NTU UEE 2020

MATHEMATICS at A – LEVEL

<u>INSTRUCTIONS</u> **Time Allowed: 2 Hours**

1. This paper consists of 4 questions and comprises 2 pages.
2. Write down your answers in the provided answer sheet.
3. Answers will be graded for content and appropriate presentation.

❖ **Question 1 (25 marks)**

(a) (i) Sketch the graphs $y = \frac{1}{a-x}$ and $y = b|a-x|$ in a single diagram, where a and b are positive constants. Label the graphs, the x-intercept(s), and the y-intercept(s) clearly.

(ii) From your answer in (i), solve the equation $\frac{1}{a-x} > b|a-x|$.

(b) (i) Express $\frac{2x^2+5x+4}{x-4} - (x+2)$ as a single simplified fraction.

(ii) Hence or otherwise, solve the inequality $\frac{2x^2+5x+4}{x-4} > (x+2)$, with the help of sign diagram.

❖ **Question 2 (25 marks)**

(a) Let $\boldsymbol{u}$ and $\boldsymbol{v}$ be vectors such that $\boldsymbol{u} = 3i - 2j + 3k$ and $\boldsymbol{v} = ai + bk$, where a and b are real numbers.

(i) Express $(\boldsymbol{u}+\boldsymbol{v}) \times (\boldsymbol{u}-\boldsymbol{v})$ in terms of a and b.

(ii) It is known that the i-component and the k-component of the vector $(\boldsymbol{u}+\boldsymbol{v}) \times (\boldsymbol{u}-\boldsymbol{v})$ are equal. Express $(\boldsymbol{u}+\boldsymbol{v}) \times (\boldsymbol{u}-\boldsymbol{v})$ in terms of a only.

(iii) Find all possible exact values of a such that the vector $(\boldsymbol{u}+\boldsymbol{v}) \times (\boldsymbol{u}-\boldsymbol{v})$ is a unit vector.

(iv) Given instead that $(\boldsymbol{u}+\boldsymbol{v}) \cdot (\boldsymbol{u}-\boldsymbol{v}) = 0$. Find the numerical value of $|\boldsymbol{v}|$.

(b) A student saves \$5 on 1 January 2019. On the next subsequent months, she saves \$2 more than the one in the previous month. In other word, she will save \$5 on 1 January 2019, \$7 on 1 February 2019, \$9 on 1 March 2019, and so on. Find the date she will first have saved \$1000 in total.

❖ **Question 3 (25 marks)**

(a) A curve C has the equation $2y^2 - 3xy + 3x^2 - 5 = 0$.

(i) Find all exact x-coordinate(s) of the stationary points of C.

(ii) Determine if the stationary point with $x > 0, y > 0$ is a maximum or a minimum.

(b) (i) Evaluate $\int x^3 \sin(nx)\, dx$, where n is a positive even integer.

(ii) Hence evaluate $\int_{\pi}^{2\pi} x^3 \sin(nx)\, dx$, expressing your answer in terms of n and π.

❖ **Question 4 (25 marks)**

(a) (i) Let a and b be real numbers such that $(a + bi)^2 = 12i - 5$. Find all possible values of $a + bi$.

(ii) Solve the roots of $\omega^2 + 4\omega = 12i - 9$, drawing the root(s) in a single Argand diagram.

(iii) Find the area of the quadrilateral enclosed by the roots and their complex conjugates.

(b) Solve the equation $z^5 + 1024 = 0$, writing all the roots in the form of $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$.

(c) A football team consisting of 1 goalkeeper, 4 defenders, 4 midfielders, and 2 attackers are going to be selected from a club with 3 goalkeepers, 8 defenders, 6 midfielders, and 8 attackers.

(i) How many different teams can be formed from the club?

(ii) It is known that one defender is a brother of one midfielder in that club. How different teams can be formed if only one of the brothers can be in the team?

- END OF PAPER -