

Nanyang Technological University

Singapore

Entrance Examination

Mathematics at A-Level

2017

Time allowed : 2 hours

Instructions

1. This paper consists of **Four (4)** questions and comprises three pages.
2. Attempt all of the questions
3. Each question worth 25 marks
4. Answer the question into the answer booklet, any working written on this question paper will not be marked
5. Do not turn over the booklet until you're told to do so

1. a. Given a function  $f(x) = \frac{8}{2x-1}$ . Determine the n-th term of the function  $f^{(n)}(x) = \frac{8}{2x-1}$ . Where n-th indicates the n-th derivative of the function. Hence find  $f^{(n)}(1)$ .

b. Given a function  $y = (\sin x)(\cos x)$ . Find the equation of the tangent of the line at  $x=\pi$ .

c. Given that  $e^{x+y} + x^2 + 1 = (2y - 1)^2$ , Find  $\frac{dy}{dx}$ .

d. Evaluate  $\int_2^3 \frac{6x^2-2x+4}{x^3-x^2+x-1} dx$

2. A family has 4 child. The probability that the child is a boy is 0.51. Find:

- (i) The probability that there are at least 2 boys
- (ii) Which combination is more likely to occur. (For example 1 boy 3 girls, etc.)
- (iii) If a school has 500 students. Find the probability of having at least 250 boys.

3. A. Given the following inequalities :

$$3x + 2y \geq 16; 5x + 7y \leq 71; -x + 6y \geq 8; -3x + y \leq -1; x \leq 7$$

$$x, y \geq 0$$

- (i) Sketch the graph and shade the area following inequalities
- (ii) Determine the range of x and y
- (iii) A store sold a muffin in a number of y packs which cost £2 each and ice cream in a number of x packs which cost £1 each. Determine the maximum value of the sales and state clearly how you get the result.

B. A series are given as follows

$$\sum_{k=0}^{\infty} \frac{2^k (a^2 - 2)^{k+1}}{(2a + 1)^{k+1}}$$

Where  $a > \sqrt{2}$

- (i) Find the ratio and determine the range of  $a$  for the series to converge
- (ii) Find the sum to infinity

4. a. Solve

$$\frac{dy}{dx} = y(\sin x) + (\sin x)(\cos x)$$

b. Given vector of  $a, b$ , and  $c$  as follows

$$a = \begin{pmatrix} 2 \\ p \\ 1 \end{pmatrix}, \quad b = \begin{pmatrix} q \\ -1 \\ 3 \end{pmatrix}, \quad \text{and} \quad c = \begin{pmatrix} p \\ 2 \\ r \end{pmatrix}$$

Find the value of  $p, q$ , and  $r$  given that  $a$  and  $b$  are perpendicular to each other,  $b$  and  $c$  are parallel to each other.