

# Simulating Voting Systems

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## Abstract

Much of the past work on voting systems focuses on *ranked voting systems*, which have a number of limitations such as Arrow's Theorem [1]. In this paper we consider ranked voting systems as well as the less commonly used class of *rated voting systems*. The systems differ in that ranked voting systems only allow the voter to order the candidates, while in rated voting systems the voter can score each candidate independently. In 2000, Warren Smith [4] evaluated ranked and rated voting systems under a Monte Carlo simulation model of voter utilities and behaviors. We replicate Smith's results with a wider selection of voting systems and voter utility distributions and conclude that range voting, a rated system where each candidate can be independently scored in the range  $[0, 1]$ , has the best performance.

This project's source code is freely available online at [https://github.com/RussellEmerine/voting\\_simulation](https://github.com/RussellEmerine/voting_simulation).



## **Acknowledgments**

My advisor is cool.



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# 1 Introduction

Many various voting systems have been proposed, used, and analyzed. Past research often evaluates properties of the voting systems, such as unexpected election outcomes or strange strategic voting behavior. Arrow's theorem [1], one of the most well-known results in the field, reveals a limitation that applies to all possible ranked voting systems, where votes can be expressed as an ordering of the candidates. Arrow's theorem does not apply to rated voting systems, where votes can give information on how much one candidate is preferred over the other. However, both these classes of voting systems have the limitation that they are susceptible to strategic voting.

Taking inspiration from Smith [4], we seek to evaluate voting systems under a statistical Monte Carlo simulation model, allowing ranked and rated systems and accounting for strategic voting. We define a utility model so that we may numerically evaluate the societal outcome of an election result. We list several voting systems and specify how honest and strategic voters behave for each. We describe the method used to generate utilities. We run Monte Carlo simulations to repeatedly produce utilities, votes, an election result, and an evaluation of that particular result. We consider those evaluations collectively in order to evaluate each voting system. We take this method of evaluation to be a good indicator of the performance of a voting system, and conclude with an analysis across voting systems.



## 2 The Utility Model

We represent and evaluate voting systems under the utility model. Each voter has a utility in  $[0, 1]$  for each candidate. When the candidate is elected, each voter receives the corresponding utility. The performance of the whole system is then the sum of the utilities (or equivalently the average utility across all voters).

Traditional analyses of ranked voting systems only allow each voter to have an ordering of the candidates by preference, on the grounds that voters cannot intrinsically know how their utilities compare to other voters' utilities. However, if we allow ourselves to believe voters can produce a reasonable comparison of their own utilities to other voters' utilities, we can model situations that ranked voting systems cannot.

Suppose 60% of voters prefer candidate  $c_1$  over candidate  $c_2$  only slightly, and 40% of voters prefer  $c_2$  over  $c_1$  very strongly. Under any ranked system,  $c_1$  always wins. Under a sufficiently flexible rated system (such as range voting), there is at least a reasonable chance that voters from the 60% submit a smaller margin of preference, resulting in  $c_2$  winning.

Furthermore, evaluating the outcome using the utility model is straightforward. Evaluating the outcome of a ranked system without using utilities is not immediately obvious, and may become impossible, or at the very least arbitrary, when Condorcet cycles are present (see the “Condorcet Least Reversal” and “Condorcet Ranked Pairs” voting systems).

If voters cannot compare their utilities to other voters' utilities (or are not confident in the accuracy of their comparisons), we can account for it in the simulation. Before deciding on votes, we linearly scale each voter's utilities so that the least favored candidate has utility 0 and the most favored candidate has utility 1. We cast votes and determine the election winner using these scaled utilities. Then, we evaluate the candidate against the voters' original unscaled utilities. We run the simulation program with and without this rescaling operation.





## 3 Voting Systems

A *voting system* is a system where a (usually large) number of *voters* cast *votes* of some kind, and the votes are aggregated in some process to select one of a (usually small, but greater than one) number of *candidates*.

Some formulations allow the output to be a set of candidates or an ordering of candidates. For simplicity, we will only consider systems that output a single winner. We will also focus on cases with three or more candidates, as with two candidates, all reasonable voting systems fall into either the ranked or rated behaviors as described in 2.

We first describe several types of voting systems, then list and describe the particular voting systems we consider in our simulation.

### 3.1 Ranked and Rated Voting Systems

#### 3.1.1 Ranked Voting Systems

In a ranked voting system, vote information can be encoded into an ordering of the candidates. For instance, plurality voting uses the most favored candidate from each ordering, while Borda voting uses the whole ordering. Ranked voting systems traditionally output an ordering of candidates (which in this section we refer to as the “outcome”); to choose our single winner we can simply choose the most favored candidate in the outcome.

Arrow’s theorem states that, if there are at least three candidates, no ranked voting system may satisfy all of the following properties [1]:

- Pareto efficiency: If every voter places  $c_i$  before  $c_j$ , then the outcome also places  $c_i$  before  $c_j$ . (This can be weakened to non-imposition, i.e. that for any  $c_i$  and  $c_j$ , there is some cast of votes such that the output places  $c_i$  before  $c_j$  [5].)
- Non-dictatorship: There is no voter whose ordering is always the same as the outcome.
- Independence of irrelevant alternatives: In two elections with the same number of voters where each pair of corresponding voters has the same relative ordering of  $c_i$  and  $c_j$ , the outcomes of both elections have the same relative ordering of  $c_i$  and  $c_j$ .

The fact that it is impossible to have all three of these very reasonable conditions is a limitation of ranked voting systems.

### 3.1.2 Rated Voting Systems

Votes need not be restricted to orderings of candidates. Rated voting systems allow a voter to provide independent scores for each candidate, which can be more informative than an ordering of the candidates. For instance, range voting allows a voter to give each candidate their own score in the range  $[0, 1]$ , thereby allowing a voter to express how much they prefer their most favored candidate over their second most favored candidate.

Since these are not ranked voting systems, Arrow’s theorem does not apply, and it is completely possible to have Pareto efficiency, non-dictatorship, and independence of irrelevant alternatives. However, they are still subject to some restrictions, notably Gibbard’s theorem, that no deterministic process of collective decision may satisfy all of the following properties [2]:

- The process has more than two possible outcomes.
- There is no voter who singlehandedly determines the outcome.
- The game-theoretically optimal vote for a voter will not depend on the voter’s beliefs of what other voters will vote.

When applied to voting systems, this implies that non-dictatorial voting systems with three or more possible outcomes (ranked, rated, or otherwise) require some kind of strategic voting that is not completely honest.

*An aside about determinism:*

Most voting systems do not have a good way of handling ties without using randomness or allowing multiple winners. However, when the number of voters is reasonably large, the chance of a tie is negligible. We will consider voting processes that are deterministic when there are no ties “good enough.”

### 3.1.3 Other Voting Systems

There are a few “obviously bad” voting systems that we consider, such as “random winner” and “worst candidate”. These are only useful as a frame of reference, and are not expected to have any of the properties discussed for ranked and rated systems.

## 3.2 COAF and Non-COAF Systems

### 3.2.1 COAF Systems

Smith’s specification of compact set based, one-vote, additive, fair voting systems describes many common ranked and rated voting systems [4]. In a COAF system with  $C$  candidates there is a compact set  $S \subseteq \mathbb{R}^C$  of allowed votes, where  $S$  is symmetric across permutations of candidates (fair). Each voter chooses one vote in  $S$  to submit. Then, the votes are added, and the candidate with the greatest sum (or equivalently, average, referred to as “score” for the following proof) is selected as the winner. For instance, plurality voting is when  $S = \{(1, 0, 0, \dots), (0, 1, 0, \dots), (0, 0, 1, \dots), \dots\}$ , and range voting is when  $S = [0, 1]^C$ .

Smith provides a formal proof of the optimality of the “moving average” strategy. This strategy generates the game theoretically optimal vote in any COAF system when given poll data in the form of a predicted ordering of candidates by likelihood to win the election (assumed to be produced by a random sample of honest pollees). Consider a COAF system with vote set  $S$  and without loss of generality label the candidates as  $c_1, c_2, \dots, c_C$  in poll order. Let  $U_1, U_2, \dots, U_C$  be the utilities of the voter for each candidate, under the utility model discussed in 2. The vote is generated as follows:

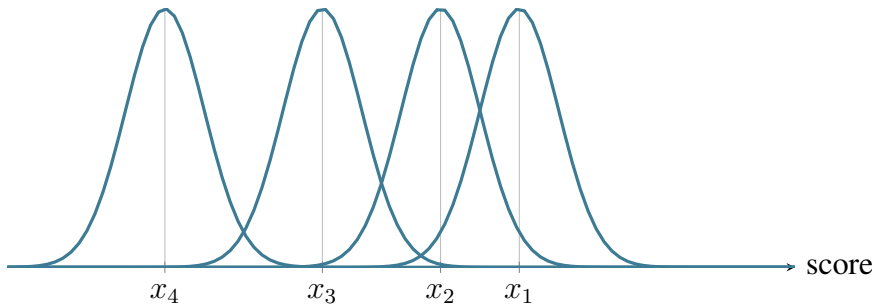
- Let the set  $X_0$  representing the set of potential votes start as  $X_0 = S$ .
- If  $U_1 > U_2$ , let  $X_1$  be the subset of  $X_0$  that maximizes the 1st component. Otherwise, let  $X_1$  be the subset of  $X_0$  that minimizes the 1st component.
- For each candidate  $c_i$  in poll order starting from  $c_2$ , if  $U_i > \frac{1}{i-1} \sum_{j=1}^{i-1} U_j$ , let  $X_i$  be the subset of  $X_{i-1}$  that maximizes the  $i$ th component. Otherwise, let  $X_i$  be the subset of  $X_{i-1}$  that minimizes the  $i$ th component.

The final set  $X_C$  will consist of exactly one vector, which will be the vote.

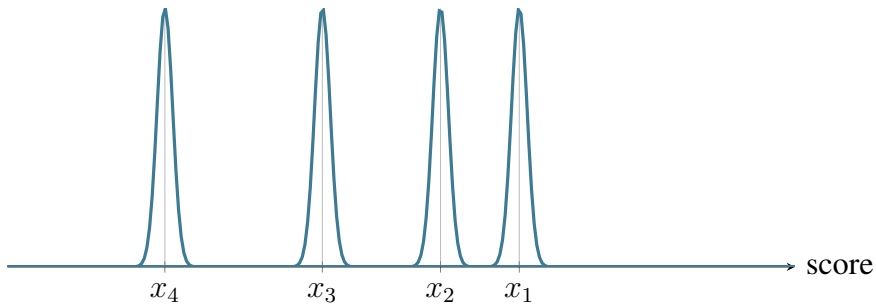
We present an alternate interpretation of Smith’s proof.

Let us say there are  $C$  candidates,  $V$  votes, and  $P$  pollees, where  $V$  and  $P$  are reasonably large. Let us say that across all votes, the  $i$ th component of the vote vector lies in a distribution with mean  $\mu_i$  and variance  $\sigma_i^2$ .  $\mu_i$  is the actual score the candidate will receive.  $\sigma_i^2$  has no direct effect on the election result but is useful for strategic analysis.

In this strategy, the only information voters have on the candidates is the polling data and their own personal associated utilities. To model this, we will assume a voter’s belief of the distribution of a candidate  $c_i$ ’s score  $\mu_i$  follows a normal distribution with the same variance as would be expected from the sample distribution, which by the central limit theorem is  $\frac{\sigma_i^2}{P}$ . These belief distributions have means we will call  $x_1, x_2, \dots, x_C$  — the numerical values of these are not important, but they are known to be in the polling order, as shown in this plot of the believed distributions of  $\mu_4, \mu_3, \mu_2$ , and  $\mu_1$ :



Since the  $P$  is large, we can assume the variance  $\frac{\sigma_i^2}{P}$  is very small:



The voter's vote can only change the outcome of the election if the values of the actual two highest scores are a “near tie”, within  $\frac{1}{V}$  of each other. Otherwise, the vote cannot change the outcome of the election — this case can be considered a fixed component of the expected utility, and so can be ignored for reasoning about the optimal vote. The following reasoning will assume there is some near tie.

Consider some fixed  $\mu_1$  near  $x_1$ . This can be within, say, 1000 standard deviations. With large  $P$ , the standard deviation is small enough that this is nowhere close to  $x_2$ . Since the probability that  $\mu_1$  is near  $x_1$  is large, its contribution dominates the expected utility, and we can ignore the case that  $\mu_1$  is not near  $x_1$ . Likewise, the probability that every other candidate  $c_i$  has  $\mu_i \leq \mu_1 + \frac{1}{V}$  is large, and we can ignore the other case. Now, the only result-determining near ties that can happen are between  $c_1$  and some other  $c_i$ . The chance of such a near tie is proportional to  $\frac{1}{V} \exp\left(-\frac{P(x_1 - x_i)^2}{2\sigma^2}\right)$ . Since  $P$  is large, this chance is much larger for  $c_2$  than for any other  $c_i$ . Therefore, the voter must optimize the vote for the case that  $c_1$  and  $c_2$  have a near tie, and so will choose some vote that maximizes or minimizes the 1st and 2nd components according to whether the voter prefers  $c_1$  over  $c_2$ . This determines the vote subsets  $X_1$  and  $X_2$ .

After narrowing down the potential votes to  $X_2$  by considering when  $c_1$  is a member of the near tie, there may still be many possible votes to make. It is already a very small chance for the optimization to  $X_2$  to affect the outcome of the election, and it is even less likely for any remaining optimization to do so — and so the restriction to  $X_2$  is already “good enough” for most purposes (which will be useful for non-COAF systems). However, further optimizations can still be determined using similar reasoning.

### 3.2.2 Non-COAF Systems

Ranked and rated systems can also be non-COAF. For instance, the “plurality with runoff” system (see 3.3.7) re-scores two candidates based on the condition that they are the frontrunners by plurality — this type of conditional cannot be modeled in an additive system.

The strategic voter behavior for non-COAF systems can be complex. For instance, strategic voting in a three-candidate plurality with runoff system may require some voters with the same preferences to vote in different ways [3]. However, as discussed in the previous section, it is very unlikely that any candidate other than the two poll frontrunners will have a chance to affect the election. We simply ensure the strategic vote optimizes for near ties between the two poll frontrunners and, under that restriction, use honest behavior for determining a unique vote. For the various ranked non-COAF systems, this is done by placing the frontrunners first and last, then keeping all other candidates in the same order.

## 3.3 A List of Voting Systems Considered

### 3.3.1 Random Winner

A candidate is chosen uniformly at random to be the winner. Useful as a frame of reference.

### 3.3.2 Random Dictator

A voter is chosen uniformly at random to be a dictator. The candidate that the dictator likes the most is the winner. Useful as a frame of reference.

### 3.3.3 Worst Candidate

The candidate that gives the worst possible sum of utilities is the winner. Useful as a frame of reference.

### 3.3.4 Range Voting

The most general COAF voting system, where  $S = [0, 1]^C$ .

Honest voters simply submit their utilities (possibly scaled as discussed in 2).

We evaluate two strategic voting behaviors for range voting. One is the moving average strategy. The other is an alternate strategy that sets a threshold at the average of the utilities of the two frontrunners, and votes 0 for candidates below the threshold and 1 for candidates above the threshold.

Note that strategic voters will always score candidates at 0 or 1, while honest voters might use other values.

### 3.3.5 Approval Voting

A COAF voting system where  $S = \{0, 1\}^C$ .

Honest voters use the average utility between all candidates as a threshold, and submit 0 for candidates below the threshold and 1 for candidates above the threshold.

Strategic voters in range voting always score candidates at 0 or 1, so their strategies work just as well in approval voting. We evaluate the same two strategies for approval voting as we do for range voting.

The only difference between approval voting and range voting is the behavior of honest voters, where range voting allows for describing one's preferences in more detail.

### 3.3.6 Plurality Voting

A COAF voting system where  $S = \{(1, 0, 0, \dots), (0, 1, 0, \dots), (0, 0, 1, \dots), \dots\}$ . In other words, each voter votes for one candidate, and the candidate with the most votes wins. This is perhaps the most commonly known voting system.

Honest voters submit their most favored candidate.

Strategic voters submit their most favored candidate among the two frontrunners from the poll ordering.

One interesting feature of plurality voting is that utility generally increases when more voters vote strategically. Consider the not uncommon case of the spoiler effect, exemplified by setting 10% of voters to have preferences  $c_3 > c_2 > c_1$ , 47% to have  $c_1 > c_2 > c_3$ , and 43% to have  $c_2 > c_1 > c_3$  (each with an equal utility difference between  $c_1$  and  $c_2$ ). If the 10% bloc votes honestly for  $c_3$ ,  $c_1$  will win, while if the 10% bloc votes strategically for  $c_2$ ,  $c_2$  will win — and  $c_2$  winning has greater total utility.

### 3.3.7 Plurality with Runoff Voting

A non-COAF system where voters first make a plurality vote (the “first stage”), then decide between the two frontrunners by a two-candidate ranked vote (the “second stage”).

For the first stage, honest voters submit their most favored candidate.

For the first stage, strategic voters submit their most favored candidate among the two frontrunners from the poll ordering.

For the second stage, honest and strategic voters submit their most favored candidate from the two allowed candidates.

This often is presented as a two-stage election, but can also be done with one vote by allowing  $C \times 2^{\binom{C}{2}}$  possible votes, consisting of a single candidate for the plurality stage, then the most favored candidate of each pair, to be used depending on the outcome of the plurality stage. (This detail does not matter for analysis or our simulation but may be useful for other implementations of simulations.)

### 3.3.8 Bullet Voting

(“Bullet voting” seems to be Smith’s terminology. We use it for consistency.)

A COAF system where  $S = \{(0, 1, 1, \dots), (1, 0, 1, \dots), (1, 1, 0, \dots), \dots\}$  (or equivalently  $\{(-1, 0, 0, \dots), (0, -1, 0, \dots), (0, 0, -1, \dots), \dots\}$ ). In other words, each voter votes *against* one candidate, and the candidate with the *fewest* votes wins.

Honest voters vote against their least favored candidate.

Strategic voters vote against their least favored candidate out of the two frontrunners from the poll ordering.

### 3.3.9 Borda Voting

A COAF system where  $S$  is the permutations of  $(0, 1, 2, \dots, C-1)$  (or equivalently  $(\frac{0}{C-1}, \frac{1}{C-1}, \dots, \frac{C-1}{C-1})$ ). In other words, each voter ranks all candidates, each candidate receives a linearly weighted score, and the candidate with the highest average score wins.

Honest voters rank the candidates honestly.

Strategic voters rank the candidates by the moving average strategy, in particular placing the more favored of the two frontrunners first in the ranking and the less favored of the two frontrunners last in the ranking.

### 3.3.10 Dabagh Voting

Also known as “Point-and-a-half” voting.

A COAF system where each vector in  $S$  has one candidate scored at 1, one candidate scored at 0.5, and all other candidates scored at 0. In other words, each voter ranks all candidates, each candidate receives a score of 1 for all voters putting them first and 0.5 for all voters putting them second, and the candidate with the highest average score wins.

Honest voters rank the candidates honestly, thus giving the score of 1 to their most favored candidate and the score of 0.5 to their second most favored candidate.

Strategic voters rank the candidates by the moving average strategy, in particular giving the score of 1 to the more favored of the two frontrunners, giving a score of 0 to the less favored of the two frontrunners, and giving the score of 0.5 to some other candidate.

### 3.3.11 Condorcet Least Reversal Voting

A non-COAF ranked system based on defeating Condorcet cycles. We consider the graph with candidates as vertices and pairwise margins of defeat as edges. For example, if 200 voters place  $c_1$  before  $c_2$  and 50 voters place  $c_2$  before  $c_1$ , then there is an edge from  $c_2$  to  $c_1$  with a weight of 150. If there are any candidates with no outgoing edges (i.e. no pairwise defeats), this candidate is the *Condorcet winner*, and wins the election. Otherwise there is a cycle in the graph called a *Condorcet cycle*. Find the set of edges of least summed weight such that flipping them identifies a Condorcet winner; that candidate wins the election.

(This specific formulation is generalizable to multiple winners or rank output. However, for the single-winner case this is in practice a matter of finding the smallest sums of weights of each vertex’s outgoing edges.)

Honest voters rank the candidates honestly.

Strategic voters rank the more favored of the two frontrunners first, the less favored of the two frontrunners last, and the rest of the candidates honestly.

### 3.3.12 Condorcet Ranked Pairs Voting

A non-COAF ranked system based on defeating Condorcet cycles. As before, we consider the graph with candidates as vertices and pairwise margins of defeat as edges. Consider each pair of candidates in order of largest to smallest margin of victory. If the pair does not form a Condorcet cycle, then add an edge between the two candidates; otherwise ignore it. At the end there will be a unique candidate that wins all pairwise comparisons under consideration.

Honest voters rank the candidates honestly.

Strategic voters rank the more favored of the two frontrunners first, the less favored of the two frontrunners last, and the rest of the candidates honestly.

### 3.3.13 Single Transferable Vote (Fewest First)

Also known as (a form of) instant runoff voting.

A non-COAF ranked system where the candidate with the fewest first-place rankings is repeatedly removed until there is one candidate remaining.

Honest voters rank the candidates honestly.

Strategic voters rank the more favored of the two frontrunners first, the less favored of the two frontrunners last, and the rest of the candidates honestly.

### 3.3.14 Single Transferable Vote (Most Last)

Also known as (a form of) instant runoff voting.

A non-COAF ranked system where the candidate with the most last-place rankings is repeatedly removed until there is one candidate remaining.

Honest voters rank the candidates honestly.

Strategic voters rank the more favored of the two frontrunners first, the less favored of the two frontrunners last, and the rest of the candidates honestly.

This version in particular is rather susceptible to strategic voting, since the strategic voting patterns described will often eliminate *both* frontrunners.

### 3.3.15 Copeland

A non-COAF ranked system based on defeating Condorcet cycles.

The candidate that wins against the greatest number of other individual candidates wins the election. Since ties are common no matter the size of the voter count (e.g. if  $c_1$  and  $c_2$  both win against 3 of their 4 opponents), some tiebreaker is necessary. Borda count is a common tiebreaker, and is what we use.

Honest voters rank the candidates honestly.

Strategic voters rank the more favored of the two frontrunners first, the less favored of the two frontrunners last, and the rest of the candidates honestly.

### 3.3.16 Bucklin

If there is a candidate  $c_i$  with more than half of the voters placing  $c_i$  in first, then  $c_i$  wins. If not, then if there is a candidate  $c_i$  with more than half of the voters placing  $c_i$  in first or second, then  $c_i$  wins. This repeats for ranks 1 through  $k$  for each  $k$ . If there are ties (i.e. if incrementing  $k$  creates multiple potential winners at once), they are broken by the count of voters placing in ranks 1 through  $k$ .

Honest voters rank the candidates honestly.

Strategic voters rank the more favored of the two frontrunners first, the less favored of the two frontrunners last, and the rest of the candidates honestly.

### 3.3.17 STAR

An acronym for “score then automatic runoff”.



A non-COAF rated system where each voter submits a rating <sup>1</sup> of all the candidates. Then, the two candidates with the highest rating (call them  $c_1$  and  $c_2$ ) are put in a runoff. If more voters score  $c_1$  higher than  $c_2$ , then  $c_1$  wins; otherwise  $c_2$  wins.

Honest voters rate the candidates honestly.

Strategic voters rate the more favored of the two frontrunners at 1, the less favored of the two frontrunners at 0, and the rest of the candidates honestly.

<sup>1</sup>Wikipedia specifies that the ratings are integers in  $\{0, 1, 2, 3, 4, 5\}$ , but I instead just use reals in  $[0, 1]$  as with standard range voting.



## 4 Utility Distributions

Discussion of utility distributions, including uniform vs. normal vs. multimodal, and random vs. issue-based.



## **5 Simulation**

Discussion of implementation of simulation.



## 6 Analysis

Analysis of some plots.





# **7 Conclusions**

Conclusions of the project.



## 8 Appendix A: Plots

All the plot images.



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