New Model for Quadruped Locomotion on Flat Ground and Back Bending

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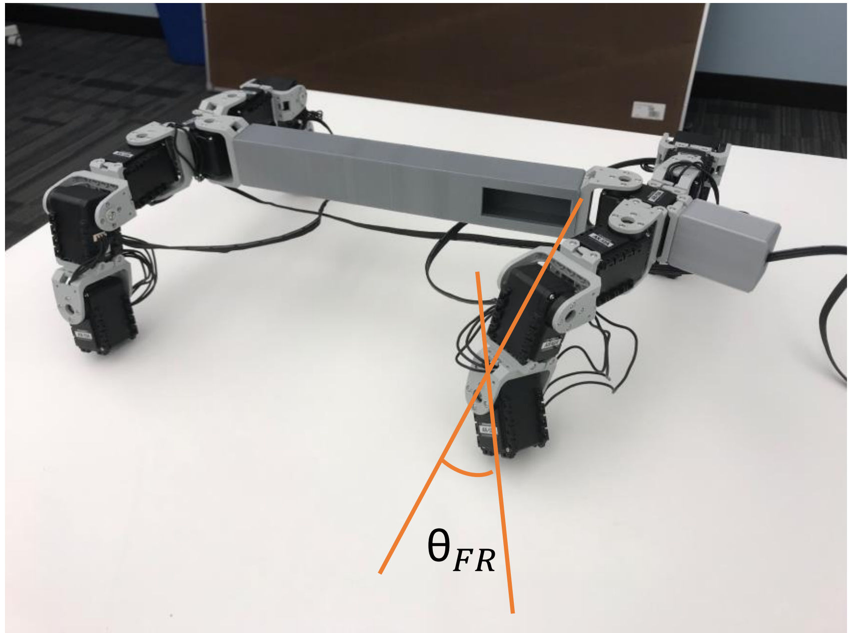
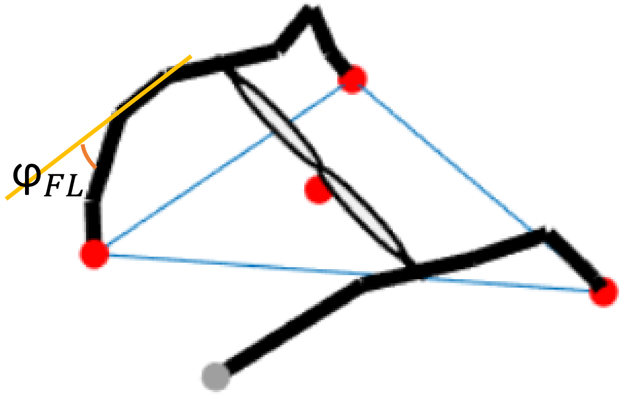
07/2018



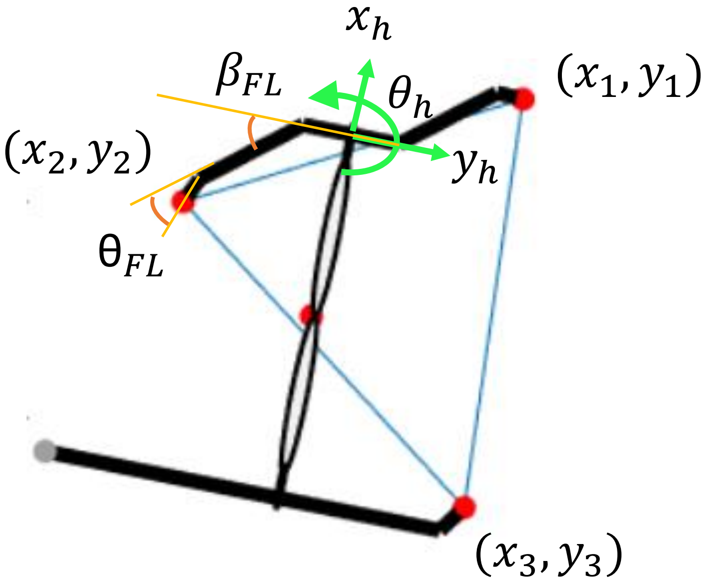
New Model



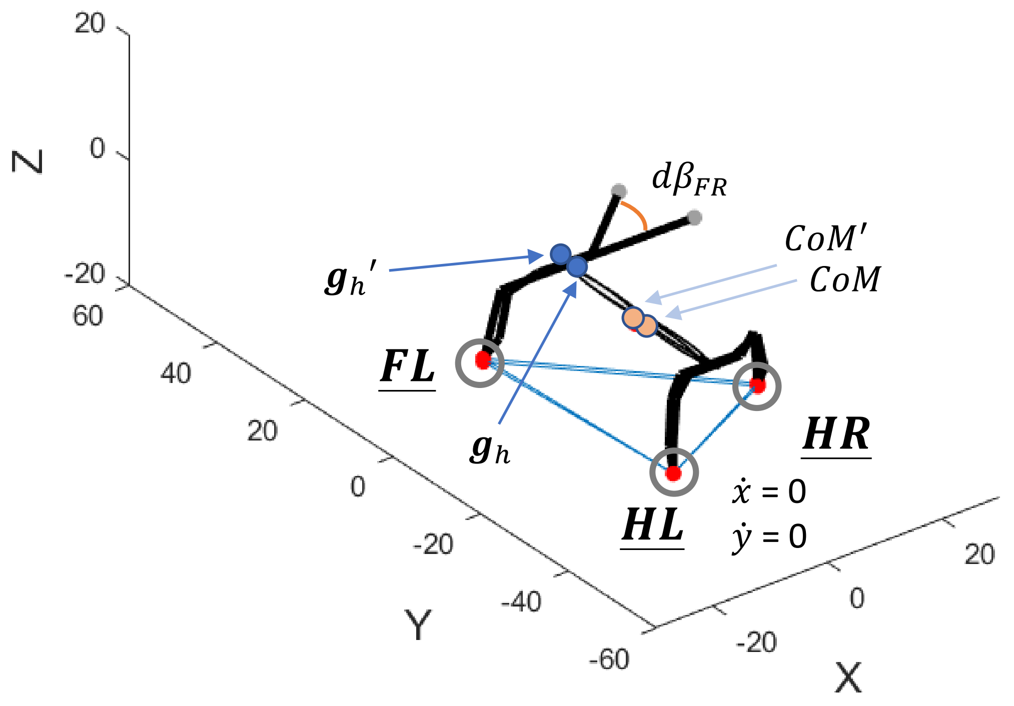
We assume, at each moment, **all legs touching the ground has zero linear velocities**, for example:

$$\begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix} = \begin{bmatrix}\dot{x}_1\\\dot{y}_1\\\dot{x}_2\\\dot{y}_2\\\dot{x}_3\\\dot{y}_3\end{bmatrix} = \omega(\boldsymbol{\beta}, \boldsymbol{\varphi}, \boldsymbol{\theta})\begin{bmatrix}\dot{\beta}\\\dot{\boldsymbol{g}}_h\end{bmatrix}$$

- leg angle $\boldsymbol{\beta} = [\beta_{FL}, \beta_{HR}, \beta_{HL}]^T$
- head position $\boldsymbol{g}_h = [x_h, y_h, \theta_h]^T$
- elbow angle $\boldsymbol{\varphi} = [\varphi_{FL}, \varphi_{HR}, \varphi_{HL}]^T$
- Knee angle $\boldsymbol{\theta} = [\theta_{FL}, \theta_{HR}, \theta_{HL}]^T$



New Model (Continued)



Optimization:

Six equations

Nine unknowns (g_h', φ', θ')

fmincon

4. With the relationship between shape changes and body velocities, we can numerically calculate the stride displacement.

e.g.,

At each Δt, β_{FL} increases by $\Delta\beta_{FL}$, β_{HR} increases by $\Delta\beta_{HR}$, β_{HL} increases by $\Delta\beta_{HL}$,

$$P_{FL} = \begin{bmatrix} x_{FL} \\ y_{FL} \end{bmatrix} = \begin{bmatrix} f_x\,(g_{head}, \beta_{FL}, \varphi_{FL}, \theta_{FL}) \\ f_y\,(g_{head}, \beta_{FL}, \varphi_{FL}, \theta_{FL}) \end{bmatrix};$$

Similar expression for P_{HR} and P_{HL}

$$P_{FL}' = \begin{bmatrix} x_{FL}' \\ y_{FL}' \end{bmatrix} = \begin{bmatrix} f_x\,(g_{head}', (\beta_{FL}+\Delta\beta_{FL}), \varphi_{FL}', \theta_{FL}') \\ f_y\,(g_{head}', (\beta_{FL}+\Delta\beta_{FL}), \varphi_{FL}', \theta_{FL}') \end{bmatrix};$$

Similar expression for P_{HR}' and P_{HL}'

$$\begin{bmatrix} P_{FL}' - P_{FL} \\ P_{HR}' - P_{HR} \\ P_{HL}' - P_{HL} \end{bmatrix} = \begin{bmatrix} x_{FL}' - x_{FL} \\ y_{FL}' - y_{FL} \\ x_{HR}' - x_{HR} \\ y_{HR}' - y_{HR} \\ x_{HL}' - x_{HL} \\ y_{HL}' - y_{HL} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (*)$$

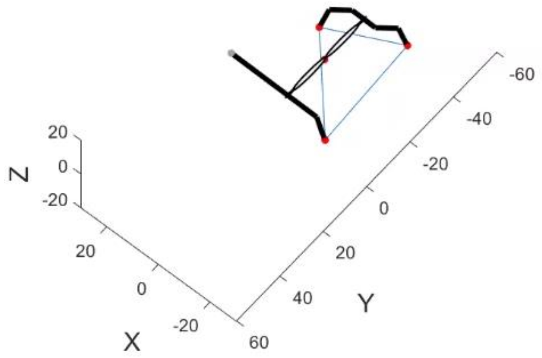
Solving equation (*) gives g_{head}' , φ' and θ',

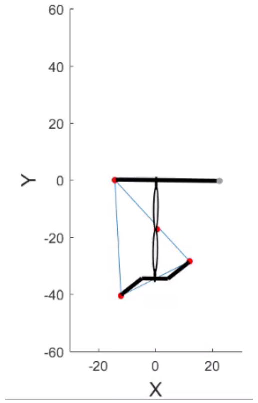
$$CoM' = h(g_{head}', \varphi', \theta'); \quad CoM = h(g_{head}, \varphi, \theta);$$

Step displacement = CoM' - CoM,

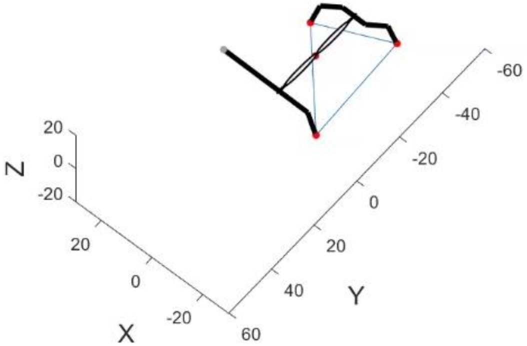
Stride displacement = sum(step displacement).

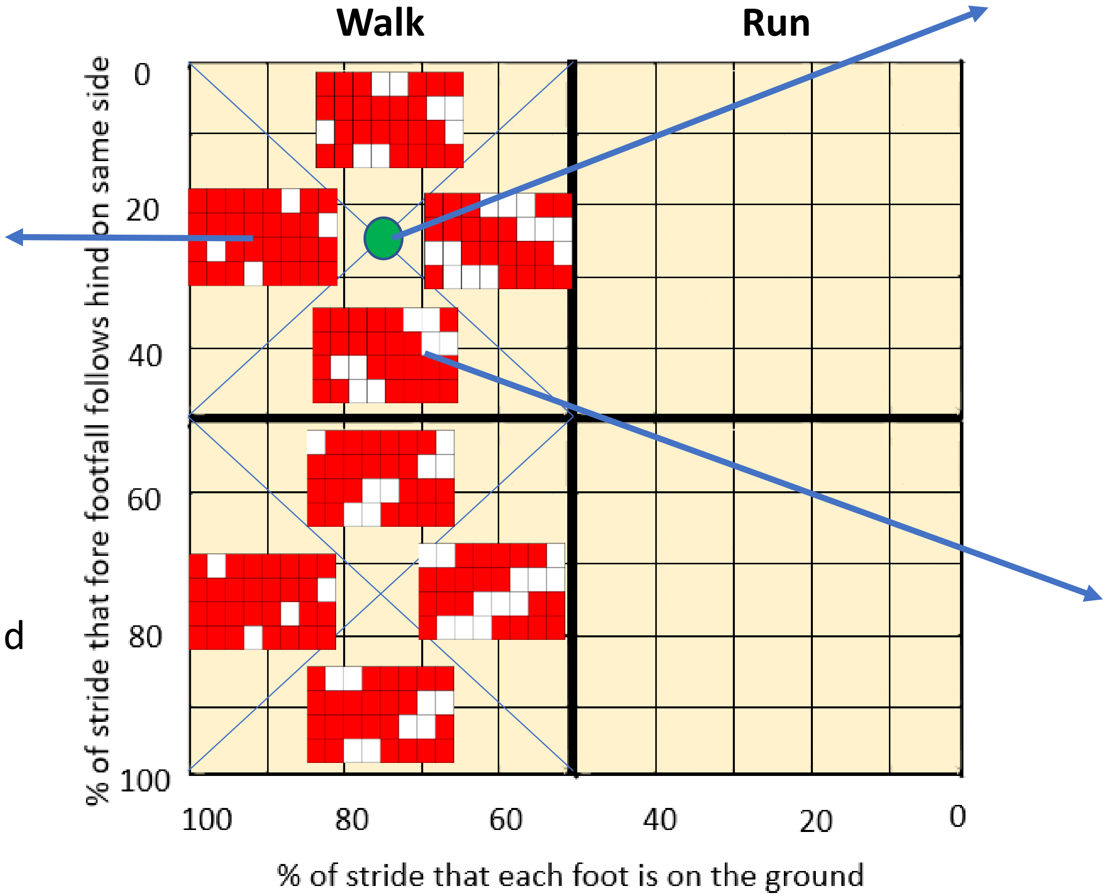
Simulation & Robot Samples

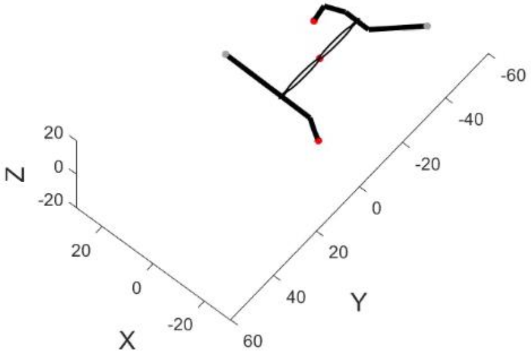




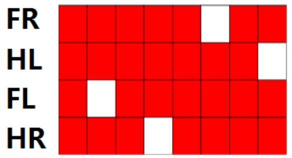
Slow motion:







Sample Gait Formula



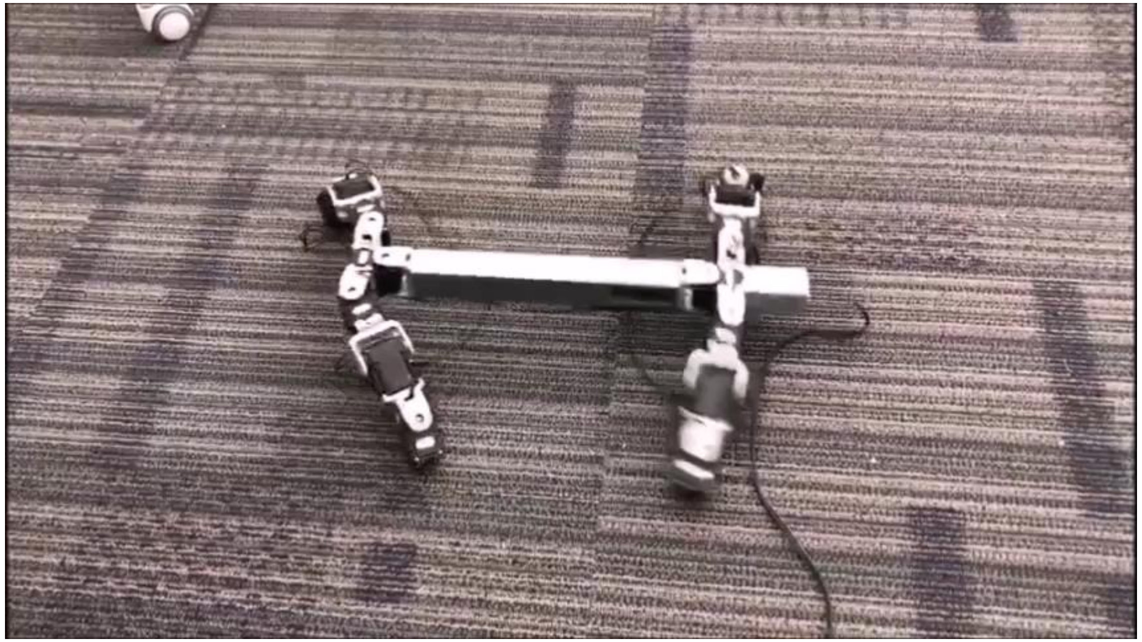
Hildebrand diagram

on the ground

in the air

Simulation & Robot Samples

A little slippage:



I guess at that moment, the last servo of the HR leg is touching the ground only with its vertex, which could not provide enough static friction force to prevent slippage.

Need a foot design

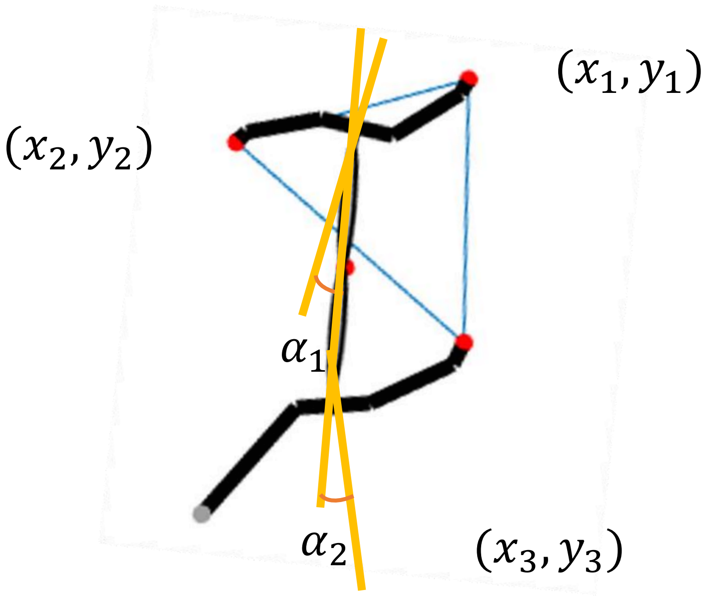
using *Universal Adjustable Joints Leveling Feet.*

Coordinate back bending to leg movement in geometric mechanics perspective

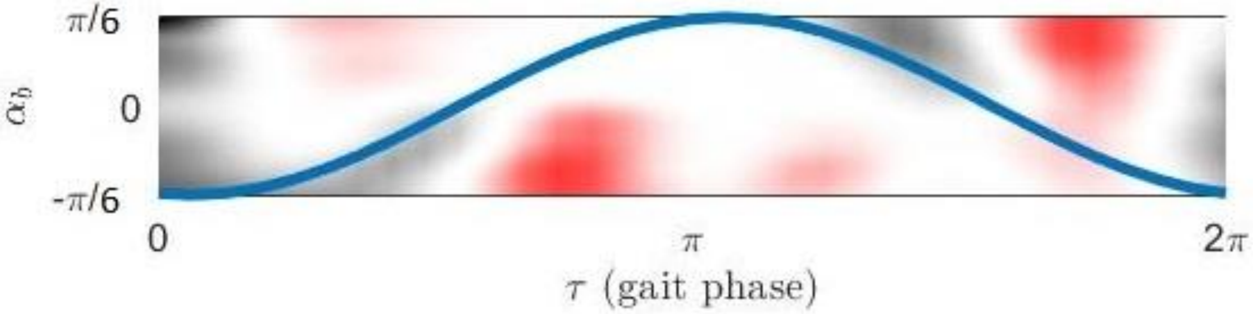
We assume, at each moment, **all legs touching the ground has zero linear velocities**, for example:

$$\begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix} = \begin{bmatrix}\dot{x}_1\\\dot{y}_1\\\dot{x}_2\\\dot{y}_2\\\dot{x}_3\\\dot{y}_3\end{bmatrix} = \omega(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\varphi}, \boldsymbol{\theta}) \begin{bmatrix}\dot{\boldsymbol{g}}_h\\\dot{\beta}\\\dot{\alpha}\end{bmatrix}$$

- body angle $\boldsymbol{\alpha} = [\alpha_1, \alpha_2]$ where $\alpha_1 = \alpha_2$
- leg angle $\boldsymbol{\beta} = [\beta_{FL}, \beta_{HR}, \beta_{HL}]^T$
- head position $\boldsymbol{g}_h = [x_h, y_h, \theta_h]^T$
- elbow angle $\boldsymbol{\varphi} = [\varphi_{FL}, \varphi_{HR}, \varphi_{HL}]^T$
- Knee angle $\boldsymbol{\theta} = [\theta_{FL}, \theta_{HR}, \theta_{HL}]^T$

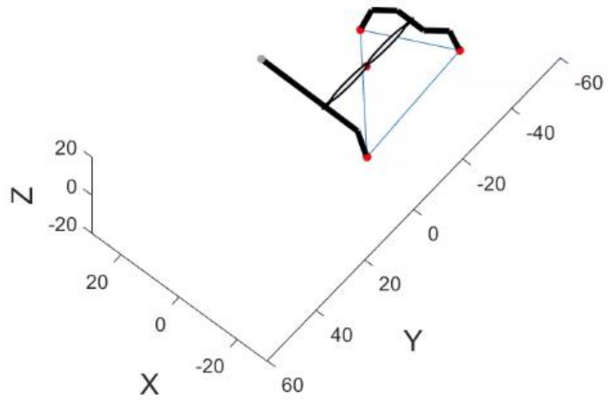


Coordinate back bending to leg movement in geometric mechanics perspective



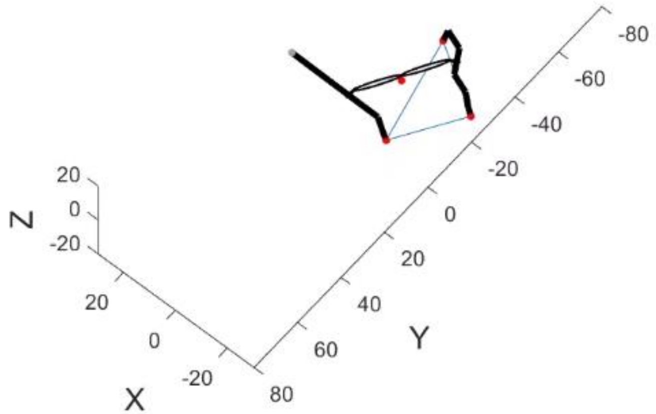
With back bending, at many moments, fmincon cannot find the solution of allowing all ground-touching legs fixed.

Without Back Bending



Displacement:
47.57cm/ 2 cycles

With Back Bending

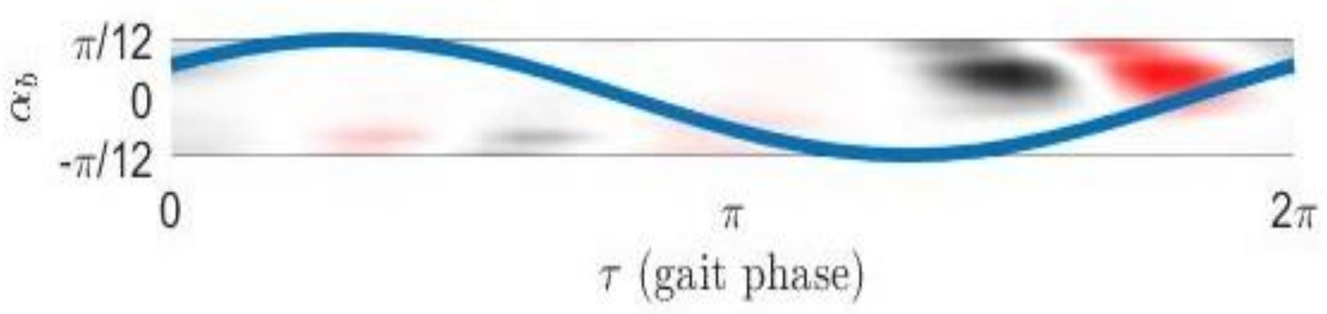


At many moments, no solution ($\boldsymbol{\theta}$=90°)

Displacement:
24.07cm/ 2 cycles

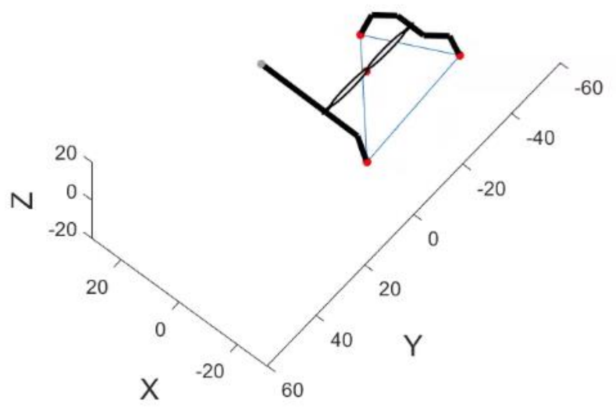
Coordinate back bending to leg movement in geometric mechanics perspective

Smaller Body Angle



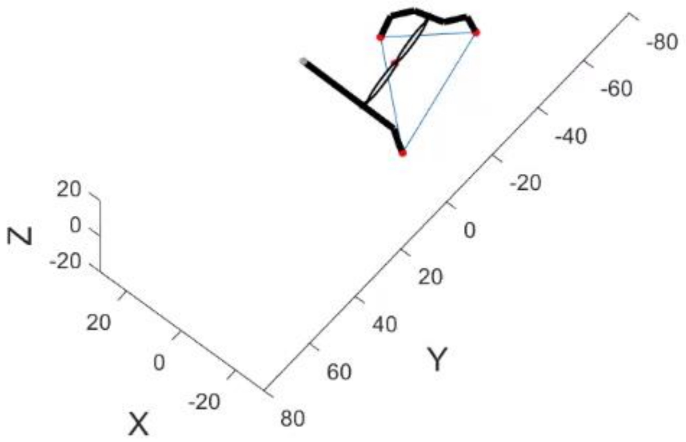
Still, at some moments, fmincon cannot find the solution of allowing all ground-touching legs fixed.

Without Back Bending



Displacement:
47.57cm/ 2 cycles

With Back Bending



At some moments, no solution ($\boldsymbol{\theta}$=90°)

Displacement:
33.40cm/ 2 cycles

Greedy is Optimal

Body angle $\alpha \in [-\frac{\pi}{6}, \frac{\pi}{6}]$, divided it into **N** segments.

Time phase t $\in [0,2\pi]$, divided it into **M** segments

At each time interval, body angle could be $\underbrace{\tilde{C} = \left\{-\frac{\pi}{6}, -\frac{\pi}{6} + \frac{\pi}{3*(N-1)}, -\frac{\pi}{6} + \frac{2*\pi}{3*(N-1)}, \ldots, \frac{\pi}{6}\right\}}_{N}$,

Computationally infeasible: A total of $\boldsymbol{O}(\boldsymbol{N}^{\boldsymbol{M}})$ different sequences of body angle choices.

To circumvent the high computational cost of straightforward enumeration,
we can try a <u>greedy algorithm</u>:

1. we choose body angle $\alpha^1 = \tilde{C}\{1\}$,
2. for the following time interval:

$$\alpha^t = \arg\max_{\alpha \in \tilde{C}} \{v^t\}$$

where α^t denoted the body angle at time step t
v^t is the forward velocity component.

Get the total body angle sequence L^1,

3. Choose body angle $\alpha^1 = \tilde{C}\{2\}, \tilde{C}\{3\}, \ldots, \tilde{C}\{N\}$, repeat step 2, get $\tilde{L} = \{L^1, L^2, \ldots, L^N\}$,
4. The global optimum:

$$L_{opti} = \arg \underbrace{\max\{stride\ displacement\}}_{L \in \tilde{L}}$$

Computation: $\boldsymbol{O}(\boldsymbol{N}^{\mathbf{2}} * \boldsymbol{M})$

Gong, Chaohui, et al. "Limbless locomotors that turn in place." *Robotics and Automation (ICRA), 2015 IEEE International Conference on*. IEEE, 2015.

Greedy is Optimal

It yields a globally optimal solution to the choice of our body reference frame (head frame) because each L yields a local optimal.

Proof:

For each L, define Δx^t as the net forward displacement of head frame in time interval Δt.

The stride displacement is

$$\Delta x = \sum_{t=1}^{N-1} \Delta x^t$$

Our objective of body angle scheduling is

$$\Delta x^* = \max_{\alpha^1,\alpha^2,\ldots\alpha^N} \left\{ \Delta x = \sum_{t=1}^{N-1} \Delta x^t \right\}$$

Since $\dot{\beta}$ is determined at each time step, Δx^t only depends on α^t,

$$\Delta x^* \leq \sum_{t=1}^{N-1} \max_{\alpha^1,\alpha^2,\ldots\alpha^N} \{\Delta x^t\} = \sum_{t=1}^{N-1} \Delta x^t$$

So it yields a local optimal solution.

Gong, Chaohui, et al. "Limbless locomotors that turn in place." *Robotics and Automation (ICRA), 2015 IEEE International Conference on*. IEEE, 2015.