Carnegie Mellon University

Tail Use in Quadruped Improves Static Stability in Diagonal Sequence Walking Gaits

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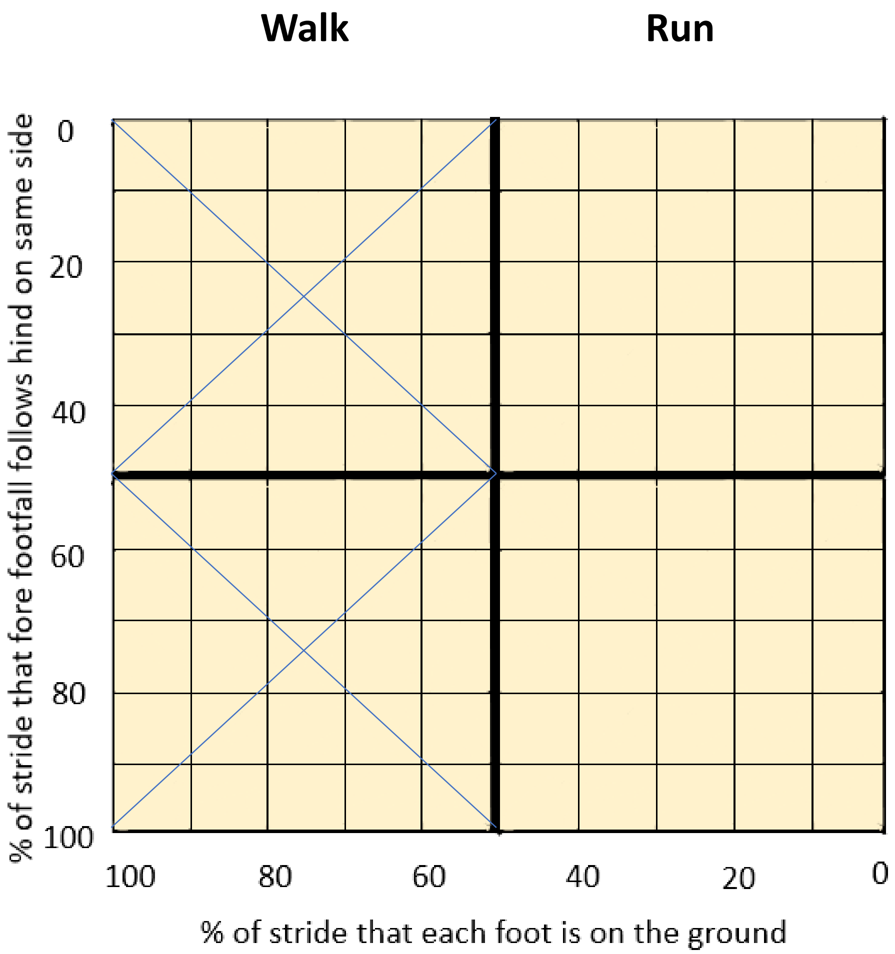
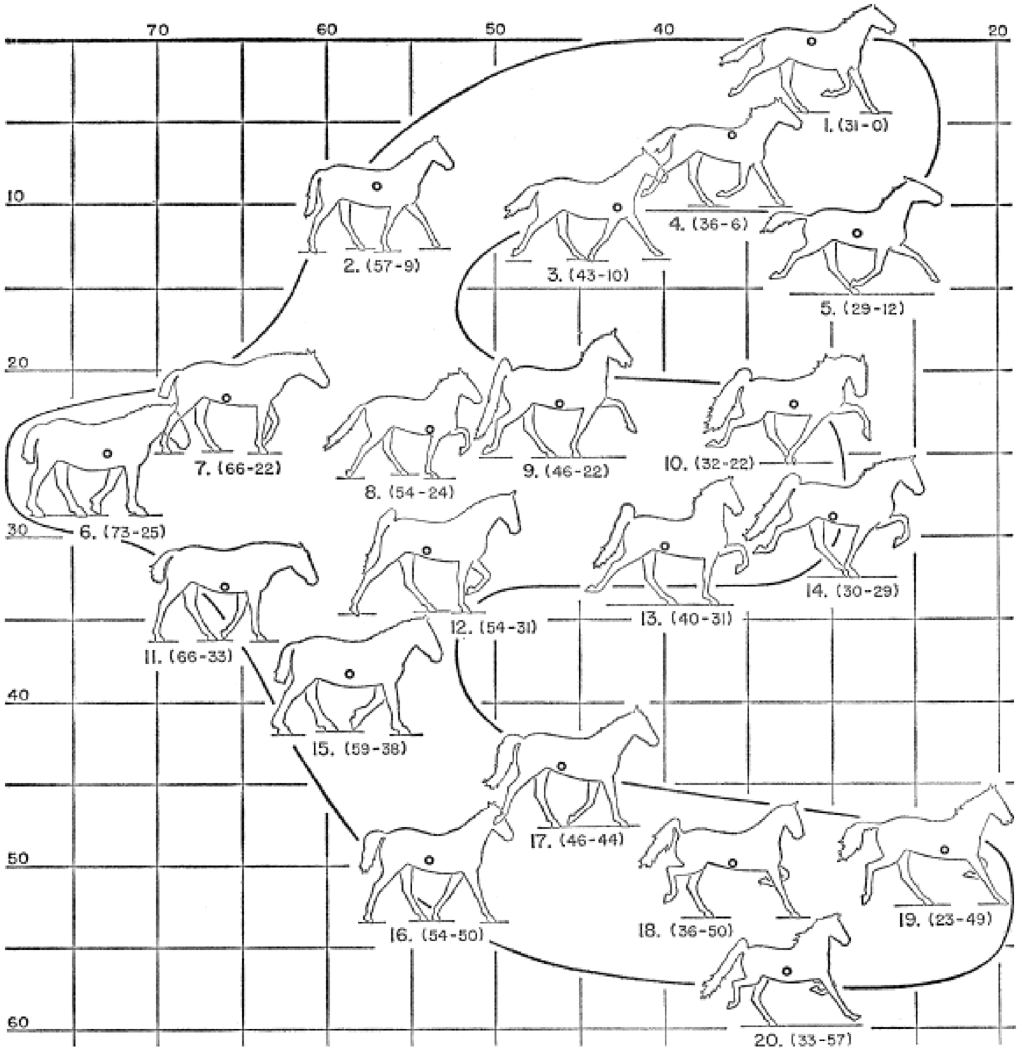
Hildebrand diagram





Hildebrand M. "Symmetrical Gaits of Horses" Source: Science, New Series, Vol. 150, No. 3697 (Nov. 5, 1965), pp. 701-708

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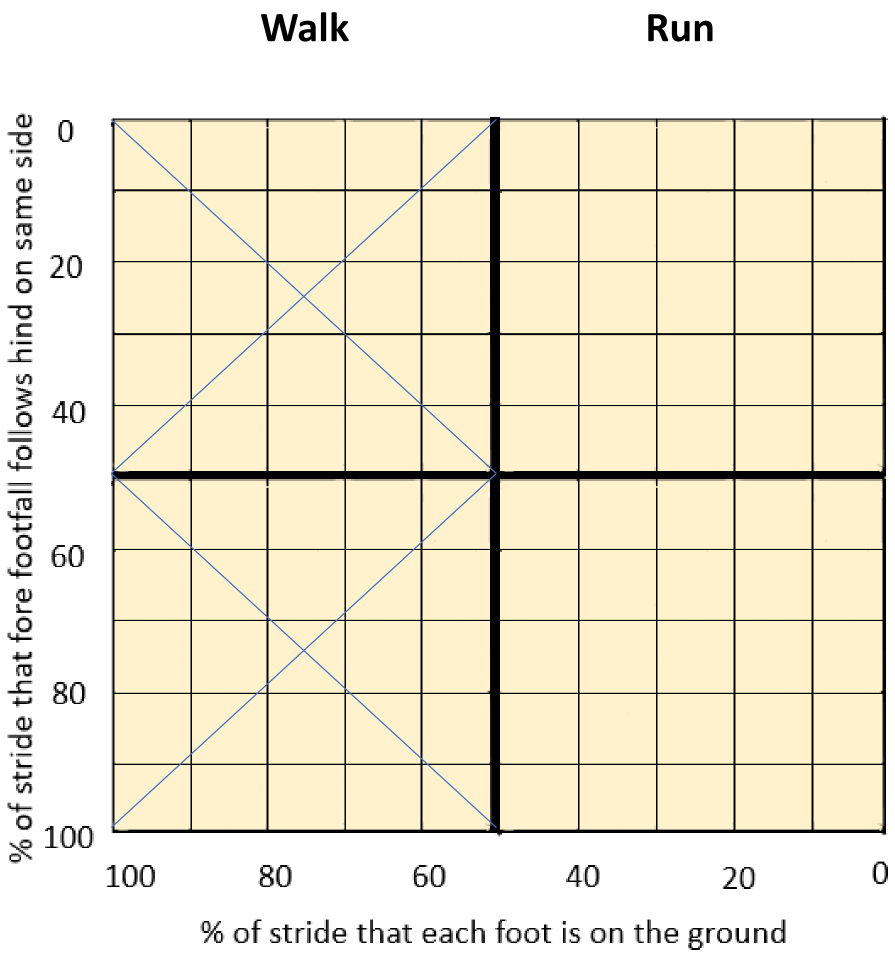
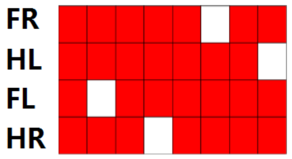
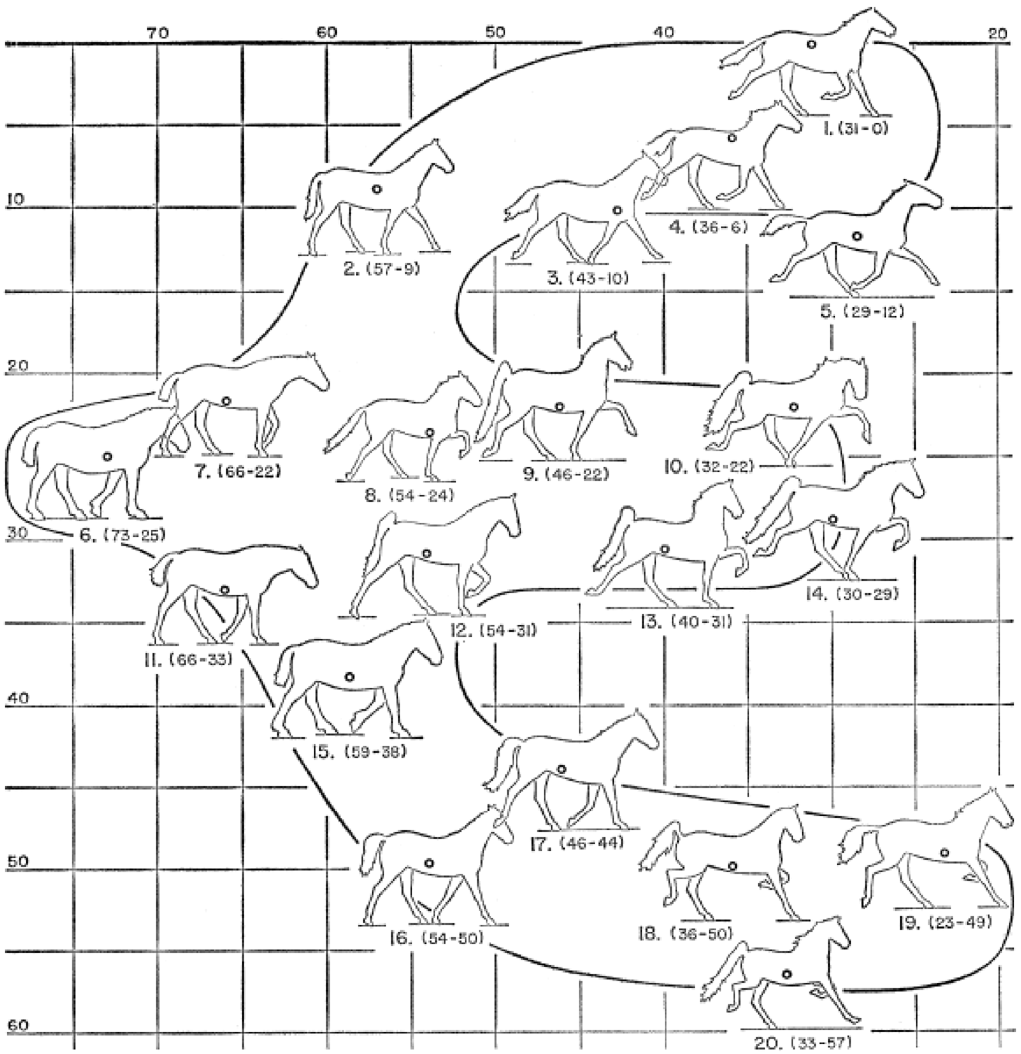


Sample Gait Formula



on the ground

in the air



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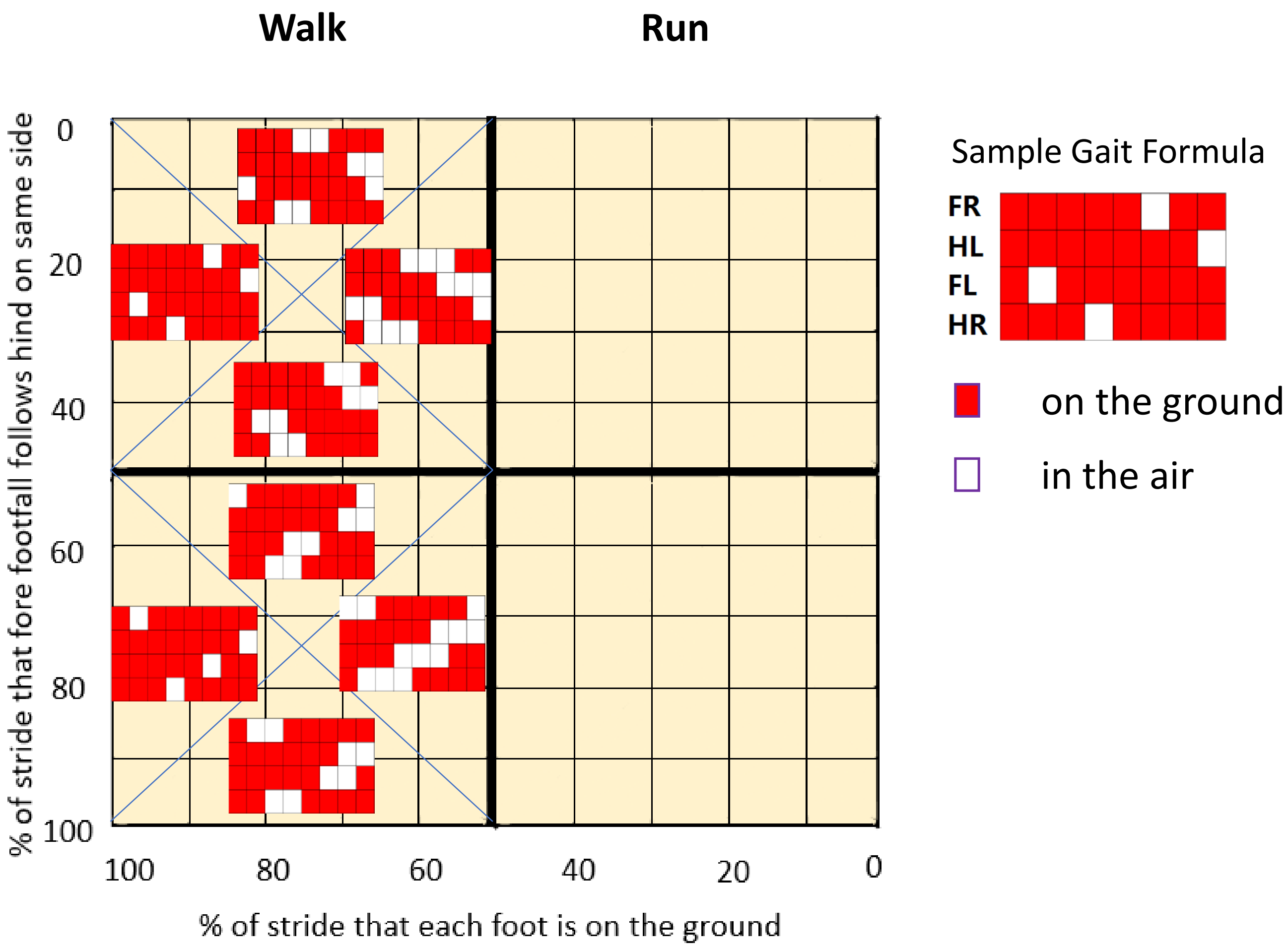

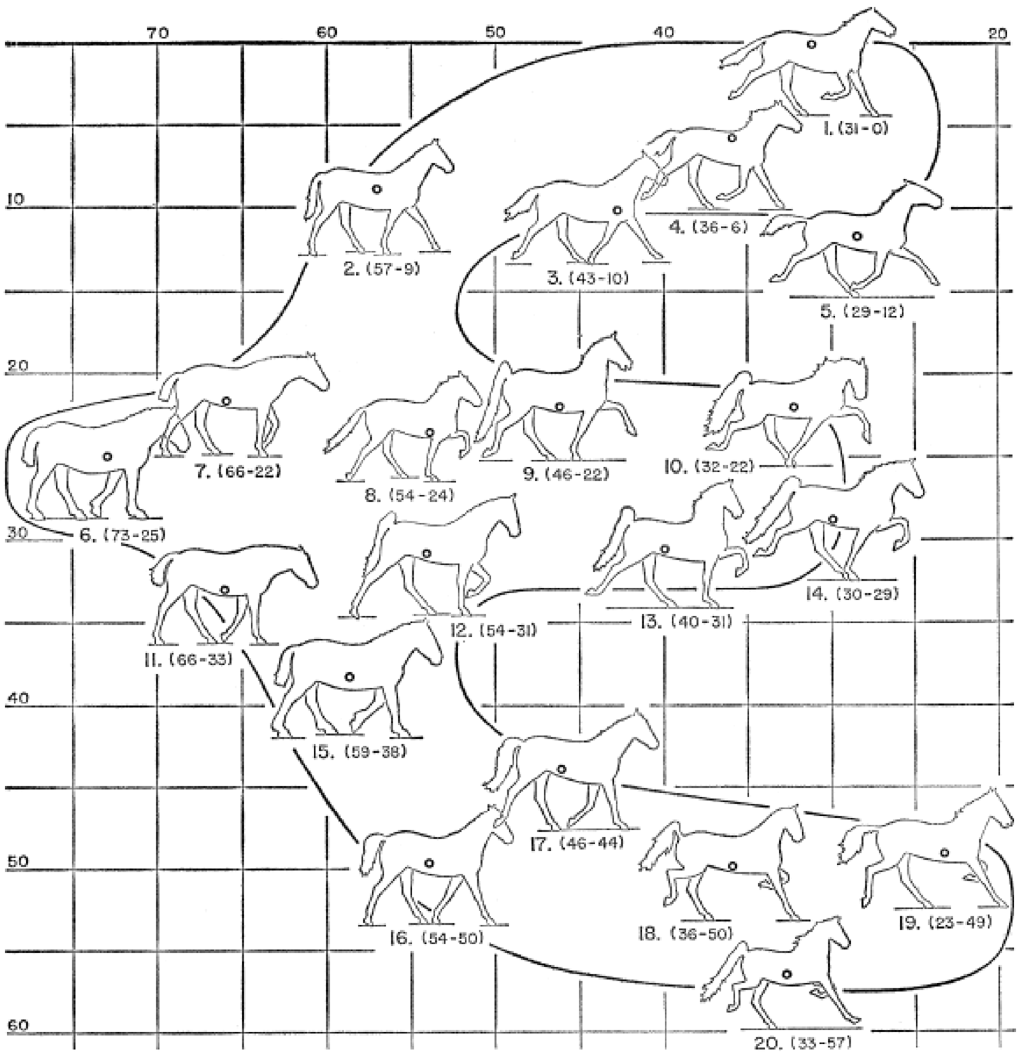

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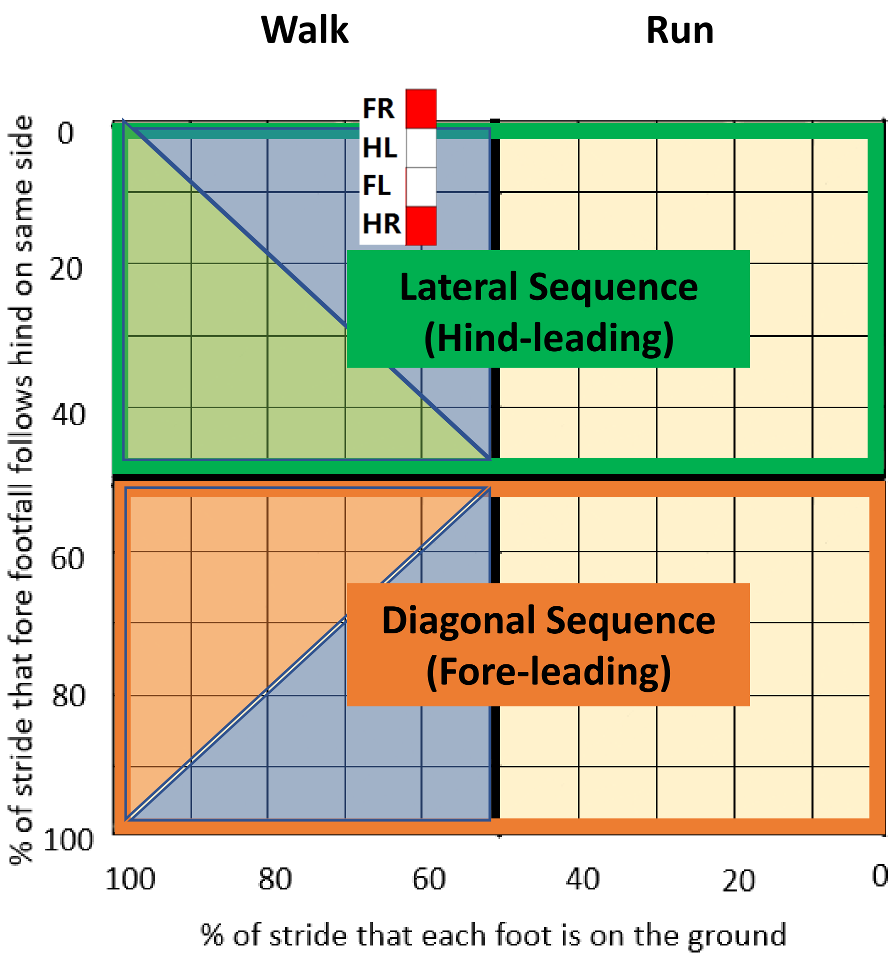
Hildebrand diagram and two types of walking gait



Note that, the gaits in blue shade have two legs on the same side lifting in the air at some moment, which is beyond the scope of this talk;

lateral sequence walking gait (green shade): the footfall of a given hind foot followed by the footfall of the forefoot on the **same** side of the body;

diagonal sequence walking gait (orange shade): the footfall of a given hind foot followed by the footfall of the forefoot on the **opposite** side of the body.

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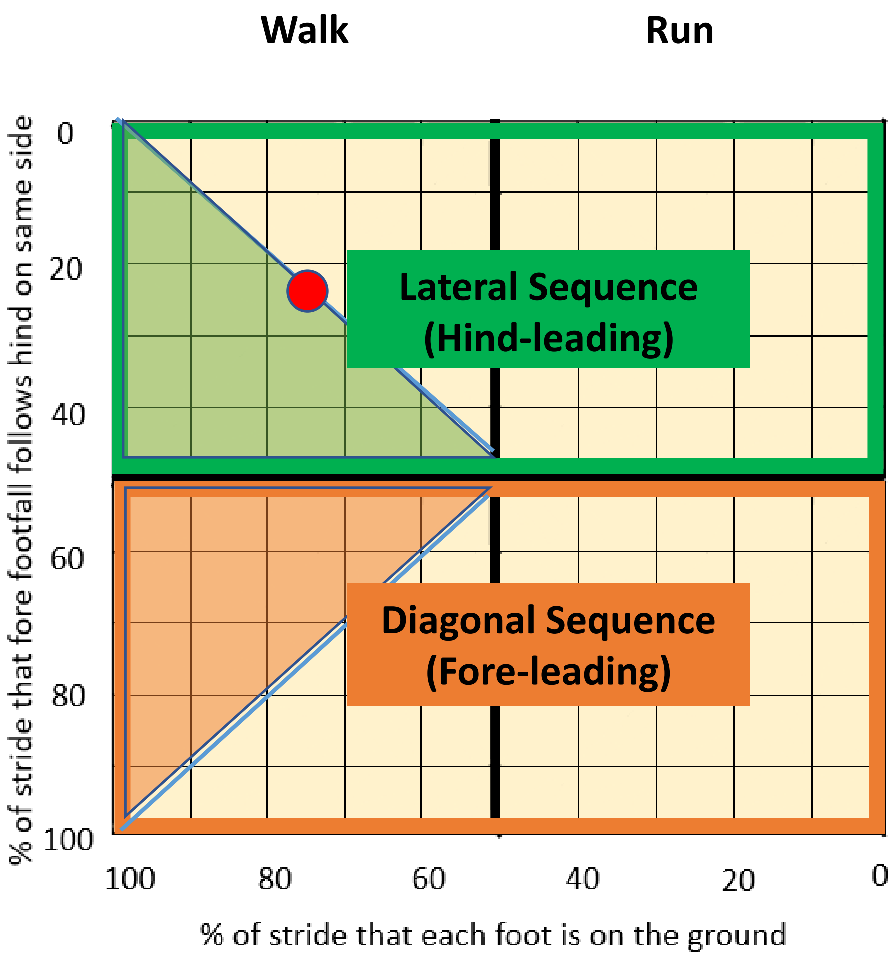
Hildebrand diagram and lateral sequence walking gait



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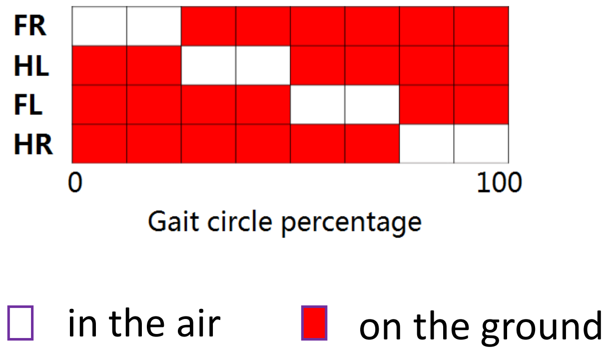
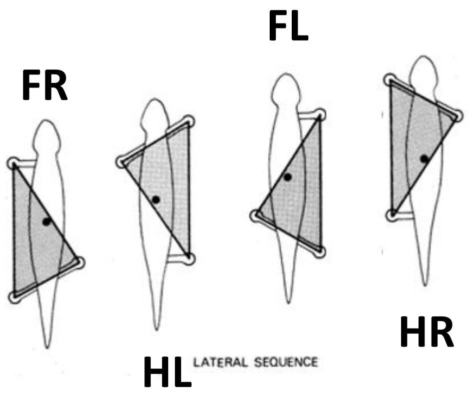
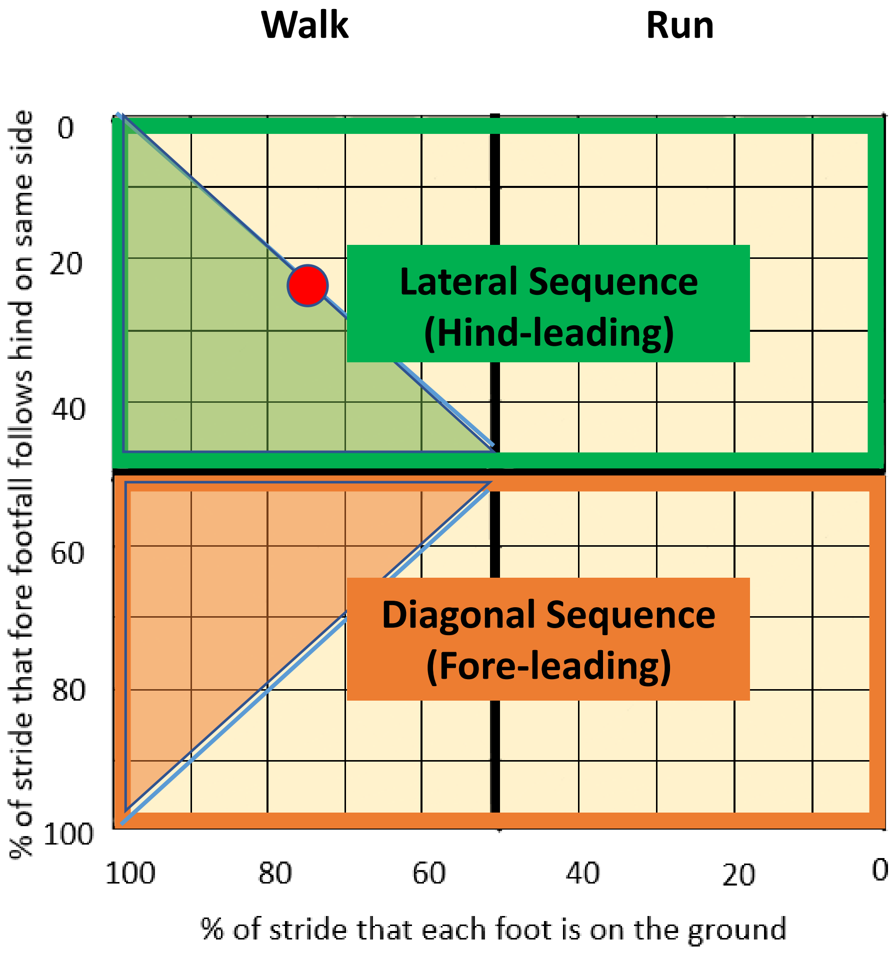




Hildebrand diagram and lateral sequence walking gait



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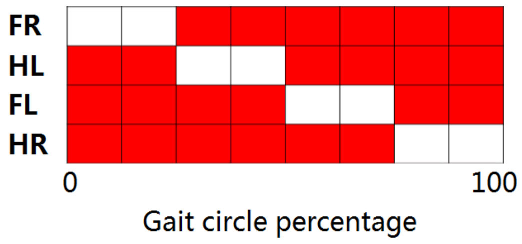
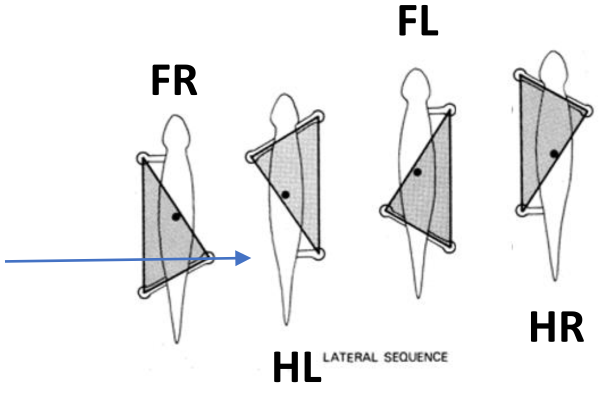
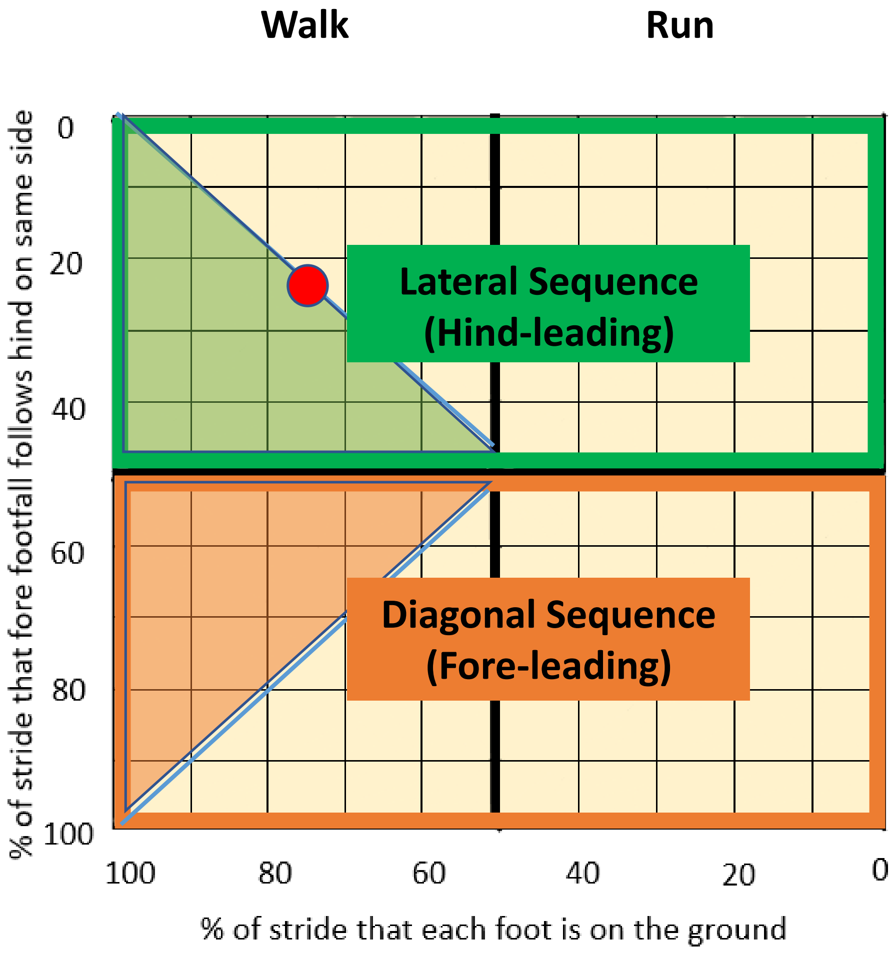


in the air   on the ground



Hildebrand diagram and lateral sequence walking gait



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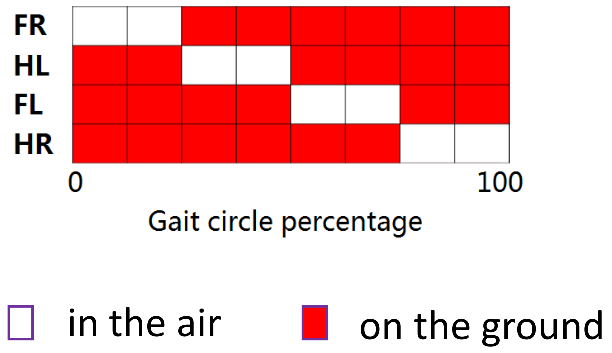
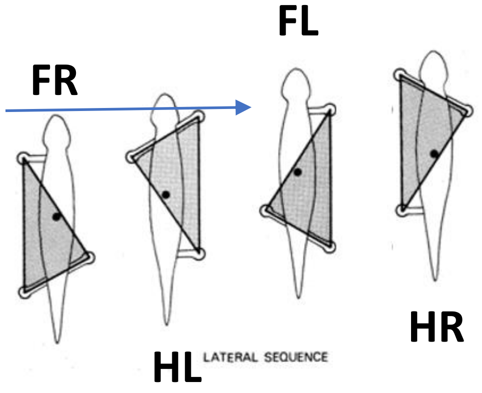
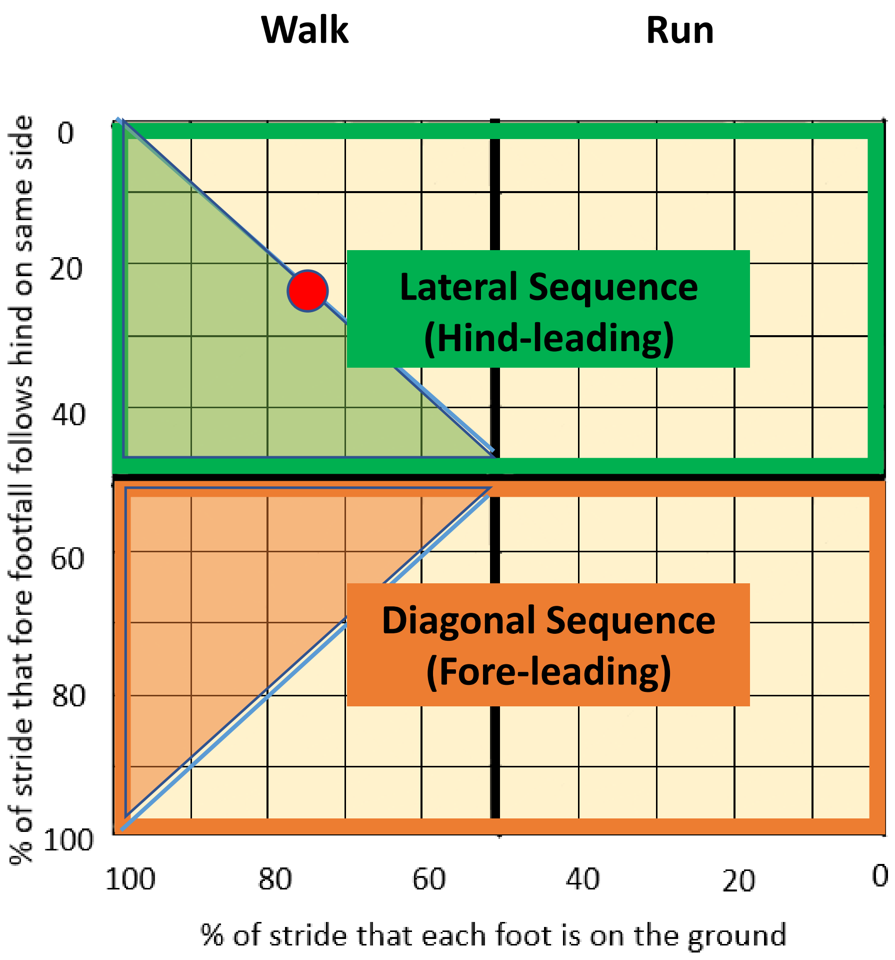




Hildebrand diagram and lateral sequence walking gait



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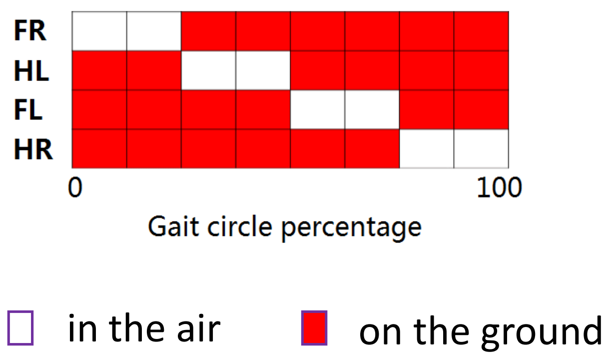
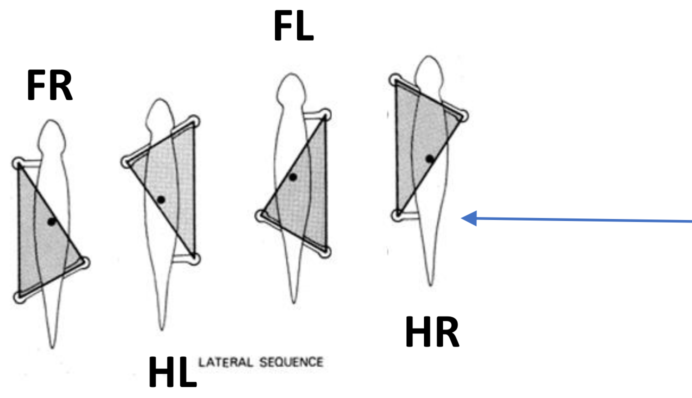
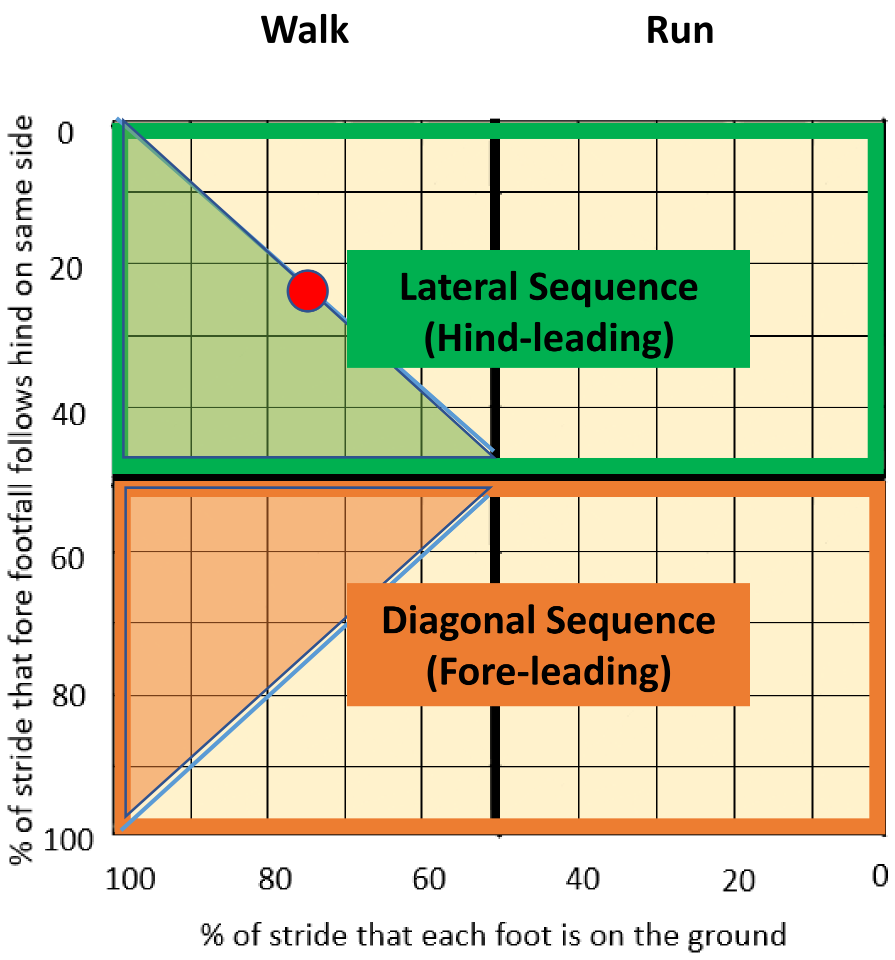




Hildebrand diagram and lateral sequence walking gait



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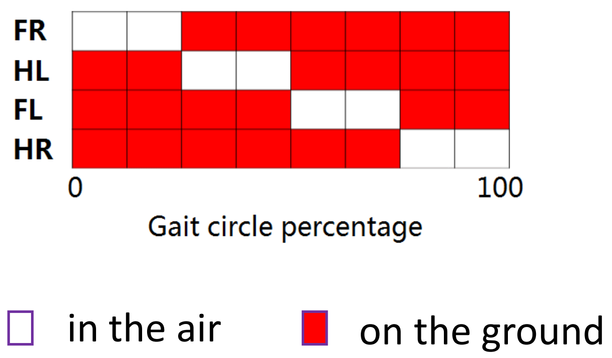
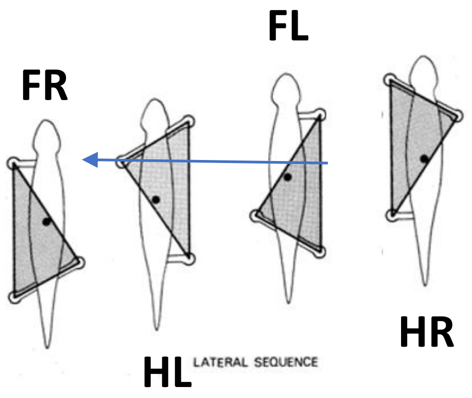
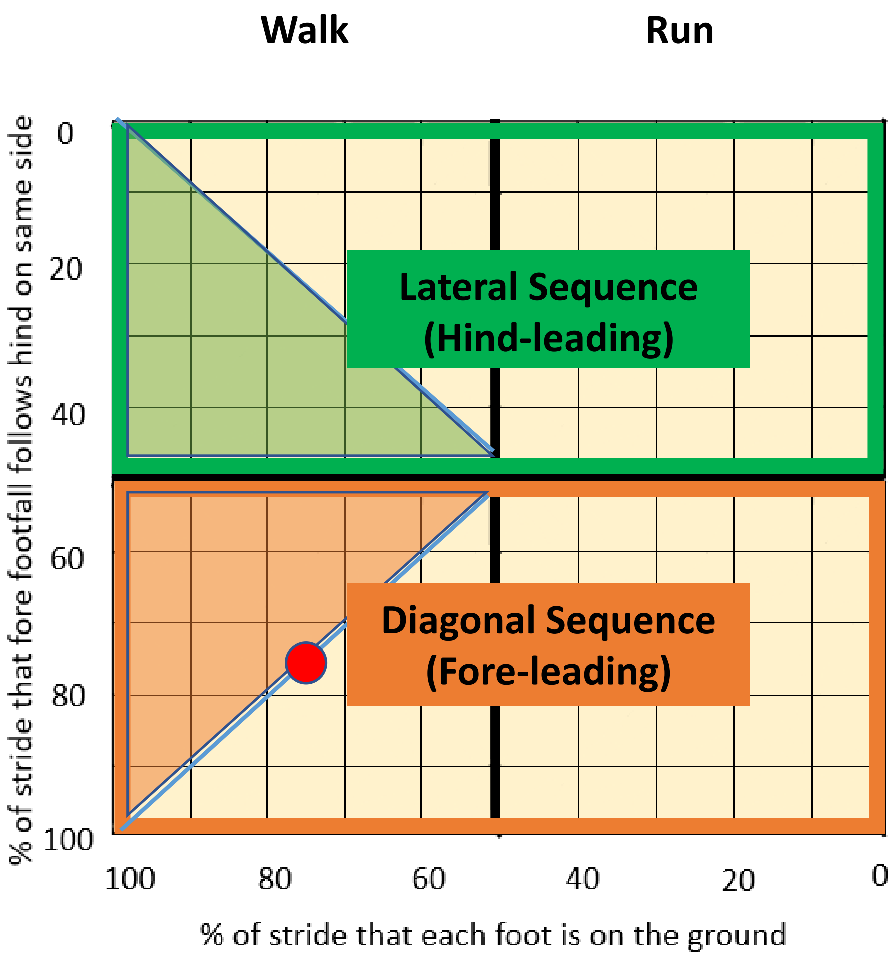
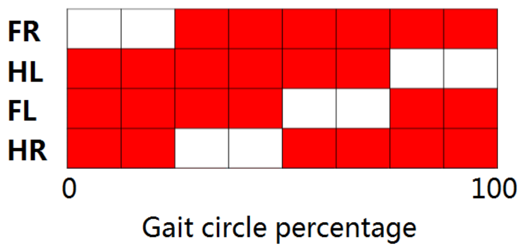
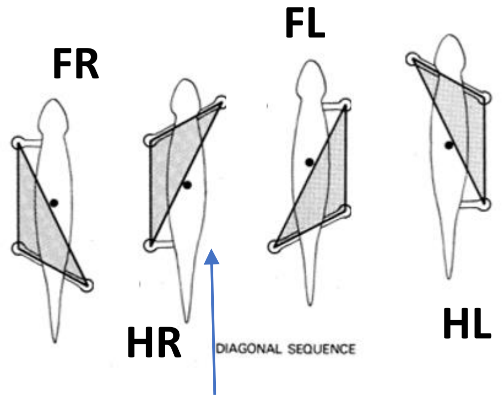




Hildebrand diagram and diagonal sequence walking gait

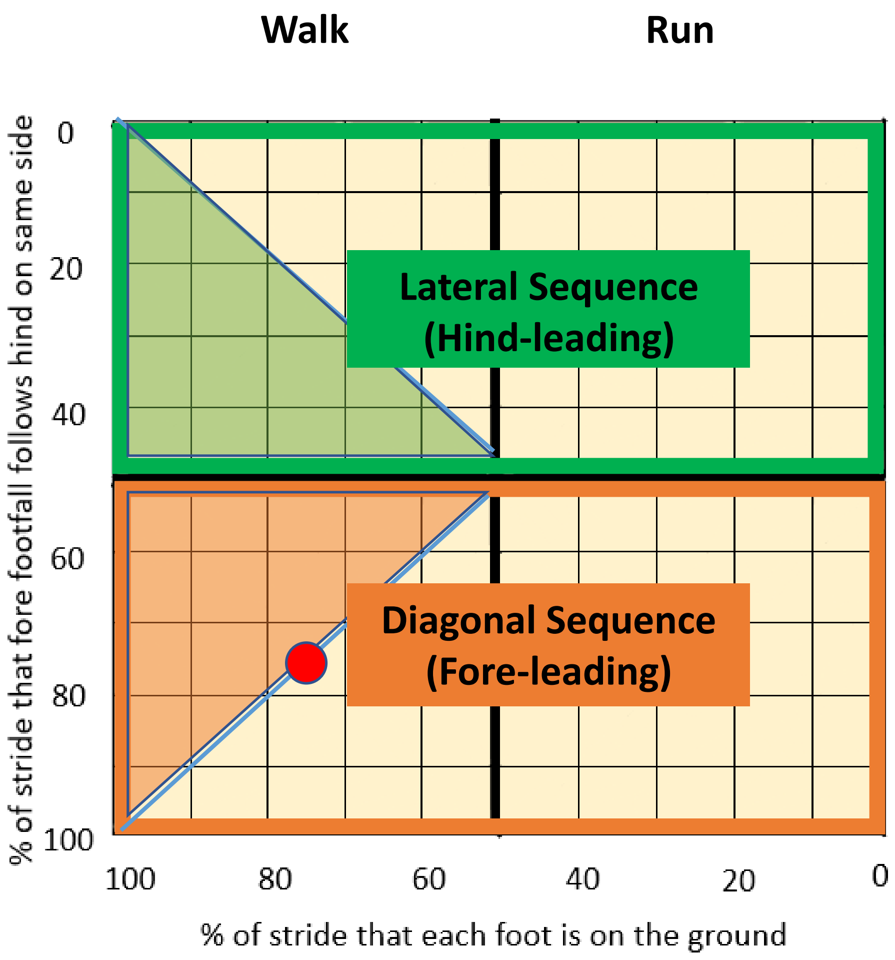


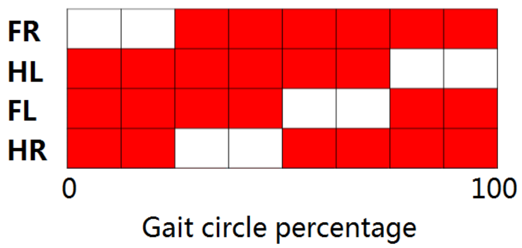


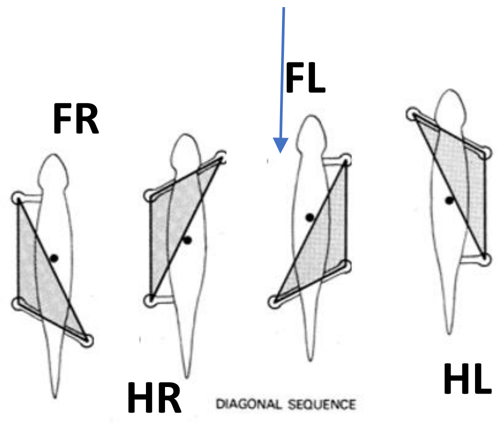


<u>diagonal sequence walking gait (orange shade): </u>the footfall of a given hind foot followed by the footfall of the forefoot on the opposite side of the body;

Hildebrand diagram and diagonal sequence walking gait







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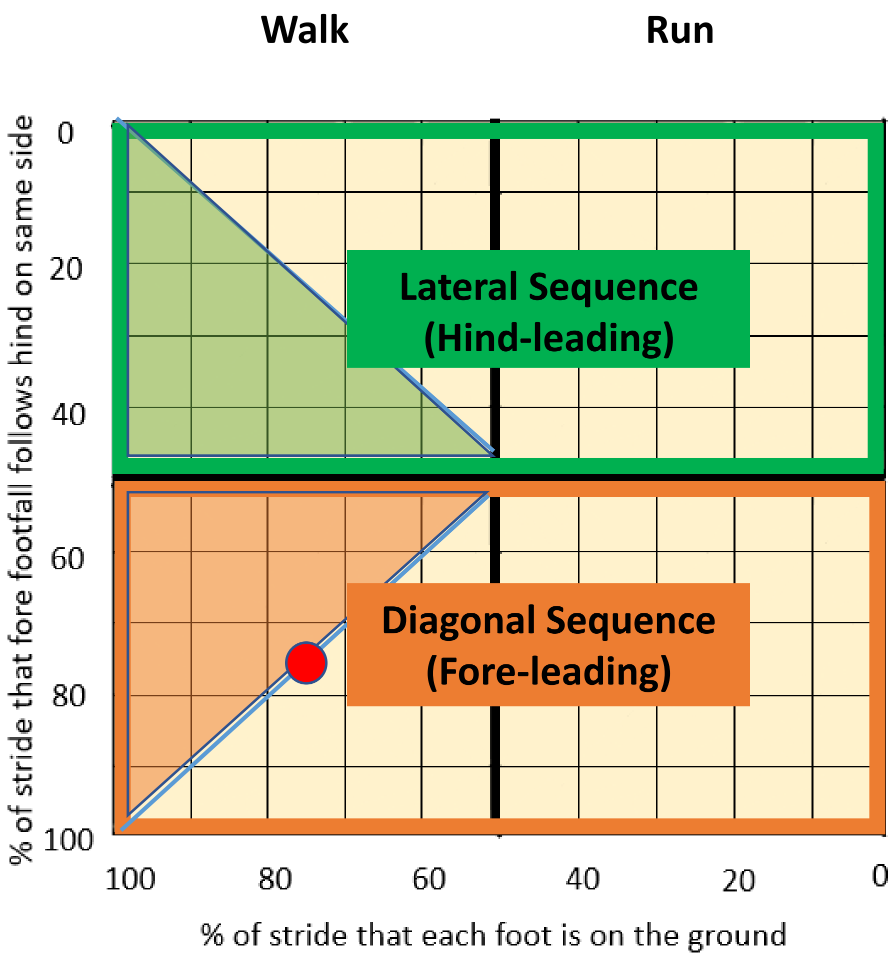
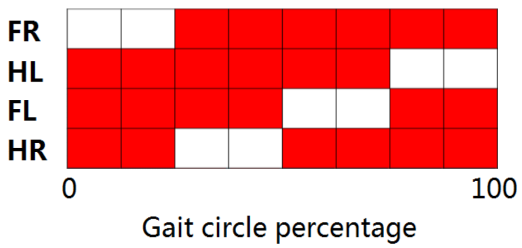
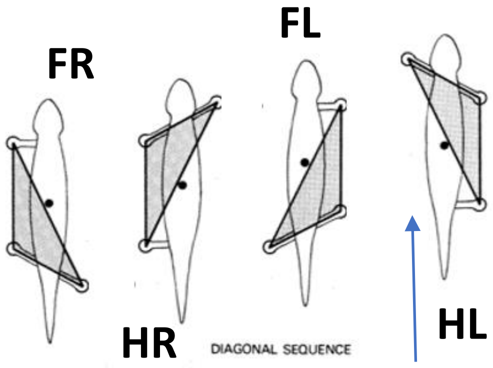
Hildebrand diagram and diagonal sequence walking gait







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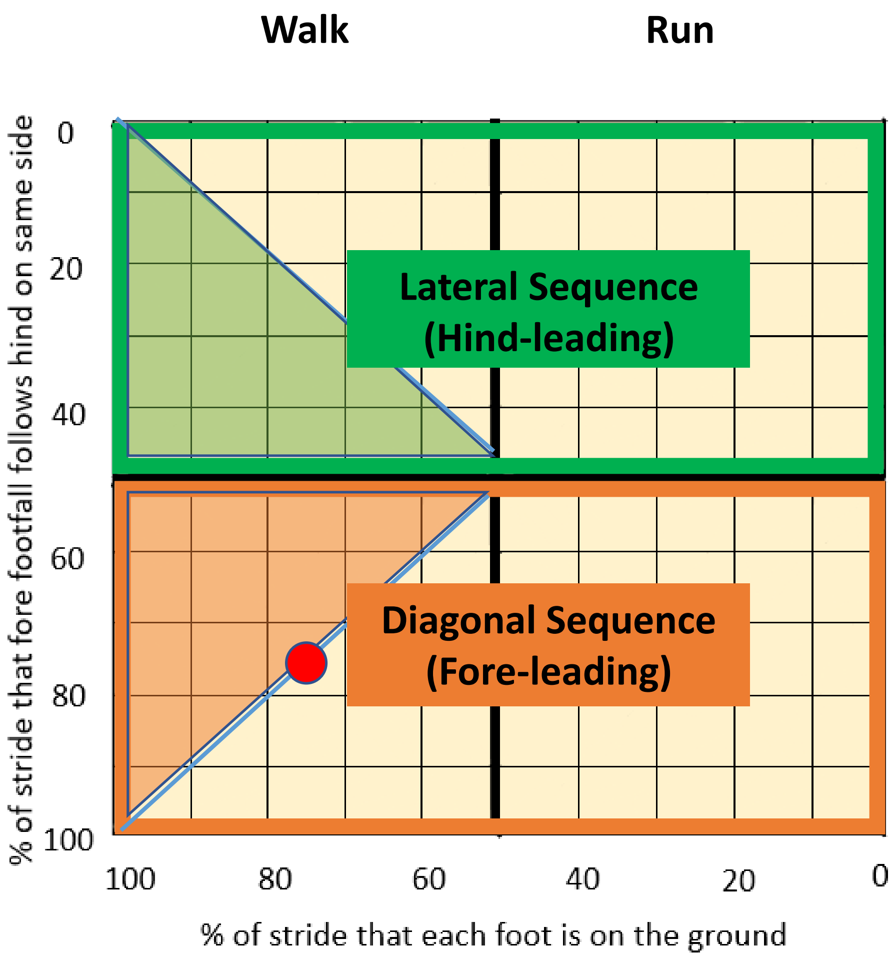
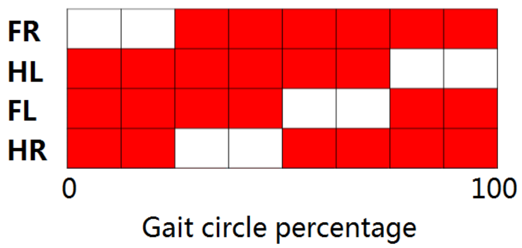
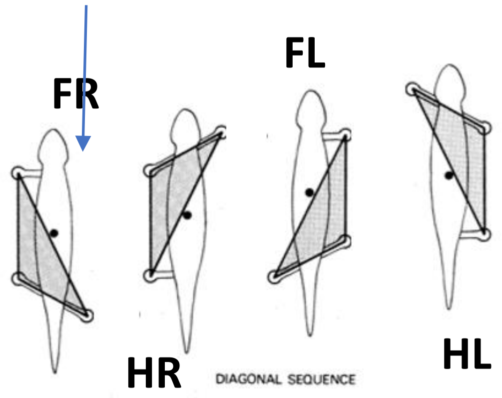
Hildebrand diagram and diagonal sequence walking gait







<u>diagonal sequence walking gait (orange shade): </u>the footfall of a given hind foot followed by the footfall of the forefoot on the **opposite** side of the body;

Lateral sequence walking gait and diagonal sequence walking gait

More stable

lateral sequence walking gait (green shade): the footfall of a given hind foot followed by the footfall of the forefoot on the **same** side of the body;

Faster

diagonal sequence walk gait (orange shade): the footfall of a given hind foot followed by the footfall of the forefoot on the **opposite** side of the body;

Lateral sequence walking gait and diagonal sequence walking gait

More stable

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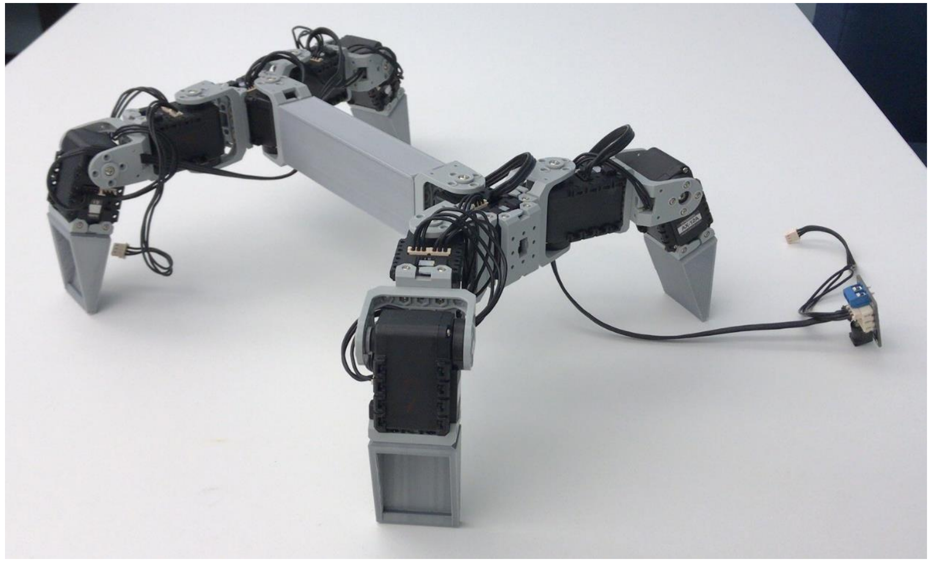
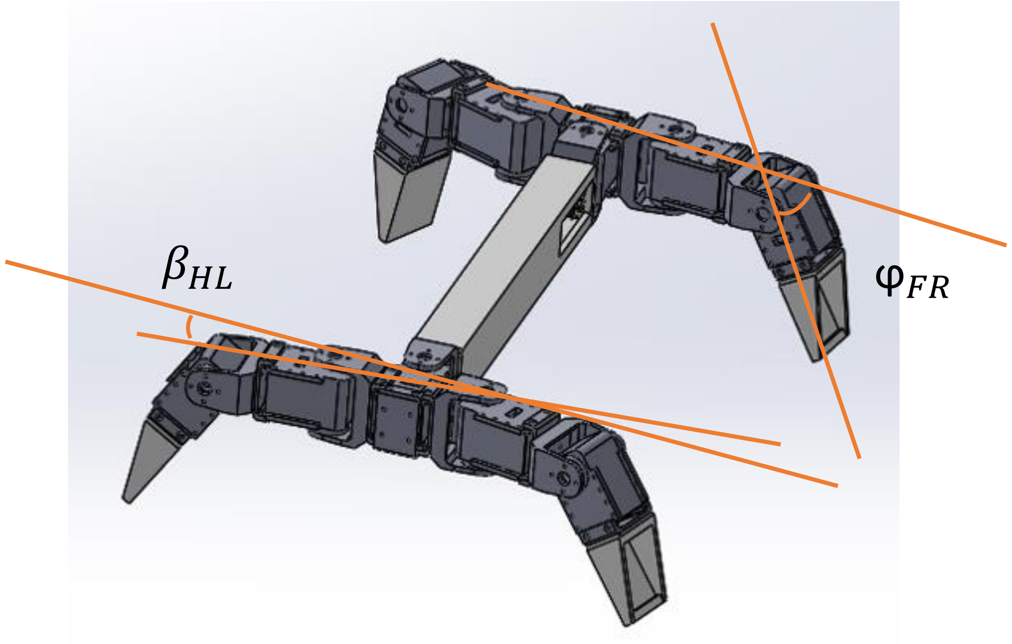
Faster

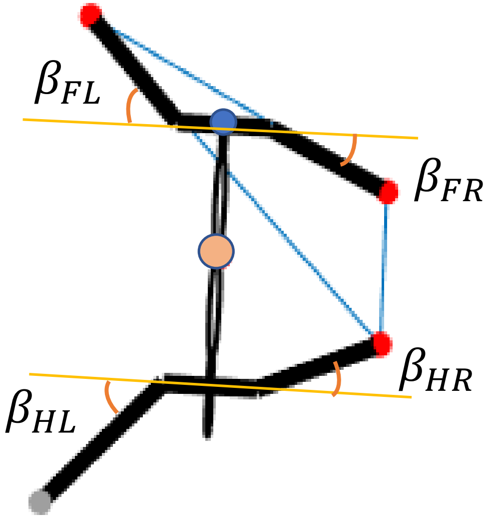
diagonal sequence walk gait (orange shade): the footfall of a given hind foot followed by the footfall of the forefoot on the **opposite** side of the body;

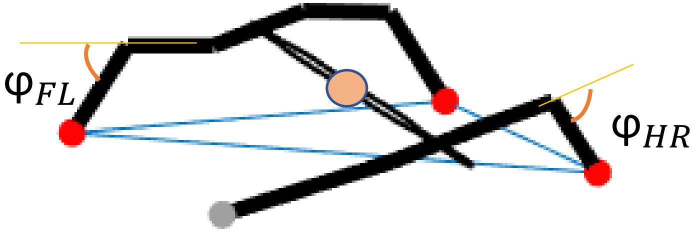
Tail improves static stability for the diagonal sequence walking gait, allowing for speed and stability.

Robot Simulation Model

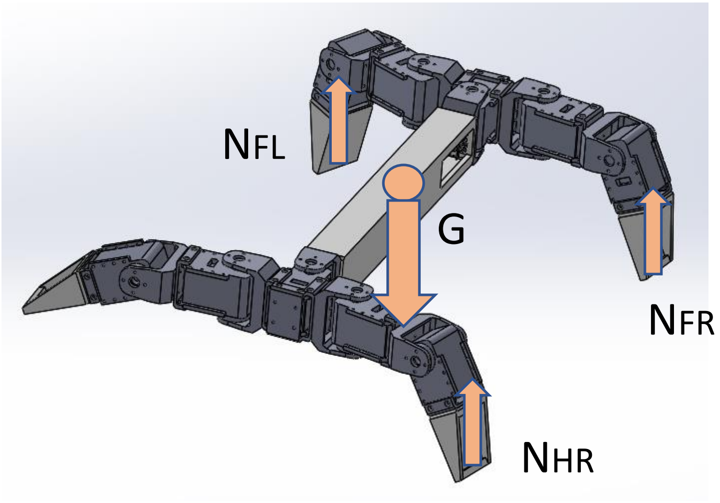


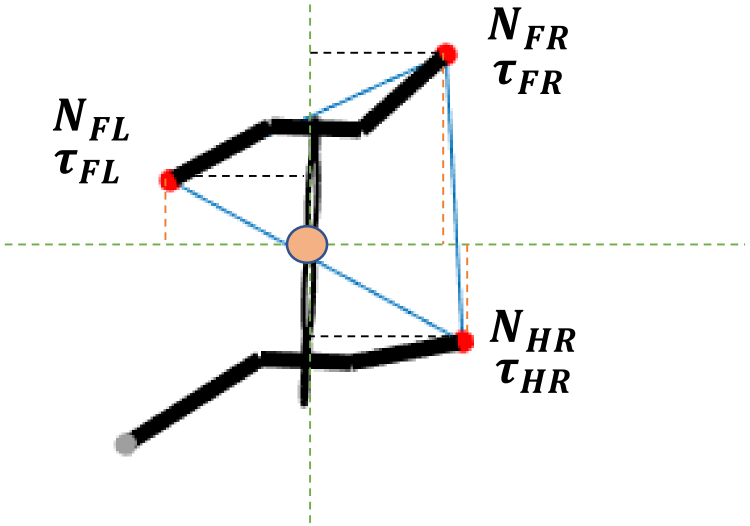






Robot Simulation Model





Coulomb Friction Model

- force/torque balance in vertical plane:

$$\boldsymbol{G} = \boldsymbol{N}_{HR} + \boldsymbol{N}_{FR} + \boldsymbol{N}_{FL}$$
$$\boldsymbol{0} = \boldsymbol{\tau}_{HR} + \boldsymbol{\tau}_{FL} + \boldsymbol{\tau}_{FR}$$

- Note that

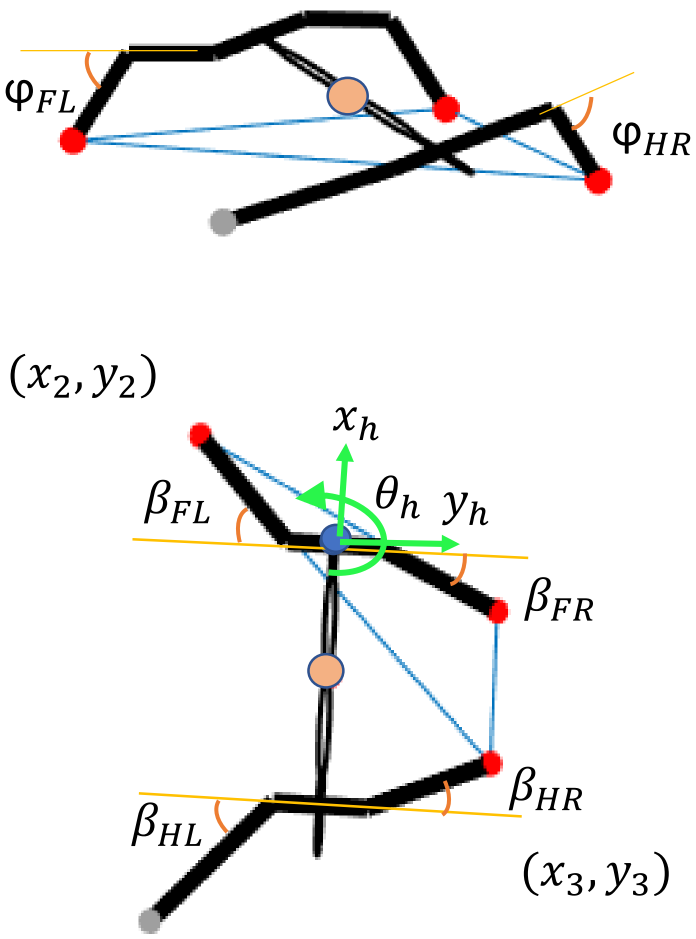
$$\boldsymbol{N}_{HR} \sim \boldsymbol{N}_{FL} > \boldsymbol{N}_{FR}$$

- Likewise, in the horizontal plane

$$(\mu_s * \boldsymbol{N}_{HR}) \sim (\mu_s * \boldsymbol{N}_{FL}) > \mu_k * \boldsymbol{N}_{FR}$$

- Therefore, if the slip exists, it would occur in the leg opposite to the lifting leg, which is the front right leg in this case.

Robot Simulation Model (Continued)



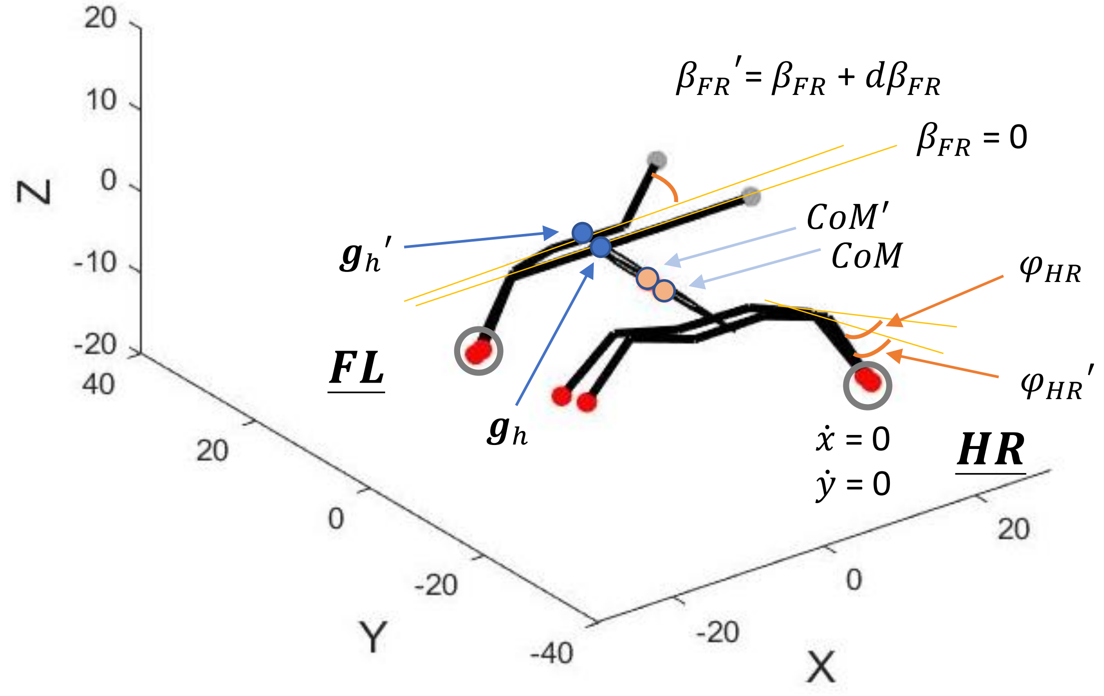
1. At every moment of a gait, shape variables β_i (leg angle) and their derivatives are known.

2. At every moment, positions of diagonal feet on ground are known. (e.g., FL (x_2, y_2) and HR (x_3, y_3)).

3. Find head position $\boldsymbol{g}_h = [x_h, y_h, \theta_h]^T$ and elbow angle φ (φ_{FL} = φ_{HR} allowing the body to stay in the horizontal plane) that ensures zero linear velocity for the diagonal legs.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{x}_2 \\ \dot{y}_2 \\ \dot{x}_3 \\ \dot{y}_3 \end{bmatrix} = \omega(\boldsymbol{r}) \begin{bmatrix} \dot{\beta} \\ \dot{\boldsymbol{g}}_h \\ \varphi \end{bmatrix}$$

Robot Simulation Model (Continued)



4. With the relationship between shape changes and body velocities, we can numerically calculate the stride displacement.

e.g.,

At each Δt, β_{FL} increases by $\Delta\beta_{FL}$, β_{HR} increases by $\Delta\beta_{HR}$,

$$P_{FL} = \begin{bmatrix} x_{FL} \\ y_{FL} \end{bmatrix} = \begin{bmatrix} f_x\,(g_{head}, \beta_{FL}, \varphi) \\ f_y\,(g_{head}, \beta_{FL}, \varphi) \end{bmatrix}; \quad P_{HR} = \begin{bmatrix} x_{HR} \\ y_{HR} \end{bmatrix} = \begin{bmatrix} f_x\,(g_{head}, \beta_{HR}, \varphi) \\ f_y\,(g_{head}, \beta_{HR}, \varphi) \end{bmatrix};$$

$$P_{FL}' = \begin{bmatrix} x_{FL}' \\ y_{FL}' \end{bmatrix} = \begin{bmatrix} f_x\,(g_{head}', (\beta_{FL}+\Delta\beta_{FL}), \varphi') \\ f_y\,(g_{head}', (\beta_{FL}+\Delta\beta_{FL}), \varphi') \end{bmatrix};$$

$$P_{HR}' = \begin{bmatrix} x_{HR}' \\ y_{HR}' \end{bmatrix} = \begin{bmatrix} f_x\,(g_{head}', (\beta_{HR}+\Delta\beta_{HR}), \varphi') \\ f_y\,(g_{head}', (\beta_{HR}+\Delta\beta_{HR}), \varphi') \end{bmatrix};$$

$$\begin{bmatrix} P_{FL}' - P_{FL} \\ P_{HR}' - P_{HR} \end{bmatrix} = \begin{bmatrix} x_{FL}' - x_{FL} \\ y_{FL}' - y_{FL} \\ x_{HR}' - x_{HR} \\ y_{HR}' - y_{HR} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (*)$$

Solving equation (*) gives g_{head}' and φ',

$$CoM' = h(g_{head}', \varphi'); \quad CoM = h(g_{head}, \varphi);$$

Step displacement = CoM' - CoM,

Stride displacement = sum(step displacement).

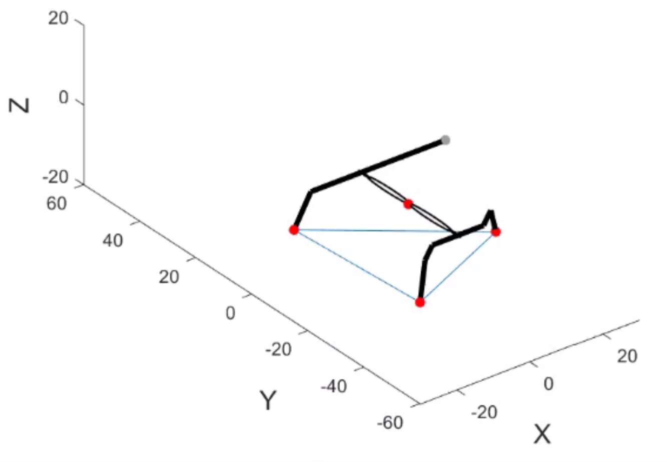
Stride displacement difference in *diagonal sequence walking gait* VS *lateral sequence walking gait*

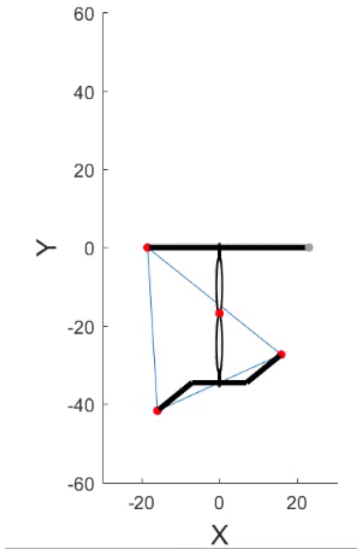
Same	Different
Leg wave amplitude (choose to be 5*pi/12 based on our observation on lizard)	The sequence of leg lifting
Leg wave frequency (from our gait design)	
Leg length (4*lizard data)	
Body length (4*lizard data)	
Leg lifting and landing phase / duty factor (same time consumed per gait cycle)	

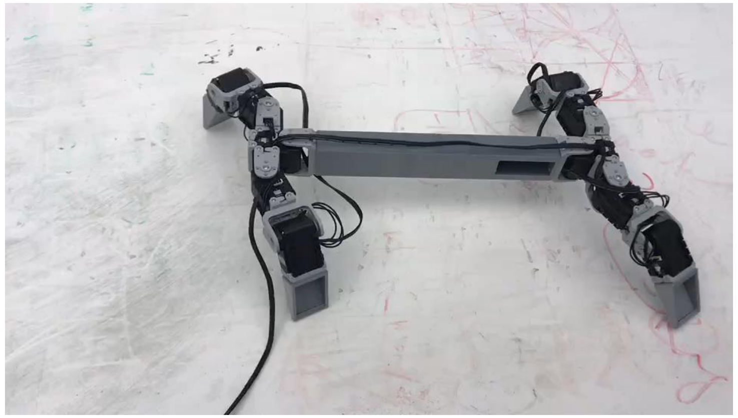
$$\text{COM velocity} = \frac{\text{COM displacement per gait cycle}}{\text{time consumed per gait cycle}}$$

Stride displacement difference in *diagonal sequence walking gait* VS *lateral sequence walking gait*

lateral sequence



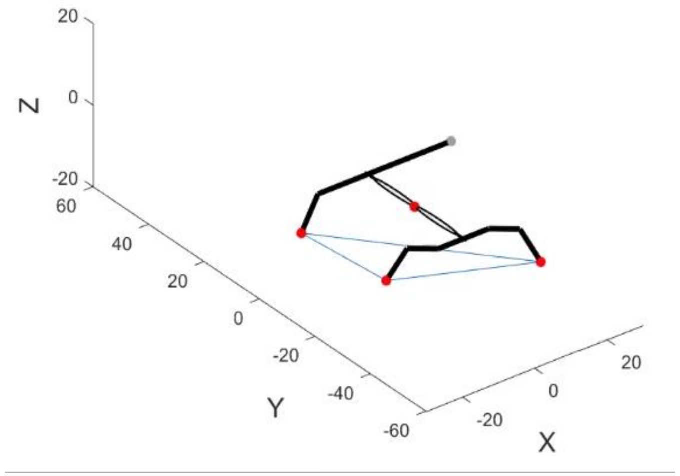


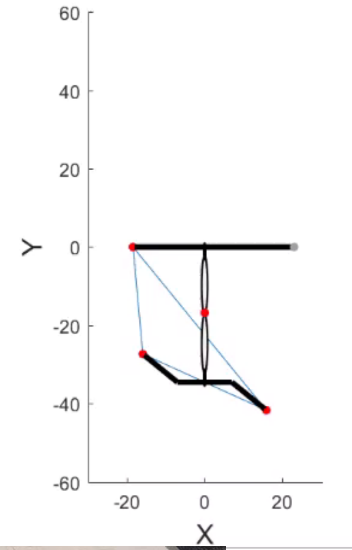


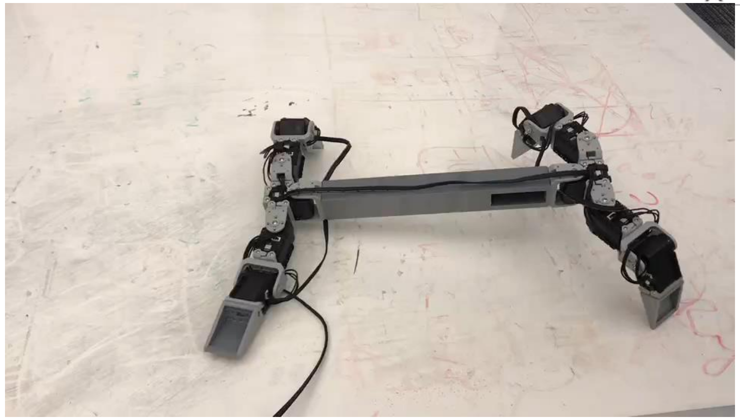
Simulation displacement (per gait cycle): 17.73 cm

Average robot test displacement (per gait cycle): 16.77 cm

diagonal sequence







Simulation displacement (per gait cycle): 23.37 cm

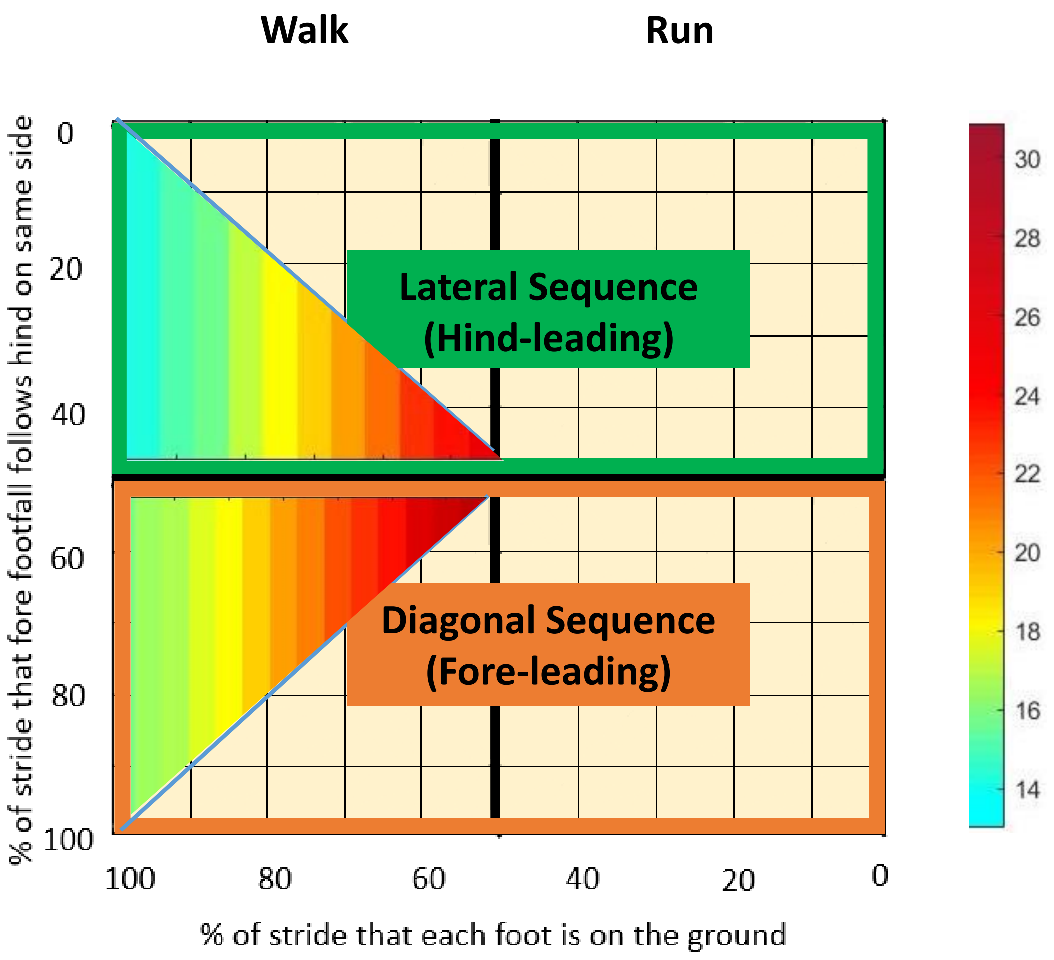
(31.8% larger)

Average robot test displacement (per gait cycle): 22.33 cm

(33.1% larger)

Stride displacement difference in

diagonal sequence walking gait VS *lateral sequence walking gait*



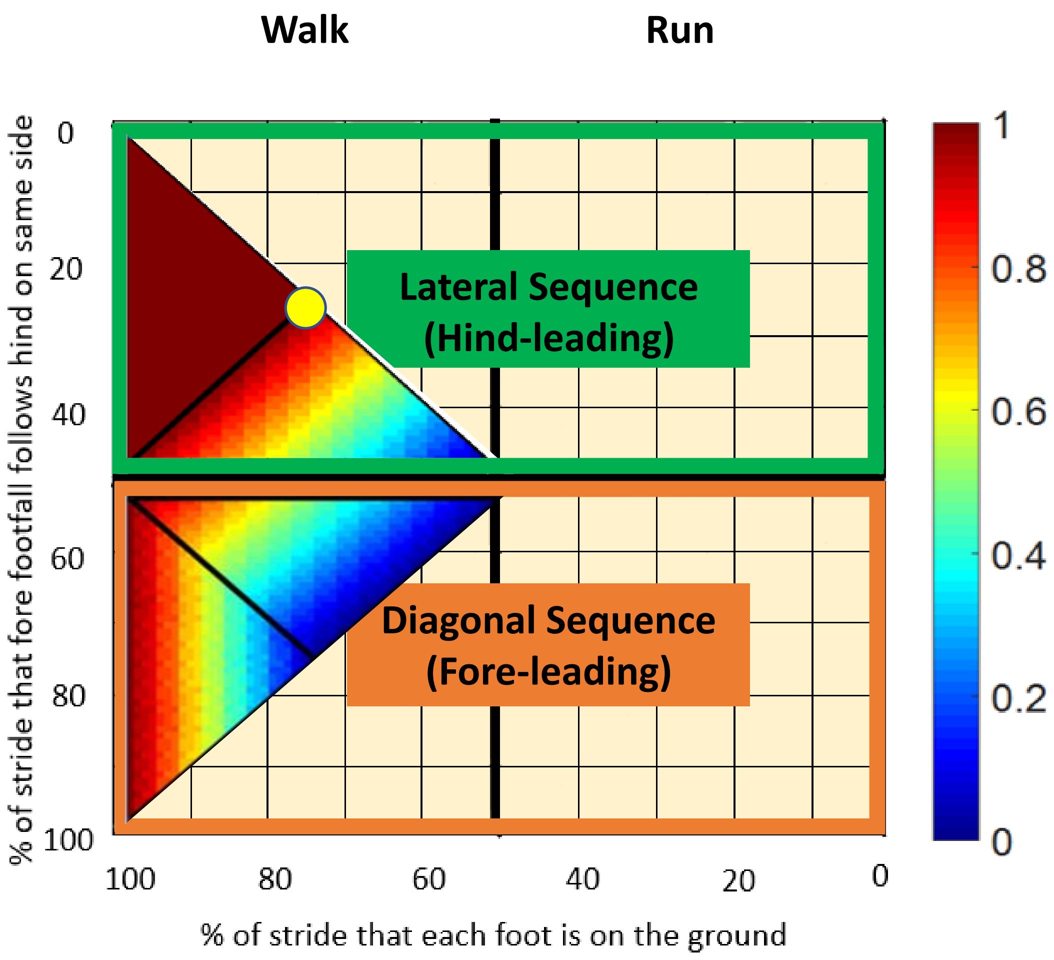
For a given % of stride that each foot is on the ground,

Diagonal sequence walk gait has
higher velocity (stride displacement);

Stride Displacement Heat Map projected onto Hildebrand Diagram

Static stability difference in

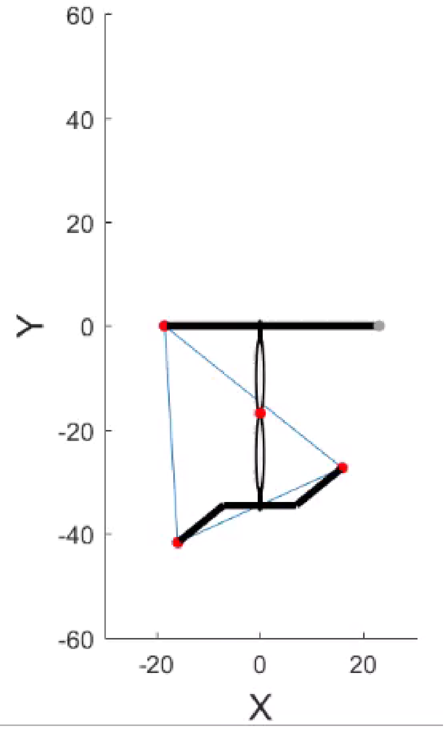
diagonal sequence walking gait VS *lateral sequence walking gait*

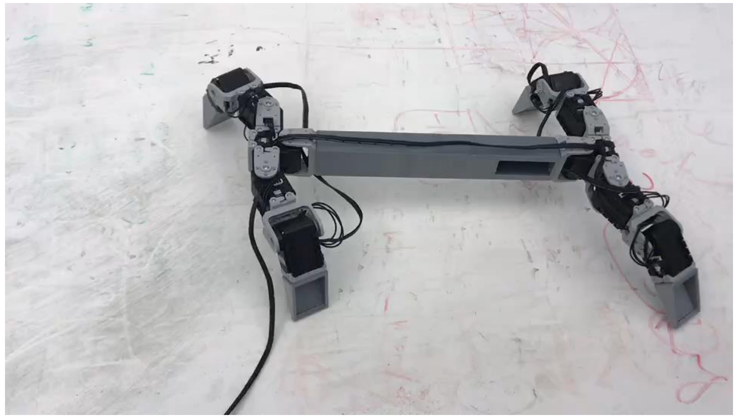


Stability Heat Map projected onto Hildebrand Diagram

lateral sequence walking gait

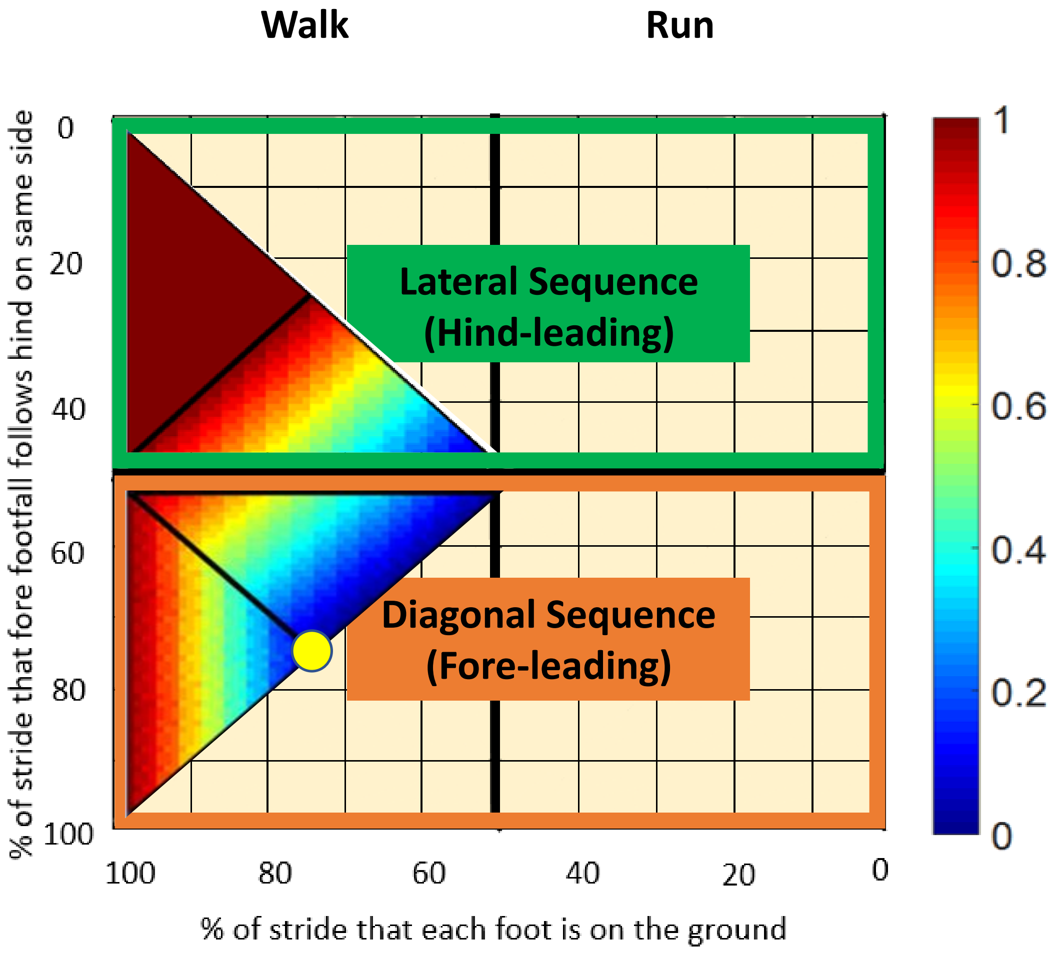
Stable because COM is always in supporting polygon





Static stability difference in

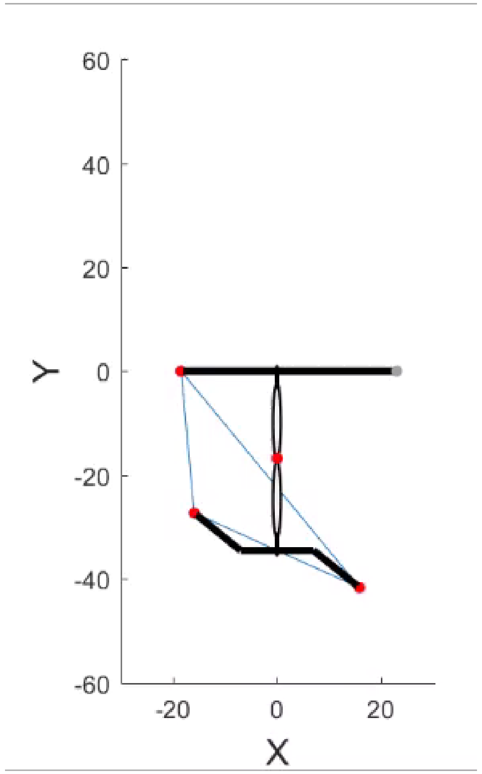
diagonal sequence walking gait VS *lateral sequence walking gait*

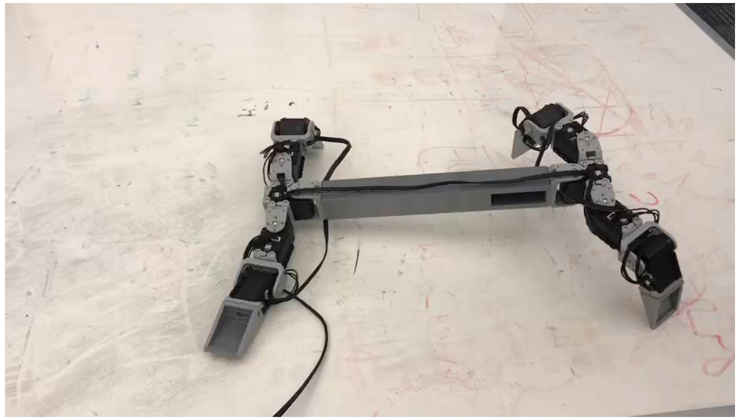


Stability Heat Map projected onto Hildebrand Diagram

diagonal sequence walking gait

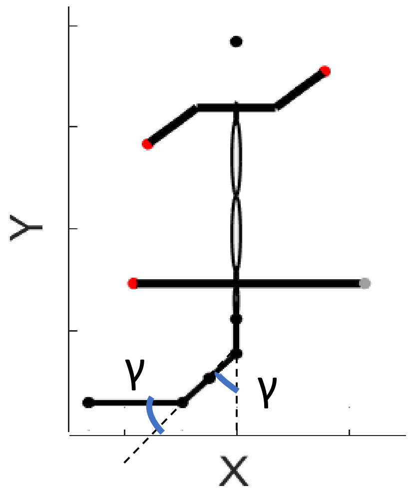
Unstable because COM is NOT in supporting polygon





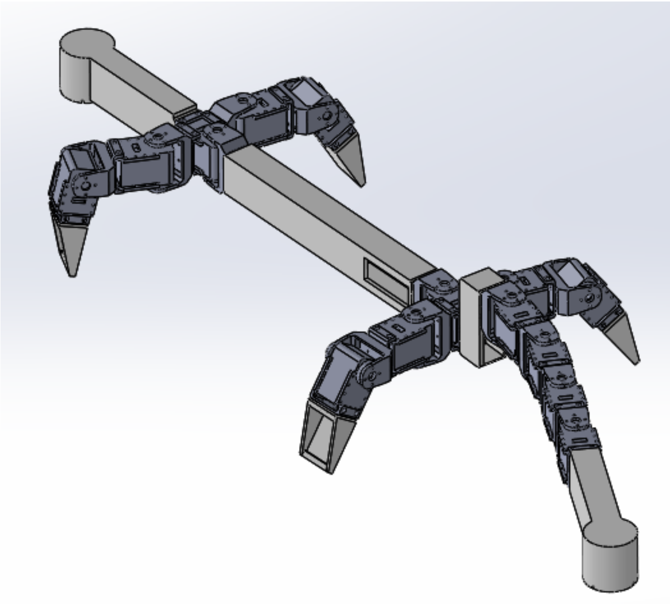
Use **an actuated tail** in *diagonal sequence walking gait*

1. Planar tail (move in the horizontal plane)



γ : tail angle allowing the tail move in the xy plane (total:4, only 2 marked)

Robot model with tail



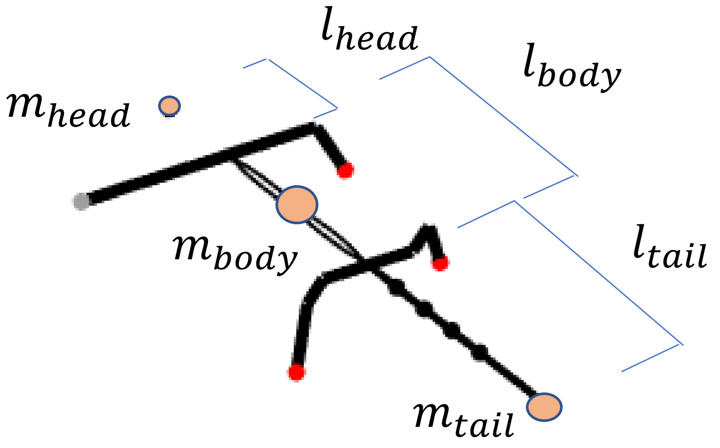
Picture of robot with tail

Since locomoting in the walking gait sequence is slow, the net displacement is a function of body shape changes and is independent of its rate. it is **OK** to design tail motion only in kinematic aspect.

Robot parameters:

$$\frac{l_{leg}}{l_{body}} \approx \frac{2}{3};\ \frac{l_{tail}}{l_{body}} \approx \frac{4}{3};\ \frac{l_{head}}{l_{body}} \approx \frac{1}{2}$$

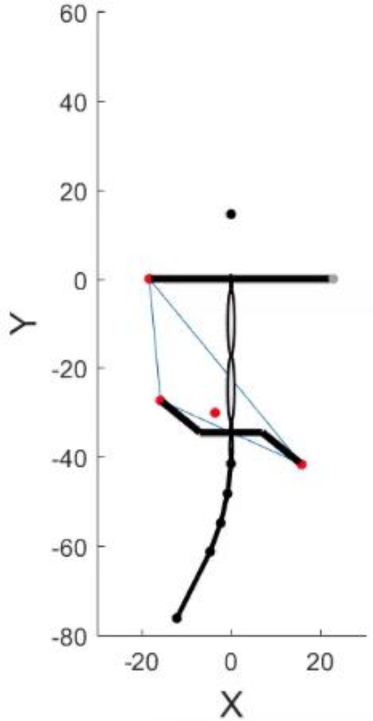
$$\frac{m_{tail}}{m_{body}} \approx \frac{5}{6};\ \frac{m_{head}}{m_{body}} \approx \frac{2}{3}$$



Use **an actuated tail** in *diagonal sequence walking gait*

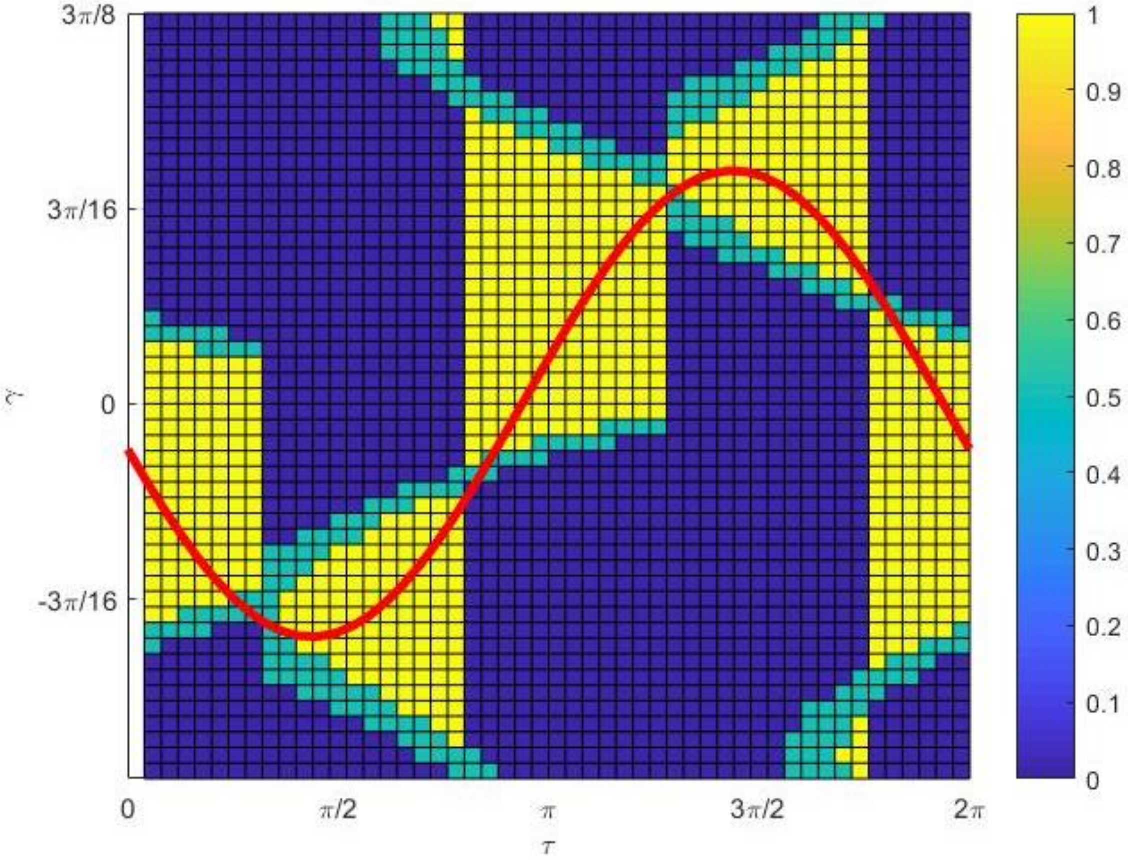
1. Planar tail (move in the horizontal plane)

The red curve could be the function of γ (sinusoidal wave), which nearly stabilize the whole gait cycle.
Below is the simulation and robot video using this γ function:



Simulation Displacement (per gait cycle):
24.16 cm (31.8% larger than lateral sequence walking gait: 18.33 cm)
AND it's a stable gait.

Stability Heat Map

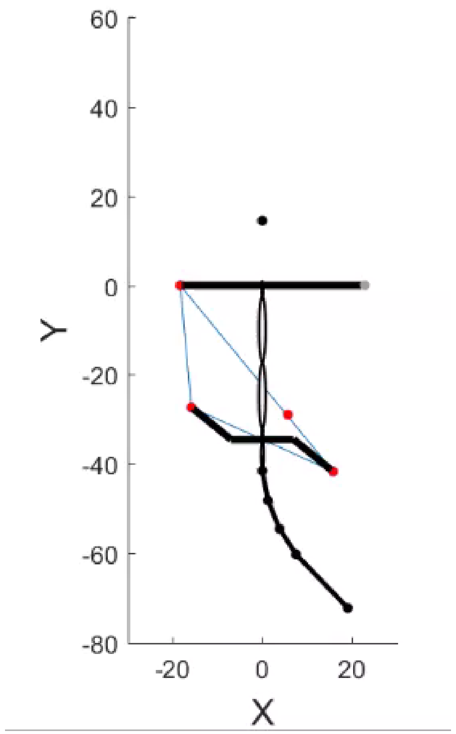


- Yellow: stable (COM in the supporting polygon)
- Light Blue: nearly stable (COM close to the polygon)
- Blue: unstable (COM far from the polygon)

Use **an actuated tail** in *diagonal sequence walking gait*

1. Planar tail (move in the horizontal plane)

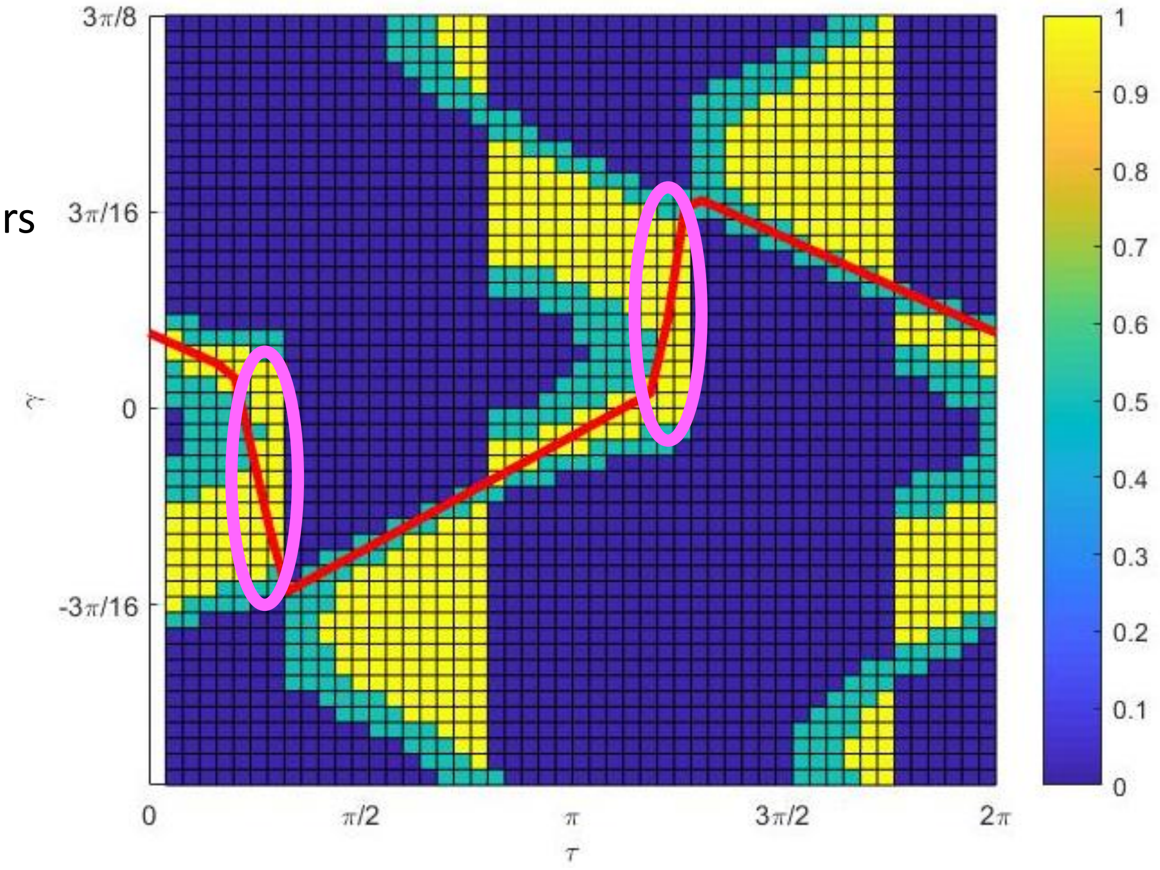
If $\frac{l_{leg}}{l_{body}}$ gets larger, e.g. $\frac{l_{leg}}{l_{body}} = 1$, and the rest of parameters remain same.

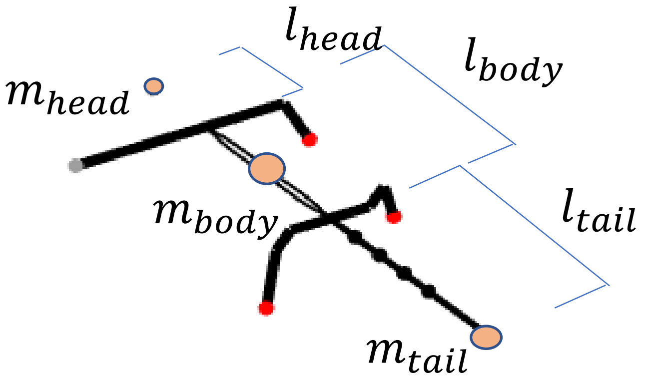


we may still be able to find a γ function to stabilize the whole gait cycle.

However, at some moment, instant change of γ is required (pink area), causing bad disturbance to the robot motion.

Stability Heat Map



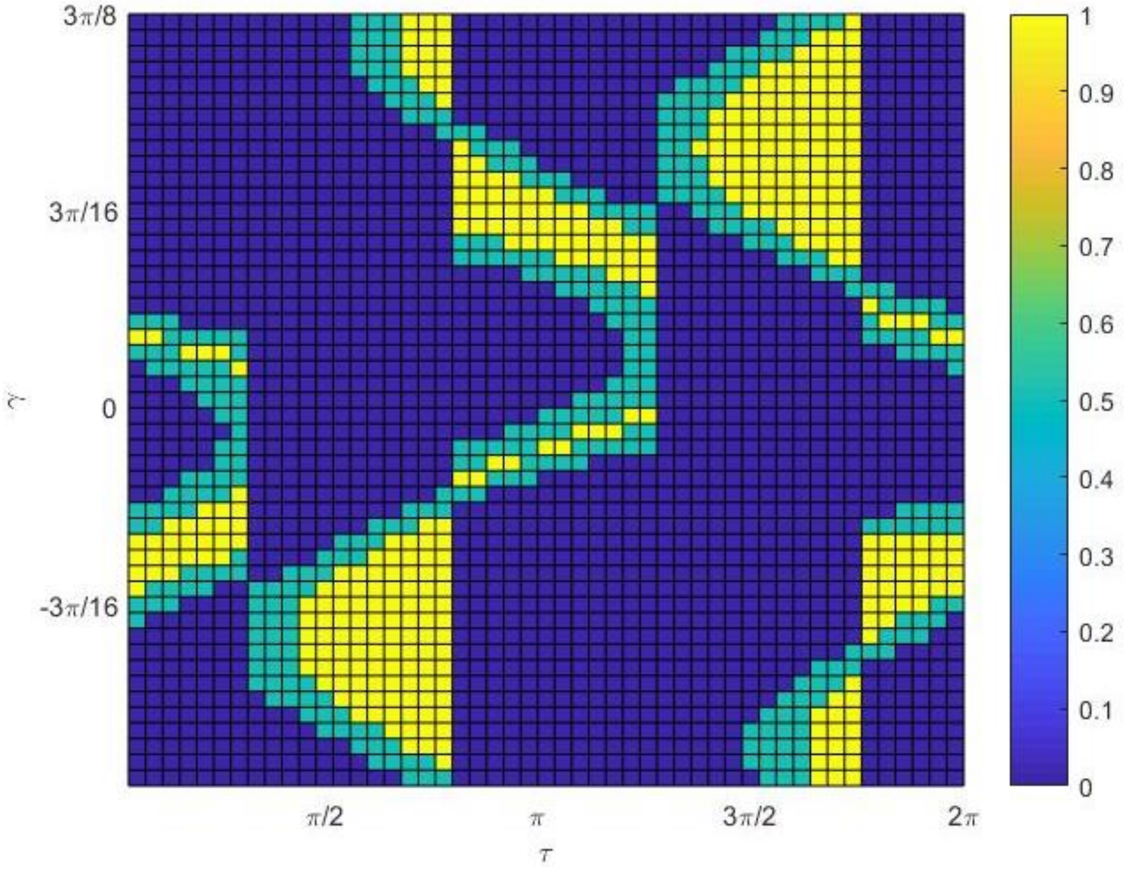


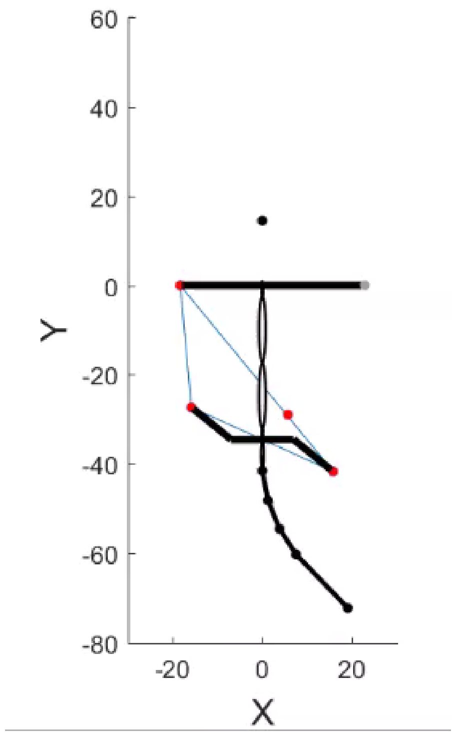
Use **an actuated tail** in *diagonal sequence walking gait*

1. Planar tail (move in the horizontal plane)

If $\frac{l_{leg}}{l_{body}}$ gets even much larger,

Stability Heat Map



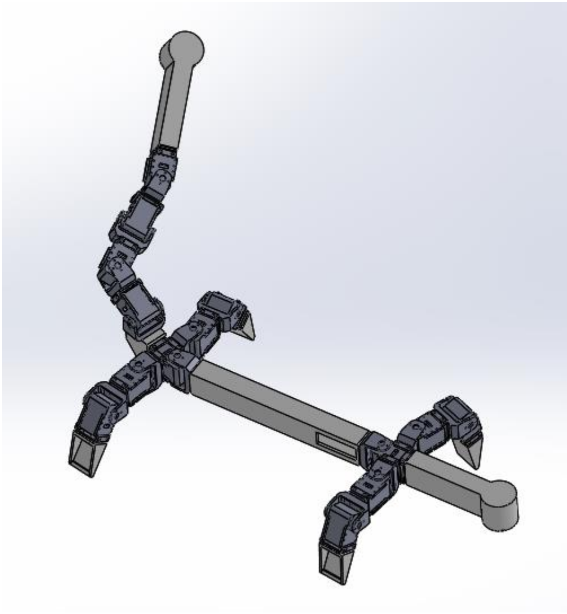


The stable region in the map becomes discontinuous. There is <u>no continuous tail motion function</u> to stabilize the whole gait cycle. The disturbance gets worse.

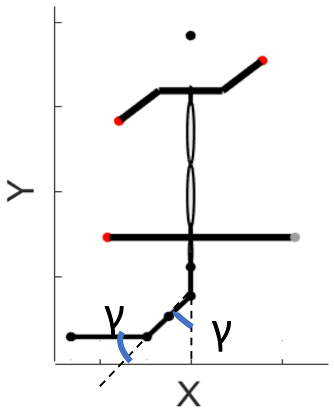
Therefore, we might consider a different tail design.

Use **an actuated tail** in *diagonal sequence walking gait*

2. **Two-Plane** tail (same tail length, two angle move in horizontal plane and two in vertical plane)

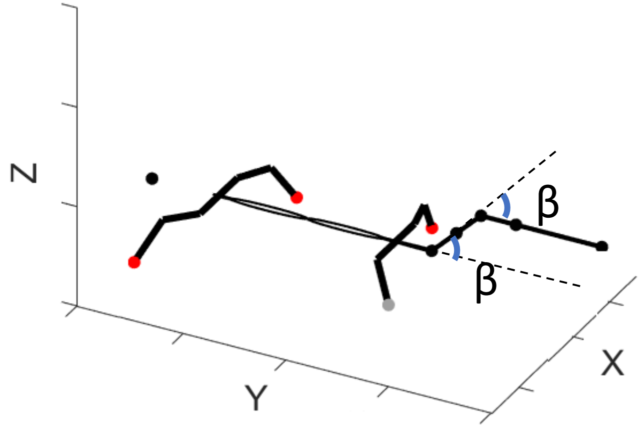


2-DoF tail has **larger reachable area** than plain tail, it could produce smooth and continuous function to keep COM in the polygon without causing disturbance while the planar tail could not.



γ : tail angle allowing the tail move in the xy plane (angle #2 and angle #4)

β: tail angle allowing the tail move in the yz plane (angle #1 and angle #3)



Use **an actuated tail** in *diagonal sequence walking gait*

2. Two-Plane tail

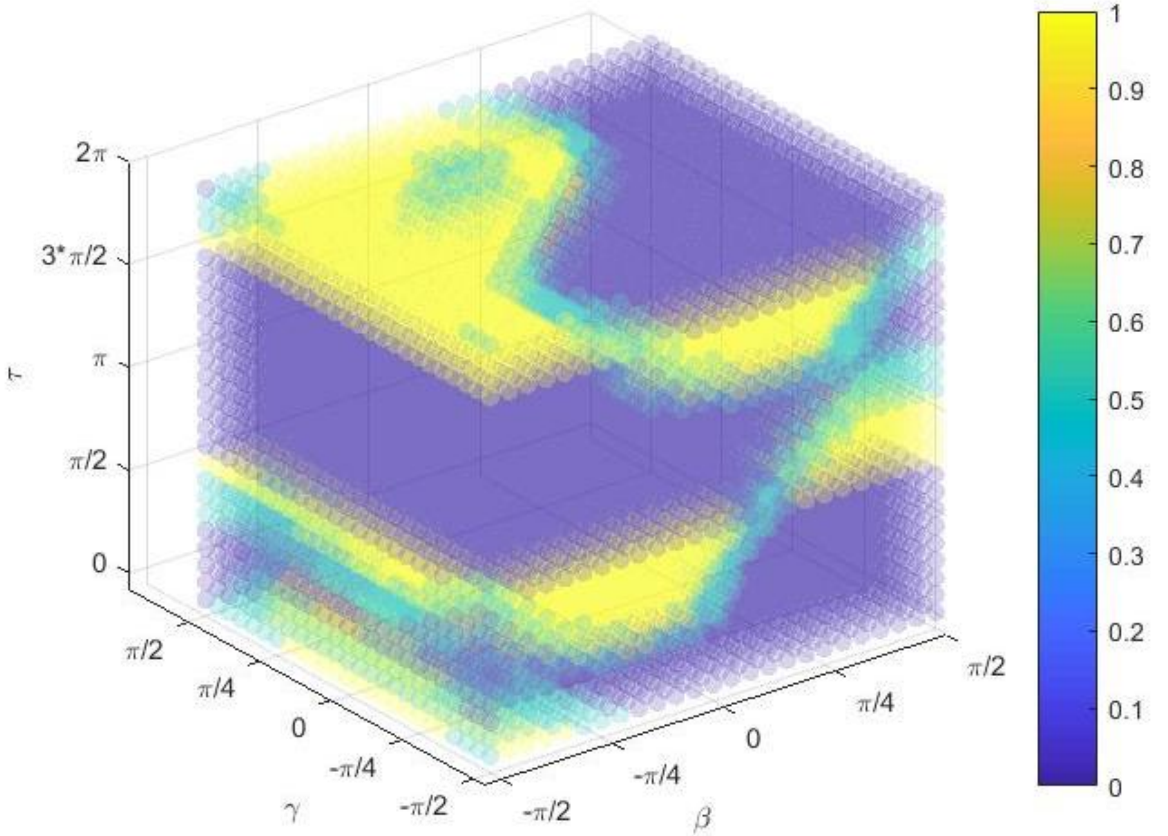
For example, $\frac{l_{leg}}{l_{body}} = 1$

Using plain tail, at some moment, immediate change of γ is required (pink area), causing bad disturbance to the robot motion.

Try two-plane tail!

τ: leg phase at each gait cycle

Stability Heat Map (4D plot)



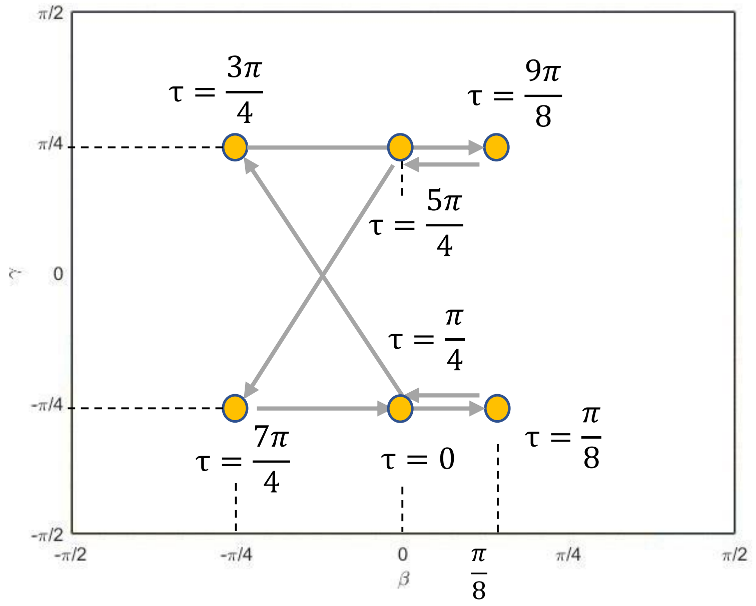
- Yellow: stable (COM in the supporting polygon)
- Light Blue: nearly stable (COM close to the polygon)
- Blue: unstable (COM far from the polygon)

Use **an actuated tail** in *diagonal sequence walking gait*

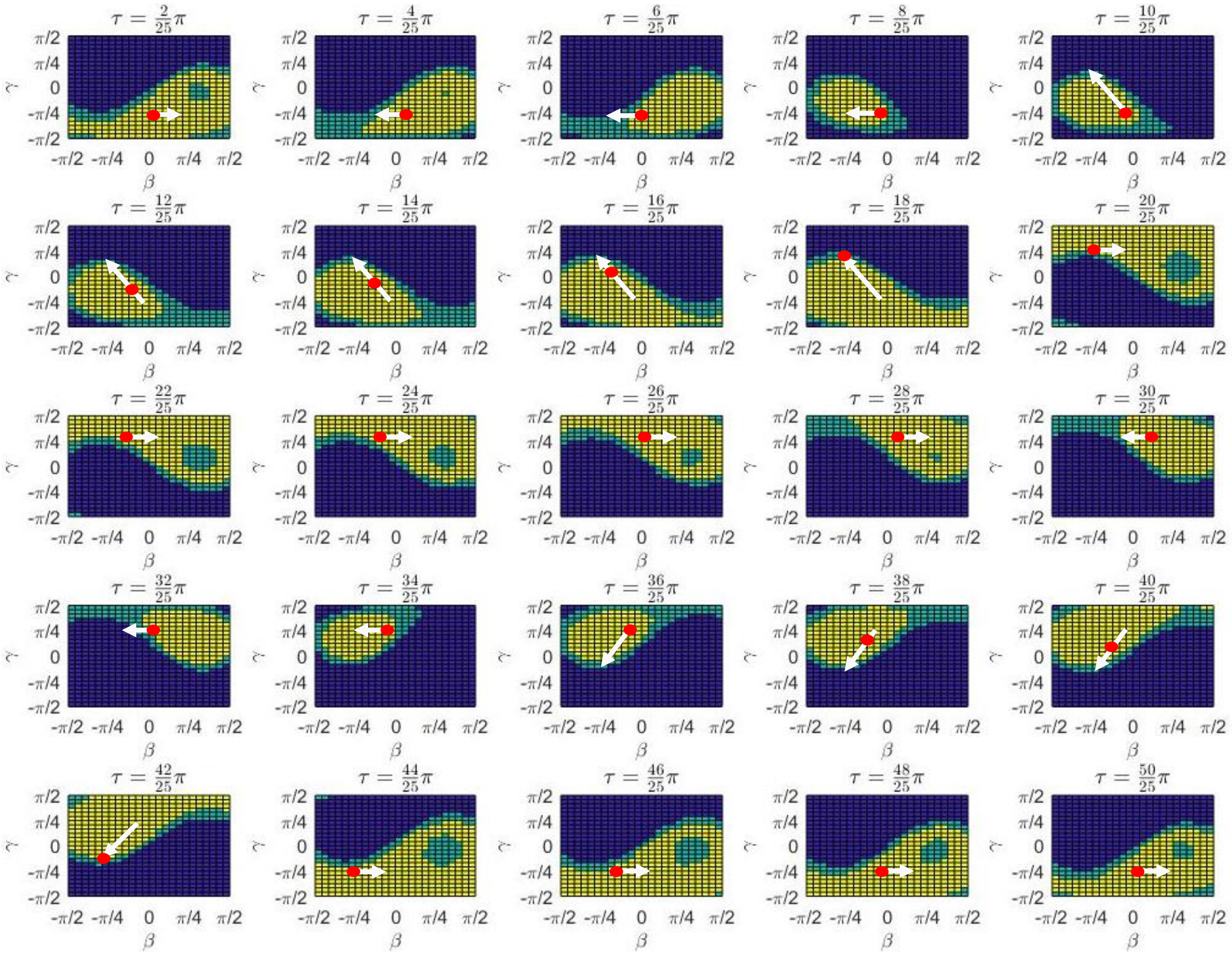
2. Two-Plane tail

Decomposing

The 4D plot is decomposed into a series of 3D plots with the phase increasing.
red dot (•, corresponding point (β,γ) at that phase τ) and white arrow (⇨): a possible trajectory nearly stabilizing the whole gait cycle.
Trajectory:

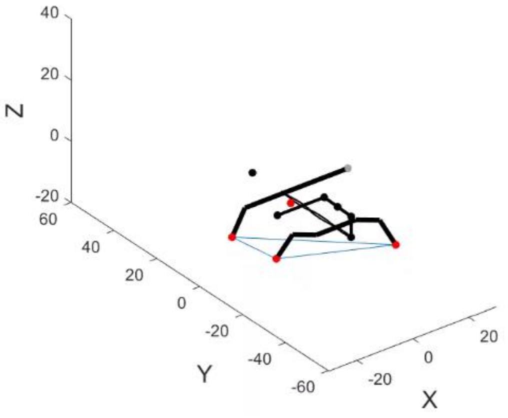


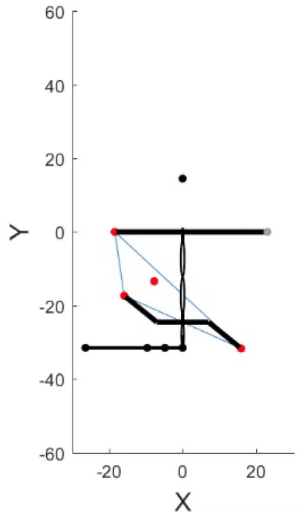
Stability Heat Map (decomposed plot)



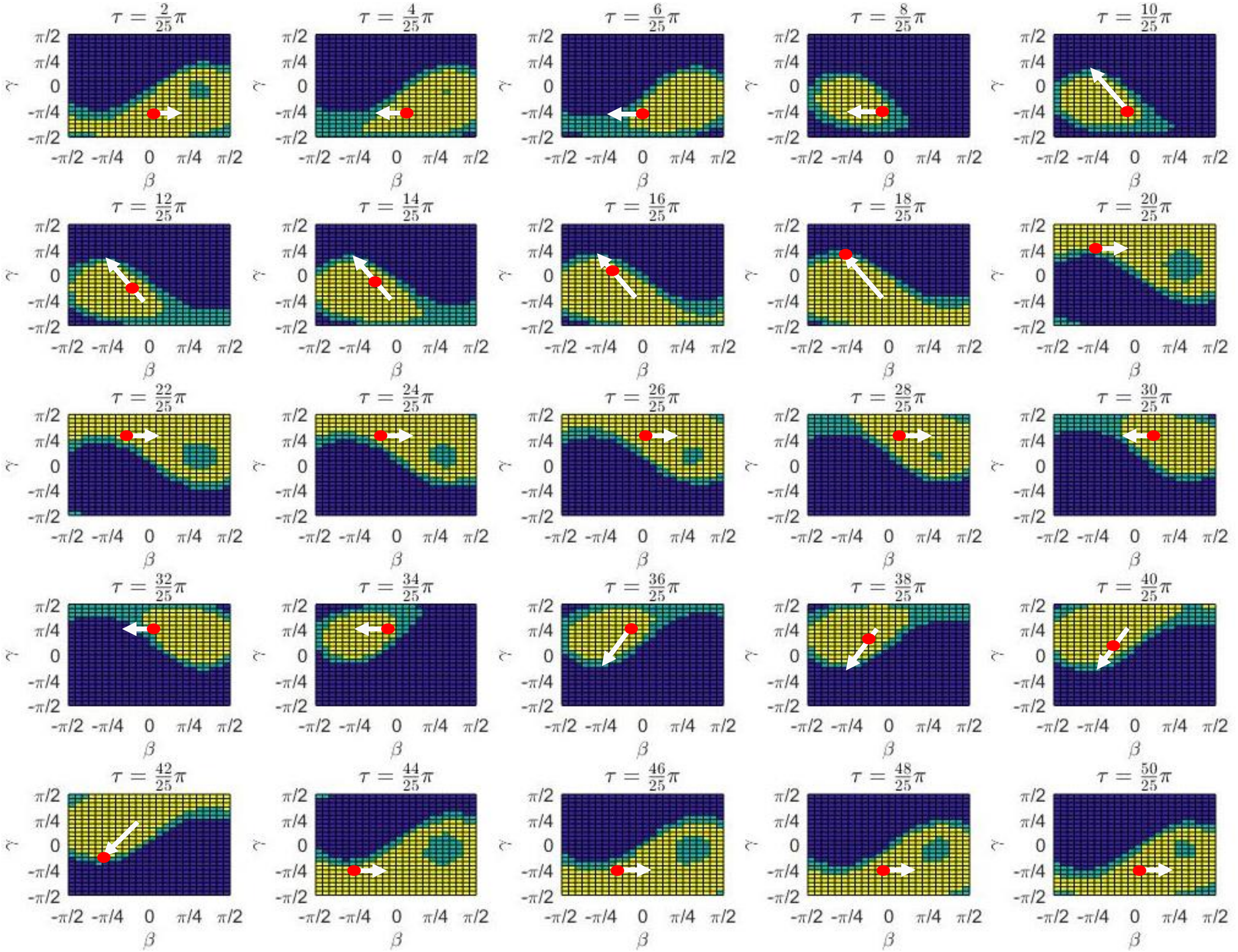
Use **an actuated tail** in *diagonal sequence walking gait*

2. Two-Plane tail





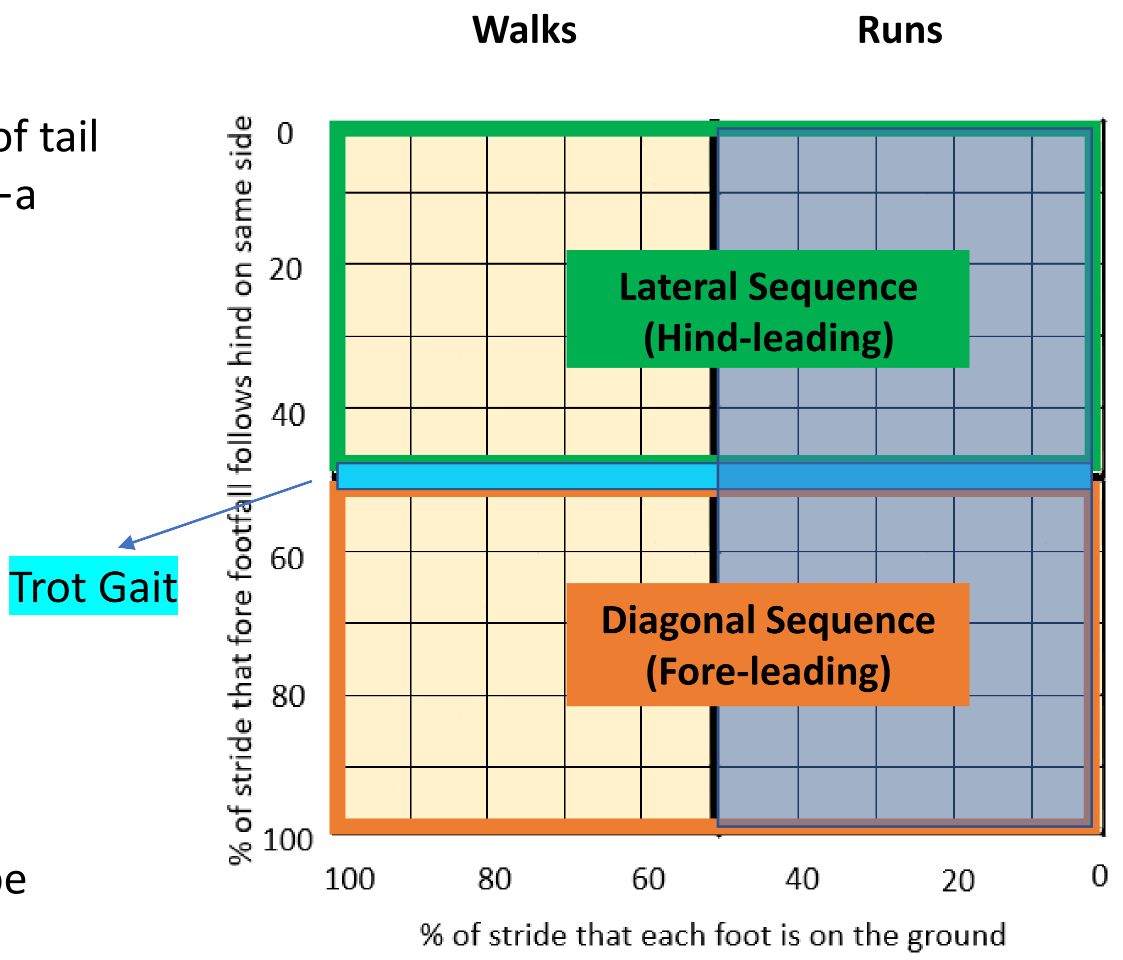
Stability Heat Map (decomposed plot)



Before

Consider the lizard walks in lateral sequence gait. The function of tail is mainly to provide an additional support for the body weight—a passive tail touching the ground.

We also tried passive tail in trot gait. The tail stabilizes the body locomotion impressively.



Future Work

Use an actuated tail to stabilize trot gait (light blue shade) and running gait (blue shade). Dynamic stability principles need to be considered. This is what we are working on.