Inertial Tail-like Appendage Use in Quadruped Improves Stability in Diagonal Sequence Walking Gaits

Haosen Xing

***Abstract*—There are two main sequences of footfall patterns for quadrupedal walking: lateral and diagonal. We observed that, compared with the lateral sequence (LS) gait, the diagonal sequence (DS) gait produces a larger stride displacement (i.e., higher average speed), but at the cost of decreased body stability. This work aims to increase the stability of the DS gait by investigating the use of an inertial tail-like appendage. Rone and Ben-Tzvi [6] analyzed the impact a planar robotic tail could have on the yaw-angle maneuvering of a quadruped robot. Here, we model the quadruped system as a self-manipulator [4] with an actuated tail in 3D which can swing in not only the yaw direction but also the pitch direction.**

I. Introduction

There are two main types of quadrupedal walking gait: the lateral sequence (LS) and the diagonal sequence (DS) gaits. In both gaits, all legs have the same duty factor, and the only difference lies in the sequence under which the foot is lifted off the ground. We observed that the LS gait has a smaller stride displacement but maintains more stability, and in practice, most quadrupedal animals adopt this gait when walking. On the other hand, the DS gait, most observed in the quadrupedal primates, produces a larger stride displacement but at the cost of stability. This work investigates the use of an inertial tail-like appendage to help stabilize DS gaits for a quadruped robot.

In nature, animals utilize tails to aid in both maneuvering and stabilization. In terms of maneuvering, tails are used by geckos to reorient while jumping and cheetahs to turn during running. In terms of stabilization, tails are used by lizards to keep balance during running and monkeys to aid in climbing. Based on these observations, a number of research groups have attempted to implement tail structures. For example, a robot car with a tail can right itself in a fall [3], a cheetah robot proves that the tail can be more effective than reaction wheels in orienting the body when space, power and time are limited [1], and a tail can also stabilize the robot when it is leaping [2]. These tail-like structures have been implemented as single-DoF (degree of freedom) pendulums. Multi-DoF tail research is more limited [5]. Inspired by the work [6] which analyzes how a six DoF serpentine planar robotic tail (Fig. 1) impacts quadruped's maneuverability in the yaw direction, we are interested in implementing an actuated multi-segment tail which can swing in the yaw and pitch directions (Fig. 2). The swinging in the yaw direction is simplified by setting $\ddot{\gamma} = 0$, which means the tail aims at impacting quadruped's stability in the pitch and roll directions. With the help of tail, a quadruped can take advantage of the higher stride displacement of the DS gait while maintaining stability.

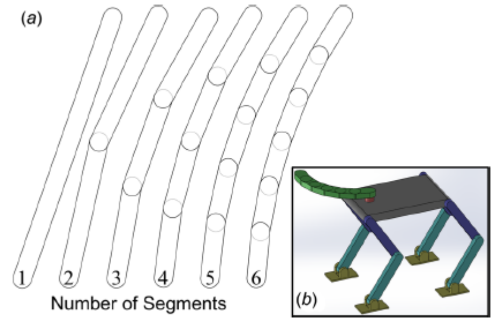

Fig. 1: **Tail structures in the referenced paper [6].**

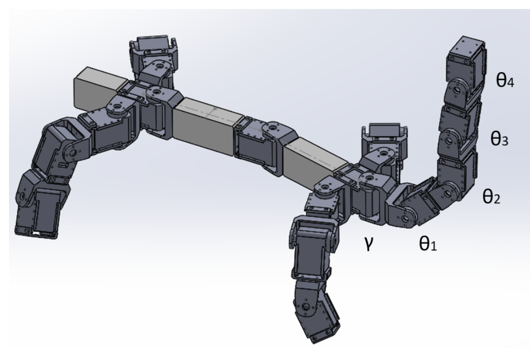

Fig. 2: **Robot CAD model with desired tail structure**

When the robot moves in a diagonal sequence walking gait, the center of mass (COM) of the system goes beyond the supporting polygon during the majority of the gait cycle. The third standing leg other than diagonal standing legs (leg 3 in Fig. 4) is lifted off the ground and the robot falls over. With Lagrangian analysis and self-manipulator-like modeling [4], we can derive tail oscillations with desired joint torques in order to keep the third leg in contact with the ground.

II. Explain

This section is a brief introduction of the planar tail model in [6]. As shown in Fig. 3, w is the world frame, followed by the object frame o, p is the quadruped body frame COM, coincident with o, t is the tail base frame and L_i is the ith tail-link frame. Model parameter $q = [\theta\ o_x\ o_y\ o_\varphi]^T$ where $\theta = [\theta_1\ \theta_2\ \cdots\ \theta_n]$ contains tail joint angles, each measuring between link $i-1$ and link i (θ_1 is the angle between frame t and link 1), and o_x, o_y and o_φ are object position. d is the distance between COM and tail base and L_L is the distance between adjacent joints on the tail. The rotation matrix of φ

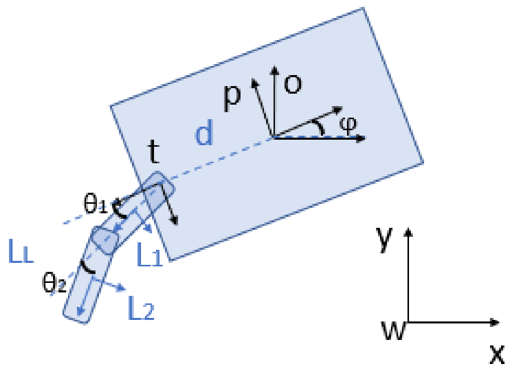

Fig. 3: **Frames and coordinates in the reference paper.**

with respect to z-axis is

$$R_1(\varphi)=\begin{bmatrix} cos(\varphi) & -sin(\varphi) & 0\\ sin(\varphi) & cos(\varphi) & 0\\ 0 & 0 & 1\end{bmatrix}. \tag{1}$$

The rigid body transformation of frame p to frame o is

$$g_{\mathrm{op}}=\begin{bmatrix} R_1(\varphi) & 0\\ 0\ 0\ 0 & 1\end{bmatrix}, \tag{2}$$

and the rigid body transformation of frame t to frame p is

$$g_{\mathrm{pt}}=\begin{bmatrix} R_1(\pi) & \begin{bmatrix}-d\\0\\0\end{bmatrix}\\ 0\ 0\ 0 & 1\end{bmatrix}. \tag{3}$$

In the tail configuration, the rigid body transformation of each tail link is,

$$g_{\mathrm{t1}}=\begin{bmatrix} R_1(\theta_1) & \begin{bmatrix}\frac{L_L}{2}\\0\\0\end{bmatrix}\\ 0\ 0\ 0 & 1\end{bmatrix}\quad if\ i=1, \tag{4}$$

$$g_{\mathrm{(i-1)i}}=\begin{bmatrix} Id & \begin{bmatrix}\frac{L_L}{2}\\0\\0\end{bmatrix}\\ 0\ 0\ 0 & 1\end{bmatrix}\begin{bmatrix} R_1(\theta_i) & \begin{bmatrix}\frac{L_L}{2}\\0\\0\end{bmatrix}\\ 0\ 0\ 0 & 1\end{bmatrix}\quad if\ i>1, \tag{5}$$

$g_{\mathrm{ti}}=g_{\mathrm{t1}}...g_{\mathrm{(i-1)i}}$ and $g_{\mathrm{pi}}=g_{\mathrm{pt}}g_{\mathrm{ti}}$, the body manipulator jacobian of ith tail-link is,

$$J^b_{pi}=\begin{bmatrix}\xi_1^\dagger & \cdots & \xi_i^\dagger & \cdots & \xi_n^\dagger\end{bmatrix}, \tag{6}$$

$$\xi_x^\dagger=\mathrm{Ad}^{-1}_{(e^{\hat\xi_x\theta_x}\cdots e^{\hat\xi_i\theta_i}g_{pi}(0))}\xi_x,\ if\ x\le i, \tag{7}$$

$$\xi_x^\dagger=0,\ if\ x>i. \tag{8}$$

The dynamic of the tail could be written in Lagrange's equations:

$$M(q)\ddot q+C(q,\dot q)\dot q+N(q,\dot q)=\Upsilon. \tag{9}$$

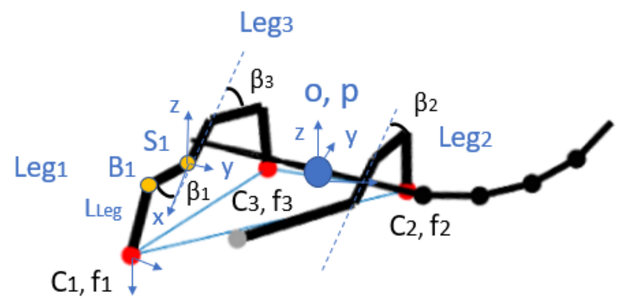

Fig. 4: **Parameters of the quadruped body with legs.**

M is the manipulator inertia matrix (tensor),

$$\hat M=\begin{bmatrix}\sum J_{pi}^{b^T}M_iJ_{pi}^b & \sum J_{pi}^{b^T}M_i\mathrm{Ad}_{g_{pi}^{-1}}\\ \sum \mathrm{Ad}^T_{g_{pi}^{-1}}M_iJ^b_{pi} & M_o+\sum\mathrm{Ad}^T_{g_{pi}^{-1}}M_i\mathrm{Ad}_{g_{pi}^{-1}}\end{bmatrix}, \tag{10}$$

$$M=\begin{bmatrix} Id & \\ & J_{op}^{b^T}\end{bmatrix}\hat M\begin{bmatrix} Id & \\ & J_{op}^{b}\end{bmatrix}, \tag{11}$$

where M_i is the mass inertia matrix of ith link, M_o is the mass inertia matrix of the body. The Coriolis matrix is

$$C_{ij}=\frac{1}{2}\sum_k\left(\frac{\partial M_{ij}}{\partial q_k}+\frac{\partial M_{ik}}{\partial q_j}-\frac{\partial M_{kj}}{\partial q_i}\right)\dot q_k, \tag{12}$$

$N=0$, and $\Upsilon=[\tau_1\ \tau_2\ \cdots\ \tau_n\ 0\ 0\ 0]^T$. An actuator is assumed to generate the required joint torque τ.

III. Implement

Our servo-driven, salamander-like robot locomotes on level grounds using its four limbs, each with three DoF (fore/aft, up/down and fore/aft). Our model assumes that the feet of the quadruped touching the ground cannot slip but are allowed to rotate, so we have

$$B_{c_i}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix}. \tag{13}$$

At each moment of a gait, standing leg angles $\beta=[\beta_1\ \beta_2\ \beta_3]$ and their derivatives are known. The changes of other two DoFs of each standing leg are negligible and the second segment of each leg, like B_1C_1, can be regarded as perpendicular to the ground, as shown in Fig. 4.

Our system is modeled as a self manipulator in 3D [4]. The quadruped body is connected to three leg links touching the ground and a tail link swinging in the air. In Fig. 5, frames w, o, p, t and L_i are similarly defined as in section II. γ is the tail joint angle in the xy plane, $\theta=[\theta_1\ \theta_2\ \theta_3\ \theta_4]$ are tail joint angles in the xz plane, $\chi=[o_x\ o_y\ o_z\ o_\alpha\ o_\beta\ o_\varphi]^T$ are object orientation. Model parameters contains $q=[\gamma\ \theta\ \beta]^T$

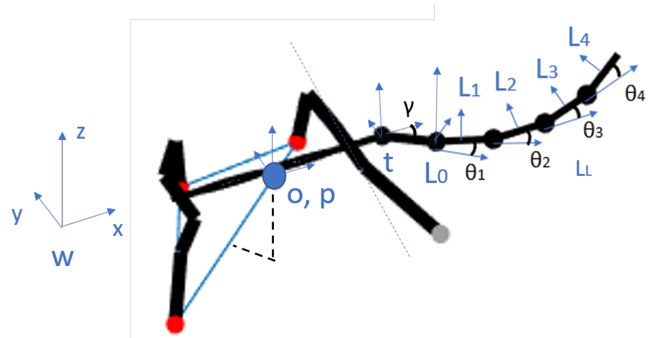

Fig. 5: **Parameters of the quadruped body with tail structure.**

and X. The rotation matrix of φ with respect to y-axis is

$$R_2(\theta) = \begin{bmatrix} cos(\theta) & 0 & sin(\theta) \\ 0 & 1 & 0 \\ -sin(\theta) & 0 & cos(\theta) \end{bmatrix}. \tag{14}$$

The movements of robot in pitch and roll directions are negligible. We can simply assume the rigid body transformation of frame p to frame o is still

$$g_{\text{op}} = \begin{bmatrix} R_1(o_\varphi) & 0 \\ 0\ 0\ 0 & 1 \end{bmatrix}. \tag{15}$$

In the tail configuration, the rigid body transformation of frame t to frame p is

$$g_{\text{pt}} = \begin{bmatrix} Id & \begin{bmatrix} d \\ 0 \\ 0 \end{bmatrix} \\ 0\ 0\ 0 & 1 \end{bmatrix}, \tag{16}$$

where d is a constant. The rigid body transformation of each tail link is

$$g_{\text{t0}} = \begin{bmatrix} R_1(\gamma) & \begin{bmatrix} \frac{L_L}{2} \\ 0 \\ 0 \end{bmatrix} \\ 0\ 0\ 0 & 1 \end{bmatrix}, \tag{17}$$

$$g_{\text{(i-1)i}} = \begin{bmatrix} Id & \begin{bmatrix} \frac{L_L}{2} \\ 0 \\ 0 \end{bmatrix} \\ 0\ 0\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_2(\theta_i) & \begin{bmatrix} \frac{L_L}{2} \\ 0 \\ 0 \end{bmatrix} \\ 0\ 0\ 0 & 1 \end{bmatrix}, \tag{18}$$

$g_{\text{p0}} = g_{\text{pt}} g_{\text{t0}}$ and $g_{\text{pi}} = g_{\text{pt}} g_{\text{t0}} ... g_{\text{(i-1)i}}$, where $i = [1,\ 2,\ 3,\ 4]$, the body manipulator Jacobian J^b_{pi} of ith tail-link (here, $i = [0,\ 1,\ 2,\ 3,\ 4]$) can be derived from Equation (6), (7) and (8). The rigid body transformation for kth leg is

$$g_{\text{pk}} = g_{\text{ps}_1} g_{\text{s}_1\text{b}_1} g_{\text{b}_1\text{c}_\text{k}}, \tag{19}$$

and the body manipulator Jacobian J^b_{pk} can be computed in the same way, where $k = [1,\ 2,\ 3]$. We define J^b_{pj} as the jth element of final body manipulator Jacobian, where j could refer to either tail links or leg links, similar in M_j, g_{pj} and $\text{Ad}_{g_{pj}}$.

The Euler-Lagrange equation could be derived and expressed in the form

$$M(q,\omega) \begin{bmatrix} \ddot{q} \\ \ddot{\text{x}} \end{bmatrix} + C \begin{bmatrix} \dot{q} \\ \dot{\text{x}} \end{bmatrix} + N + A^T \lambda = \Upsilon. \tag{20}$$

where

$$\hat{M} = \begin{bmatrix} \sum J^{b^T}_{pj} M_j J^b_{pj} & \sum J^{b^T}_{pj} M_j \text{Ad}_{g^{-1}_{pj}} \\ \sum \text{Ad}^T_{g^{-1}_{pj}} M_j J^b_{pj} & M_o + \sum \text{Ad}^T_{g^{-1}_{pj}} M_j \text{Ad}_{g^{-1}_{pj}} \end{bmatrix}, \tag{21}$$

$$M = \begin{bmatrix} Id & \\ & J^{b^T}_{op} \end{bmatrix} \hat{M} \begin{bmatrix} Id & \\ & J^b_{op} \end{bmatrix}. \tag{22}$$

C can be derived from Equation (12), $N = 0$, and $A = \begin{bmatrix} -J_h & \bar{G}^T_s \end{bmatrix}^T$ where

$$J_h = \text{diag}(0,\ \dots\ 0,\ B^T_{c_1} \text{Ad}_{g^{-1}_{s_1 c_1}} J^s_{s_1 f_1}, \dots\ B^T_{c_3} \text{Ad}_{g^{-1}_{s_3 c_3}} J^s_{s_3 f_3}), \tag{23}$$

$$\bar{G}^T_s = \begin{bmatrix} \text{Ad}^T_{g^{-1}_{oc_1}} B_{c_1} & \text{Ad}^T_{g^{-1}_{oc_2}} B_{c_2} & \text{Ad}^T_{g^{-1}_{oc_3}} B_{c_3} \end{bmatrix}^T J^b_{op}, \tag{24}$$

Υ contains tail joint torques τ while the rest is assumed to be 0. In order to prevent the robot from falling over, the dynamic equation (20) is required to make sure λs related to leg angles β are larger than 0.

IV. Extend

Compared to the tail structure in [6], our tail model has five DoFs with the first one (Fig. 5, γ) swinging in yaw direction (zero acceleration) and the following four (Fig. 5, $\theta1, \theta2, \theta3, \theta4$) in pitch direction. Assuming $\ddot{\gamma} = 0$, the tail can impact and only impact stabilization in pitch and roll directions.

The referenced paper derived dynamic equations from the kinematic model. Using the Newton-Euler equations, it formulated the tail loading by calculating the tail joints forces and moments, transferred tail loading to the quadruped base and related system accelerations to tail loading and feet loading. While in a more simplified and straightforward way, we assume the system to be a self-manipulator [4]. The body is connected with three leg links, which keep in contact with the ground, and a tail link, which swings in the air.

V. Conclusion

With an actuated tail, the quadruped can walk in DS gait for higher stride displacement while maintaining stability. Our model works well when the robot walks on the level ground which is coarse enough to provide enough friction forces, but performs poor on the slop, rough terrain or frictionless surfaces.

In the future, in order to adapt the model to various environments, we may consider using a new friction model. We can also modify $\ddot{\gamma}$ to impact the yaw-angle maneuvering (turning). Instead of walking gaits, further work could be using the tail to balance dynamic quadrupedal trot gaits.

References

[1] Randall Briggs, Jongwoo Lee, Matt Haberland, and Sangbae Kim. Tails in biomimetic design: Analysis, simulation, and experiment. In *Intelligent Robots and Systems (IROS), 2012 IEEE/RSJ International Conference on*, pages 1473–1480. IEEE, 2012.

[2] Anna Brill, Avik De, Aaron Johnson, and Daniel Koditschek. Tail-assisted rigid and compliant legged leaping. 2015.

[3] Evan Chang-Siu, Thomas Libby, Masayoshi Tomizuka, and Robert J Full. A lizard-inspired active tail enables rapid maneuvers and dynamic stabilization in a terrestrial robot. In *Intelligent Robots and Systems (IROS), 2011 IEEE/RSJ International Conference on*, pages 1887–1894. IEEE, 2011.

[4] Aaron M Johnson and Daniel E Koditschek. Legged self-manipulation. *IEEE Access*, 1:310–334, 2013.

[5] Alan Mutka, Matko Orsag, and Zdenko Kovacic. Stabilizing a quadruped robot locomotion using a two degree of freedom tail. In *Control & Automation (MED), 2013 21st Mediterranean Conference on*, pages 1336–1342. IEEE, 2013.

[6] William Rone and Pinhas Ben-Tzvi. Dynamic modeling and simulation of a yaw-angle quadruped maneuvering with a planar robotic tail. *Journal of Dynamic Systems, Measurement, and Control*, 138(8):084502, 2016.