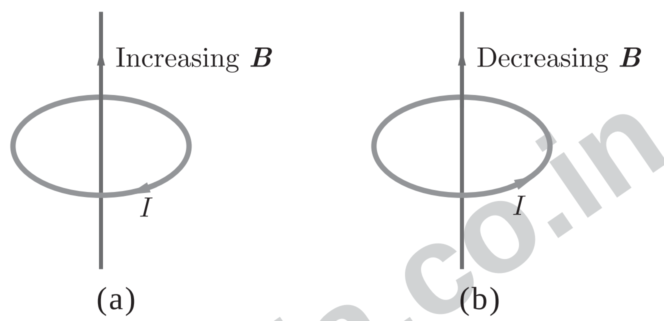


Figure 6.1: Determination of Direction of Induced Current in a Loop according to Lenz's Law (a) B in Upward Direction Increasing with Time (b) B in Upward Direction Decreasing with Time

METHODOLOGY: TO DETERMINE THE POLARITY OF INDUCED EMF

To determine the polarity of induced emf (direction of induced current), we may follow the steps given below.

Step 1: Obtain the direction of magnetic flux density through the loop. In both the Figures 6.1(a),(b) the magnetic field is directed upward.

Step 2: Deduce whether the field is increasing or decreasing with time along its direction. In Figure 6.1(a), the magnetic field directed upward is increasing, whereas in Figure 6.1(b), the magnetic field directed upward is decreasing with time.

Step 3: For increasing field assign the direction of induced current in the loop such that it produces the field opposite to the given magnetic field direction. Whereas for decreasing field assign the direction of induced current in the loop such that it produces the field in the same direction that of the given magnetic field. In Figure 6.1(a), using right hand rule we conclude that any current flowing in clockwise direction in the loop will cause a magnetic field directed downward and hence, opposes the increase in flux (i.e. opposes the field that causes it). Similarly in Figure 6.1(b), using right hand rule, we conclude that any current flowing in anti-clockwise direction in the loop will cause a magnetic field directed upward and hence, opposes the decrease in flux (i.e. opposes the field that causes it).

Step 4: Assign the polarity of induced emf in the loop corresponding to the obtained direction of induced current.

6.4 MOTIONAL AND TRANSFORMER EMFS

According to Faraday's law, for a flux variation through a loop, there will be induced emf in the loop. The variation of flux with time may be caused in following three ways:

6.4.1 Stationary Loop in a Time Varying Magnetic Field

For a stationary loop located in a time varying magnetic field, the induced emf in the loop is given by

$$V_{\text{emf}} = \oint_L E \cdot dL = -\int_S \frac{\partial B}{\partial t} \cdot dS$$

This emf is induced by the time-varying current (producing time-varying magnetic field) in a stationary loop is called *transformer emf*.

6.4.2 Moving Loop in Static Magnetic Field

When a conducting loop is moving in a static field, an emf is induced in the loop. This induced emf is called *motional emf* and given by

$$V_{\text{emf}} = \oint_L E_m \cdot dL = \oint_L (u \times B) \cdot dL$$

where u is the velocity of loop in magnetic field. Using Stoke's theorem in above equation, we get

$$\nabla \times E_m = \nabla \times (u \times B)$$

6.4.3 Moving Loop in Time Varying Magnetic Field

This is the general case of induced emf when a conducting loop is moving in time varying magnetic field. Combining the above two results, total emf induced is

$$V_{\text{emf}} = \oint_L E \cdot dL = -\int_S \frac{\partial B}{\partial t} \cdot dS + \oint_L (u \times B) \cdot dL$$

or, $$V_{\text{emf}} = \underbrace{-\int_S \frac{\partial B}{\partial t} \cdot dS}_{\text{transformer emf}} + \underbrace{\oint_L (u \times B) \cdot dL}_{\text{motional emf}}$$

Using Stoke's theorem, we can write the above equation in differential form as

$$\nabla \times E = -\frac{\partial B}{\partial t} + \nabla \times (u \times B)$$

6.5 INDUCTANCE

An inductance is the inertial property of a circuit caused by an induced reverse voltage that opposes the flow of current when a voltage is applied. A circuit or a part of circuit that has inductance is called an inductor. A device can have either self inductance or mutual inductance.

6.5.1 Self Inductance

Consider a circuit carrying a varying current which produces varying magnetic field which in turn produces induced emf in the circuit to oppose the change in flux. The emf induced is called emf of self-induction because the change in flux is produced by the circuit itself. This phenomena is called self-induction and the property of the circuit to produce self-induction is known as *self inductance*.

Self Inductance of a Coil

Suppose a coil with N number of turns carrying current I. Let the current induces the total magnetic flux Φ passing through the loop of the coil. Thus, we have

$$N\Phi \propto I$$

or $$N\Phi = LI$$

or $$L = \frac{N\Phi}{I}$$

where L is a constant of proportionality known as self inductance.

Expression for Induced EMF in terms of Self Inductance

If a variable current i is introduced in the circuit, then magnetic flux linked with the circuit also varies depending on the current. So, the self-inductance of the circuit can be written as

$$L = \frac{d\Phi}{di} \qquad \text{...(6.4)}$$

Since, the change in flux through the coil induces an emf in the coil given by

$$V_{emf} = -\frac{d\Phi}{dt} \qquad \text{...(6.5)}$$

So, from equations (6.4) and (6.5), we get

$$V_{emf} = -L\frac{di}{dt}$$

6.5.2 Mutual Inductance

Mutual inductance is the ability of one inductor to induce an emf across another inductor placed very close to it. Consider two coils carrying current I_1 and I_2 as shown in Figure 6.2. Let B_2 be the magnetic flux density produced due to the current I_2 and S_1 be the cross sectional area of coil 1. So, the magnetic flux due to B_2 will link with the coil 1, that is, it will pass through the surface S_1. Total magnetic flux produced by coil 2 that passes through coil 1 is called mutual flux and given as

$$\Phi_{12} = \int_{S_1} B_2 \cdot dS$$

We define the mutual inductance M_{12} as the ratio of the flux linkage on coil 1 to current I_2, i.e.

$$M_{12} = \frac{N_1\Phi_{12}}{I_2}$$

where N_1 is the number turns in coil 1. Similarly, the mutual inductance M_{21} is defined as the ratio of flux linkage on coil 2 (produced by current in coil 1) to current I_1, i.e.

$$M_{21} = \frac{N_2\Phi_{21}}{I_1}$$

The unit of mutual inductance is Henry (H). If the medium surrounding the circuits is linear, then

$$M_{12} = M_{21}$$

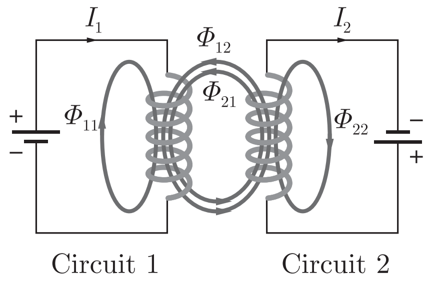


Figure 6.2 : Mutual Inductance between Two Current Carrying Coils

Expression for Induced EMF in terms of Mutual Inductance

If a variable current i_2 is introduced in coil 2 then, the magnetic flux linked with coil 1 also varies depending on current i_2. So, the mutual inductance can be given as

$$M_{12} = \frac{d\Phi_{12}}{di_2} \qquad ...(6.6)$$

The change in the magnetic flux linked with coil 1 induces an emf in coil 1 given as

$$(V_{emf})_1 = -\frac{d\Phi_{12}}{dt} \qquad ...(6.7)$$

So, from equations (6.6) and (6.7) we get

$$(V_{emf})_1 = -M_{12}\frac{di_2}{dt}$$

This is the induced emf in coil 1 produced by the current i_2 in coil 2. Similarly, the induced emf in the coil 2 due to a varying current in the coil 1 is given as

$$(V_{emf})_2 = -M_{21}\frac{di_1}{dt}$$

6.6 MAXWELL'S EQUATIONS

The set of four equations which have become known as Maxwell's equations are those which are developed in the earlier chapters and associated with them the name of other investigators. These equations describe the sources and the field vectors in the broad fields to electrostatics, magnetostatics and electro-magnetic induction.

6.6.1 Maxwell's Equations for Time Varying Fields

The four Maxwell's equation include Faraday's law, Ampere's circuital law, Gauss's law, and conservation of magnetic flux. There is no guideline for giving numbers to the various Maxwell's equations. However, it is customary to call the Maxwell's equation derived from Faraday's law as the first Maxwell's equation.

Maxwell's First Equation : Faraday's Law

The electromotive force around a closed path is equal to the time derivative of the magnetic displacement through any surface bounded by the path.

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad \text{(Differential form)}$$

or

$$\oint_L \boldsymbol{E} \cdot d\boldsymbol{L} = -\frac{\partial}{\partial t}\int_S \boldsymbol{B} \cdot d\boldsymbol{S} \qquad \text{(Integral form)}$$

Maxwell's Second Equation: Modified Ampere's Circuital law

The magnetomotive force around a closed path is equal to the conduction plus the time derivative of the electric displacement through any surface bounded by the path. i.e.

$$\nabla \times H = J + \frac{\partial D}{\partial t} \qquad \text{(Differential form)}$$

$$\oint_L H \cdot dL = \int_S \left(J + \frac{\partial D}{\partial t}\right) \cdot dS \qquad \text{(Integral form)}$$

Maxwell's Third Equation : Gauss's Law for Electric Field

The total electric displacement through any closed surface enclosing a volume is equal to the total charge within the volume. i.e.,

$$\nabla \cdot D = \rho_v \qquad \text{(Differential form)}$$

or,
$$\oint_S D \cdot dS = \int_v \rho_v dv \qquad \text{(Integral form)}$$

This is the Gauss' law for static electric fields.

Maxwell's Fourth Equation : Gauss's Law for Magnetic Field

The net magnetic flux emerging through any closed surface is zero. In other words, the magnetic flux lines do not originate and end anywhere, but are continuous. i.e.,

$$\nabla \cdot B = 0 \qquad \text{(Differential form)}$$

or,
$$\oint_S B \cdot dS = 0 \qquad \text{(Integral form)}$$

This is the Gauss' law for static magnetic fields, which confirms the non-existence of magnetic monopole. Table 6.1 summarizes the Maxwell's equation for time varying fields.

Table 6.1: Maxwell's Equation for Time Varying Field

S.N.	Differential form	Integral form	Name
1.	$\nabla \times E = -\frac{\partial B}{\partial t}$	$\oint_L E \cdot dL = -\frac{\partial}{\partial t}\int_S B \cdot dS$	Faraday's law of electromagnetic induction
2.	$\nabla \times H = J + \frac{\partial D}{\partial t}$	$\oint_L H \cdot dL = \int_S \left(J + \frac{\partial D}{\partial t}\right) \cdot dS$	Modified Ampere's circuital law
3.	$\nabla \cdot D = \rho_v$	$\oint_S D \cdot dS = \int_v \rho_v dv$	Gauss' law of Electrostatics
4.	$\nabla \cdot B = 0$	$\oint_S B \cdot dS = 0$	Gauss' law of Magnetostatic (non-existence of magnetic mono-pole)

6.6.2 Maxwell's Equations for Static Fields

For static fields, all the field terms which have time derivatives are zero, i.e.

$$\frac{\partial B}{\partial t} = 0$$

and
$$\frac{\partial D}{\partial t} = 0$$

Therefore, for a static field the four Maxwell's equations described above reduces to the following form.

Table 6.2: Maxwell's equation for static field

S.N.	Differential Form	Integral form	Name
1.	$\nabla \times E = 0$	$\oint_L E \cdot dL = 0$	Faraday's law of electromagnetic induction
2.	$\nabla \times H = J$	$\oint_L H \cdot dL = \int_S J \cdot dS$	Modified Ampere's circuital law
3.	$\nabla \cdot D = \rho_v$	$\oint_S D \cdot dS = \int_v \rho_v dv$	Gauss' law of Electrostatics
4.	$\nabla \cdot B = 0$	$\oint_S B \cdot dS = 0$	Gauss' law of Magnetostatic (non-existence of magnetic mono-pole)

6.6.3 Maxwell's Equations in Phasor Form

In a time-varying field, the field quantities $E(x,y,z,t)$, $D(x,y,z,t)$, $B(x,y,z,t)$, $H(x,y,z,t)$, $J(x,y,z,t)$ and $\rho_v(x,y,z,t)$ can be represented in their respective phasor forms as below:

$$E = \text{Re}\{E_s e^{j\omega t}\} \quad ...(6.8a)$$
$$D = \text{Re}\{D_s e^{j\omega t}\} \quad ...(6.8b)$$
$$B = \text{Re}\{B_s e^{j\omega t}\} \quad ...(6.8c)$$
$$H = \text{Re}\{H_s e^{j\omega t}\} \quad ...(6.8d)$$
$$J = \text{Re}\{J_s e^{j\omega t}\} \quad ...(6.8e)$$

and
$$\rho_v = \text{Re}\{\rho_{vs} e^{j\omega t}\} \quad ...(6.8f)$$

where E_s, D_s, B_s, H_s, J_s and ρ_{vs} are the phasor forms of respective field quantities. Using these relations, we can directly obtain the phasor form of Maxwell's equations as described below.

Maxwell's First Equation: Faraday's Law

In time varying field, first Maxwell's equation is written as

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad ...(6.9)$$

Now, from equation (6.8a) we can obtain

$$\nabla \times E = \nabla \times \text{Re}\{E_s e^{j\omega t}\} = \text{Re}\{\nabla \times E_s e^{j\omega t}\}$$

and using equation (6.8c) we get

$$\frac{\partial B}{\partial t} = \frac{\partial}{\partial t}\text{Re}\{B_s e^{j\omega t}\} = \text{Re}\{j\omega B_s e^{j\omega t}\}$$

Substituting the two results in equation (6.9) we get

$$\text{Re}\{\nabla \times E_s e^{j\omega t}\} = -\text{Re}\{j\omega B_s e^{j\omega t}\}$$

Hence,
$$\nabla \times E_s = -j\omega B_s \quad \text{(Differential form)}$$

or,
$$\oint_L E_s \cdot dL = -j\omega \int_S B_s \cdot dS \quad \text{(Integral form)}$$

Maxwell's Second Equation: Modified Ampere's Circuital Law

In time varying field, second Maxwell's equation is written as

$$\nabla \times H = J + \frac{\partial D}{\partial t} \quad ...(6.10)$$

From equation (6.8d) we can obtain

$$\nabla \times H = \nabla \times \text{Re}\{H_s e^{j\omega t}\} = \text{Re}\{\nabla \times H_s e^{j\omega t}\}$$

From equation (6.8e), we have

$$J = \mathrm{Re}\{J_s e^{j\omega t}\}$$

and using equation (6.8b) we get

$$\frac{\partial D}{\partial t} = \frac{\partial}{\partial t}\mathrm{Re}\{D_s e^{j\omega t}\} = \mathrm{Re}\{j\omega D_s e^{j\omega t}\}$$

Substituting these results in equation (6.10) we get

$$\mathrm{Re}\{\nabla \times H_s e^{j\omega t}\} = \mathrm{Re}\{J_s e^{j\omega t} + j\omega D_s e^{j\omega t}\}$$

Hence, $$\nabla \times H_s = J_s + j\omega D_s \qquad \text{(Differential form)}$$

or, $$\oint_L H_s \cdot dL = \int_S (J_s + j\omega D_s) \cdot dS \qquad \text{(Integral form)}$$

Maxwell's Third Equation : Gauss's Law for Electric Field

In time varying field, third Maxwell's equation is written as

$$\nabla \cdot D = \rho_v \qquad ...(6.11)$$

From equation (6.8b) we can obtain

$$\nabla \cdot D = \nabla \cdot \mathrm{Re}\{D_s e^{j\omega t}\} = \mathrm{Re}\{\nabla \cdot D_s e^{j\omega t}\}$$

and from equation (6.8f) we have

$$\rho_v = \mathrm{Re}\{\rho_{vs} e^{j\omega t}\}$$

Substituting these two results in equation (6.11) we get

$$\mathrm{Re}\{\nabla \cdot D_s e^{j\omega t}\} = \mathrm{Re}\{\rho_{vs} e^{j\omega t}\}$$

Hence, $$\nabla \cdot D_s = \rho_{vs} \qquad \text{(Differential form)}$$

or, $$\oint_S D_s \cdot dS = \int_v \rho_{vs} dv \qquad \text{(Integral form)}$$

Maxwell's Fourth Equation : Gauss's Law for Magnetic Field

$$\nabla \cdot B = 0 \qquad ...(6.12)$$

From equation (6.8c) we can obtain

$$\nabla \cdot B = \nabla \cdot \mathrm{Re}\{B_s e^{j\omega t}\} = \mathrm{Re}\{\nabla \cdot B_s e^{j\omega t}\}$$

Substituting it in equation (6.12) we get

$$\mathrm{Re}\{\nabla \cdot B_s e^{j\omega t}\} = 0$$

Hence, $$\nabla \cdot B_s = 0 \qquad \text{(Differential form)}$$

or, $$\oint_S B_s \cdot dS = 0 \qquad \text{(Integral form)}$$

Table 6.3 summarizes the Maxwell's equations in phasor form.

Table 6.3: Maxwell's Equations in Phasor Form

S.N.	Differential form	Integral form	Name
1.	$\nabla \times E_s = -j\omega B_s$	$\oint_L E_s \cdot dL = -j\omega \int_S B_s \cdot dS$	Faraday's law of electromagnetic induction
2.	$\nabla \times H_s = J_s + j\omega D_s$	$\oint_L H_s \cdot dL = \int_S (J_s + j\omega D_s) \cdot dS$	Modified Ampere's circuital law
3.	$\nabla \cdot D_s = \rho_{vs}$	$\oint_S D_s \cdot dS = \int_v \rho_{vs} dv$	Gauss' law of Electrostatics
4.	$\nabla \cdot B_s = 0$	$\oint_S B_s \cdot dS = 0$	Gauss' law of Magnetostatic (non-existence of magnetic mono-pole)

6.7 MAXWELL'S EQUATIONS IN FREE SPACE

For electromagnetic fields, free space is characterised by the following parameters:

1. Relative permittivity, $\varepsilon_r = 1$
2. Relative permeability, $\mu_r = 1$
3. Conductivity, $\sigma = 0$
4. Conduction current density, $J = 0$
5. Volume charge density, $\rho_v = 0$

As we have already obtained the four Maxwell's equations for time-varying fields, static fields, and harmonic fields; these equations can be easily written for the free space by just replacing the variables to their respective values in free space.

6.7.1 Maxwell's Equations for Time Varying Fields in Free Space

By substituting the parameters, $J = 0$ and $\rho_v = 0$ in the Maxwell's equations given in Table 6.1, we get the Maxwell's equation for time-varying fields in free space as summarized below:

Table 6.4: Maxwell's Equations for Time Varying Fields in Free Space

S.N.	Differential form	Integral form	Name
1.	$\nabla \times E = -\frac{\partial B}{\partial t}$	$\oint_L E \cdot dL = -\frac{\partial}{\partial t}\int_S B \cdot dS$	Faraday's law of electromagnetic induction
2.	$\nabla \times H = \frac{\partial D}{\partial t}$	$\oint_L H \cdot dL = \int_S \frac{\partial D}{\partial t} \cdot dS$	Modified Ampere's circuital law
3.	$\nabla \cdot D = 0$	$\oint_S D \cdot dS = 0$	Gauss' law of Electrostatics
4.	$\nabla \cdot B = 0$	$\oint_S B \cdot dS = 0$	Gauss' law of Magnetostatic (non-existence of magnetic mono-pole)

6.7.2 Maxwell's Equations for Static Fields in Free Space

Substituting the parameters, $J = 0$ and $\rho_v = 0$ in the Maxwell's equation given in Table 6.2, we get the Maxwell's equation for static fields in free space as summarized below.

Table 6.5: Maxwell's Equations for Static Fields in Free Space

S.N.	Differential Form	Integral Form	Name
1.	$\nabla \times E = 0$	$\oint_L E \cdot dL = 0$	Faraday's law of electromagnetic induction
2.	$\nabla \times H = 0$	$\oint_L H \cdot dL = 0$	Modified Ampere's circuital law
3.	$\nabla \cdot D = 0$	$\oint_S D \cdot dS = 0$	Gauss' law of Electrostatics
4.	$\nabla \cdot B = 0$	$\oint_S B \cdot dS = 0$	Gauss' law of Magnetostatic (non-existence of magnetic mono-pole)

Thus, all the four Maxwell's equation vanishes for static fields in free space.

6.7.3 Maxwell's Equations for Time Harmonic Fields in Free Space

Again, substituting the parameters, $J = 0$ and $\rho_v = 0$ in the Maxwell's equations given in Table 6.3, we get the Maxwell's equation for time harmonic fields in free space as summarized below.

Table 6.6 : Maxwell's Equations for Time-Harmonic Fields in Free Space

S.N.	Differential form	Integral form	Name
1.	$\nabla \times E_s = -j\omega B_s$	$\oint_L E_s \cdot dL = -j\omega \int_S B_s \cdot dS$	Faraday's law of electromagnetic induction
2.	$\nabla \times H_s = j\omega D_s$	$\oint_L H_s \cdot dL = \int_S j\omega D_s \cdot dS$	Modified Ampere's circuital law
3.	$\nabla \cdot D_s = 0$	$\oint_S D_s \cdot dS = 0$	Gauss' law of Electrostatics
4.	$\nabla \cdot B_s = 0$	$\oint_S B_s \cdot dS = 0$	Gauss' law of Magnetostatic (non-existence of magnetic mono-pole)

****

EXERCISE 6.1

MCQ 6.1.1 A perfect conducting sphere of radius r is such that it's net charge resides on the surface. At any time t, magnetic field $B(r,t)$ inside the sphere will be

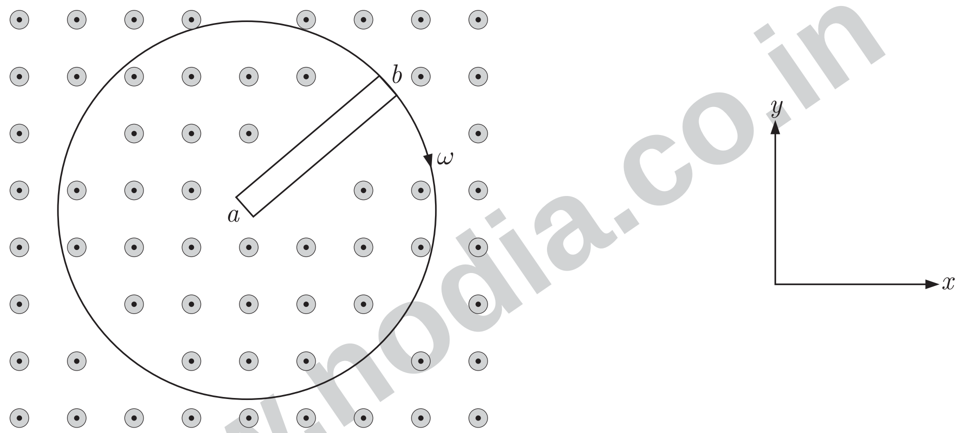
(A) 0

(B) uniform, independent of r

(C) uniform, independent of t

(D) uniform, independent of both r and t

MCQ 6.1.2 A straight conductor ab of length l lying in the xy plane is rotating about the centre a at an angular velocity ω as shown in the figure.



If a magnetic field B is present in the space directed along a_z then which of the following statement is correct ?

(A) V_{ab} is positive

(B) V_{ab} is negative

(C) V_{ba} is positive

(D) V_{ba} is zero

MCQ 6.1.3 **Assertion (A) :** A small piece of bar magnet takes several seconds to emerge at bottom when it is dropped down a vertical aluminum pipe where as an identical unmagnetized piece takes a fraction of second to reach the bottom.
Reason (R) : When the bar magnet is dropped inside a conducting pipe, force exerted on the magnet by induced eddy current is in upward direction.

(A) Both A and R are true and R is correct explanation of A.

(B) Both A and R are true but R is not the correct explanation of A.

(C) A is true but R is false.

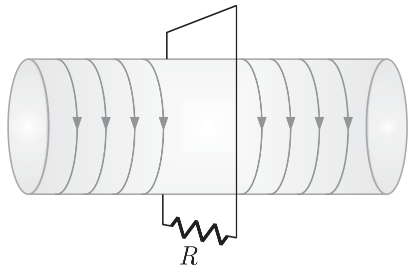
(D) A is false but R is true.

MCQ 6.1.4 Self inductance of a long solenoid having n turns per unit length will be proportional to

(A) n (B) $1/n$

(C) n^2 (D) $1/n^2$

MCQ 6.1.5 A wire with resistance R is looped on a solenoid as shown in figure.

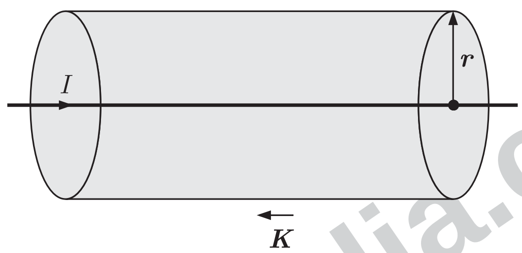


If a constant current is flowing in the solenoid then the induced current flowing in the loop with resistance R will be

(A) non uniform (B) constant

(C) zero (D) none of these

MCQ 6.1.6 A long straight wire carries a current $I = I_0\cos(\omega t)$. If the current returns along a coaxial conducting tube of radius r as shown in figure then magnetic field and electric field inside the tube will be respectively.

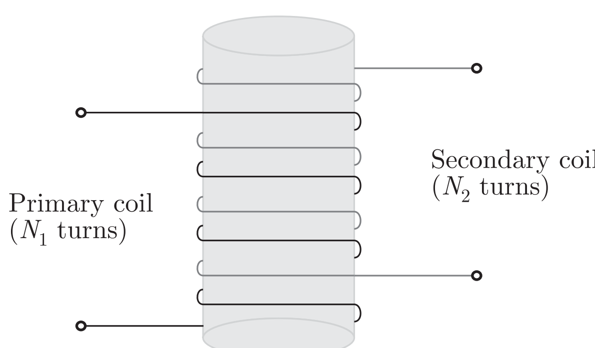


(A) radial, longitudinal (B) circumferential, longitudinal

(C) circumferential, radial (D) longitudinal, circumferential

MCQ 6.1.7 Assertion (A) : Two coils are wound around a cylindrical core such that the primary coil has N_1 turns and the secondary coils has N_2 turns as shown in figure. If the same flux passes through every turn of both coils then the ratio of emf induced in the two coils is

$$\frac{V_{\text{emf}\,2}}{V_{\text{emf}\,1}} = \frac{N_2}{N_1}$$



Reason (R) : In a primitive transformer, by choosing the appropriate no. of turns, any desired secondary emf can be obtained.

(A) Both A and R are true and R is correct explanation of A.

(B) Both A and R are true but R is not the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

MCQ 6.1.8 In a non magnetic medium electric field $E = E_0 \cos \omega t$ is applied. If the permittivity of medium is ε and the conductivity is σ then the ratio of the amplitudes of the conduction current density and displacement current density will be

(A) $\mu_0 / \omega\varepsilon$
(B) $\sigma / \omega\varepsilon$
(C) $\sigma\mu_0 / \omega\varepsilon$
(D) $\omega\varepsilon / \sigma$

MCQ 6.1.9 In a medium, the permittivity is a function of position such that $\frac{\nabla \varepsilon}{\varepsilon} \approx 0$. If the volume charge density inside the medium is zero then $\nabla \cdot \boldsymbol{E}$ is roughly equal to

(A) $\varepsilon \boldsymbol{E}$
(B) $-\varepsilon \boldsymbol{E}$
(C) 0
(D) $-\nabla \varepsilon \cdot \boldsymbol{E}$

MCQ 6.1.10 In free space, the electric field intensity at any point (r, θ, ϕ) in spherical coordinate system is given by

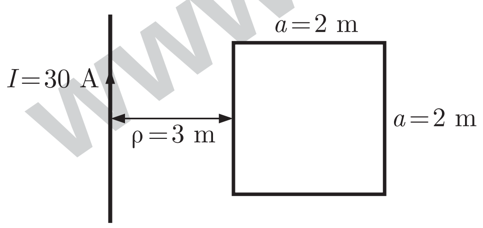
$$\boldsymbol{E} = \frac{\sin \theta \cos(\omega t - kr)}{r} \boldsymbol{a}_\theta$$

The phasor form of magnetic field intensity in the free space will be

(A) $\frac{k \sin \theta}{\omega \mu_0 r} e^{-jkr} \boldsymbol{a}_\phi$
(B) $-\frac{k \sin \theta}{\omega \mu_0 r} e^{-jkr} \boldsymbol{a}_\phi$
(C) $\frac{k \omega \mu_0}{r} e^{-jkr} \boldsymbol{a}_\phi$
(D) $\frac{k \sin \theta}{r} e^{-jkr} \boldsymbol{a}_\phi$

Common Data For Q. 11 and 12 :

A conducting wire is formed into a square loop of side 2 m. A very long straight wire carrying a current $I = 30$ A is located at a distance 3 m from the square loop as shown in figure.



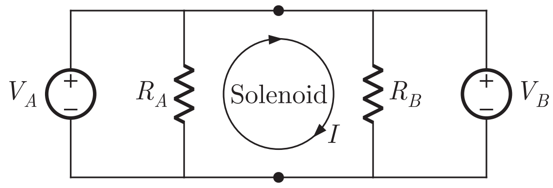
MCQ 6.1.11 If the loop is pulled away from the straight wire at a velocity of 5 m/s then the induced e.m.f. in the loop after 0.6 sec will be

(A) 5 μvolt
(B) 2.5 μvolt
(C) 25 μvolt
(D) 5 mvolt

MCQ 6.1.12 If the loop is pulled downward in the parallel direction to the straight wire, such that distance between the loop and wire is always 3 m then the induced e.m.f. in the loop at any time t will be

(A) linearly increasing with t
(B) always 0
(C) linearly decreasing with t
(D) always constant but not zero.

MCQ 6.1.13 Two voltmeters A and B with internal resistances R_A and R_B respectively is connected to the diametrically opposite points of a long solenoid as shown in figure. Current in the solenoid is increasing linearly with time. The correct relation between the voltmeter's reading V_A and V_B will be

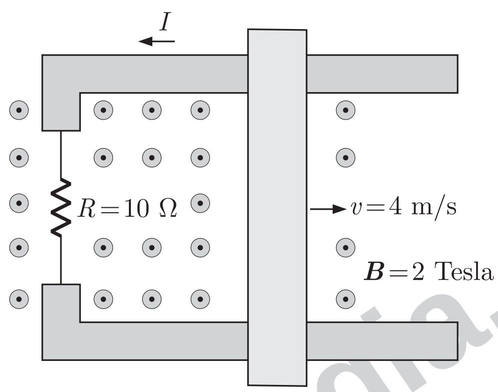


(A) $V_A = V_B$ (B) $V_A = - V_B$

(C) $\frac{V_A}{V_B} = \frac{R_A}{R_B}$ (D) $\frac{V_A}{V_B} = -\frac{R_A}{R_B}$

Common Data For Q. 14 and 15 :

Two parallel conducting rails are being placed at a separation of 5 m with a resistance $R = 10\,\Omega$ connected across it's one end. A conducting bar slides frictionlessly on the rails with a velocity of 4 m/s away from the resistance as shown in the figure.



MCQ 6.1.14 If a uniform magnetic field $B = 2$ Tesla pointing out of the page fills entire region then the current I flowing in the bar will be

(A) 0 A (B) -40 A

(C) 4 A (D) -4 A

MCQ 6.1.15 The force exerted by magnetic field on the sliding bar will be

(A) 4 N, opposes it's motion

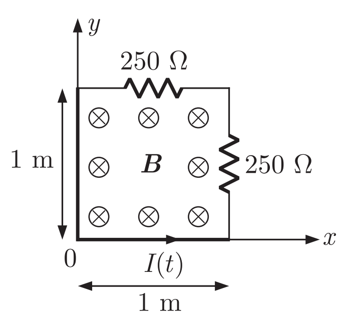
(B) 40 N, opposes it's motion

(C) 40 N, in the direction of it's motion

(D) 0

MCQ 6.1.16 Two small resistor of 250 Ω each is connected through a perfectly conducting filament such that it forms a square loop lying in x-y plane as shown in the figure. Magnetic flux density passing through the loop is given as

$$B = -7.5\cos(120\pi t - 30^\circ)\,a_z$$

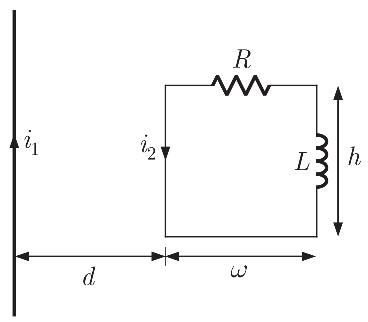


The induced current $I(t)$ in the loop will be

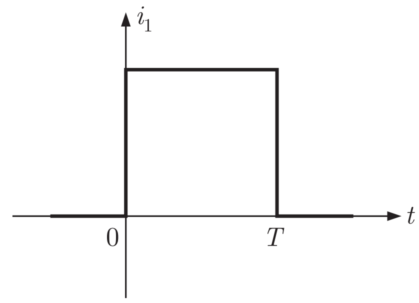
(A) $0.02\sin(120\pi t - 30^\circ)$ (B) $2.8\times10^{3}\sin(120\pi t - 30^\circ)$

(C) $-5.7\sin(120\pi t - 30^\circ)$ (D) $5.7\sin(120\pi t - 30^\circ)$

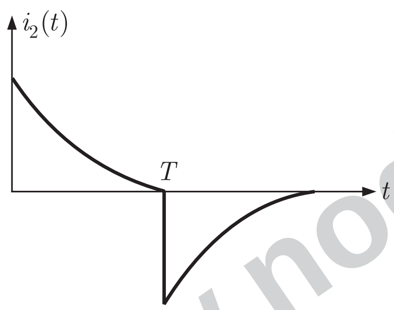
MCQ 6.1.17 A rectangular loop of self inductance L is placed near a very long wire carrying current i_1 as shown in figure (a). If i_1 be the rectangular pulse of current as shown in figure (b) then the plot of the induced current i_2 in the loop versus time t will be (assume the time constant of the loop, $\tau \gg L/R$)

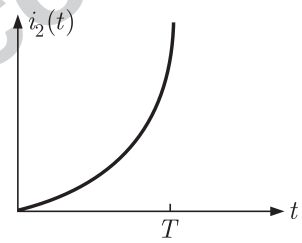


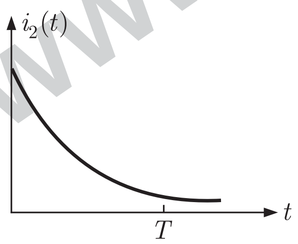
(a)

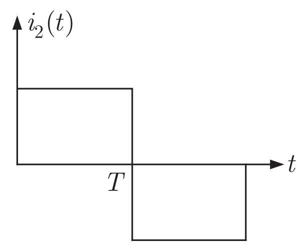


(b)

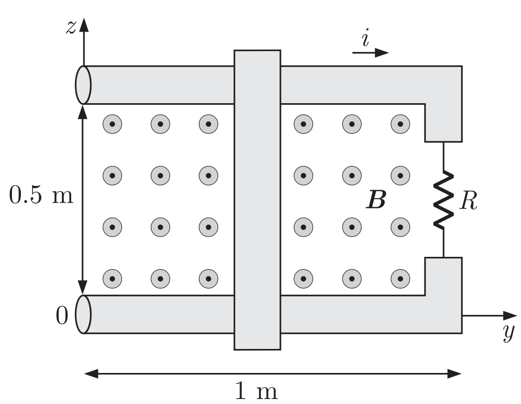
(A)


(B)


(C)


(D)


MCQ 6.1.18 Two parallel conducting rails is placed in a varying magnetic field $B = 0.2\cos\omega t a_x$. A conducting bar oscillates on the rails such that it's position is given by $y = 0.5(1-\cos\omega t)$ m. If one end of the rails are terminated in a resistance $R = 5\,\Omega$, then the current i flowing in the rails will be



(A) $0.01\omega \sin \omega t(1 + 2\cos \omega t)$ (B) $-0.01\omega \sin \omega t(1 + 2\cos \omega t)$
(C) $0.01\omega \cos \omega t(1 + 2\sin \omega t)$ (D) $0.05\omega \sin \omega t(1 + 2\sin \omega t)$

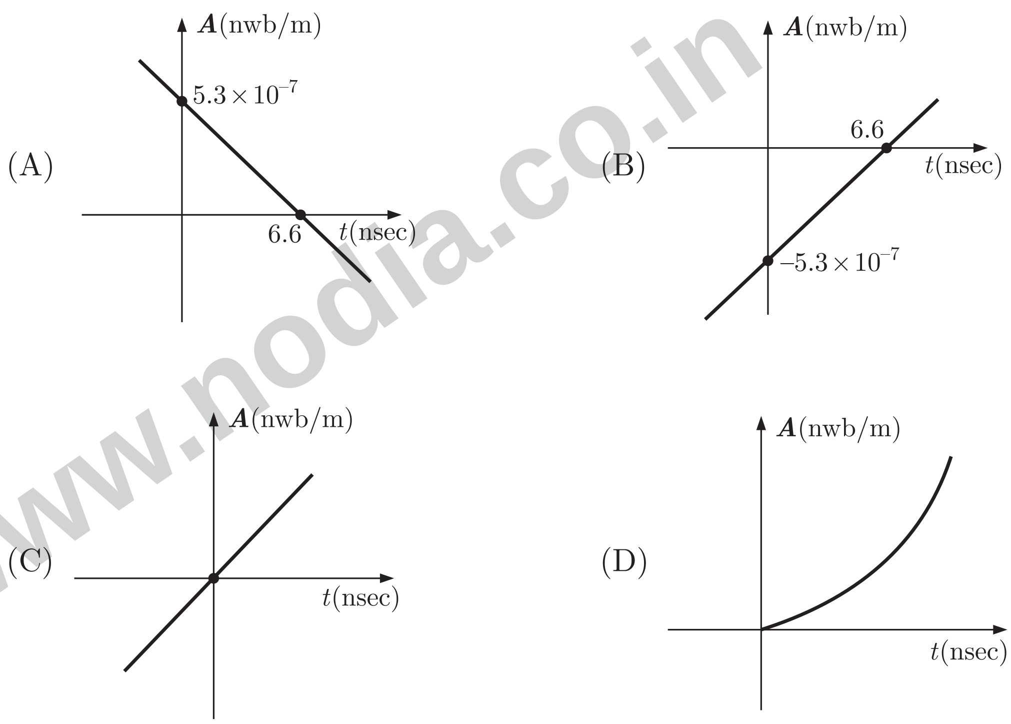
MCQ 6.1.19 Electric flux density in a medium ($\varepsilon_r = 10$, $\mu_r = 2$) is given as

$$D = 1.33\sin(3 \times 10^8 t - 0.2x)\, a_y \;\mu\text{C/m}^2$$

Magnetic field intensity in the medium will be

(A) $10^{-5}\sin(3 \times 10^8 t - 0.2x)\, a_y$ A/m
(B) $2\sin(3 \times 10^8 t - 0.2x)\, a_y$ A/m
(C) $-4\sin(3 \times 10^8 t - 0.2x)\, a_y$ A/m
(D) $4\sin(3 \times 10^8 t - 0.2x)\, a_y$ A/m

MCQ 6.1.20 A current filament located on the x-axis in free space with in the interval $-0.1 < x < 0.1$ m carries current $I(t) = 8t$ A in a_x direction. If the retarded vector potential at point $P(0, 0, 2)$ be $A(t)$ then the plot of $A(t)$ versus time will be



Common Data For Q. 21 and 22 :

In a region of electric and magnetic fields E and B, respectively, the force experienced by a test charge qC are given as follows for three different velocities.

Velocity m/sec	Force, N
a_x	$q(a_y + a_z)$
a_y	qa_y
a_z	$q(2a_y + a_z)$

MCQ 6.1.21 What will be the magnetic field B in the region ?

(A) a_x (B) $a_x - a_y$
(C) a_z (D) $a_y - a_z$

MCQ 6.1.22 What will be electric field E in the region ?

(A) $a_x - a_z$ (B) $a_y - a_z$

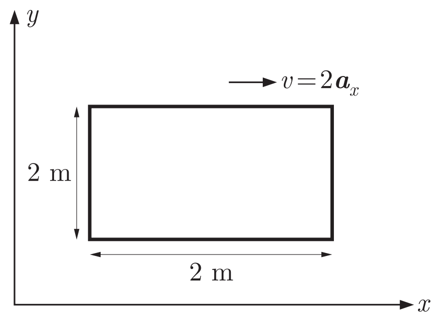
(C) $a_y + a_z$ (D) $a_y + a_z - a_x$

MCQ 6.1.23 In a non-conducting medium ($\sigma = 0$, $\mu_r = \varepsilon_r = 1$), the retarded potentials are given as $V = y(x - ct)$ volt and $A = y(\frac{x}{c} - t)a_x$ Wb/m where c is velocity of waves in free space. The field (electric and magnetic) inside the medium satisfies Maxwell's equation if

(A) $J = 0$ only (B) $\rho_v = 0$ only

(C) $J = \rho_v = 0$ (D) Can't be possible

MCQ 6.1.24 In Cartesian coordinates magnetic field is given by $B = -2/x\, a_z$. A square loop of side 2 m is lying in xy plane and parallel to the y-axis. Now, the loop is moving in that plane with a velocity $v = 2a_x$ as shown in the figure.



What will be the circulation of the induced electric field around the loop ?

(A) $\frac{16}{x(x+2)}$ (B) $\frac{8}{x}$

(C) $\frac{8}{x(x+2)}$ (D) $\frac{x(x+2)}{16}$

Common Data For Q. 25 to 27 :

In a cylindrical coordinate system, magnetic field is given by

$$B = \begin{cases} 0 & \text{for } \rho < 4\text{ m} \\ 2\sin\omega t\, a_z & \text{for } 4 < \rho < 5\text{ m} \\ 0 & \text{for } \rho > 5\text{ m} \end{cases}$$

MCQ 6.1.25 The induced electric field in the region $\rho < 4$ m will be

(A) 0 (B) $\frac{2\omega\cos\omega t}{\rho}a_\phi$

(C) $-2\cos\omega t a_\phi$ (D) $\frac{1}{2\sin\omega t}a_\phi$

MCQ 6.1.26 The induced electric field at $\rho = 4.5$ m is

(A) 0 (B) $-\frac{17\omega\cos\omega t}{18}$

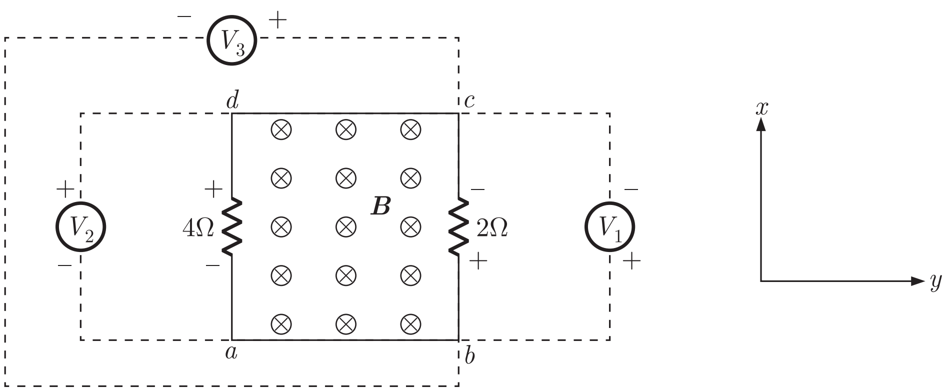
(C) $\frac{4\omega\cos\omega t}{9}$ (D) $-\frac{17\omega\cos\omega t}{4}$

MCQ 6.1.27 The induced electric field in the region $\rho > 5$ m is

(A) $-\frac{18}{\rho}\omega\cos\omega t a_\phi$ (B) $\frac{-9\omega\cos\omega t}{\rho}a_\phi$

(C) $-9\rho\cos\omega t a_\phi$ (D) $\frac{9\omega\cos\omega t}{\rho}a_\phi$

MCQ 6.1.28 Magnetic flux density, $B = 0.1t\, a_z$ Tesla threads only the loop *abcd* lying in the plane *xy* as shown in the figure.

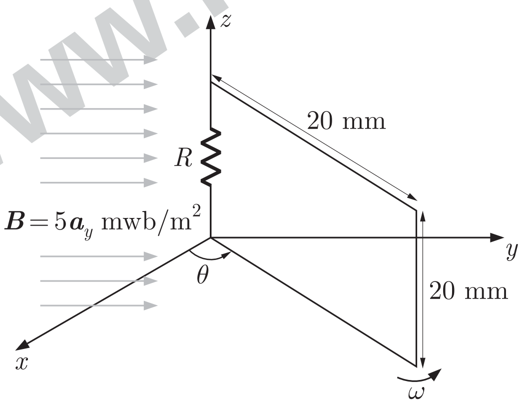


Consider the three voltmeters V_1, V_2 and V_3, connected across the resistance in the same *xy* plane. If the area of the loop *abcd* is $1\ \text{m}^2$ then the voltmeter readings are

	V_1	V_2	V_3
(A)	66.7 mV	33.3 mV	66.7 mV
(B)	33.3 mV	66.7 mV	33.3 mV
(C)	66.7 mV	66.7 mV	33.3 mV
(D)	33.3 mV	66.7 mV	66.7 mV

Common Data For Q. 29 and 30 :

A square wire loop of resistance R rotated at an angular velocity ω in the uniform magnetic field $B = 5a_y\ \text{mWb/m}^2$ as shown in the figure.



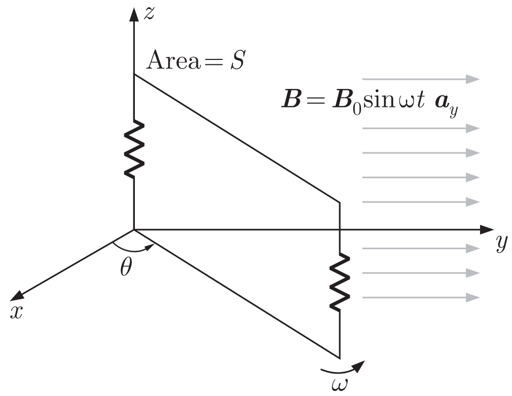
MCQ 6.1.29 If the angular velocity, $\omega = 2\ \text{rad/sec}$ then the induced e.m.f. in the loop will be

(A) $2\sin\theta\ \mu\text{V/m}$
(B) $2\cos\theta\ \mu\text{V/m}$
(C) $4\cos\theta\ \mu\text{V/m}$
(D) $4\sin\theta\ \mu\text{V/m}$

MCQ 6.1.30 If resistance, $R = 40\ \text{m}\Omega$ then the current flowing in the square loop will be

(A) $0.2\sin\theta$ mA
(B) $0.1\sin\theta$ mA
(C) $0.1\cos\theta$ mA
(D) $0.5\sin\theta$ mA

MCQ 6.1.31 In a certain region magnetic flux density is given as $B = B_0 \sin\omega t\, a_y$. A rectangular loop of wire is defined in the region with it's one corner at origin and one side along z-axis as shown in the figure.



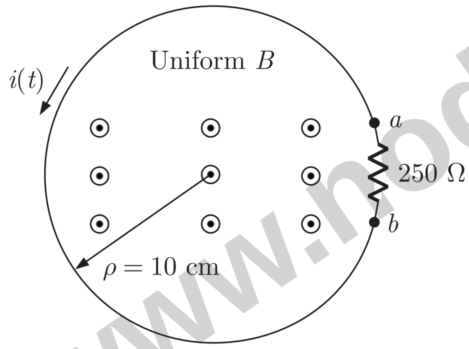
If the loop rotates at an angular velocity ω (same as the angular frequency of magnetic field) then the maximum value of induced e.m.f in the loop will be

(A) $\frac{1}{2}B_0S\omega$ (B) $2B_0S\omega$

(C) $B_0S\omega$ (D) $4B_0S\omega$

Common Data For Q. 32 and 33 :

Consider the figure shown below. Let $B = 10\cos 120\pi t$ Wb/m^2 and assume that the magnetic field produced by $i(t)$ is negligible



MCQ 6.1.32 The value of v_{ab} is

(A) $-118.43\cos 120\pi t$ V (B) $118.43\cos 120\pi t$ V

(C) $-118.43\sin 120\pi t$ V (D) $118.43\sin 120\pi t$ V

MCQ 6.1.33 The value of $i(t)$ is

(A) $-0.47\cos 120\pi t$ A (B) $0.47\cos 120\pi t$ A

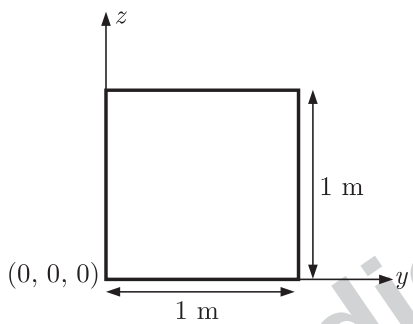
(C) $-0.47\sin 120\pi t$ A (D) $0.47\sin 120\pi t$ A

****

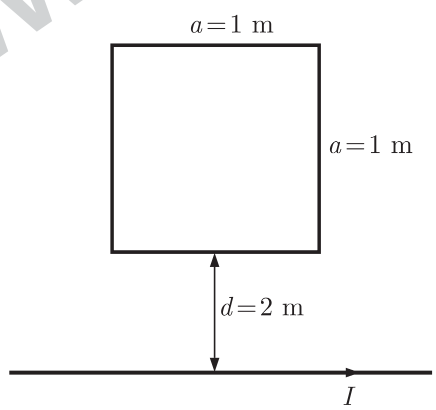
EXERCISE 6.2

QUES 6.2.1 A small conducting loop is released from rest with in a vertical evacuated cylinder. What is the voltage induced (in mV) in the falling loop ?
(Assume earth magnetic field $= 10^{-6}$ T at a constant angle of $10°$ below the horizontal)

QUES 6.2.2 A square loop of side 1 m is located in the plane $x = 0$ as shown in figure. A non-uniform magnetic flux density through it is given as $B = 4z^3 t^2 \boldsymbol{a}_x$, The emf induced in the loop at time $t = 2$ sec will be ____ Volt.



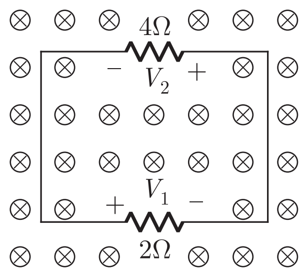
QUES 6.2.3 A very long straight wire carrying a current $I = 5$ A is placed at a distance of 2 m from a square loop as shown. If the side of the square loop is 1 m then the total flux passing through the square loop will be ____ $\times 10^{-7}$ wb



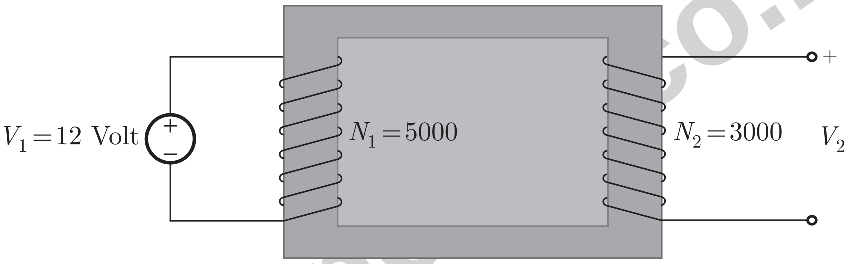
QUES 6.2.4 In a medium where no D.C. field is present, the conduction current density at any point is given as $J_d = 20\cos(1.5 \times 10^8 t)\boldsymbol{a}_y$ A/m^2. Electric flux density in the medium will be $D_0 \sin(1.5 \times 10^8 t)\boldsymbol{a}_y$ nC/m^2 such that $D_0 =$ ____

QUES 6.2.5 A conducting medium has permittivity, $\varepsilon = 4\varepsilon_0$ and conductivity, $\sigma = 1.14 \times 10^8$ s/m. The ratio of magnitude of displacement current and conduction current in the medium at 50 GHz will be _____ $\times 10^{-8}$.

QUES 6.2.6 In a certain region magnetic flux density is given as $B = 0.1 t a_z$ Wb/m^2. An electric loop with resistance $2\,\Omega$ and $4\,\Omega$ is lying in x-y plane as shown in the figure. If the area of the loop is $1\,\text{m}^2$ then, the voltage drop V_1 across the $2\,\Omega$ resistance is ____ mV.



QUES 6.2.7 A magnetic core of uniform cross section having two coils (Primary and secondary) wound on it as shown in figure. The no. of turns of primary coil is 5000 and no. of turns of secondary coil is 3000. If a voltage source of 12 volt is connected across the primary coil then what will be the voltage (in Volt) across the secondary coil ?

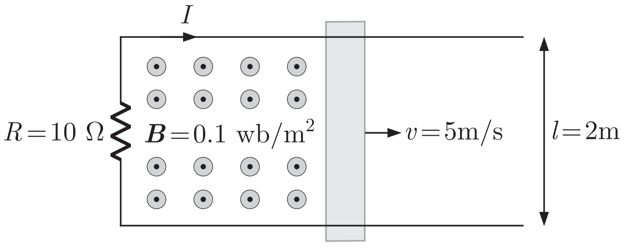


QUES 6.2.8 Magnetic field intensity in free space is given as

$$H = 0.1\cos(15\pi y)\sin(6\pi \times 10^9 t - bx) a_z \text{ A/m}$$

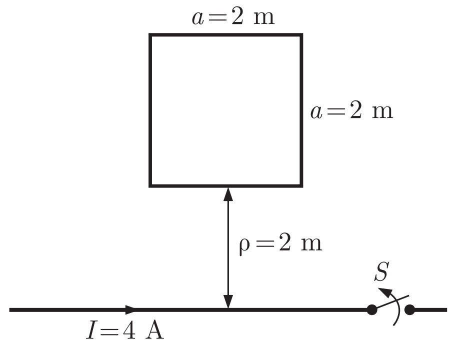
It satisfies Maxwell's equation when $|b| =$ ____

QUES 6.2.9 Two parallel conducting rails are being placed at a separation of 2 m as shown in figure. One end of the rail is being connected through a resistor $R = 10\,\Omega$ and the other end is kept open. A metal bar slides frictionlessly on the rails at a speed of 5 m/s away from the resistor. If the magnetic flux density $B = 0.1$ Wb/m^2 pointing out of the page fills entire region then the current I flowing in the resistor will be ____ Ampere.

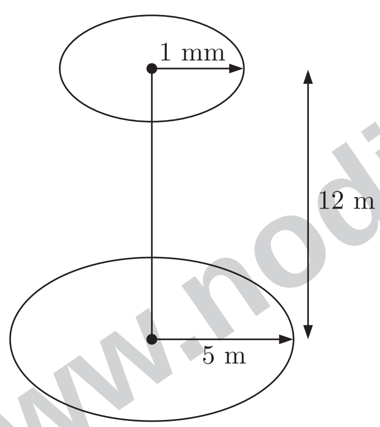


QUES 6.2.10 An infinitely long straight wire with a closed switch S carries a uniform current $I = 4$ A as shown in figure. A square loop of side $a = 2$ m and resistance

$R = 4\,\Omega$ is located at a distance $2\,\text{m}$ from the wire. Now at any time $t = t_0$ the switch is open so the current I drops to zero. What will be the total charge (in nC) that passes through a corner of the square loop after $t = t_0$?



QUES 6.2.11 A circular loop of radius $5\,\text{m}$ carries a current $I = 2\,\text{A}$. If another small circular loop of radius $1\,\text{mm}$ lies a distance $12\,\text{m}$ above the large circular loop such that the planes of the two loops are parallel and perpendicular to the common axis as shown in figure then total flux through the small loop will be ____ fermi-weber.



QUES 6.2.12 A non magnetic medium at frequency $f = 1.6 \times 10^8\,\text{Hz}$ has permittivity $\varepsilon = 54\varepsilon_0$ and resistivity $\rho = 0.77\,\Omega\text{-m}$. What will be the ratio of amplitudes of conduction current to the displacement current ?

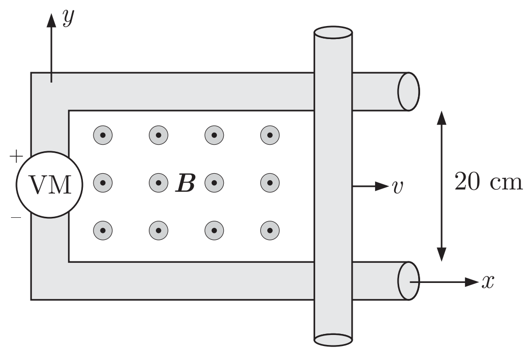
QUES 6.2.13 In a certain region a test charge is moving with an angular velocity $2\,\text{rad/sec}$ along a circular path of radius $2\,\text{m}$ centred at origin in the x-y plane. If the magnetic flux density in the region is $\boldsymbol{B} = 2\boldsymbol{a}_z\,\text{Wb/m}^2$ then the electric field viewed by an observer moving with the test charge is ____ V/m in $\boldsymbol{a}_\rho$ direction.

Common Data For Q. 13 and 14 :

In a non uniform magnetic field $\boldsymbol{B} = 8x^2\boldsymbol{a}_z\,\text{Tesla}$, two parallel rails with a separation of $20\,\text{cm}$ and connected with a voltmeter at it's one end is located in x-y plane as shown in figure. The Position of the bar which is sliding on the rails is given as

$$x = t(1 + 0.4t^2)$$

QUES 6.2.14 What will be the voltmeter reading (in volt) at $t = 0.4$ sec ?



QUES 6.2.15 What will be the voltmeter reading (in volt) at $x = 12$ cm ?

QUES 6.2.16 In a non conducting medium ($\sigma = 0$) magnetic field intensity at any point is given by $H = \cos(10^{10}t - bx)a_z$ A/m . The permittivity of the medium is $\varepsilon = 0.12$ nF/m and permeability of the medium is $\mu = 3\times10^{-5}$ H/m. D.C. field is not present in medium. Field satisfies Maxwell's equation, if $|b| =$ ____

QUES 6.2.17 Electric field in free space in given as

$$E = 5\sin(10\pi y)\cos(6\pi\times10^9 - bx)a_z$$

It satisfies Maxwell's equation for $|b| = ?$

QUES 6.2.18 8 A current is flowing along a straight wire from a point charge situated at the origin to infinity and passing through the point $(2,2,2)$. The circulation of the magnetic field intensity around the closed path formed by the triangle having the vertices $(2,0,0)$, $(0,2,0)$ and $(0,0,2)$ is equal to ____ Ampere.

QUES 6.2.19 A 50 turn rectangular loop of area 64 cm^2 rotates at 60 revolution per seconds in a magnetic field $B = 0.25\sin377t$ Wb/m^2 directed normal to the axis of rotation. What is the rms value of the induced voltage (in volt) ?

****

EXERCISE 6.3

MCQ 6.3.1 Match List I with List II and select the correct answer using the codes given below (Notations have their usual meaning)

	List-I		List-II
a	Ampere's circuital law	1.	$\nabla \cdot D = \rho_v$
b	Faraday's law	2.	$\nabla \cdot B = 0$
c	Gauss's law	3.	$\nabla \times E = -\frac{\partial B}{\partial t}$
d	Non existence of isolated magneticharge	4.	$\nabla \times H = J + \frac{\partial D}{\partial t}$

Codes :

	a	b	c	d
(A)	4	3	2	1
(B)	4	1	3	2
(C)	2	3	1	4
(D)	4	3	1	2

MCQ 6.3.2 Magneto static fields is caused by

(A) stationary charges

(B) steady currents

(C) time varying currents

(D) none of these

MCQ 6.3.3 Let A be magnetic vector potential and E be electric field intensity at certain time in a time varying EM field. The correct relation between E and A is

(A) $E = -\frac{\partial A}{\partial t}$ (B) $A = -\frac{\partial E}{\partial t}$

(C) $E = \frac{\partial A}{\partial t}$ (D) $A = \frac{\partial E}{\partial t}$

MCQ 6.3.4 A closed surface S defines the boundary line of magnetic medium such that the field intensity inside it is B. Total outward magnetic flux through the closed surface will be

(A) $B \cdot S$ (B) 0

(C) $B \times S$ (D) none of these

MCQ 6.3.5 The total magnetic flux through a conducting loop having electric field $E = 0$ inside it will be

(A) 0

(B) constant

(C) varying with time only

(D) varying with time and area of the surface both

MCQ 6.3.6 A cylindrical wire of a large cross section made of super conductor carries a current I. The current in the superconductor will be confined.

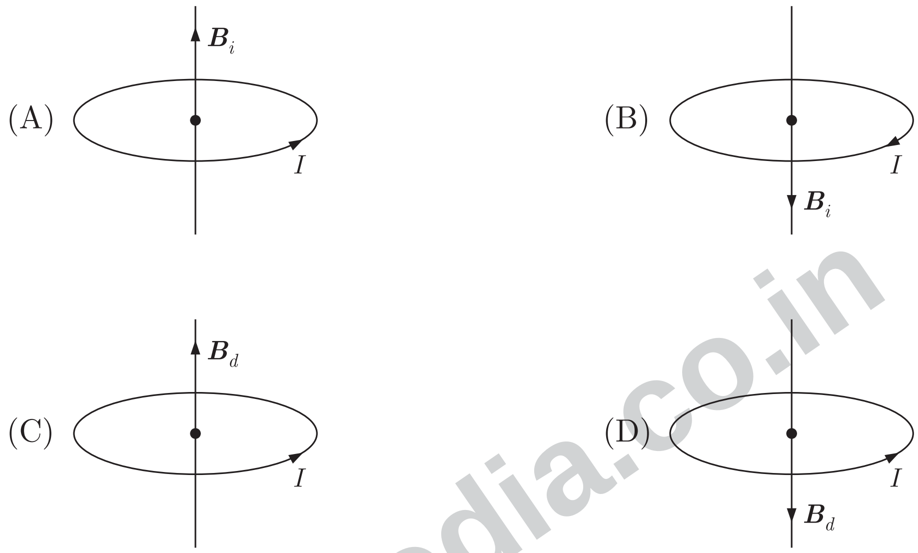
(A) inside the wire

(B) to the axis of cylindrical wire

(C) to the surface of the wire

(D) none of these

MCQ 6.3.7 If B_i denotes the magnetic flux density increasing with time and B_d denotes the magnetic flux density decreasing with time then which of the configuration is correct for the induced current I in the stationary loop ?



MCQ 6.3.8 A circular loop is rotating about z-axis in a magnetic field $B = B_0 \cos \omega t a_y$. The total induced voltage in the loop is caused by

(A) Transformer emf

(B) motion emf.

(C) Combination of (A) and (B)

(D) none of these

MCQ 6.3.9 For static magnetic field,

(A) $\nabla \times B = \rho$ (B) $\nabla \times B = \mu J$

(C) $\nabla \cdot B = \mu_0 J$ (D) $\nabla \times B = 0$

MCQ 6.3.10 Displacement current density is

(A) D (B) J

(C) $\partial D / \partial t$ (D) $\partial J / \partial t$

MCQ 6.3.11 The time varying electric field is

(A) $E = - \nabla V$ (B) $E = - \nabla V - \dot{A}$

(C) $E = - \nabla V - B$ (D) $E = - \nabla V - D$

MCQ 6.3.12 A field can exist if it satisfies

(A) Gauss's law

(B) Faraday's law

(C) Coulomb's law

(D) All Maxwell's equations

MCQ 6.3.13 Maxwell's equations give the relations between

(A) different fields

(B) different sources

(C) different boundary conditions

(D) none of these

MCQ 6.3.14 If E is a vector, then $\nabla \cdot \nabla \times E$ is

(A) 0 (B) 1

(C) does not exist (D) none of these

MCQ 6.3.15 The Maxwell's equation, $\nabla \cdot B = 0$ is due to

(A) $B = \mu H$ (B) $B = \frac{H}{\mu}$

(C) non-existence of a mono pole (D) none of these

MCQ 6.3.16 For free space,

(A) $\sigma = \infty$ (B) $\sigma = 0$

(C) $J \neq 0$ (D) none of these

MCQ 6.3.17 For time varying EM fields

(A) $\nabla \times H = J$ (B) $\nabla \times H = \dot{D} + J$

(C) $\nabla \times E = 0$ (D) none of these

****

EXERCISE 6.4

MCQ 6.4.1 A magnetic field in air is measured to be $B = B_0\left(\frac{x}{x^2+y^2}a_y - \frac{y}{x^2+y^2}a_x\right)$ What current distribution leads to this field ?
[Hint : The algebra is trivial in cylindrical coordinates.]

(A) $J = \frac{B_0 z}{\mu_0}\left(\frac{1}{x^2+y^2}\right), r \neq 0$

(B) $J = -\frac{B_0 z}{\mu_0}\left(\frac{2}{x^2+y^2}\right), r \neq 0$

(C) $J = 0, r \neq 0$

(D) $J = \frac{B_0 z}{\mu_0}\left(\frac{1}{x^2+y^2}\right), r \neq 0$

MCQ 6.4.2 For static electric and magnetic fields in an inhomogeneous source-free medium, which of the following represents the correct form of Maxwell's equations ?

(A) $\nabla \cdot E = 0$, $\nabla \times B = 0$

(B) $\nabla \cdot E = 0$, $\nabla \cdot B = 0$

(C) $\nabla \times E = 0$, $\nabla \times B = 0$

(D) $\nabla \times E = 0$, $\nabla \cdot B = 0$

MCQ 6.4.3 If C is closed curve enclosing a surface S, then magnetic field intensity H, the current density J and the electric flux density D are related by

(A) $\iint_S H \cdot dS = \oint_C \left(J + \frac{\partial D}{\partial t}\right) \cdot dl$

(B) $\int_S H \cdot dl = \oiint_S \left(J + \frac{\partial D}{\partial t}\right) \cdot dS$

(C) $\oiint_S H \cdot dS = \int_C \left(J + \frac{\partial D}{\partial t}\right) \cdot dl$

(D) $\oint_C H \cdot dl = \iint_S \left(J + \frac{\partial D}{\partial t}\right) \cdot dS$

MCQ 6.4.4 The unit of $\nabla \times H$ is

(A) Ampere

(B) Ampere/meter

(C) Ampere/meter^2

(D) Ampere-meter

MCQ 6.4.5 The Maxwell equation $\nabla \times H = J + \frac{\partial D}{\partial t}$ is based on

(A) Ampere's law

(B) Gauss' law

(C) Faraday's law

(D) Coulomb's law

MCQ 6.4.6 A loop is rotating about they y-axis in a magnetic field $B = B_0 \cos(\omega t + \phi)\, a_x$ T. The voltage in the loop is

(A) zero

(B) due to rotation only

(C) due to transformer action only

(D) due to both rotation and transformer action

MCQ 6.4.7 The credit of defining the following current is due to Maxwell

(A) Conduction current

(B) Drift current

(C) Displacement current

(D) Diffusion current

MCQ 6.4.8 A varying magnetic flux linking a coil is given by $\Phi = 1/3\lambda t^3$. If at time $t = 3$ s, the emf induced is 9 V, then the value of λ is.

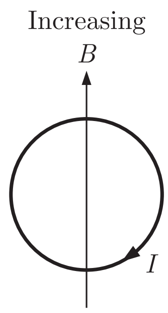
(A) zero

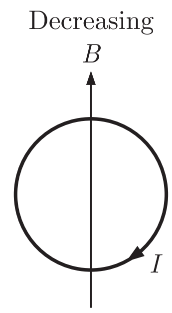
(B) 1 Wb/s^2

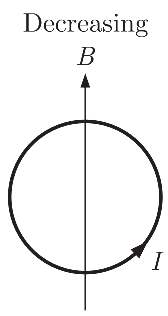
(C) -1 Wb/s^2

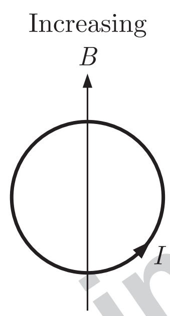
(D) 9 Wb/s^2

MCQ 6.4.9 Assuming that each loop is stationary and time varying magnetic field B, induces current I, which of the configurations in the figures are correct ?

1. 

2. 

3. 

4. 

(A) 1, 2, 3 and 4 (B) 1 and 3 only
(C) 2 and 4 only (D) 3 and 4 only

MCQ 6.4.10 Assertion (A) : For time varying field the relation $E = -\nabla V$ is inadequate.
Reason (R) : Faraday's law states that for time varying field $\nabla \times E = 0$
(A) Both Assertion (A) and Reason (R) are individually true and Reason (R) is the correct explanation of Assertion (A)
(B) Both Assertion (A) and Reason (R) are individually true but Reason (R) is not the correct explanation of Assertion (A)
(C) Assertion (A) is true but Reason (R) is false
(D) Assertion (A) is false but Reason (R) is true

MCQ 6.4.11 Who developed the concept of time varying electric field producing a magnetic field ?
(A) Gauss (B) Faraday
(C) Hertz (D) Maxwell

MCQ 6.4.12 A single turn loop is situated in air, with a uniform magnetic field normal to its plane. The area of the loop is 5 m^2 and the rate of charge of flux density is $2\text{ Wb/m}^2\text{/s}$. What is the emf appearing at the terminals of the loop ?
(A) -5 V (B) -2 V
(C) -0.4 V (D) -10 V

MCQ 6.4.13 Which of the following equations results from the circuital form of Ampere's law ?
(A) $\nabla \times E = -\frac{\partial B}{\partial t}$ (B) $\nabla \cdot B = 0$
(C) $\nabla \cdot D = \rho$ (D) $\nabla \times H = J + \frac{\partial D}{\partial t}$

MCQ 6.4.14 Assertion (A) : Capacitance of a solid conducting spherical body of radius a is given by $4\pi\varepsilon_0 a$ in free space.

Reason (R) : $\nabla \times H = j\omega\varepsilon E + J$

(A) Both A and R are individually true and R is the correct explanation of A.

(B) Both A and R are individually true but R is not the correct explanation of A.

(C) A is true but R is false

(D) A is false but R is true

MCQ 6.4.15 Two conducting thin coils X and Y (identical except for a thin cut in coil Y) are placed in a uniform magnetic field which is decreasing at a constant rate. If the plane of the coils is perpendicular to the field lines, which of the following statement is correct ? As a result, emf is induced in

(A) both the coils (B) coil Y only

(C) coil X only (D) none of the two coils

MCQ 6.4.16 Assertion (A) : Time varying electric field produces magnetic fields.
Reason (R) : Time varying magnetic field produces electric fields.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

MCQ 6.4.17 Match List I (Electromagnetic Law) with List II (Different Form) and select the correct answer using the code given below the lists :

	List-I		List-II
a.	Ampere's law	1.	$\nabla \cdot D = \rho_v$
b.	Faraday's law	2.	$\nabla \cdot J = -\frac{\partial h}{\partial t}$
c.	Gauss law	3.	$\nabla \times H = J + \frac{\partial D}{\partial t}$
d.	Current	4.	$\nabla \times E = -\frac{\partial B}{\partial t}$

Codes :

	a	b	c	d
(A)	1	2	3	4
(B)	3	4	1	2
(C)	1	4	3	2
(D)	3	2	1	4

MCQ 6.4.18 Two metal rings 1 and 2 are placed in a uniform magnetic field which is decreasing with time with their planes perpendicular to the field. If the rings are identical except that ring 2 has a thin air gap in it, which one of the following statements is correct ?

(A) No e.m.f is induced in ring 1

(B) An e.m.f is induced in both the rings

(C) Equal Joule heating occurs in both the rings

(D) Joule heating does not occur in either ring.

MCQ 6.4.19 Which one of the following Maxwell's equations gives the basic idea of radiation ?

(A) $\left.\begin{array}{l}\nabla \times H = \partial D/\partial t \\ \nabla \times E = \partial B/\partial t\end{array}\right\}$

(B) $\left.\begin{array}{l}\nabla \times E = -\partial B/\partial t \\ \nabla \cdot D = -\partial B/\partial t\end{array}\right\}$

(C) $\left.\begin{array}{l}\nabla \cdot D = \rho \\ \nabla \cdot D = 0\end{array}\right\}$

(D) $\left.\begin{array}{l}\nabla \cdot B = \rho \\ \nabla \times H = (\partial D/\partial t)\end{array}\right\}$

MCQ 6.4.20 Which one of the following is NOT a correct Maxwell equation ?

(A) $\nabla \times H = \frac{\partial D}{\partial t} + J$

(B) $\nabla \times E = \frac{\partial H}{\partial t}$

(C) $\nabla \cdot D = \rho$

(D) $\nabla \cdot B = 0$

MCQ 6.4.21 Match List I (Maxwell equation) with List II (Description) and select the correct answer :

List I

a. $\oint B \cdot dS = 0$

b. $\oint D \cdot dS = \int_v \rho_v dv$

c. $\oint E \cdot dl = -\int \frac{\partial B}{\partial t} \cdot dS$

d. $\oint H \cdot dl = \int \frac{\partial (D + J)}{\partial t} \cdot dS$

List II

1. The mmf around a closed path is equal to the conduction current plus the time derivative of the electric displacement current through any surface bounded by the path.
2. The emf around a closed path is equal to the time derivative is equal to the time derivative of the magnetic displacement through any surface bounded by the path.
3. The total electric displacement through the surface enclosing a volume is equal to total charge within the volume
4. The net magnetic flux emerging through any closed surface is zero.

Codes :

	a	b	c	d
(A)	1	3	2	4
(B)	4	3	2	1
(C)	4	2	3	1
(D)	1	2	3	4

MCQ 6.4.22 The equation of continuity defines the relation between

(A) electric field and magnetic field

(B) electric field and charge density

(C) flux density and charge density

(D) current density and charge density

MCQ 6.4.23 What is the generalized Maxwell's equation $\nabla \times H = J_c + \frac{\partial D}{\partial t}$ for the free space ?

(A) $\nabla \times H = 0$ (B) $\nabla \times H = J_c$

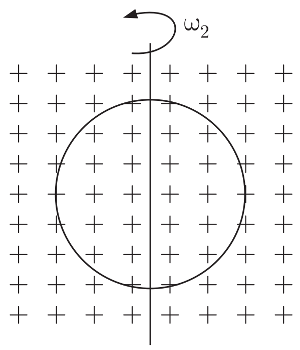
(C) $\nabla \times H = \frac{\partial D}{\partial t}$ (D) $\nabla \times H = D$

MCQ 6.4.24 Magnetic field intensity is $H = 3a_x + 7ya_y + 2xa_z$ A/m. What is the current density J A/m^2 ?

(A) $-2a_y$ (B) $-7a_z$

(C) $3a_x$ (D) $12a_y$

MCQ 6.4.25 A circular loop placed perpendicular to a uniform sinusoidal magnetic field of frequency ω_1 is revolved about an axis through its diameter at an angular velocity ω_2 rad/sec ($\omega_2 < \omega_1$) as shown in the figure below. What are the frequencies for the e.m.f induced in the loop ?



(A) ω_1 and ω_2 (B) $\omega_1, \omega_2 + \omega_2$ and ω_2

(C) $\omega_2, \omega_1 - \omega_2$ and ω_2 (D) $\omega_1 - \omega_2$ and $\omega_1 + \omega_2$

MCQ 6.4.26 Which one of the following is not a Maxwell's equation ?

(A) $\nabla \times H = (\sigma + j\omega\varepsilon)E$ (B) $F = Q(E + v \times B)$

(C) $\oint_c H \cdot dl = \oint_s J \cdot dS + \oint_s \frac{\partial D}{\partial t} \cdot dS$ (D) $\oint_S B \cdot dS = 0$

MCQ 6.4.27 Consider the following three equations :

1. $\nabla \times E = -\frac{\partial B}{\partial t}$

2. $\nabla \times H = J + \frac{\partial D}{\partial t}$

3. $\nabla \cdot B = 0$

Which of the above appear in Maxwell's equations ?

(A) 1, 2 and 3 (B) 1 and 2

(C) 2 and 3 (D) 1 and 3

MCQ 6.4.28 In free space, if $\rho_v = 0$, the Poisson's equation becomes

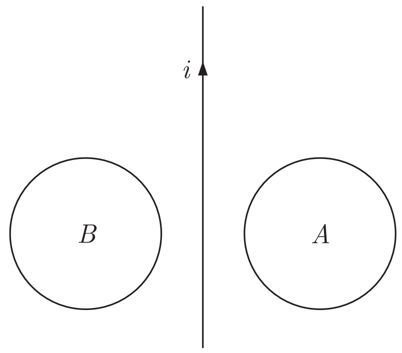
(A) Maxwell's divergence equation $\nabla \cdot B = 0$

(B) Laplacian equation $\nabla^2 V = 0$

(C) Kirchhoff's voltage equation $\Sigma V = 0$

(D) None of the above

MCQ 6.4.29 A straight current carrying conductor and two conducting loops A and B are shown in the figure given below. What are the induced current in the two loops ?



(A) Anticlockwise in A and clockwise in B
(B) Clockwise in A and anticlockwise in B
(C) Clockwise both in A and B
(D) Anticlockwise both in A and B

MCQ 6.4.30 Which one of the following equations is not Maxwell's equation for a static electromagnetic field in a linear homogeneous medium ?

(A) $\nabla \cdot B = 0$ (B) $\nabla \times D = \vec{0}$
(C) $\oint_c B \cdot dl = \mu_0 I$ (D) $\nabla^2 A = \mu_0 J$

MCQ 6.4.31 Match List I with List II and select the correct answer using the codes given below :

	List I		List II
a	Continuity equation	1.	$\nabla \times H = J + \frac{\partial D}{\partial t}$
b	Ampere's law	2.	$J = \frac{\partial D}{\partial t}$
c	Displacement current	3.	$\nabla \times E = -\frac{\partial B}{\partial t}$
d	Faraday's law	4.	$\nabla \times J = -\frac{\partial \rho_v}{\partial t}$

Codes :

	a	b	c	d
(A)	4	3	2	1
(B)	4	1	2	3
(C)	2	3	4	1
(D)	2	1	4	3

MCQ 6.4.32 The magnetic flux through each turn of a 100 turn coil is $(t^3 - 2t)$ milli-Webers where t is in seconds. The induced e.m.f at $t = 2$ s is

(A) 1 V (B) -1 V
(C) 0.4 V (D) -0.4 V

MCQ 6.4.33 Match List I (Type of field denoted by A) with List II (Behaviour) and select the correct answer using the codes given below :

	List I		List II
a	A static electric field in a charge free region	1.	$\nabla \cdot A = 0$, $\nabla \times A \neq 0$
b	A static electric field in a charged region	2.	$\nabla \cdot A \neq 0$, $\nabla \times A = 0$
c	A steady magnetic field in a current carrying conductor	3.	$\nabla \cdot A \neq 0$, $\nabla \times A \neq 0$
d	A time-varying electric field in a charged medium with time-varying magnetic field	4.	$\nabla \cdot A = 0$, $\nabla \times A = 0$

Codes :

	a	b	c	d
(A)	4	2	3	1
(B)	4	2	1	3
(C)	2	4	3	1
(D)	2	4	1	3

MCQ 6.4.34 Which one of the following pairs is not correctly matched ?

(A) Gauss Theorem : $\oint_s D \cdot ds = \oint_v \nabla \cdot Ddv$

(B) Gauss's Law : $\oint D \cdot ds = \oint_v \rho dv$

(C) Coulomb's Law : $V = -\frac{d\phi_m}{dt}$

(D) Stoke's Theorem : $\oint \xi \cdot dl = \oint_s (\nabla \times \xi) \cdot ds$

MCQ 6.4.35 Maxwell equation $\nabla \times E = -(\partial B/\partial t)$ is represented in integral form as

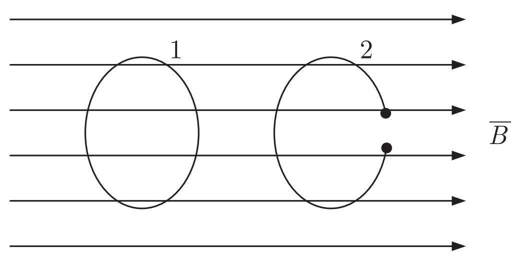
(A) $\oint E \cdot dl = -\frac{\partial}{\partial t}\oint B \cdot dl$

(B) $\oint E \cdot dl = -\frac{\partial}{\partial t}\int_s B \cdot ds$

(C) $\oint E \times dl = -\frac{\partial}{\partial t}\oint B \cdot dl$

(D) $\oint E \times dl = -\frac{\partial}{\partial t}\int_s B \cdot dl$

MCQ 6.4.36 Two conducting coils 1 and 2 (identical except that 2 is split) are placed in a uniform magnetic field which decreases at a constant rate as in the figure. If the planes of the coils are perpendicular to the field lines, the following statements are made :



1. an e.m.f is induced in the split coil 2
2. e.m.fs are induced in both coils

3. equal Joule heating occurs in both coils
4. Joule heating does not occur in any coil

Which of the above statements is/are true ?

(A) 1 and 4
(B) 2 and 4
(C) 3 only
(D) 2 only

MCQ 6.4.37

For linear isotropic materials, both E and H have the time dependence $e^{j\omega t}$ and regions of interest are free of charge. The value of $\nabla \times H$ is given by

(A) σE
(B) $j\omega\varepsilon E$
(C) $\sigma E + j\omega\varepsilon E$
(D) $\sigma E - j\omega\varepsilon E$

MCQ 6.4.38

Which of the following equations is/are not Maxwell's equations(s) ?

(A) $\nabla \cdot J = -\frac{\partial\rho_v}{\partial t}$
(B) $\nabla \cdot D = \rho_v$
(C) $\nabla \cdot E = -\frac{\partial B}{\partial t}$
(D) $\oint H \cdot dl = \int_s \left(\sigma E + \varepsilon\frac{\partial E}{\partial t}\right) \cdot ds$

Select the correct answer using the codes given below :

(A) 2 and 4
(B) 1 alone
(C) 1 and 3
(D) 1 and 4

MCQ 6.4.39

Assertion (A) : The relationship between Magnetic Vector potential A and the current density J in free space is

$$\nabla \times (\nabla \times A) = \mu_0 J$$

For a magnetic field in free space due to a dc or slowly varying current is $\nabla^2 A = -\mu_0 J$

Reason (R) : For magnetic field due to dc or slowly varying current $\nabla \cdot A = 0$.

(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

MCQ 6.4.40

Given that $\nabla \times H = J + \frac{\partial D}{\partial t}$

Assertion (A) : In the equation, the additional term $\frac{\partial D}{\partial t}$ is necessary.

Reason (R) : The equation will be consistent with the principle of conservation of charge.

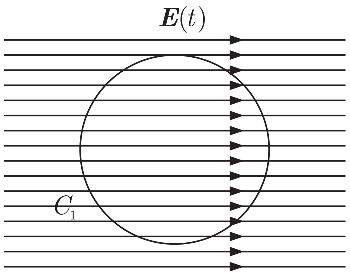
(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

MCQ 6.4.41

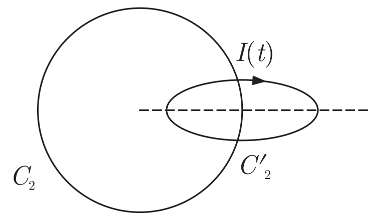
A circular loop is rotating about the y-axis as a diameter in a magnetic field $B = B_0 \sin\omega t a_x$ Wb/m^2. The induced emf in the loop is

(A) due to transformer emf only
(B) due to motional emf only
(C) due to a combination of transformer and motional emf
(D) zero

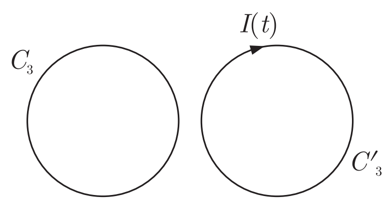
MCQ 6.4.42 Consider coils C_1, C_2, C_3 and C_4 (shown in the given figures) which are placed in the time-varying electric field $E(t)$ and electric field produced by the coils C'_2, C'_3 and C'_4 carrying time varying current $I(t)$ respectively :

1.


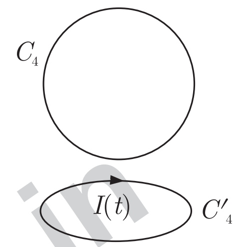
Time varying electric field $\boldsymbol{E}(t)$ parallel to the plane of coil C_1

2.


Coil planes are orthogonal

3.


Co-planer coils

4.


Coil planes are orthogonal

The electric field will induce an emf in the coils

(A) C_1 and C_2 (B) C_2 and C_3

(C) C_1 and C_3 (D) C_2 and C_4

MCQ 6.4.43 Match List I (Law/quantity) with List II (Mathematical expression) and select the correct answer :

	List I		List II
a.	Gauss's law	1.	$\nabla \cdot D = \rho$
b.	Ampere's law	2.	$\nabla \times E = -\frac{\partial B}{\partial t}$
c.	Faraday's law	3.	$\mathcal{P} = E \times H$
d.	Poynting vector	4.	$F = q(E + v \times B)$
		5.	$\nabla \times H = J_c + \frac{\partial D}{\partial t}$

Codes :

	a	b	c	d
(A)	1	2	4	3
(B)	3	5	2	1
(C)	1	5	2	3
(D)	3	2	4	1

****

SOLUTIONS 6.1

SOL 6.1.1 Option (C) is correct.

From Faraday's law, the relation between electric field and magnetic field is

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

Since the electric field inside a conducting sphere is zero.

i.e. $E = 0$

So the rate of change in magnetic flux density will be

$$\frac{\partial B}{\partial t} = -(\nabla \times E) = 0$$

Therefore $B(r,t)$ will be uniform inside the sphere and independent of time.

SOL 6.1.2 Option (A) is correct.

Electric field intensity experienced by the moving conductor ab in the presence of magnetic field B is given as

$$E = v \times B \qquad \text{where } v \text{ is the velocity of the conductor.}$$

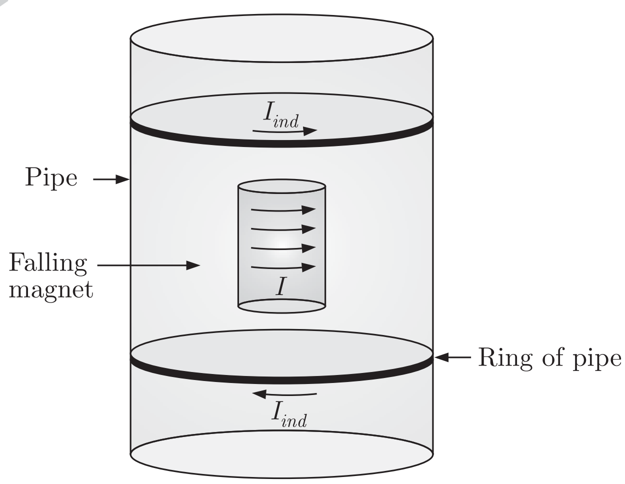
So, electric field will be directed from b to a as determined by right hand rule for the cross vector. Therefore, the voltage difference between the two ends of the conductor is given as

$$V_{ab} = -\int_a^b E \cdot dl$$

Thus, the positive terminal of voltage will be a and V_{ab} will be positive.

SOL 6.1.3 Option (A) is correct.

Consider a magnet bar being dropped inside a pipe as shown in figure.



Suppose the current I in the magnet flows counter clockwise (viewed from above) as shown in figure. So near the ends of pipe, it's field points upward. A ring of pipe below the magnet experiences an increasing upward flux as the magnet approaches and hence by Lenz's law a current will be induced in it such as to produce downward flux.

Thus, I_{ind} must flow clockwise which is opposite to the current in the magnet.

Since opposite currents repel each other so, the force exerted on the magnet due to the induced current is directed upward. Meanwhile a ring above the magnet experiences a decreasing upward flux; so it's induced current parallel to I and it attracts magnet upward. And flux through the rings next to the magnet bar is constant. So no current is induced in them.
Thus, for all we can say that the force exerted by the eddy current (induced current according to Lenz's law) on the magnet is in upward direction which causes the delay to reach the bottom. Whereas in the cases of unmagnetized bar no induced current is formed. So it reaches in fraction of time.
Thus, A and R both true and R is correct explanation of A.

SOL 6.1.4 Option (C) is correct.
The magnetic flux density inside a solenoid of n turns per unit length carrying current I is defined as

$$B = \mu_0 nI$$

Let the length of solenoid be l and its cross sectional radius be r. So, the total magnetic flux through the solenoid is

$$\Phi = (\mu_0 nI)(\pi r^2)(nl) \qquad (1)$$

Since the total magnetic flux through a coil having inductance L and carrying current I is given as

$$\Phi = LI$$

So comparing it with equation (1) we get,

$$L = \mu_0 n^2 I\pi^2 l$$

and as for a given solenoid, radius r and length l is constant therefore

$$L \propto n^2$$

SOL 6.1.5 Option (C) is correct.
The magnetic flux density inside the solenoid is defined as

$$B = \mu_0 nI$$

where $n \rightarrow$ no. of turns per unit length
$I \rightarrow$ current flowing in it.

So the total magnetic flux through the solenoid is

$$\Phi = \int B \cdot dS = (\mu_0 nI)(\pi a^2)$$

where $a \rightarrow$ radius of solenoid

Induced emf in a loop placed in a magnetic field is defined as

$$V_{emf} = -\frac{d\Phi}{dt}$$

where Φ is the total magnetic flux passing through the loop. Since the resistance R is looped over the solenoid so total flux through the loop will be equal to the total flux through the solenoid and therefore the induced emf in the loop of resistance will be

$$V_{emf} = -\pi a^2 \mu_0 n\frac{dI}{dt}$$

Since current I flowing in the solenoid is constant so, the induced emf is

$$V_{emf} = 0$$

and therefore the induced current in the loop will be zero.

SOL 6.1.6 Option (B) is correct.

It will be similar to the current in a solenoid.

So, the magnetic field will be in circumferential while the electric field is longitudinal.

SOL 6.1.7 Option (B) is correct.

In Assertion (A) the magnetic flux through each turn of both coils are equal So, the net magnetic flux through the two coils are respectively

$$\Phi_1 = N_1\Phi$$

and $$\Phi_2 = N_2\Phi$$

where Φ is the magnetic flux through a single loop of either coil and N_1, N_2 are the total no. of turns of the two coils respectively.

Therefore the induced emf in the two coils are

$$V_{\text{emf}\,1} = -\frac{d\Phi_1}{dt} = -N_1\frac{d\Phi}{dt}$$

$$V_{\text{emf}\,2} = -\frac{d\Phi_2}{dt} = -N_2\frac{d\Phi}{dt}$$

Thus, the ratio of the induced emf in the two loops are

$$\frac{V_{\text{emf}\,2}}{V_{\text{emf}\,1}} = \frac{N_2}{N_1}$$

Now, in Reason (R) : a primitive transformer is similar to the cylinder core carrying wound coils. It is the device in which by choosing the appropriate no. of turns, any desired secondary emf can be obtained.

So, both the statements are correct but R is not the explanation of A.

SOL 6.1.8 Option (B) is correct.

Electric flux density in the medium is given as

$$D = \varepsilon E = \varepsilon E_0\cos\omega t \qquad (E = E_0\cos\omega t)$$

Therefore the displacement current density in the medium is

$$J_d = \frac{\partial D}{\partial t} = -\omega\varepsilon E_0\sin\omega t$$

and the conduction current density in the medium is

$$J_c = \sigma E = \sigma E_0\cos\omega t$$

So, the ratio of amplitudes of conduction current density and displacement current density is

$$\frac{|J_c|}{|J_d|} = \frac{\sigma}{\omega\varepsilon}$$

SOL 6.1.9 Option (C) is correct.

Given the volume charge density, $\rho_v = 0$

So, from Maxwell's equation we have

$$\nabla\cdot D = \rho_v$$

$$\nabla\cdot D = 0 \qquad (1)$$

Now, the electric flux density in a medium is defined as

$$D = \varepsilon E \qquad \text{(where } \varepsilon \text{ is the permittivity of the medium)}$$

So, putting it in equation (1) we get,

$$\nabla\cdot(\varepsilon E) = 0$$

or, $$E\cdot(\nabla\varepsilon) + \varepsilon(\nabla\cdot E) = 0$$

and since $$\frac{\nabla\varepsilon}{\varepsilon} \approx 0 \Rightarrow \nabla\varepsilon \approx 0 \qquad \text{(given)}$$

Therefore, $$\nabla\cdot E \approx 0$$

SOL 6.1.10 Option (A) is correct.
Given the electric field intensity in time domain as

$$E = \frac{\sin\theta\cos(\omega t - kr)}{r}\boldsymbol{a}_\theta$$

So, the electric field intensity in phasor form is given as

$$E_s = \frac{\sin\theta}{r}e^{-jkr}\boldsymbol{a}_\theta$$

and $$\nabla \times E_s = \frac{1}{r}\frac{\partial}{kdr}(rE_{\theta s})\boldsymbol{a}_\phi = (-jk)\frac{\sin\theta}{r}e^{-jkr}\boldsymbol{a}_\phi$$

Therefore, from Maxwell's equation we get the magnetic field intensity as

$$H_s = -\frac{\nabla \times E_s}{j\omega r_0} = \frac{k}{\omega r_0}\frac{\sin\theta}{r}e^{-jkr}\boldsymbol{a}_\phi$$

SOL 6.1.11 Option (B) is correct.
Magnetic flux density produced at a distance ρ from a long straight wire carrying current I is defined as

$$B = \frac{\mu_0 I}{2\pi\rho}\boldsymbol{a}_\phi$$

where $\boldsymbol{a}_\phi$ is the direction of flux density as determined by right hand rule. So, the magnetic flux density produced by the straight conducting wire linking through the loop is normal to the surface of the loop.
Now consider a strip of width $d\rho$ of the square loop at distance ρ from the wire for which the total magnetic flux linking through the square loop is given as

$$\Phi = \int_S B \cdot dS$$

$$= \frac{\mu_0 I}{2\pi}\int_\rho^{\rho+a}\frac{1}{\rho}(ad\rho) \qquad \text{(area of the square loop is } dS = ad\rho)$$

$$= \frac{\mu_0 Ia}{2\pi}\ln\left(\frac{\rho+a}{\rho}\right)$$

The induced emf due to the change in flux (when pulled away) is given as

$$V_{\text{emf}} = -\frac{d\Phi}{dt} = -\frac{\mu_0 Ia}{2\pi}\frac{d}{dt}\left[\ln\left(\frac{\rho+a}{\rho}\right)\right]$$

Therefore, $$V_{\text{emf}} = -\frac{\mu_0 Ia}{2\pi}\left(\frac{1}{\rho+a}\frac{d\rho}{dt} - \frac{1}{\rho}\frac{d\rho}{dt}\right)$$

Given $$\frac{d\rho}{dt} = \text{velocity of loop} = 5\,\text{m/s}$$

and since the loop is currently located at $3\,\text{m}$ distance from the straight wire, so after $0.6\,\text{sec}$ it will be at

$$\rho = 3 + (0.6)\times v \qquad (v \rightarrow \text{velocity of the loop})$$

$$= 3 + 0.6\times 5 = 6\,\text{m}$$

So, $$V_{\text{emf}} = -\frac{\mu_0\times(30)\times 2}{2\pi}\left[\frac{1}{8}(5) - \frac{1}{6}(5)\right] \qquad (a = 2\,\text{m}, I = 30\,\text{A})$$

$$= 25\times 10^{-7}\,\text{volt} = 2.5\,\mu\text{volt}$$

SOL 6.1.12 Option (B) is correct.
Since total magnetic flux through the loop depends on the distance from the straight wire and the distance is constant. So the flux linking through the loop will be constant, if it is pulled parallel to the straight wire. Therefore the induced emf in the loop is

$$V_{\text{emf}} = -\frac{d\Phi}{dt} = 0 \qquad (\Phi \text{ is constant})$$

SOL 6.1.13 Option (D) is correct.

Total magnetic flux through the solenoid is given as

$$\Phi = \mu_0 nI$$

where n is the no. of turns per unit length of solenoid and I is the current flowing in the solenoid.

Since the solenoid carries current that is increasing linearly with time

i.e. $I \propto t$

So the net magnetic flux through the solenoid will be

$$\Phi \propto t$$

or, $\Phi = kt$ where k is a constant.

Therefore the emf induced in the loop consisting resistances R_A, R_B is

$$V_{emf} = -\frac{d\Phi}{dt}$$

$$V_{emf} = -k$$

and the current through R_1 and R_2 will be

$$I_{ind} = -\frac{k}{R_1 + R_2}$$

Now according to Lenz's law the induced current I in a loop flows such as to produce a magnetic field that opposes the change in $B(t)$.

i.e. the induced current in the loop will be opposite to the direction of current in solenoid (in anticlockwise direction).

So, $V_A = I_{ind} R_A = -\frac{kR_A}{R_A + R_B}$

and $V_B = -I_{ind} R_B = \left(\frac{kR_B}{R_A + R_B}\right)$

Thus, the ratio of voltmeter readings is

$$\frac{V_A}{V_B} = -\frac{R_A}{R_B}$$

SOL 6.1.14 Option (D) is correct.

Induced emf in the conducting loop formed by rail, bar and the resistor is given by

$$V_{emf} = -\frac{d\Phi}{dt}$$

where Φ is total magnetic flux passing through the loop.

The bar is located at a distance x from the resistor at time t. So the total magnetic flux passing through the loop at time t is

$\Phi = \int B \cdot dS = \mathrm{B}lx$ where l is separation between the rails

Now the induced emf in a loop placed in magnetic field is defined as

$$V_{emf} = -\frac{d\Phi}{dt}$$

where Φ is the total magnetic flux passing through the loop. Therefore the induced emf in the square loop is

$$V_{emf} = -\frac{d}{dt}(\mathrm{B}lx) = -\mathrm{B}l\frac{dx}{dt} \qquad (\Phi = \mathrm{B}lx)$$

Since from the given figure, we have

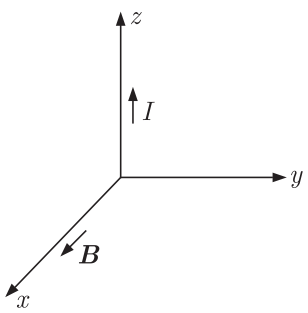
$$l = 5\text{ m}$$

$$B = 2\text{ T}$$

and $$dx/dt \to \text{velocity of bar} = 4\text{ m/s}$$
So, induced emf is
$$V_{emf} = -(2)(5)(4) = -40\text{ volt}$$
Therefore the current in the bar loop will be
$$I = \frac{V_{emf}}{R} = -\frac{40}{10} = -4\text{ A}$$

SOL 6.1.15 Option (B) is correct.

As obtained in the previous question the current flowing in the sliding bar is
$$I = -4\text{ A}$$
Now we consider magnetic field acts in $\boldsymbol{a}_x$ direction and current in the sliding bar is flowing in $+\boldsymbol{a}_z$ direction as shown in the figure.



Therefore, the force exerted on the bar is
$$\boldsymbol{F} = \int I d\boldsymbol{l} \times \boldsymbol{B} = \int_0^5 (-4dz\boldsymbol{a}_z) \times (2\boldsymbol{a}_x)$$
$$= -8\boldsymbol{a}_y[z]_0^5 = -40\boldsymbol{a}_y\text{ N}$$
i.e. The force exerted on the sliding bar is in opposite direction to the motion of the sliding bar.

SOL 6.1.16 Option (C) is correct.

Given the magnetic flux density through the square loop is
$$\boldsymbol{B} = 7.5\cos(120\pi t - 30^\circ)\boldsymbol{a}_z$$
So the total magnetic flux passing through the loop will be
$$\Phi = \oint_S \boldsymbol{B}\cdot d\boldsymbol{S}$$
$$= [-7.5\cos(120\pi t - 30^\circ)\boldsymbol{a}_z](1\times 1)(-\boldsymbol{a}_z)$$
$$= 7.5\cos(120\pi t - 30^\circ)$$
Now, the induced emf in the square loop is given by
$$V_{emf} = -\frac{d\Phi}{dt} = 7.5\times 120\pi\sin(120\pi t - 30^\circ)$$
The polarity of induced emf (according to Lenz's law) will be such that induced current in the loop will be in opposite direction to the current $I(t)$ shown in the figure. So we have
$$I(t) = -\frac{V_{emf}}{R}$$
$$= -\frac{7.5\times 120\pi}{500}\sin(120\pi t - 30^\circ) \qquad (R = 250 + 250 = 500\,\Omega)$$
$$= -5.7\sin(120\pi t - 30^\circ)$$

SOL 6.1.17 Option (A) is correct.

Consider the mutual inductance between the rectangular loop and straight wire be M. So applying KVL in the rectangular loop we get,

$$M\frac{di_1}{dt} = L\frac{di_2}{dt} + Ri_2 \qquad ...(1)$$

Now from the shown figure (b), the current flowing in the straight wire is given as

$$i_1 = I_1u(t) - I_1u(t-T) \quad (I_1 \text{ is amplitude of the current})$$

or, $$\frac{di_1}{dt} = I_1\delta(t) - I_1\delta(t-T) \qquad (2)$$

So, at $t = 0$ $$\frac{di_1}{dt} = I_1$$

and $$MI_1 = L\frac{di_2}{dt} + Ri_2 \qquad \text{(from equation (1))}$$

Solving it we get

$$i_2 = \frac{M}{L}I_1e^{-(R/L)t} \qquad \text{for } 0 < t < T$$

Again in equation (2) at $t = T$ we have

$$\frac{di_1}{dt} = -I_1$$

and $$-MI_1 = L\frac{di_2}{dt} + Ri_2 \qquad \text{(from equation (1))}$$

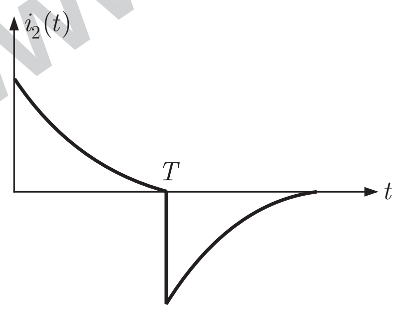
Solving it we get

$$i_2 = -\frac{M}{L}I_1e^{-(R/L)(t-T)} \qquad \text{for } t > T$$

Thus, the current in the rectangular loop is

$$i_2 = \begin{cases} \frac{M}{L}I_1e^{-(R/L)t} & 0 < t < T \\ -\frac{M}{L}I_1e^{-(R/L)(t-T)} & t > T \end{cases}$$

Plotting i_2 versus t we get



SOL 6.1.18 Option (A) is correct.

Total magnetic flux passing through the loop formed by the resistance, bar and the rails is given as:

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$= \mathbf{B} \cdot \mathbf{S} = [0.2\cos\omega t\mathbf{a}_x] \cdot [0.5(1-y)\mathbf{a}_x]$$

$$= 0.1[1 - 0.5(1-\cos\omega t)]\cos\omega t \quad (y = 0.5(1-\cos\omega t)\,\text{m})$$

$$= 0.05\cos\omega t(1+\cos\omega t) = 0.05(\cos\omega t + \cos^2\omega t)$$

So, the induced emf in the loop is

$$V_{emf} = -\frac{d\Phi}{dt}$$

and as determined by Lenz's law, the induced current will be flowing in opposite direction to the current i. So the current i in the loop will be

$$\begin{aligned} i &= -\frac{V_{emf}}{R} = -\frac{1}{R}\left(-\frac{d\Phi}{dt}\right) \\ &= \frac{0.05}{5}[-\omega\sin\omega t - 2\omega\cos\omega t\sin\omega t] \\ &= -0.01\omega\sin\omega t(1+2\cos\omega t) \end{aligned}$$

SOL 6.1.19 Option (D) is correct.

Given the electric flux density in the medium is

$$D = 1.33\sin(3\times10^8 t - 0.2x)\boldsymbol{a}_y\ \mu\text{C/m}^2$$

So, the electric field intensity in the medium is given as

$$E = \frac{D}{\varepsilon} \qquad \text{where } \varepsilon \text{ is the permittivity of the medium}$$

or,
$$E = \frac{D}{\varepsilon_r\varepsilon_0} = \frac{1.33\times10^{-6}\sin(3\times10^8 t - 0.2x)}{10\times8.85\times10^{-12}}\boldsymbol{a}_y \quad (\varepsilon_r = 10)$$
$$= 1.5\times10^4\sin(3\times10^8 t - 0.2x)\boldsymbol{a}_y$$

Now, from maxwell's equation we have

$$\nabla\times E = -\frac{\partial B}{\partial t}$$

or,
$$\begin{aligned} \frac{\partial B}{\partial t} &= -\nabla\times E \\ &= -\frac{\partial E_y}{\partial x}\boldsymbol{a}_z \\ &= -(-0.2)\times(1.5\times10^4)\cos(3\times10^8 t - 0.2x)\boldsymbol{a}_y \\ &= 3\times10^3\cos(3\times10^8 t - 0.2x)\boldsymbol{a}_y \end{aligned}$$

Integrating both sides, we get the magnetic flux density in the medium as

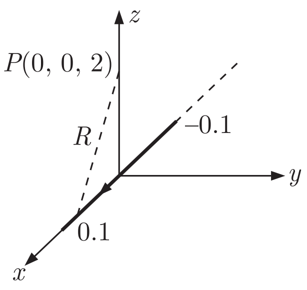
$$\begin{aligned} B &= \int 3\times10^3\cos(3\times10^8 t - 0.2x)\boldsymbol{a}_y \\ &= \frac{3\times10^3}{3\times10^8}\sin(3\times10^8 t - 0.2x)\boldsymbol{a}_y \\ &= 10^{-5}\sin(3\times10^8 t - 0.2x)\boldsymbol{a}_y \text{ Tesla} \end{aligned}$$

Therefore the magnetic field intensity in the medium is

$$H = \frac{B}{\mu} = \frac{B}{\mu_r\mu_0} = \frac{10^{-5}\sin(3\times10^8 t - 0.2x)}{2\times4\pi\times10^{-7}} \qquad \mu_r = 2$$

Thus
$$H = 4\sin(3\times10^8 t - 0.2x)\boldsymbol{a}_y \text{ A/m}$$

SOL 6.1.20 Option (B) is correct.



The magnetic vector potential for a direct current flowing in a filament is given as

$$A = \int \frac{\mu_0 I}{4\pi R}\boldsymbol{a}_x dx$$

Here current $I(t)$ flowing in the filament shown in figure is varying with time as

$$I(t) = 8t \text{ A}$$

So, the retarded vector potential at the point P will be given as

$$A = \int \frac{\mu_0 I(t - R/c)}{4\pi R} a_x dx$$

where R is the distance of any point on the filamentary current from P as shown in the figure and c is the velocity of waves in free space. So, we have

$$R = \sqrt{x^2 + 4} \text{ and } c = 3 \times 10^8 \text{ m/s}$$

Therefore, $$A = \int_{x=-0.1}^{0.1} \frac{\mu_0 8(t - R/c)}{4\pi R} a_x dx$$

$$= \frac{8\mu_0}{4\pi}\left[\int_{-0.1}^{0.1} \frac{t}{\sqrt{x^2+4}} dx - \int_{-0.1}^{0.1} \frac{1}{c} dx\right]$$

$$= 8 \times 10^{-7} t\left[\ln\left(x + \sqrt{x^2+4}\right)\right]_{-0.1}^{0.1} - \frac{8 \times 10^{-7}}{3 \times 10^8}[x]_{-0.1}^{0.1}$$

$$= 8 \times 10^{-7} t \ln\left(\frac{0.1 + \sqrt{4.01}}{-0.1 + \sqrt{4.01}}\right) - 0.53 \times 10^{-15}$$

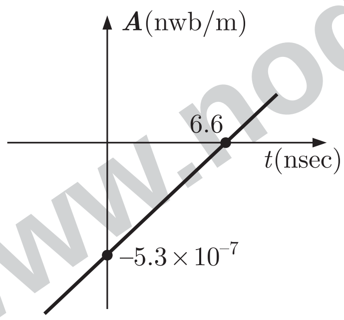
$$= 8 \times 10^{-8} t - 0.53 \times 10^{-15}$$

or, $$A = (80t - 5.3 \times 10^{-7}) a_x \text{ nWb/m} \qquad (1)$$

So, when $A = 0$ $\qquad t = 6.6 \times 10^{-9} = 6.6 \text{ n sec}$

and when $t = 0$ $\qquad A = -5.3 \times 10^{-7} \text{ nWb/m}$

From equation (1) it is clear that A will be linearly increasing with respect to time. Therefore the plot of A versus t is



NOTE :
Time varying potential is usually called the retarded potential.

SOL 6.1.21 Option (A) is correct.

The force experienced by a test charge q in presence of both electric field E and magnetic field B in the region will be evaluated by using Lorentz force equation as

$$F = q(E + v \times B)$$

So, putting the given three forces and their corresponding velocities in above equation we get the following relations

$$q(a_y + a_z) = q(E + a_x \times B) \qquad (1)$$

$$qa_y = q(E + a_y \times B) \qquad (2)$$

$$q(2a_y + a_z) = q(E + a_z \times B) \qquad (3)$$

Subtracting equation (2) from (1) we get

$$a_z = (a_x - a_y) \times B \qquad (4)$$

and subtracting equation (1) from (3) we get

$$a_y = (a_z - a_x) \times B \qquad (5)$$

Now we substitute $B = B_x a_x + B_y a_y + B_z a_z$ in eq (4) to get

$$a_z = B_y a_z - B_z a_y + B_x a_z - B_z a_x$$

So, comparing the x, y and z components of the two sides we get

$$B_x + B_y = 1$$

and $$B_z = 0$$

Again by substituting $B = B_x a_x + B_y a_y + B_z a_z$ in eq (5), we get

$$a_y = B_x a_y - B_y a_x - B_y a_z + B_z a_y$$

So, comparing the x, y and z components of the two sides we get

$$B_x + B_z = 1$$

and $$B_y = 0$$

as calculated above $B_z = 0$, therefore $B_x = 1$

Thus, the magnetic flux density in the region is

$$B = a_x \text{ Wb/m}^2 \qquad (B_x = 1,\ B_y = B_z = 0)$$

SOL 6.1.22 Option (C) is correct.

As calculated in previous question the magnetic flux density in the region is

$$B = a_x \text{ Wb/m}^2$$

So, putting it in Lorentz force equation we get

$$F = q(E + V \times B)$$

or, $$q(a_y + a_z) = q(E + a_x \times a_x)$$

Therefore, the electric field intensity in the medium is

$$E = a_y + a_z \text{ V/m}$$

SOL 6.1.23 Option (C) is correct.

Given

Retarded scalar potential, $\quad V = y(x - ct)$ volt

and retarded vector potential, $\quad A = y\left(\frac{x}{c} - t\right)a_x$ Wb/m

Now the magnetic flux density in the medium is given as

$$B = \nabla \times A$$

$$= -\frac{\partial A_y}{\partial y} a_z = \left(t - \frac{x}{c}\right)a_z \text{ Tesla} \qquad (1)$$

So, the magnetic field intensity in the medium is

$$H = \frac{B}{\mu_0} \qquad (\mu_0 \text{ is the permittivity of the medium})$$

$$= \frac{1}{\mu_0}\left(t - \frac{x}{c}\right)a_z \text{ A/m} \qquad (2)$$

and the electric field intensity in the medium is given as

$$E = -\nabla V - \frac{\partial A}{\partial t}$$

$$= -(x - ct)a_y - y a_x + y a_x = (ct - x)a_y \qquad (3)$$

So, the electric flux density in the medium is

$$D = \varepsilon_0 E \qquad (\varepsilon_0 \text{ is the permittivity of the medium})$$

$$= \varepsilon_0 (ct - x) a_y \text{ C/m}^2 \qquad (4)$$

Now we determine the condition for the field to satisfy all the four Maxwell's equation.

(a) $$\nabla \cdot D = \rho_v$$

or, $$\rho_v = \nabla \cdot [\varepsilon_0 (ct - x) a_y] \qquad \text{(from equation (4))}$$

$$= 0$$

It means the field satisfies Maxwell's equation if $\rho_v = 0$.

(b) $\nabla \cdot B = 0$

Now, $\nabla \cdot B = \nabla \cdot \left[\left(t - \frac{x}{c}\right)a_z\right] = 0$ (from equation (1))

So, it already, satisfies Maxwell's equation

(c) $\nabla \times H = J + \frac{\partial D}{\partial t}$

Now, $\nabla \times H = -\frac{\partial H_z}{\partial x} a_y = \frac{1}{\mu_0 c} a_y = \sqrt{\frac{\varepsilon_0}{\mu_0}} a_y$ (from equation (2))

and from equation (4) we have

$$\frac{\partial D}{\partial t} = \varepsilon_0 c a_y = \sqrt{\frac{\varepsilon_0}{\mu_0}} a_y \qquad \left(\text{Since in free space } c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right)$$

Putting the two results in Maxwell's equation, we get the condition

$$J = 0$$

(d) $\nabla \times E = -\frac{\partial B}{\partial t}$

Now $\nabla \times E = \frac{\partial E_y}{\partial x} a_z = -a_z$

$$\frac{\partial B}{\partial t} = a_z$$

So, it already satisfies Maxwell's equation. Thus, by combining all the results we get the two required conditions as $J = 0$ and $\rho_v = 0$ for the field to satisfy Maxwell's equation.

SOL 6.1.24 Option (A) is correct.

Given the magnetic flux density through the loop is

$$B = -2/x\, a_z$$

So the total magnetic flux passing through the loop is given as

$$\Phi = \int B \cdot dS = \int_x^{x+2}\int_y^{y+2}\left(-\frac{2}{x}a_z\right)\cdot(-dxdya_z)$$

$$= \left(2\ln\frac{x+2}{x}\right)(2) = 4\ln\left(\frac{x+2}{x}\right)$$

Therefore, the circulation of induced electric field in the loop is

$$\oint_C E \cdot dl = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left[4\ln\left(\frac{x+2}{x}\right)\right]$$

$$= -\frac{4}{\left(\frac{x+2}{x}\right)}\frac{d}{dt}\left(\frac{x+2}{x}\right)$$

$$= -\frac{4x}{x+2}\left(-\frac{2}{x^2}\frac{dx}{dt}\right)$$

$$= \frac{8}{x(x+2)}(2) = \frac{16}{x(x+2)} \qquad \left(\frac{dx}{dt} = v = 2a_x\right)$$

SOL 6.1.25 Option (A) is correct.

As the magnetic flux density for $\rho < 4$ is $B = 0$ so, the total flux passing through the closed loop defined by $\rho = 4$ m is

$$\Phi = \int B \cdot dS = 0$$

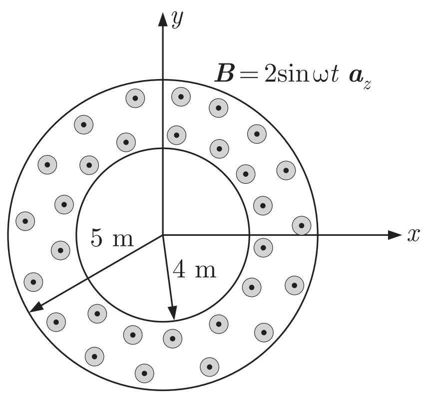
So, the induced electric field circulation for the region $\rho < 4$ m is given as

$$\oint_C E \cdot dl = -\frac{d\Phi}{dt} = 0$$

or, $E = 0$ for $\rho < 4\text{ m}$

SOL 6.1.26 Option (B) is correct.

As the magnetic field for the region $\rho < 4\text{ m}$ and $\rho > 5\text{ m}$ is zero so we get the distribution of magnetic flux density as shown in figure below.



At any distance ρ from origin in the region $4 < \rho < 5\text{ m}$, the circulation of induced electric field is given as

$$\oint_C \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left(\int \boldsymbol{B} \cdot d\boldsymbol{S}\right)$$

$$= -\frac{d}{dt}\left[2\sin\omega t(\pi\rho^2 - \pi 4^2)\right]$$

$$= -2\omega\cos\omega t(\pi\rho^2 - 16\pi)$$

or,
$$E(2\pi\rho) = -2\omega\cos\omega t(\pi\rho^2 - 16\pi)$$

$$E = -\frac{2(\rho^2 - 16)\omega\cos\omega t}{2\rho}$$

So, the induced electric field intensity at $\rho = 4.5\text{ m}$ is

$$\boldsymbol{E} = -\frac{2}{4.5}((4.5)^2 - 16)\omega\cos\omega t$$

$$= -\frac{17}{18}\omega\cos\omega t$$

SOL 6.1.27 Option (B) is correct.

For the region $\rho > 5\text{ m}$ the magnetic flux density is 0 and so the total magnetic flux passing through the closed loop defined by $\rho = 5\text{ m}$ is

$$\Phi = \int_0^5 \boldsymbol{B} \cdot d\boldsymbol{S} = \int_0^4 \boldsymbol{B} \cdot d\boldsymbol{S} + \int_4^5 \boldsymbol{B} \cdot d\boldsymbol{S}$$

$$= 0 + \int_4^5 (2\sin\omega t)\boldsymbol{a}_z \cdot d\boldsymbol{S}$$

$$= (2\sin\omega t)\left[\pi(5)^2 - \pi(4)^2\right] = 18\pi\sin\omega t$$

So, the circulation of magnetic flux density for any loop in the region $\rho > 5\text{ m}$ is

$$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d\psi}{dt}$$

$$E(2\pi\rho) = -\frac{d}{dt}(18\pi\sin\omega t)$$

$$= -18\pi\omega\cos\omega t$$

So, the induced electric field intensity in the region $\rho > 5\text{ m}$ is

$$\boldsymbol{E} = \frac{-18\pi\omega\cos\omega t}{2\pi\rho}\boldsymbol{a}_\phi$$

$$= -\frac{9}{\rho}\omega\cos\omega t\boldsymbol{a}_\phi$$

SOL 6.1.28 Option (D) is correct.

The distribution of magnetic flux density and the resistance in the circuit are same as given in section A (Q. 31) so, as calculated in the question, the two voltage drops in the loop due to magnetic flux density $\boldsymbol{B} = 0.1t\,\boldsymbol{a}_z$ are

$$V_1 = 33.3\text{ mV}$$

and $$V_2 = 66.67\text{ mV} = 66.7\text{ mV}$$

Now V_3 (voltmeter) which is directly connected to terminal cd is in parallel to both V_2 and V_1. It must be kept in mind that the loop formed by voltmeter V_3 and resistance $2\,\Omega$ also carries the magnetic flux density crossing through it. So, in this loop the induced emf will be produced which will be same as the field produced in loop $abcd$ at the enclosed fluxes will be same.

Therefore as calculated above induced emf in the loop of V_3 is

$$V_{\text{emf}} = 100\text{ mV}$$

According to lenz's law it's polarity will be opposite to V_3 and so

$$-V_{\text{emf}} = V_1 + V_3$$

or, $$V_3 = 100 - 33.3 = 66.7\text{ mV}$$

SOL 6.1.29 Option (D) is correct.

The induced emf in a closed loop is defined as

$$V_{\text{emf}} = -\frac{d\Phi}{dt}$$

where Φ is the total magnetic flux passing through the square loop

At any time t, angle between $\boldsymbol{B}$ and $d\boldsymbol{S}$ is θ since $\boldsymbol{B}$ is in $\boldsymbol{a}_y$ direction so the total magnetic flux passing through the square loop is

$$\Phi = \int \boldsymbol{B}\cdot d\boldsymbol{S}$$
$$= (B)(S)\cos\theta$$
$$= (5\times10^{-3})(20\times10^{-3}\times20\times10^{-3})\cos\theta$$
$$= 2\times10^{-6}\cos\theta$$

Therefore the induced emf in the loop is

$$V_{\text{emf}} = -\frac{d\Phi}{dt}$$
$$= -2\times10^{-6}\frac{d}{dt}(\cos\theta)$$
$$= 2\times10^{-6}\sin\theta\frac{d\theta}{dt}$$

and as $$\frac{d\theta}{dt} = \text{angular velocity} = 2\text{ rad/sec}$$

So, $$V_{\text{emf}} = (2\times10^{-6})\sin\theta(2)$$
$$= 4\times10^{-6}\sin\theta\text{ V/m} = 4\sin\theta\,\mu\text{V/m}$$

SOL 6.1.30 Option (B) is correct.

As calculated in previous question the induced emf in the closed square loop is

$$V_{\text{emf}} = 4\sin\theta\,\mu\text{V/m}$$

So the induced current in the loop is

$$I = \frac{V_{\text{emf}}}{R}$$ where R is the resistance in the loop.

$$= \frac{4\sin\theta \times 10^{-6}}{40 \times 10^{-3}} \qquad (R = 40\ \text{m}\Omega)$$
$$= 0.1\sin\theta\ \text{mA}$$

SOL 6.1.31 Option (C) is correct.
The total magnetic flux through the square loop is given as
$$\Phi = \oint B \cdot dS = (B_0 \sin\omega t)(S)\cos\theta$$
So, the induced emf in the loop is
$$V_{\text{emf}} = -\frac{d\Phi}{dt} = -\frac{d}{dt}[(B_0\sin\omega t)(S)\cos\theta]$$
$$= -B_0 S\frac{d}{dt}[\sin\omega t\cos\omega t] \qquad (\theta = \omega t)$$
$$= -B_0 S\cos 2\omega t$$
Thus, the maximum value of induced emf is
$$|V_{emf}| = B_0 S\omega$$

SOL 6.1.32 Option (C) is correct.
e.m.f. induced in the loop due to the magnetic flux density is given as
$$V_{emf} = -\frac{\partial\Phi}{\partial t} = -\frac{\partial}{\partial t}(10\cos 120\pi t)(\pi\rho^2)$$
$$= -\pi(10\times 10^{-2})^2 \times (120\pi)(-10\sin 120\pi t)$$
$$= 12\pi^2\sin 120\pi t$$
As determined by Lenz's law the polarity of induced e.m.f will be such that b is at positive terminal with respect to a.
i.e. $$V_{ba} = V_{emf} = 12\pi^2\sin 120\pi t$$
or $$V_{ab} = -12\pi^2\sin 120\pi t$$
$$= -118.43\sin 120\pi t\ \text{Volt}$$

SOL 6.1.33 Option (D) is correct.
As calculated in previous question, the voltage induced in the loop is
$$V_{ab} = -12\pi^2\sin 120\pi t$$
Therefore, the current flowing in the loop is given as
$$I(t) = -\frac{V_{ab}}{250} = \frac{12\pi^2\sin 120\pi t}{250}$$
$$= 0.47\sin 120\pi t$$

****

SOLUTIONS 6.2

SOL 6.2.1 Correct answer is 0.

As the conducting loop is falling freely So, the flux through loop will remain constant. Therefore, the voltage induced in the loop will be zero.

SOL 6.2.2 Correct answer is -4.

The magnetic flux density passing through the loop is given as

$$B = 4z^3t^2\mathbf{a}_x$$

Since the flux density is directed normal to the plane $x = 0$ so the total magnetic flux passing through the square loop located in the plane $x = 0$ is

$$\Phi = \int B \cdot dS = \int_{y=0}^{1}\int_{z=0}^{1}(4z^3t^2)\,dydz = t^2 \qquad (dS = (dydz)\,\mathbf{a}_x)$$

Induced emf in a loop placed in magnetic field is defined as

$$V_{\text{emf}} = -\frac{d\Phi}{dt}$$

where Φ is the total magnetic flux passing through the loop. So the induced emf in the square loop is

$$V_{\text{emf}} = -\frac{d(t^2)}{dt} = -2t \qquad (\Phi = t^2)$$

Therefore at time $t = 2$ sec the induced emf is

$$V_{\text{emf}} = -4 \text{ volt}$$

SOL 6.2.3 Correct answer is 4.05 .

Magnetic flux density produced at a distance ρ from a long straight wire carrying current I is defined as

$$B = \frac{\mu_0 I}{2\pi\rho}\mathbf{a}_\phi$$

where $\mathbf{a}_\phi$ is the direction of flux density as determined by right hand rule. So the flux density produced by straight wire at a distance ρ from it is

$$B = \frac{\mu_0 I}{2\pi\rho}\mathbf{a}_n \qquad (\mathbf{a}_n \text{ is unit vector normal to the loop})$$

Therefore the total magnet flux passing through the loop is

$$\Phi = \int B \cdot dS = \int_d^{d+a}\frac{\mu_0 I}{2\pi\rho}\,a\,d\rho \qquad (dS = a\,d\rho\,\mathbf{a}_n)$$

where $d\rho$ is width of the strip of loop at a distance ρ from the straight wire. Thus,

$$\Phi = \int_2^3\left(\frac{\mu_0 I}{2\pi}\right)\frac{d\rho}{\rho} = \frac{\mu_0 I}{2\pi}\ln\left(\frac{3}{2}\right) = \frac{\mu_0(5)}{2\pi}\ln(1.5)$$
$$= (2\times10^{-7})(5)\ln(1.5) = 4.05\times10^{-7}\text{ Wb}$$

SOL 6.2.4 Correct answer is 133.3 .

The displacement current density in a medium is equal to the rate of change in electric flux density in the medium.

$$J_d = \frac{\partial D}{\partial t}$$

Since the displacement current density in the medium is given as

$$J_d = 20\cos(1.5\times10^8 t)\mathbf{a}_y \text{ A/m}^2$$

So, the electric flux density in the medium is

$$D = \int J_d\,dt + C \qquad (C \to \text{constant})$$
$$= \int 20\cos(1.5\times10^8 t)\mathbf{a}_y\,dt + C$$

As there is no D.C. field present in the medium so, we get $C = 0$ and thus,

$$D = \frac{20\sin(1.5\times10^8 t)}{1.5\times10^8}\mathbf{a}_y = 1.33\times10^{-7}\sin(1.5\times10^8 t)\mathbf{a}_y$$
$$= 133.3\sin(1.5\times10^8 t)\mathbf{a}_y \text{ nC/m}^2$$

Since, from the given problem we have the flux density

$$D = D_0\sin(1.5\times10^8 t)\mathbf{a}_y \text{ nC/m}^2$$

So, we get

$$D_0 = 133.3$$

SOL 6.2.5 Correct answer is 9.75 .

The ratio of magnitudes of displacement current to conduction current in any medium having permittivity ε and conductivity σ is given as

$$\left|\frac{\text{Displacement current}}{\text{Conduction current}}\right| = \frac{\omega\varepsilon}{\sigma}$$

where ω is the angular frequency of the current in the medium.

Given frequency, $f = 50\text{ GHz}$

Permittivity, $\varepsilon = 4\varepsilon_0 = 4\times8.85\times10^{-12}$

Conductivity, $\sigma = 1.14\times10^8\text{ s/m}$

So, $\omega = 2\pi f = 2\pi\times50\times10^9 = 100\pi\times10^9$

Therefore, the ratio of magnitudes of displacement current to the conduction current is

$$\left|\frac{I_d}{I_c}\right| = \frac{100\pi\times10^9\times4\times8.85\times10^{-12}}{1.14\times10^8} = 9.75\times10^{-8}$$

SOL 6.2.6 Correct answer is 33.3 .

Given magnetic flux density through the square loop is

$$B = 0.1t\mathbf{a}_z \text{ Wb/m}^2$$

So, total magnetic flux passing through the loop is

$$\Phi = B\cdot dS = (0.1t)(1) = 0.1t$$

The induced emf (voltage) in the loop is given as

$$V_{\text{emf}} = -\frac{d\phi}{dt} = -0.1\text{ Volt}$$

As determined by Lenz's law the polarity of induced emf will be such that

$$V_1 + V_2 = -V_{\text{emf}}$$

Therefore, the voltage drop in the $2\,\Omega$ resistance is

$$V_1 = \left(\frac{2}{2+4}\right)(-V_{\text{emf}}) = \frac{0.1}{3} = 33.3\text{ mV}$$

SOL 6.2.7 Correct answer is 7.2 .

Voltage, $V_1 = -N_1\dfrac{d\Phi}{dt}$

where Φ is total magnetic flux passing through it.

Again $$V_2 = -N_2\frac{d\Phi}{dt}$$

Since both the coil are in same magnetic field so, change in flux will be same for both the coil.

Comparing the equations (1) and (2) we get

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$V_2 = V_1\frac{N_2}{N_1} = (12)\frac{3000}{5000} = 7.2\text{ volt}$$

SOL 6.2.8 Correct answer is 41.6 .

In phasor form the magnetic field intensity can be written as

$$\boldsymbol{H}_s = 0.1\cos(15\pi y)e^{-jbx}\boldsymbol{a}_z\text{ A/m}$$

Similar as determined in MCQ 42 using Maxwell's equation we get the relation

$$(15\pi)^2 + b^2 = \omega^2\pi_0\varepsilon_0$$

Here $$\omega = 6\pi\times10^9$$

So, $$(15\pi)^2 + b^2 = \left(\frac{6\pi\times10^9}{3\times10^8}\right)^2$$

$$(15\pi)^2 + b^2 = 400\pi^2$$

$$b^2 = 175\pi^2 \quad\Rightarrow\quad b = \pm 41.6\text{ rad/m}$$

So, $$|b| = 41.6\text{ rad/m}$$

SOL 6.2.9 Correct answer is 0.01 .

Induced emf. in the conducting loop formed by rail, bar and the resistor is given by

$$V_{emf} = -\frac{d\Phi}{dt}$$

where Φ is total magnetic flux passing through the loop.

Consider the bar be located at a distance x from the resistor at time t. So the total magnetic flux passing through the loop at time t is

$$\Phi = \int \boldsymbol{B}\cdot d\boldsymbol{S} = Blx \qquad \text{(area of the loop is } S = lx\text{)}$$

Now the induced emf in a loop placed in magnetic field is defined as

$$V_{emf} = -\frac{d\Phi}{dt}$$

where Φ is the total magnetic flux passing through the loop. Therefore the induced emf in the square loop is

$$V_{emf} = -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt} \qquad (\Phi = Blx)$$

Since from the given figure, we have

$$l = 2\text{ m} \quad\text{and}\quad B = 0.1\text{ Wb/m}^2$$

and $$dx/dt = \text{velocity of bar} = 5\text{ m/s}$$

So, induced emf is

$$V_{emf} = -(0.1)(2)(5) = -1\text{ volt}$$

According to Lenz's law the induced current I in a loop flows such as to produce magnetic field that opposes the change in $B(t)$. As the bar moves away from the resistor the change in magnetic field will be out of the page so the induced current will be in the same direction of I shown in figure.

Thus, the current in the loop is

$$I = -\frac{V_{\text{emf}}}{R} = -\frac{(-1)}{10} = 0.01\text{ A} \qquad (R = 10\Omega)$$

SOL 6.2.10 Correct answer is 277.

Magnetic flux density produced at a distance ρ from a long straight wire carrying current I is defined as

$$B = \frac{\mu_0 I}{2\pi\rho}\boldsymbol{a}_{\phi}$$

where $\boldsymbol{a}_{\phi}$ is the direction of flux density as determined by right hand rule. Since the direction of magnetic flux density produced at the loop is normal to the surface of the loop So, total flux passing through the loop is given by

$$\Phi = \int_S B \cdot dS = \int_{\rho=2}^{4}\left(\frac{\mu_0 I}{2\pi\rho}\right)(ad\rho) \qquad (dS = ad\rho)$$

$$= \frac{\mu_0 Ia}{2\pi}\int_2^4 \frac{d\rho}{\rho}$$

$$= \frac{\mu_0 I2}{2\pi}\ln 2 = \frac{\mu_0 I}{\pi}\ln(2)$$

The current flowing in the loop is I_{loop} and induced e.m.f. is V_{emf}.

So, $$V_{\text{emf}} = I_{loop}R = -\frac{d\Phi}{dt}$$

$$\frac{dQ}{dt}(R) = -\frac{\mu_0}{\pi}\ln(2)\frac{dI}{dt}$$

where Q is the total charge passing through a corner of square loop.

$$\frac{dQ}{dt} = -\frac{\mu_0}{4\pi}\ln(2)\frac{dI}{dt} \qquad (R = 4\,\Omega)$$

$$dQ = -\frac{\mu_0}{4\pi}\ln(2)\,dI$$

Therefore the total charge passing through a corner of square loop is

$$Q = -\frac{\mu_0}{4\pi}\ln(2)\int_4^0 dI$$

$$= -\frac{\mu_0}{4\pi}\ln(2)(0-4)$$

$$= \frac{4\times 4\pi\times 10^{-7}}{4\pi}\ln(2)$$

$$= 2.77\times 10^{-7}\text{ C} = 277\text{ nC}$$

SOL 6.2.11 Correct answer is 44.9 .

Since the radius of small circular loop is negligible in comparison to the radius of the large loop. So, the flux density through the small loop will be constant and equal to the flux on the axis of the loops.

So, $$B = \frac{\mu_0 I}{2}\frac{R^2}{(z^2+R^2)^{3/2}}\boldsymbol{a}_z$$

where $R \to$ radius of large loop $= 5$ m

$z \to$ distance between the loops $= 12$ m

$$B = \frac{\mu_0\times 2}{2}\times\frac{(5)^2}{[(12)^2+(5)^2]^{3/2}}\boldsymbol{a}_z = \frac{25\mu_0}{(13)^3}\boldsymbol{a}_z$$

Therefore, the total flux passing through the small loop is

$$\Phi = \int B\cdot dS = \frac{25\mu_0}{(13)^3}\times\pi r^2$$ where r is radius of small circular loop.

$$= \frac{25\times 4\pi\times 10^{-7}}{(13)^3}\times\pi(10^{-3})^2 = 44.9\text{ fWb}$$

SOL 6.2.12 Correct answer is 2.7 .

Electric field in any medium is equal to the voltage drop per unit length.

i.e. $$E = \frac{V}{d}$$

where $V \rightarrow$ potential difference between two points.

$d \rightarrow$ distance between the two points.

The voltage difference between any two points in the medium is

$$V = V_0 \cos 2\pi ft$$

So the conduction current density in the medium is given as

$$J_c = \sigma E \qquad (\sigma \rightarrow \text{conductivity of the medium})$$

$$= \frac{E}{\rho} \qquad (\rho \rightarrow \text{resistivity of the medium})$$

$$= \frac{V}{\rho d} = \frac{V_0 \cos 2\pi ft}{\rho d} \qquad (V = V_0 \cos 2\pi ft)$$

or, $$|J_c| = \frac{V_0}{\rho d}$$

and displacement current density in the medium is given as

$$J_d = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} = \varepsilon \frac{\partial}{\partial t}\left[\frac{V_0 \cos(2\pi ft)}{d}\right] \qquad (V = V_0 \cos 2\pi ft)$$

$$= \frac{\varepsilon V_0}{d}[-2\pi ft \sin(2\pi ft)]$$

or, $$|J_d| = \frac{2\pi f \varepsilon V_0}{d}$$

Therefore, the ratio of amplitudes of conduction current and displacement current in the medium is

$$\frac{|I_c|}{|I_d|} = \frac{|J_C|}{|J_d|} = \frac{(V_0)/(\rho d)}{(d)/(2\pi f \varepsilon V_0)} = \frac{1}{2\pi f \varepsilon \rho}$$

$$= \frac{1}{2\pi \times (1.6 \times 10^8) \times (54 \times 8.85 \times 10^{-12}) \times 0.77}$$

$$= 2.7$$

SOL 6.2.13 Correct answer is 8.

Let the test charge be q coulomb So the force presence of experienced by the test charge in the presence of magnetic field is

$$F = q(v \times B)$$

and the force experienced can be written in terms of the electric field intensity as

$$F = qE$$

Where E is field viewed by observer moving with test charge.

Putting it in Eq. (i)

$$qE = q(v \times B)$$

$$E = (\omega \rho \boldsymbol{a}_\phi) \times (2\boldsymbol{a}_z)$$

where ω is angular velocity and ρ is radius of circular loop.

$$= (2)(2)(2)\boldsymbol{a}_\rho = 8\boldsymbol{a}_\rho \text{ V/m}$$

SOL 6.2.14 Correct answer is -0.35 .

As shown in figure the bar is sliding away from origin.

Now when the bar is located at a distance dx from the voltmeter, then, the vector area of the loop formed by rail and the bar is

$$dS = (20 \times 10^{-2})(dx)\,\boldsymbol{a}_z$$

So, the total magnetic flux passing through the loop is

$$\Phi = \int_S B \cdot dS$$
$$= \int_0^x (8x^2 a_z)(20 \times 10^{-2} dx a_z)$$
$$= \frac{1.6\left[t(1+0.4t^2)\right]^3}{3}$$

Therefore, the induced e.m.f. in the loop is given as

$$V_{emf} = -\frac{d\Phi}{dt} = -\frac{1.6}{3} \times 3(t+0.4t^3)^2 \times (1+1.2t^2)$$
$$V_{emf} = -1.6\left[(0.4)+(0.4)^4\right]^2 \times \left[1+(1.2)(0.4)^2\right] \qquad (t = 0.4\text{ sec})$$
$$= -0.35\text{ volt}$$

Since the voltmeter is connected in same manner as the direction of induced emf (determined by Lenz's law).
So the voltmeter reading will be

$$V = V_{emf} = -0.35\text{ volt}$$

SOL 6.2.15 Correct answer is -23.4 .
Since the position of bar is give as

$$x = t(1+0.4t^2)$$

So for the position $x = 12\text{ cm}$ we have

$$0.12 = t(1+0.4t^2)$$

or, $$t = 0.1193\text{ sec}$$

As calculated in previous question, the induced emf in the loop at a particular time t is

$$V_{emf} = -(1.6)\left[t+0.4t^3\right]^2(1+1.2t^2)$$

So, at $t = 0.1193\text{ sec}$,

$$V_{emf} = -1.6\left[(0.1193)+0.4(0.1193)^3\right]^2\left[1+(1.2)(0.1193)^2\right]$$
$$= -0.02344 = -23.4\text{ mV}$$

Since the voltmeter is connected in same manner as the direction of induced emf as determined by Lenz's law. Therefore, the voltmeter reading at $x = 12\text{ cm}$ will be

$$V = V_{emf} = -23.4\text{ mvolt}$$

SOL 6.2.16 Correct answer is ± 600.
Given the magnetic field intensity in the medium is

$$H = \cos(10^{10}t - bx)a_z\text{ A/m}$$

Now from the Maxwell's equation, we have

$$\nabla \times H = \frac{\partial D}{\partial t}$$

or, $$\frac{\partial D}{\partial t} = -\frac{\partial H_z}{\partial x}a_y = -b\sin(10^{10}t - bx)a_y$$

$$D = \int -b\sin(10^{10}t - bx)dt + C \qquad \text{where } C \text{ is a constant.}$$

Since no D.C. field is present in the medium so, we get $C = 0$ and therefore,

$$D = \frac{b}{10^{10}}\cos(10^{10}t - bx)a_y\text{ C/m}^2$$

and the electric field intensity in the medium is given as

$$E = \frac{D}{\varepsilon} = \frac{b}{0.12 \times 10^{-9} \times 10^{10}}\cos(10^{10t} - bx)a_y \quad (\varepsilon = 0.12\text{ nF/m})$$

Again From the Maxwell's equation

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

or,
$$\frac{\partial B}{\partial t} = -\nabla \times \left[\frac{b}{1.2}\cos(10^{10t} - bx)a_y\right]$$

$$= -\frac{b^2}{1.2}\sin(10^{10t} - bx)a_z$$

So, the magnetic flux density in the medium is

$$B = -\int \frac{b^2}{1.2}\sin(10^{10t} - bx)a_z dt$$

$$= \frac{b^2}{(1.2)\times 10^{10}}\cos(10^{10}t - bx)a_z \qquad (1)$$

We can also determine the value of magnetic flux density as :

$$B = \mu H$$

$$= (3\times 10^{-5})\cos(10^{10}t - bx)a_z \qquad (2)$$

Comparing the results of equation (1) and (2) we get,

$$\frac{b^2}{(1.2)\times 10^{10}} = 3\times 10^{-5}$$

$$b^2 = 3.6\times 10^5$$

$$b = \pm 600 \text{ rad/m}$$

SOL 6.2.17 Correct answer is 54.414 .

Given the electric field in time domain as

$$E = 5\sin(10\pi y)\cos(6\pi\times 10^9 - bx)a_z$$

Comparing it with the general equation for electric field intensity given as

$$E = E_0\cos(\omega t - \beta x)a_z$$

We get,
$$\omega = 6\pi\times 10^9$$

Now in phasor form, the electric field intensity is

$$E_s = 5\sin(10\pi y)e^{-jbx}a_z \qquad (1)$$

From Maxwell's equation we get the magnetic field intensity as

$$H_s = -\frac{1}{j\omega\mu_0}(\nabla\times E_s) = \frac{j}{\omega\mu_0}\left[\frac{\partial E_{sz}}{\partial y}a_x - \frac{\partial E_{sz}}{\partial x}a_y\right]$$

$$= \frac{j}{\omega\mu_0}[50\pi\cos(10\pi y)e^{-jbx}a_x + j5b\sin(10\pi y)a_y]e^{-jbx}$$

Again from Maxwell's equation we have the electric field intensity as

$$E_s = \frac{1}{j\omega\varepsilon_0}(\nabla\times H_s) = \frac{1}{j\omega\varepsilon_0}\left[\frac{\partial H_{sy}}{\partial x} - \frac{\partial H_{sx}}{\partial y}\right]a_z$$

$$= \frac{1}{\omega^2\mu_0\varepsilon_0}[(j5b)(-jb)\sin(10\pi y)e^{-jbx} + (50\pi)(10\pi)\sin(10\pi y)e^{-jbx}]a_z$$

$$= \frac{1}{\omega^2\mu_0\varepsilon_0}[5b^2 + 500\pi^2]\sin 10\pi y e^{-jbx}a_z$$

Comparing this result with equation (1) we get

$$\frac{1}{\omega^2\mu_0\varepsilon_0}(5b^2 + 500\pi^2) = 5$$

or,
$$b^2 + 100\pi^2 = \omega^2\mu_0\varepsilon_0$$

$$b^2 + 100\pi^2 = (6\pi\times 10^9)^2\times\frac{1}{(3\times 10^8)^2} \quad \left(\omega = 6\pi\times 10^9, \sqrt{\mu_0\varepsilon_0} = \frac{1}{c}\right)$$

$$b^2 + 100\pi^2 = 400\pi^2$$

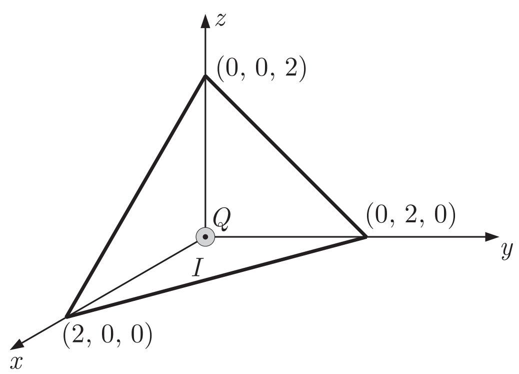
$$b^2 = 300\pi^2$$

$$b = \pm\sqrt{300}\,\pi \text{ rad/m}$$

So, $$|b| = \sqrt{300}\,\pi = 54.414 \text{ rad/m}$$

SOL 6.2.18 Correct answer is 7.

Let the point change located at origin be Q and the current I is flowing out of the page through the closed triangular path as shown in the figure.



As the current I flows away from the point charge along the wire, the net charge at origin will change with increasing time and given as

$$\frac{dQ}{dt} = -I$$

So the electric field intensity will also vary through the surface and for the varying field circulation of magnetic field intensity around the triangular loop is defined as

$$\oint H \cdot dl = [I_d]_{enc} + [I_c]_{enc}$$

where $[I_c]_{enc}$ is the actual flow of charge called enclosed conduction current and $[I_d]_{enc}$ is the current due to the varying field called enclosed displacement current which is given as

$$[I_d]_{enc} = \frac{d}{dt}\int_S (\varepsilon_0 E) \cdot dS = \frac{d}{dt}\int_S D \cdot dS \qquad (1)$$

From symmetry the total electric flux passing through the triangular surface is

$$\int_S D \cdot dS = \frac{Q}{8}$$

So, $$[I_d]_{enc} = \frac{d}{dt}\left(\frac{Q}{8}\right) = \frac{1}{8}\frac{dQ}{dt} = -\frac{I}{8} \qquad \text{(from equation (1))}$$

whereas $$[I_c]_{enc} = I$$

So, the net circulation of the magnetic field intensity around the closed triangular loop is

$$\oint_C H \cdot dl = [I_d]_{enc} + [I_c]_{enc}$$

$$= -\frac{I}{8} + I = \frac{7}{8}(8) = 7 \text{ A} \qquad (I = 8 \text{ A})$$

SOL 6.2.19 Correct answer is 21.33 .

As calculated in previous question the maximum induced voltage in the rotating loop is given as

$$|V_{emf}| = B_0 S\omega$$

From the given data, we have

$$B_0 = 0.25 \text{ Wb/m}^2$$

$$S = 64 \text{ cm}^2 = 64 \times 10^{-4} \text{ m}^2$$

and $\omega = 60 \times 2\pi = 377$ rad/sec (In one revolution 2π radian is covered)

So, the r.m.s. value of the induced voltage is

$$\begin{aligned}[V_{emf}]_{r.m.s} &= \frac{1}{\sqrt{2}}|V_{emf}| = \frac{1}{\sqrt{2}}B_0 S\omega \\ &= \frac{1}{\sqrt{2}}(0.25 \times 64 \times 10^{-4} \times 377) \\ &= 0.4265\end{aligned}$$

Since the loop has 50 turns so net induced voltage will be 50 times the calculated value.

i.e.
$$\begin{aligned}[V_{emf}]_{r.m.s} &= 50 \times (0.4265) \\ &= 21.33 \text{ volt}\end{aligned}$$

****

SOLUTIONS 6.3

SOL 6.3.1 Option (D) is correct.

SOL 6.3.2 Option (B) is correct.
The line integral of magnetic field intensity along a closed loop is equal to the current enclosed by it.

i.e. $$\int H \cdot dl = I_{enc}$$

So, for the constant current, magnetic field intensity will be constant i.e. magnetostatic field is caused by steady currents.

SOL 6.3.3 Option (A) is correct.
From Faraday's law the electric field intensity in a time varying field is defined as

$$\nabla \times E = -\frac{\partial B}{\partial t}$$ where B is magnetic flux density in the EM field.

and since the magnetic flux density is equal to the curl of magnetic vector potential

i.e. $$B = \nabla \times A$$

So, putting it in equation (1), we get

$$\nabla \times E = -\frac{\partial}{\partial t}(\nabla \times A)$$

or $$\nabla \times E = \nabla \times \left(-\frac{\partial}{\partial t} A\right)$$

Therefore, $$E = -\frac{\partial A}{\partial t}$$

SOL 6.3.4 Option (B) is correct.
Since total magnetic flux through a surface S is defined as

$$\Phi = \int_S B \cdot dS$$

From Maxwell's equation it is known that curl of magnetic flux density is zero

$$\nabla \cdot B = 0$$

$$\int_S B \cdot dS = \int_v (\nabla \cdot B) dv = 0 \qquad \text{(Stokes Theorem)}$$

Thus, net outwards flux will be zero for a closed surface.

SOL 6.3.5 Option (B) is correct.
From the integral form of Faraday's law we have the relation between the electric field intensity and net magnetic flux through a closed loop as

$$\oint E \cdot dl = -\frac{d\Phi}{dt}$$

Since electric field intensity is zero ($E = 0$) inside the conducting loop. So, the rate of change in net magnetic flux through the closed loop is

$$\frac{d\Phi}{dt} = 0$$

i.e. Φ is constant and doesn't vary with time.

SOL 6.3.6 Option (C) is correct.

A superconductor material carries zero magnetic field and zero electric field inside it.

i.e. $B = 0$ and $E = 0$

Now from Ampere-Maxwell equation we have the relation between the magnetic flux density and electric field intensity as

$$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

So, $J = 0$ $\quad (B = 0,\ E = 0)$

Since the net current density inside the superconductor is zero so all the current must be confined at the surface of the wire.

SOL 6.3.7 Option (C) is correct.

According to Lenz's law the induced current I in a loop flows such as to produce a magnetic field that opposes the change in $B(t)$.

Now the configuration shown in option (A) and (B) for increasing magnetic flux B_i, the change in flux is in same direction to B_i as well as the current I flowing in the loop produces magnetic field in the same direction so it does not follow the Lenz's law.

For the configuration shown in option (D), as the flux B_d is decreasing with time so the change in flux is in opposite direction to B_d as well as the current I flowing in the loop produces the magnetic field in opposite direction so it also does not follow the Lenz's law.

For the configuration shown in option (C), the flux density B_d is decreasing with time so the change in flux is in opposite direction to B_d but the current I flowing in the loop produces magnetic field in the same direction to B_d (opposite to the direction of change in flux density). Therefore this is the correct configuration.

SOL 6.3.8 Option (C) is correct.

Induced emf in a conducting loop is given by

$$V_{emf} = -\frac{d\Phi}{dt}$$ where Φ is total magnetic flux passing through the loop.

Since, the magnetic field is non-uniform so the change in flux will be caused by it and the induced emf due to it is called transformer emf.

Again the field is in a_y direction and the loop is rotating about z-axis so flux through the loop will also vary due to the motion of the loop. This causes the emf which is called motion emf. Thus, total induced voltage in the rotating loop is caused by the combination of both the transformer and motion emf.

SOL 6.3.9 Option (B) is correct.

SOL 6.3.10 Option (C) is correct.

SOL 6.3.11 Option (B) is correct.

SOL 6.3.12 Option (D) is correct.

SOL 6.3.13 Option (A) is correct.

SOL 6.3.14 Option (A) is correct.

SOL 6.3.15 Option (C) is correct.

SOL 6.3.16 Option (B) is correct.

SOL 6.3.17 Option (B) is correct.

****

SOLUTIONS 6.4

SOL 6.4.1

Option (C) is correct.

Given, the magnetic flux density in air as

$$B = B_0\left(\frac{x}{x^2+y^2}a_y - \frac{y}{x^2+y^2}a_x\right) \quad ...(1)$$

Now, we transform the expression in cylindrical system, substituting

$$x = r\cos\phi \quad \text{and} \quad y = r\sin\phi$$

$$a_x = \cos\phi a_r - \sin\phi a_\phi$$

and $$a_y = \sin\phi a_r + \cos\phi a_\phi$$

So, we get $$B = B_0 a_\phi$$

Therefore, the magnetic field intensity in air is given as

$$H = \frac{B}{\mu_0} = \frac{B_0 a_\phi}{\mu_0}, \text{ which is constant}$$

So, the current density of the field is

$$J = \nabla\times H = 0 \quad \text{(since } H \text{ is constant)}$$

SOL 6.4.2

Option (D) is correct.

Maxwell equations for an EM wave is given as

$$\nabla\cdot B = 0$$

$$\nabla\cdot E = \frac{\rho_v}{\varepsilon}$$

$$\nabla\times E = -\frac{\partial B}{\partial t}$$

$$\nabla\times H = \frac{\partial D}{\partial t} + J$$

So, for static electric magnetic fields

$$\nabla\cdot B = 0$$

$$\nabla\cdot E = \rho_v/\varepsilon$$

$$\nabla\times E = 0 \quad \left(\frac{\partial B}{\partial t} = 0\right)$$

$$\nabla\times H = J \quad \left(\frac{\partial D}{\partial t} = 0\right)$$

SOL 6.4.3

Option (D) is correct.

$$\nabla\times H = J + \frac{\partial D}{\partial t} \quad \text{Maxwell Equations}$$

$$\iint_S(\nabla\times H)\cdot dS = \iint_S\left(J + \frac{\partial D}{\partial t}\right)\cdot dS \quad \text{Integral form}$$

$$\oint H\cdot dl = \iint_S\left(J + \frac{\partial D}{\partial t}\right)\cdot dS \quad \text{Stokes Theorem}$$

SOL 6.4.4

Option (C) is correct.

From Maxwells equations we have

$$\nabla\times H = \frac{\partial D}{\partial t} + J$$

Thus, $\nabla\times H$ has unit of current density J (i.e., A/m^2)

SOL 6.4.5 Option (A) is correct.

This equation is based on Ampere's law as from Ampere's circuital law we have

$$\oint_l H \cdot dl = I_{\text{enclosed}}$$

or, $$\oint_l H \cdot dl = \int_S J \cdot dS$$

Applying Stoke's theorem we get

$$\int_S (\nabla \times H) \cdot dS = \int_S J \cdot dS$$

$$\nabla \times H = J$$

Then, it is modified using continuity equation as

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

SOL 6.4.6 Option (D) is correct.

When a moving circuit is put in a time varying magnetic field induced emf have two components. One due to time variation of magnetic flux density B and other due to the motion of circuit in the field.

SOL 6.4.7 Option (C) is correct.

From maxwell equation we have

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

The term $\frac{\partial D}{\partial t}$ defines displacement current.

SOL 6.4.8 Option (C) is correct.

Emf induced in a loop carrying a time varying magnetic flux Φ is defined as

$$V_{emf} = -\frac{d\Phi}{dt}$$

$$9 = -\frac{d}{dt}\left(\frac{1}{3}\lambda t^3\right)$$

$$9 = -\lambda t^2$$

at time, $t = 3\,\text{s}$, we have

$$9 = -\lambda (3)^2$$

$$\lambda = -1\,\text{Wb/s}^2$$

SOL 6.4.9 Option (B) is correct.

According to Lenz's law the induced emf (or induced current) in a loop flows such as to produce a magnetic field that opposed the change in B. The direction of the magnetic field produced by the current is determined by right hand rule.

Now, in figure (1), B directed upwarded increases with time where as the field produced by current I is downward so, it obey's the Lenz's law.

In figure (2), B directed upward is decreasing with time whereas the field produced by current I is downwards (i.e. additive to the change in B) so, it doesn't obey Lenz's law.

In figure (3), B directed upward is decreasing with time where as current I produces the field directed upwards (i.e. opposite to the change in B) So, it also obeys Lenz's law.

In figure (4), B directed upward is increasing with time whereas current I produces field directed upward (i.e. additive to the change in B) So, it

doesn't obey Lenz's law.
Thus, the configuration 1 and 3 are correct.

SOL 6.4.10 Option (C) is correct.
Faraday's law states that for time varying field,

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

Since, the curl of gradient of a scalar function is always zero

i.e. $\nabla \times (\nabla V) = 0$

So, the expression for the field, $E = -\nabla V$ must include some other terms is

$$E = -\nabla V - \frac{\partial A}{\partial t}$$

i.e. A is true but R is false.

SOL 6.4.11 Option (B) is correct.
Faraday develops the concept of time varying electric field producing a magnetic field. The law he gave related to the theory is known as Faraday's law.

SOL 6.4.12 Option (D) is correct.
Given, the area of loop

$$S = 5\text{ m}^2$$

Rate of change of flux density,

$$\frac{\partial B}{\partial t} = 2\text{ Wb/m}^2/\text{S}$$

So, the emf in the loop is

$$V_{\text{emf}} = -\frac{\partial}{\partial t}\int B \cdot dS$$
$$= (5)(-2) = -10\text{ V}$$

SOL 6.4.13 Option (D) is correct.
The modified Maxwell's differential equation.

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

This equation is derived from Ampere's circuital law which is given as

$$\oint H \cdot dl = I_{enc}$$
$$\oint (\nabla \times H) \cdot dS = \int J dS$$
$$\nabla \times H = J$$

SOL 6.4.14 Option (B) is correct.
Electric potential of an isolated sphere is defined as

$$C = 4\pi\varepsilon_0 a \qquad \text{(free space)}$$

The Maxwell's equation in phasor form is written as

$$\nabla \times H = j\omega\varepsilon E + \sigma E = j\omega\varepsilon E + J \qquad (J = \sigma E)$$

So A and R both are true individually but R is not the correct explanation of A.

SOL 6.4.15 Option (A) is correct.
If a coil is placed in a time varying magnetic field then the e.m.f. will induce in coil. So here in both the coil e.m.f. will be induced.

SOL 6.4.16 Option (B) is correct.
Both the statements are individually correct but R is not explanation of A.

SOL 6.4.17 Option (B) is correct.

Ampere's law	$\nabla \times H = J + \frac{\partial D}{\partial t}$	(a → 3)
Faraday' law	$\nabla \times E = \frac{\partial B}{\partial t}$	(b → 4)
Gauss law	$\nabla \cdot D = \rho_v$	(c → 1)
Current continuity	$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$	(d → 2)

SOL 6.4.18 Option (B) is correct.
Since, the magnetic field perpendicular to the plane of the ring is decreasing with time so, according to Faraday's law emf induced in both the ring is

$$V_{\text{emf}} = -\frac{\partial}{\partial t}\int B \cdot dS$$

Therefore, emf will be induced in both the rings.

SOL 6.4.19 Option (A) is correct.
The Basic idea of radiation is given by the two Maxwell's equation

$$\nabla \times H = \frac{\partial D}{\partial t}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

SOL 6.4.20 Option (B) is correct.
The correct Maxwell's equation are

$$\nabla \times H = J + \frac{\partial D}{\partial t} \qquad \nabla \cdot D = \rho$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad \nabla \cdot B = 0$$

SOL 6.4.21 Option (B) is correct.
In List I

a. $\oint B \cdot dS = 0$

The surface integral of magnetic flux density over the closed surface is zero or in other words, net outward magnetic flux through any closed surface is zero. (a → 4)

b. $\oint D \cdot dS = \int_v \rho_v dv$

Total outward electric flux through any closed surface is equal to the charge enclosed in the region. (b → 3)

c. $\int E \cdot dl = -\int \frac{\partial B}{\partial t} dS$

i.e. The line integral of the electric field intensity around a closed path is equal to the surface integral of the time derivative of magnetic flux density (c → 2)

d. $\oint H \cdot dS = \int \left(\frac{\partial D}{\partial t} + J\right) da$

i.e. The line integral of magnetic field intensity around a closed path is equal to the surface integral of sum of the current density and time derivative of electric flux density. (d → 1)

SOL 6.4.22 Option (D) is correct.

The continuity equation is given as

$$\nabla \cdot J = -\rho_v$$

i.e. it relates current density (J) and charge density ρ_v.

SOL 6.4.23 Option (C) is correct.

Given Maxwell's equation is

$$\nabla \times H = J_c + \frac{\partial D}{\partial t}$$

For free space, conductivity, $\sigma = 0$ and so,

$$J_c = \sigma E = 0$$

Therefore, we have the generalized equation

$$\nabla \times H = \frac{\partial D}{\partial t}$$

SOL 6.4.24 Option (A) is correct.

Given the magnetic field intensity,

$$H = 3a_x + 7ya_y + 2xa_z$$

So from Ampere's circuital law we have

$$J = \nabla \times H$$

$$= \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3 & 7y & 2x \end{vmatrix}$$

$$= a_x(0) - a_y(2-0) + a_z(0) = -2a_y$$

SOL 6.4.25 Option (A) is correct.

The emf in the loop will be induced due to motion of the loop as well as the variation in magnetic field given as

$$V_{\text{emf}} = -\int \frac{\partial B}{\partial t} dS + \oint (v \times B) dl$$

So, the frequencies for the induced e.m.f. in the loop is ω_1 and ω_2.

SOL 6.4.26 Option (B) is correct.

$$F = Q(E + v \times B) \text{ is Lorentz force equation.}$$

SOL 6.4.27 Option (A) is correct.

All of the given expressions are Maxwell's equation.

SOL 6.4.28 Option (B) is correct.

Poission's equation for an electric field is given as

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

where, V is the electric potential at the point and ρ_v is the volume charge density in the region. So, for $\rho_v = 0$ we get,

$$\nabla^2 V = 0$$

Which is Laplacian equation.

SOL 6.4.29 Option (A) is correct.

The direction of magnetic flux due to the current 'i' in the conductor is determined by right hand rule. So, we get the flux through A is pointing into the paper while the flux through B is pointing out of the paper.

According to Lenz's law the induced e.m.f. opposes the flux that causes it.

So again by using right hand rule we get the direction of induced e.m.f. is anticlockwise in A and clockwise in B.

SOL 6.4.30 Option (D) is correct.

$$\nabla^2 A = -\mu_0 J$$

This is the wave equation for static electromagnetic field.
i.e. It is not Maxwell's equation.

SOL 6.4.31 Option (B) is correct.

Continuity equation $\nabla \times J = -\frac{\partial \rho_v}{\partial t}$ (a → 4)

Ampere's law $\nabla \times H = J + \frac{\partial D}{\partial t}$ (b → 1)

Displacement current $J = \frac{\partial D}{\partial t}$ (c → 2)

Faraday' law $\nabla \times E = -\frac{\partial B}{\partial t}$ (d → 3)

SOL 6.4.32 Option (B) is correct.
Induced emf in a coil of N turns is defined as

$$V_{emf} = -N\frac{d\Phi}{dt}$$

where Φ is flux linking the coil. So, we get

$$\begin{aligned} V_{emf} &= -100\frac{d}{dt}(t^3 - 2t) \\ &= -100(3t^2 - 2) \\ &= -100(3(2)^2 - 2) = -1000\text{ mV} \qquad (\text{at } t = 2\text{ s}) \\ &= -1\text{ V} \end{aligned}$$

SOL 6.4.33 Option (B) is correct.
A static electric field in a charge free region is defined as

$$\nabla \cdot E = 0 \qquad (a \to 4)$$

and $\nabla \times E = 0$

A static electric field in a charged region have

$$\nabla \cdot E = \frac{\rho_v}{\varepsilon} \neq 0 \qquad (b \to 2)$$

and $\nabla \times E = 0$

A steady magnetic field in a current carrying conductor have

$$\nabla \cdot B = 0 \qquad (c \to 1)$$

$$\nabla \times B = \mu_0 J \neq 0$$

A time varying electric field in a charged medium with time varying magnetic field have

$$\nabla \times E = -\frac{\partial B}{\partial t} \neq 0 \qquad (d \to 3)$$

$$\nabla \cdot E = \frac{\rho_v}{\varepsilon} \neq 0$$

SOL 6.4.34 Option (C) is correct.

$$V = -\frac{d\Phi_m}{dt}$$

It is Faraday's law that states that the change in flux through any loop induces e.m.f. in the loop.

SOL 6.4.35

Option (B) is correct.
From stokes theorem, we have

$$\int (\nabla \times E) \cdot dS = \oint E \cdot dl \qquad (1)$$

Given, the Maxwell's equation

$$\nabla \times E = -(\partial B / \partial t)$$

Putting this expression in equation (1) we get,

$$\oint E \cdot dl = -\frac{\partial}{\partial t}\int_s B \cdot dS$$

SOL 6.4.36

Option (D) is correct.
Since, the flux linking through both the coil is varying with time so, emf are induced in both the coils.
Since, the loop 2 is split so, no current flows in it and so joule heating does not occur in coil 2 while the joule heating occurs in closed loop 1 as current flows in it.
Therefore, only statement 2 is correct.

SOL 6.4.37

Option (C) is correct.
The electric field intensity is

$$E = E_0 e^{j\omega t} \qquad \text{where } E_0 \text{ is independent of time}$$

So, from Maxwell's equation we have

$$\nabla \times H = J + \frac{\varepsilon\,\partial E}{\partial t}$$

$$= \sigma E + \varepsilon(j\omega)E_0 e^{j\omega t} = \sigma E + j\omega\varepsilon E$$

SOL 6.4.38

Option (C) is correct.
Equation (1) and (3) are not the Maxwell's equation.

SOL 6.4.39

Option (A) is correct.
From the Maxwell's equation for a static field (DC) we have

$$\nabla \times B = \mu_0 J$$

$$\nabla \times (\nabla \times A) = \mu_0 J$$

$$\nabla(\nabla \cdot A) - \nabla^2 A = \mu_0 J$$

For static field (DC), $\nabla \cdot A = 0$

therefore we have, $\nabla^2 A = -\mu_0 J$

So, both A and R are true and R is correct explanation of A.

SOL 6.4.40

Option (A) is correct.
For a static field, Maxwells equation is defined as

$$\nabla \times H = J$$

and since divergence of the curl is zero

i.e. $\nabla \cdot (\nabla \times H) = 0$

$$\nabla \cdot J = 0$$

but in the time varying field, from continuity equation (conservation of charges)

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t} \neq 0$$

So, an additional term is included in the Maxwell's equation.

i.e. $\nabla \times H = J + \dfrac{\partial D}{\partial t}$

where $\frac{\partial D}{\partial t}$ is displacement current density which is a necessary term.

Therefore A and R both are true and R is correct explanation of A.

SOL 6.4.41 Option (C) is correct.

Since, the circular loop is rotating about the y-axis as a diameter and the flux lines is directed in $\boldsymbol{a}_x$ direction. So, due to rotation magnetic flux changes and as the flux density is function of time so, the magnetic flux also varies w.r.t time and therefore the induced e.m.f. in the loop is due to a combination of transformer and motional e.m.f. both.

SOL 6.4.42 Option (A) is correct.

For any loop to have an induced e.m.f., magnetic flux lines must link with the coil.

Observing all the given figures we conclude that loop C_1 and C_2 carries the flux lines through it and so both the loop will have an induced e.m.f.

SOL 6.4.43 Option (C) is correct.

Gauss's law	$\nabla \cdot D = \rho$	(a → 1)
Ampere's law	$\nabla \times H = J_c + \frac{\partial D}{\partial t}$	(b → 5)
Faraday's law	$\nabla \times E = -\frac{\partial B}{\partial t}$	(c → 2)
Poynting vector	$\mathcal{P} = E \times H$	(d → 3)

****