Average Value of a Function
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1.

$$h(x) = \begin{cases} 3x+2 & \text{for } x \le 2 \\ 4x & \text{for } x2 < x < 5 \\ 20 & \text{for } x \ge 5 \end{cases}$$

Find the average value of $h(x)$ on the interval $x\epsilon(0,10)$.

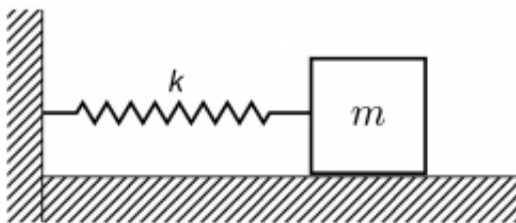| x	1	4	7	9
$g(x)$	20	25	35	47

2. Given the above values of g(x) at selected points, use the trapezoidal rule to estimate the average value of g(x) on the interval, $x\epsilon(1,9)$.

3.

$$f(x) = \frac{1}{x^2 - 1} \tag{1}$$

(a) Find the average value, if it exists, of $f(x)$ on the interval (-1, 1).

(b) Find the average value, if it exists, of $f(x)$ on the interval (0,2).

4.

A 5 kg mass is in an equilibrium position at $x = 0$. The mass is pulled outward to the right, and released at time $t = 0$, where t is measured in seconds. It begins to move in a sinusoidal motion about the equilibrium position; the motion can be modeled by the equation $x(t) = 2cos(\omega t)$ where the frequency ω is equal to $\sqrt{\frac{k}{m}}$, the spring constant k is 125 N/m, and x represents the horizontal distance from equilibrium.

Note: a positive x value means that the mass is to the right of equilibrium, and a negative x value means that the mass is to the left of equilibrium.

(a) What is the average value of the mass's x-position for the first $\frac{2\pi}{5}$ seconds of motion?

(b) What is the average value of the mass's absolute distance from equilibrium for the first $\frac{2\pi}{5}$ seconds of motion?

(c) What is the average value of the mass's velocity for the first $\frac{2\pi}{5}$ seconds of motion?