1.
$$\frac{4x^2+10}{|x^2+2|-1} < |x^2+2|+1$$

2.
$$\frac{1}{4x^2+5}+\frac{1}{6x^2+7} > \frac{1}{8x^2+9}+\frac{1}{10x^2+11}$$

3.
$$\frac{1}{x+3}-\frac{1}{x-5} < \frac{1}{x+4}-\frac{1}{x-6}$$

4.
$$\frac{4x^2-10}{|x^2-2|-1} > |x^2-2|+1$$

5.
$$\frac{\log(x^2-7x+12)}{\log(\sqrt{x}-1)} < \frac{\log(x^2-x+6)}{\log(x-1)}; x > 2$$

6.
$$\frac{x^2+3x}{|x+3|+10} > \frac{x^2+4x}{|x+4|+9}$$

7. The function of $f(x)$ is given by,
$$f(x)=\frac{mx^2-4\sqrt{3}x+n}{x^2+1}$$
Find the value of m and n if the maximum value of the function $f(x)$ is 7 and the minimum value of $f(x)$ is -1.

8. The function $f(x)$ is given by,
$$f(x)=\frac{e^x}{x}-a(x-\ln x)$$
Find the interval where the function $f(x)$ is decreasing monotonically.

9. The function $f(x)=x^m+(m-1)x^n$ can be categorized as an even function if and only if both m and n is divisible by 2. Suppose there exist another function $g(x)=x^{m_i}+(m_i-1)x^{n_i}$ where $m_i, n_i \in \mathbb{Z}$ are not divisible by 2. Determine whether the function
$$h(x)=\frac{d}{dx}f\Big(g(x)\Big)$$
is an even or an odd function.

10. Let x_1, x_2, and x_3 be the root of the following cubic equation,
$$f(x)=2x^3+12x^2-9x-5$$
find the value of $x_1x_2x_3^2+x_1x_2x_3^{-2}+x_1-x_2+x_3$, if $x_1>x_2>x_3$.

11. The picture below show the graph of function, $f^2(x) = 6(x-3)^{-1} + 4x + 6$

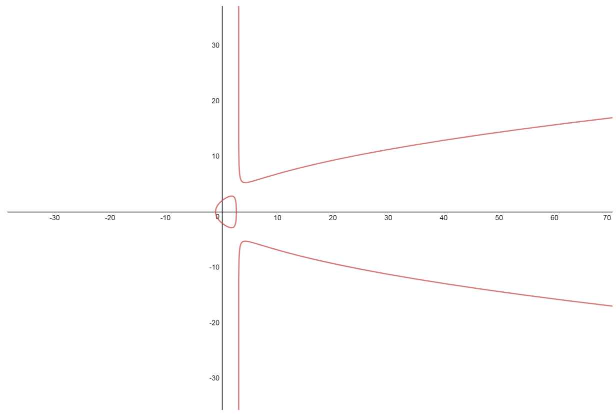

FIGURE 1 – Graph of $f^2(x)$

Sketch the graph of $f(|x|); f(|x|) > 0$

12. Sketch the graph of the function,

$$f(x) = \frac{x+5}{x-10} - 2$$

and in the same diagram show the sketch of the function $g(x) = f(x+10)$.

13. Sketch the graph of the function,

$$f(x) = \frac{x-3}{x+3} + 3$$

and in the same diagram show the sketch of the function $g(x) = f^2(x-3)$.

14. Sketch the graph of the function,

$$f(x) = \frac{x^2+3x-4}{4x-5}$$

Determine and show clearly on your diagram, the asymptotes, intercepts with the co-ordinate axes, and the co-ordinates of stationary point.

15. Sketch the graph of the function,

$$f(x) = \frac{2x^2-23x+51}{4|x|-8}$$

Determine and show clearly on your diagram, the asymptotes, intercepts with the co-ordinate axes, and the co-ordinates of stationary point.

16. Prove by induction that, $\forall b > a > 1; a, b \in \mathbb{Z}$, the function

$$f(x) = \frac{x^a x^b}{x^b - x^a}$$

will never decrease at the interval $[a, b)$

17. For a smooth function $f(x)$, the n- th derivative of $f(x)$ is defined as :

$$f^{(0)}(x) = f(x); f^{(n)}(x) = \frac{d}{dx} f^{(n-1)}(x), n > 1$$

Prove by mathematical induction that,

$$\Big(f(x)g(x)\Big)^{(n)} = \sum_{k=0}^{n} \binom{n}{k} f^{(k)}(x) g^{(n-k)}(x)$$

18. Prove by induction that,

$$\sum_{k=1}^{n} \frac{1}{k^2} \leq 2 - \frac{1}{n}; \forall n > 1$$

Hence prove that,

$$\frac{1}{(k+1)^2} - \frac{1}{k} \leq -\frac{1}{k+1}$$

19. Prove by induction that,

$$f(x, n) = \sum_{k=1}^{n} kx^k = \frac{nx^{n+2} - (n+1)x^{n+1} + x}{(x-1)^2}; \forall x > 1; n > 1$$

Hence calculate the value of $f(\pi, 14)$

20. Given that :

$$\left(\frac{\sqrt{3} + i}{2}\right)^n = -i^n$$

for some $n \in \mathbb{Z}$. Use the De Moivre's theorem, or otherwise,to show that $n - 3$ is divisible by 6.

21. Sketch in an Argand diagram the locus l of the points that represent complex number z satisfying $\arg(z - 2i) = \frac{\pi}{3}$. Let z_0 and z_i be the points in the above Argand diagram that represent the complex number $2i$ and $\sqrt{3} + 5i$, respectively. Find the distance $z_0 z_i$ and show that point z_i lies on locus l

22. z is a variable complex number such that $|z| = 1$ and $u = 3z - z^{-1}$. Show that the locus of the point in Argand plane representing u is an ellipse and find the equation of the ellipse.

23. Find $a \in \mathbb{R}$ such that $z = -i$ is a root for polynomial

$$P(z) = z^3 - z^2 + z + 1 + a$$

Furthermore, for such value of a find the factors of $P(z)$ in $\mathbb{R}$ and $\mathbb{C}$.

24. z_1 and z_2 are the roots for the following complex quadratic equation,

$$(6 + 8i)z^2 + 5z + 24 - 10i = 0$$

find the value of $|\arg(z_1 z_2) + \arg(z_1 + z_2)|$

25. A complex number z_1 is given by $z_1 = 5 + 12i$, suppose there exist another complex number z_2, z_3, and z_4. The coordinate of z_2, z_3, and z_4 lies on the loci of z_n, given that $z_2 = 18 + 12i$, find z_3 and z_4, such that $\angle z_2 z_n z_3 = \angle z_3 z_n z_4 = \angle z_4 z_n z_2$, the loci of z_n is given by $|z_n - z_1| = 13$

26. The complex number w is given by $w = 2 + 2i$. Express w in the polar form where $r > 0, -\pi < \phi_w < \pi$. In the Argand diagram, the points P, Q, and R represent the complex number w, w^2,and $4w^{-1}$ respectively. The point K on the positive real axis such that QRK is an isosceles triangle with $|RQ| = |RK|$. Find the complex number k represented by K.

27. $$\int \tan x \sec^4 x dx = ...$$

28. $$\int \frac{dx}{x^2 - 4x + 13} = ...$$

29. $$\int \cos(\ln x) dx = ...$$

30. $$\int \ln(\cos x) dx = ...$$

31. $$\int \frac{2x^4}{x^3 - x^2 + x - 1} dx = ...$$

32. $$\int \frac{\cos 4x + 1}{\cot x - \tan x} dx = ...$$

33. $$\int \frac{dx}{\cos x + 2\sin x + 2} = ...$$

34. $$\int \sqrt{\frac{x}{1-x}} dx = ...$$

35.
$$\int \frac{1+e^{2x}}{1+e^{3x}}dx = \ldots$$

36.
$$\int \frac{dx}{x^2\sqrt[4]{(x^4+1)^3}} = \ldots$$

37.
$$\int \frac{\sin x^2}{x^2}dx = \ldots$$

38. Find the general solution of the following differential equation
$$\frac{dy}{dx} + \frac{xy\sin x + y\cos x}{x\cos x} = \frac{1}{x\cos x}$$

39. Find the general solution of the following differential equation
$$\frac{dy}{dx} = e^{y-x}(1+x^2)\sec y$$
Sketch the solution curve for which $y(0) = 0$

40. Find the general solution of the following differential equation
$$\frac{dy}{dx} + \frac{2y}{x} = \frac{\sin x}{x^2}$$
Expressing y in term of x. Sketch the solution curve for which $y(\pi) = 1$

41. Find the general solution of the following differential equation
$$(1+x)\frac{dy}{dx} - xy = xe^{-x}$$
Given that $y(0) = 1$.

42. Find the value of k such that the following differential equation has an integrating factor equals to x^2y^k.
$$x(x^2+2y) + 2y(x^2+y)\frac{dy}{dx} = 0$$

43. The equation for logistic growth is given below.
$$\frac{dx}{dt} = kx(L-x)$$
Prove that the differential equation above has the solution :
$$x = \frac{Lx_0}{x_0 + (L-x_0)e^{-Lkt}}$$

44. Suppose that curve C_1 and C_2 intersect at point (x_0, y_0) with slopes m_1 and m_2.Then the positive angle from C_1 (i.e., from the tangent line to C_1 at (x_0, y_0)) to C_2 satisfies.

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

Find the angle of circle with equation $x^2 + y^2 = 1$ to the circle $(x-1)^2 + y^2 = 1$ at the 2 point of intersection.

45. Let

$$f(x) = \frac{x}{1 - x - x^2}$$

Show that the Taylor approximation of $f(x)$ when $x \to 0$ is given by :

$$f(x) = a_1 x + a_2 x^2 + ... + a_n x^n + o(x^n)$$

Where $a_1, a_2, a_3, ..., a_n$ is the Fibonacci sequence, given by :

$$a_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$$

46. A boy standing at a point A on the shore of a circular lake, with radius 200 m, wants to reach a point C on the diametrically opposite shore of the lake. He can row at the speed of 200 m per minute and walk at speed of 400 m per minute. Given that he first row in a straight line to a point B on the shore and then walks along the shore of the lake to C, and that angle BAC is θ, prove that the total time T(minutes) is given by :

$$T = 2\cos\theta + \theta; \left(0 < \theta < \frac{1}{2}\pi\right)$$

Hence find the angle θ that will maximize T and find this value of T, giving your answer in exact form.

47. In the late 1830s, the French physiologist Jean Poiseuille discovered the formula we use today to predict how much the radius of partially clogged artery has to be expanded to restore normal flow. His formula,

$$V = kr^4$$

Says that the volume V of fluid flowing through a small pipe or tube in unit of time at a fixed pressure is a constant times the fourth power of the tube radius. How will a 10% increase in r affect the V?